

**Euler’s Theorem**

This theorem emerges from theories concerned with differential equations. The theorem finds many applications in thermodynamics. In particular the theorem concerned with homogeneous functions of the first degree is important. This theorem can be stated as follows [1].

\[ f(k \cdot x, k \cdot y, k \cdot z) = k \cdot f(x, y, z) \quad (a) \]

By way of illustration we consider a liquid mixture volume \( V \) prepared using \( n_1 \) and \( n_2 \) moles of liquid 1 and 2. If we had used \( 2.n_1 \) and \( 2.n_2 \) moles of liquids 1 and 2, then the final volume would have been \( 2.V \).

The important theorem allows us to set down the following descriptions.

For a system comprising \( i \)-chemical substances, it follows that

\[ V = \sum_{j=1}^{i} n_j \cdot V_j \quad (b) \]

where partial molar volume \( V_j = \left( \frac{\partial V}{\partial n_j} \right)_{T,p,n(i \neq j)} \) \( (c) \)

It should be noted that some thermodynamic monographs, when citing equation (b), include the phrase ‘at constant temperature and pressure’. Other monographs do not include this phrase on the grounds that the isobaric - isothermal condition is included in the definition of the partial derivative in equation (c). In practice nothing is lost by including this phrase simply to indicate that the analysis is concerned with the properties of systems in the \( T – p \) - composition domain.

A similar analysis in the context of Gibbs energies leads to the following two equations and the definition of chemical potentials.

\[ G = \sum_{j=1}^{i} n_j \cdot \mu_j \quad (d) \]

where chemical potential \( \mu_j = \left( \frac{\partial G}{\partial n_j} \right)_{T,p,n(i \neq j)} \) \( (e) \)

**Footnotes**