**Equilibrium: Depression of Freezing Point of a Solvent by a Solute**

A given homogeneous liquid system (at pressure $p$) comprises solvent $i$ and solute $j$ at temperature $T$ and pressure $p$. In the absence of solute $j$, the freezing point of the solvent is $T_i^0$. But in the presence of solute $j$ the freezing point is temperature $T$ where $T < T_i^0$. The depression of freezing point $\theta \equiv T_i^0 - T$ is recorded for a solution where the mole fraction of solvent is $x_i(sln)$. If the solution is dilute, we can assume that the thermodynamic properties of the solution are ideal. From the Schroeder-van Laar equation,

$$-\ln[x_i(s\ln)] = \frac{\Delta_i H_i^0(T)}{R} \left[ \frac{1}{T} - \frac{1}{T_i^0} \right]$$  \tag{a}

$$-\ln[x_i(s\ln)] = \frac{\Delta_i H_i^0}{R} \cdot \frac{\theta}{(T_i^0 - \theta) \cdot T_i^0}$$  \tag{b}

If $T_i^0 - \theta \equiv T_i^0$, \quad $-\ln[x_i(s\ln)] = \frac{\Delta_i H_i^0}{R} \cdot \frac{\theta}{(T_i^0)^2}$ \tag{c}

Or,

$$\ln \left[ \frac{1}{x_i(s\ln)} \right] = \frac{\Delta_i H_i^0}{R} \cdot \frac{\theta}{(T_i^0)^2}$$  \tag{d}

Hence \[2\] \quad $\theta = \left[ \frac{R \cdot (T_i^0)^2 \cdot M_i}{\Delta_i H_i^0} \right] \cdot m_j \quad \tag{e}$

The quantity enclosed in the [...] brackets is characteristic of the solvent.

**Footnote**

[1] \quad $\theta = T_i^0 - T$ ; \quad $\frac{1}{T} - \frac{1}{T_i^0} = \frac{T_i^0 - T}{T \cdot T_i^0} = \frac{T_i^0 - T}{(T_i^0 - \theta) \cdot T_i^0}$

[2] \quad $\frac{1}{x_i} = \frac{1}{1 - x_j} = \frac{1}{1 - [n_j/(n_i + n_j)]} = \frac{n_i + n_j}{n_i + n_j - n_j}$

For a solution where the molality of solute $j = m_j$

$m_j = \frac{n_j}{n_i \cdot M_i}$ \quad Then, \quad $\frac{1}{x_i} = \frac{n_i + n_j \cdot M_i \cdot m_j}{n_i}$

$$-\ln[x_i(s\ln)] = -\ln[1 + M_i \cdot m_j]$$

$$-\ln[x_i(s\ln)] = -\ln[1 - x_j(s\ln)] \approx x_j$$
\[ x_j = \frac{m_j}{(1/M_j) + m_j} \approx m_j \cdot M_j \]