**Compressions: Ratio: Isentropic and Isothermal**

Using a calculus operation, we obtain equations relating isothermal and isentropic dependencies of volume on pressure.

Thus,

$$\left( \frac{\partial V}{\partial p} \right)_T = -(\partial T / \partial p)_V \cdot (\partial V / \partial T)_p$$

$$\left( \frac{\partial V}{\partial p} \right)_S = -(\partial S / \partial p)_V \cdot (\partial V / \partial S)_p$$

And,

$$\left( \frac{\partial V}{\partial p} \right)_T = -(\partial T / \partial p)_V \cdot (\partial V / \partial T)_p \cdot (\partial S / \partial T)_V$$

Then,

$$\left( \frac{\partial V}{\partial p} \right)_T / \left( \frac{\partial V}{\partial p} \right)_S = \frac{(\partial S / \partial T)_p}{(\partial S / \partial T)_V}$$

The Gibbs-Helmholtz equation requires that $H = G - T \cdot (\partial G / \partial T)_p$

Also

$$\left( \frac{\partial H}{\partial T} \right)_p = C_p = -T \cdot (\partial^2 G / \partial T^2)_p = T \cdot (\partial S / \partial T)_p$$

Similarly,

$$\left( \frac{\partial U}{\partial T} \right)_V = C_v = T \cdot (\partial S / \partial T)_V$$

Hence,

$$K_T / K_S = C_p / C_v$$