ALTERNATIVE METHODS OF SEASONAL ADJUSTMENT

by D.S.G. Pollock and Emi Mise (University of Leicester)

We examine two alternative methods of seasonal adjustment, which operate, respectively, in the time domain and the frequency domain.

The time-domain method relies on a simplified Wiener–Kolmogorov filter, of which the parameters may be determined by the user. The frequency-domain method requires the user to identify the frequency bands in which the seasonal elements reside, which become the stop bands of the filter.

It is observed that the essential information in macroeconomic time series typically resides in a limited low-frequency band. The rest of the spectrum, apart from the seasonal component, is liable to be dead space.

Therefore, instead of attempting to remove only the seasonal elements, it is proposed that one should attempt to isolate the low-frequency component.

1. THE SEAT–TRAMO FILTERS

The SEATS-TRAMO method of seasonal adjustment is based upon an estimated seasonal ARIMA model. A partial fraction decomposition of the autocovariance generating function of the model will serve to isolate the parts belonging to the trend, the seasonal component and the irregular residue.

According to the principal of canonical decompositions, noise is removed from the trend and the seasonal component and assigned to the irregular component.

The filters to extract the components are formed, according to the Wiener–Kolmogorov principal, from the ratio of the autocovariance function of the component to that of the process as a whole

The trend-extraction filter serves as a means of seasonal adjustment as much as the seasonal-adjustment filter, but it also suppresses the high-frequency elements. In the absence of significant high-frequency elements, the two filters should generate similar outputs.

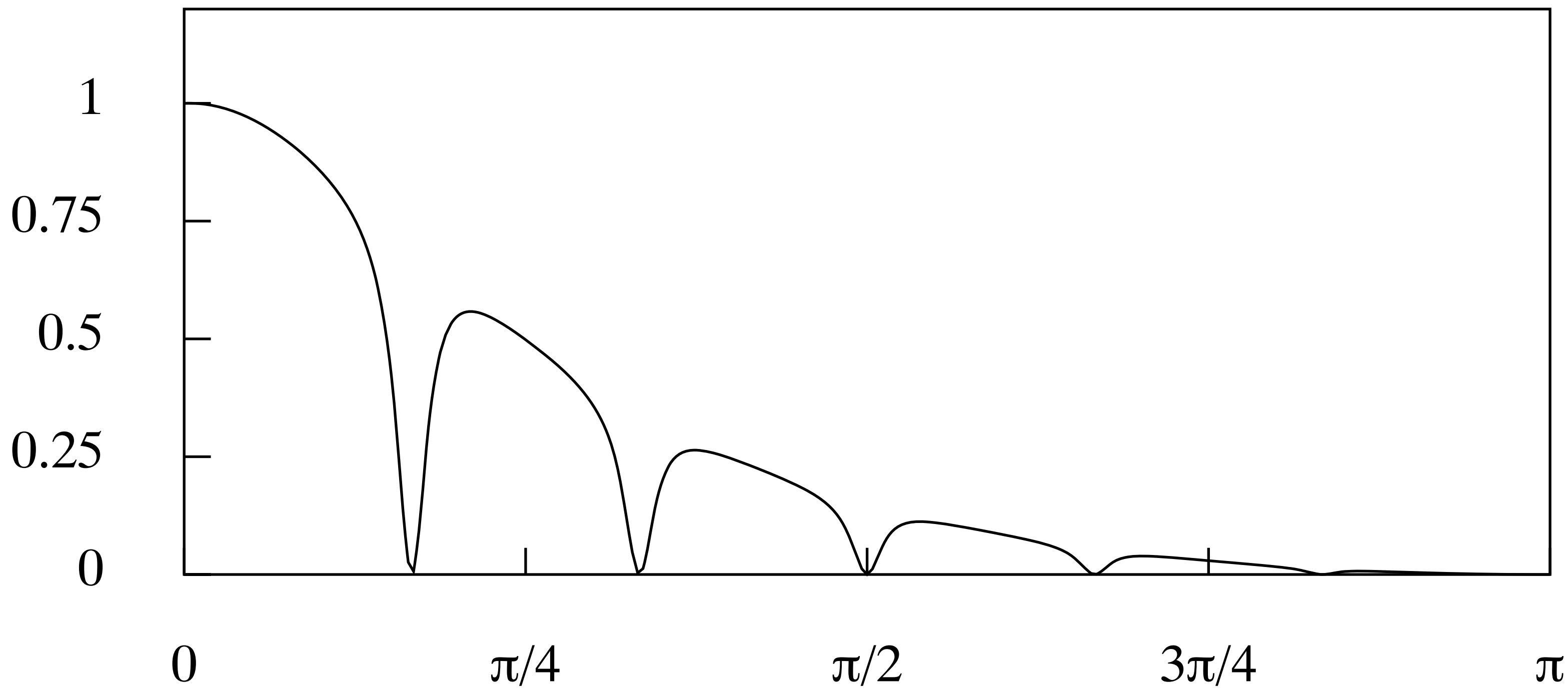

Figure 1. The gain of the canonical trend-extraction filter of SEATS–TRAMO associated with the airline passenger model.

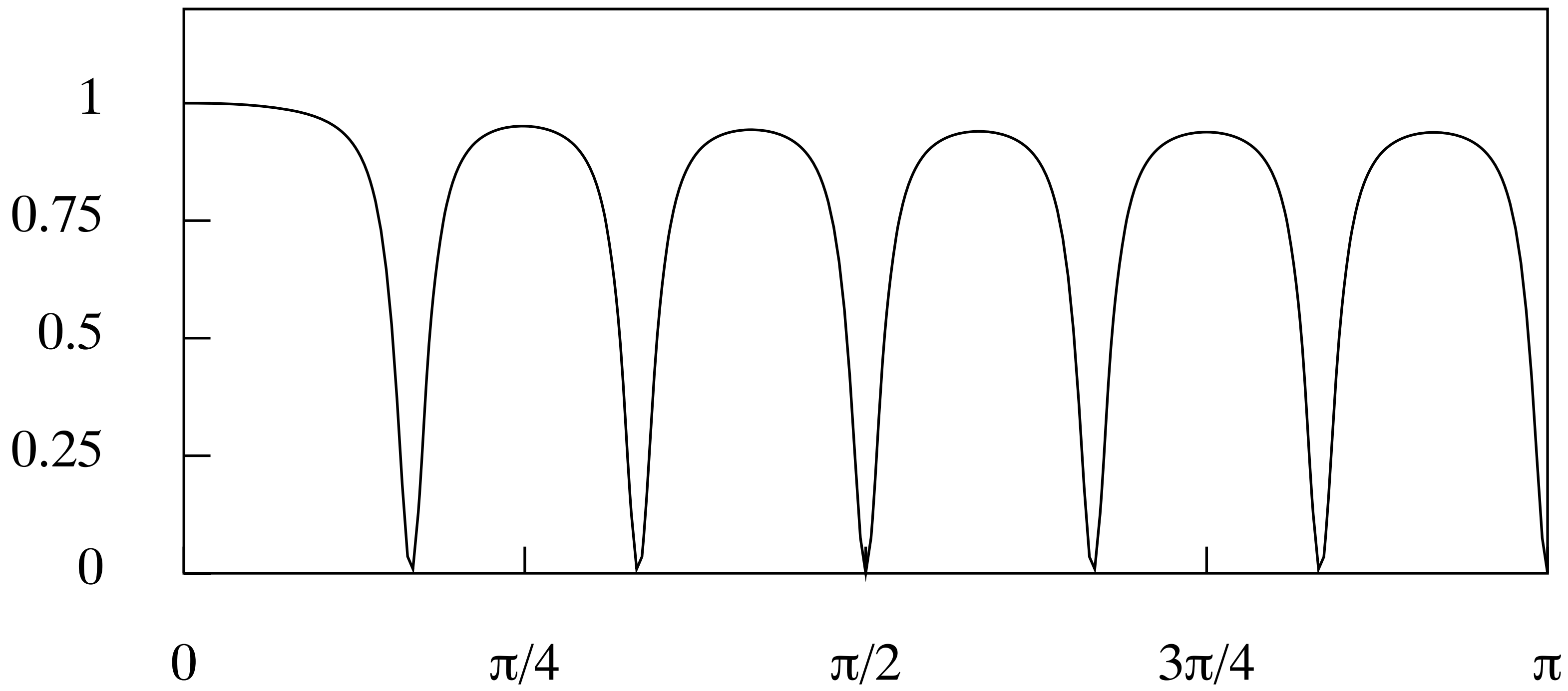

Figure 2. The gain of the seasonal-adjustment filter of SEATS–TRAMO associated with the airline passenger model

2. THE SIMPLIFIED WIENER–KOLMOGOROV FILTER

Consider the model

$$y(z) = \frac{R(z)}{\Sigma(z)}\nu(z) + \eta(z).$$

Here, $\nu(z)$ and $\eta(z)$ are the z-transforms of independent white-noise sequences, $\Sigma(z) = 1 + z + z^2 + \cdots + z^{s-1}$ is the seasonal summation operator, and

$$R(z) = 1 + \rho z + \rho^2 z^2 + \cdots + \rho^{s-1} z^{s-1},$$

with $\rho < 1$, serves to limit the effect of the poles of $\Sigma(z)$ to the vicinities of the seasonal frequencies.

A simple seasonal-adjustment filter is derived from the following function:

$$\beta_C(z) = \frac{\sigma_\eta^2 \Sigma(z)\Sigma(z^{-1})}{\sigma_\eta^2 \Sigma(z)\Sigma(z^{-1}) + \sigma_\nu^2 R(z)R(z^{-1})}.$$

Setting $z = \exp\{-\mathrm{i}\omega\}$ and letting ω run from 0 to π generates the frequency response of the filter.

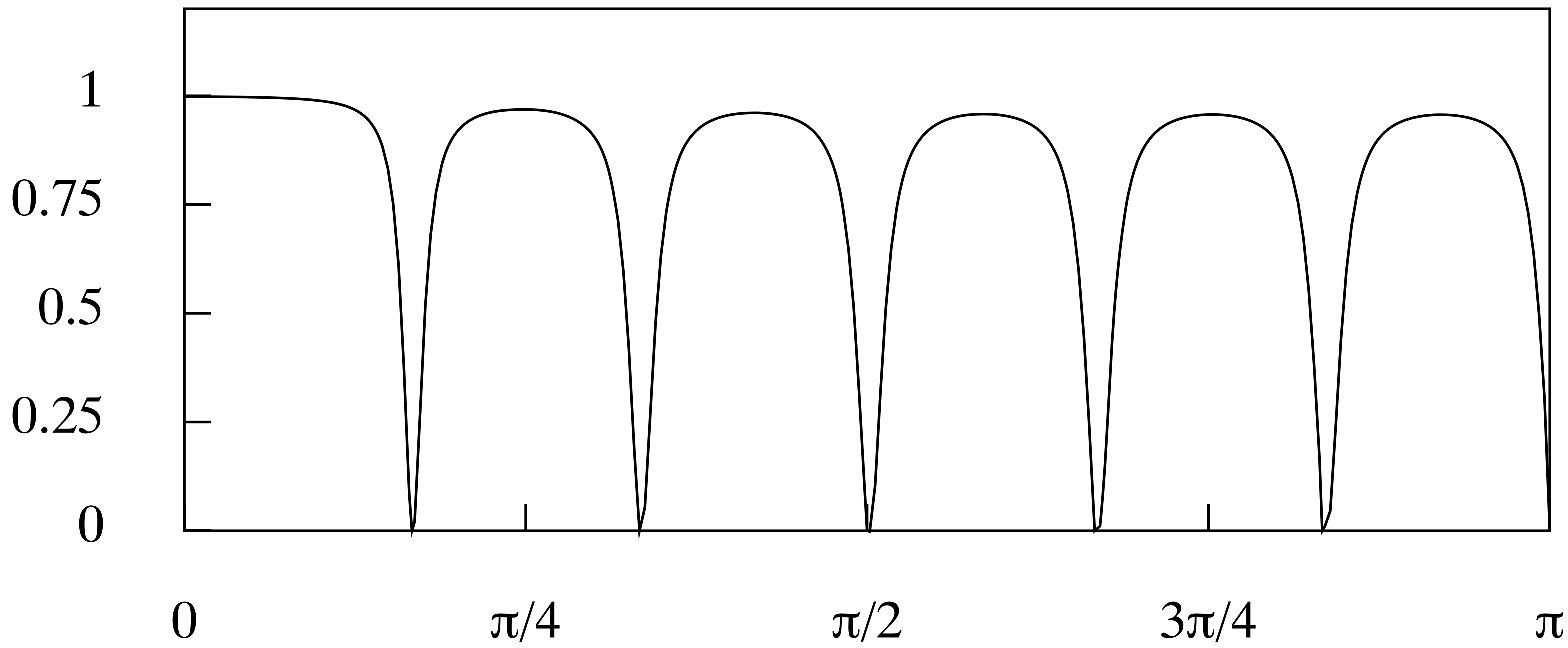

Figure 3. The gain of the simplified Wiener–Kolmogorov seasonal-adjustment filter.

3. THE ANALYSIS OF UK CONSUMPTION

To determine the spectral structure of an macroeconomic data sequence, one must first remove the trend.

The differencing operator nullifies the element at zero frequency and it radically attenuates adjacent low-frequency elements, thereby rendering the low-frequency component invisible in the periodogram of the differenced data.

The low-frequency spectral structure is visible in the periodogram of the residuals of a polynomial regression. Let Q be the second-order matrix difference operator, effective in removing a linear trend from the data vector y. Then, the formula for the vector of residuals is

$$g = Q(Q'Q)^{-1}Q'y.$$

Examples of the effects of detrending are provided by the analysis of the U.K. consumption data. The periodogram of the residuals reveals a band-limited structure with wide dead spaces.

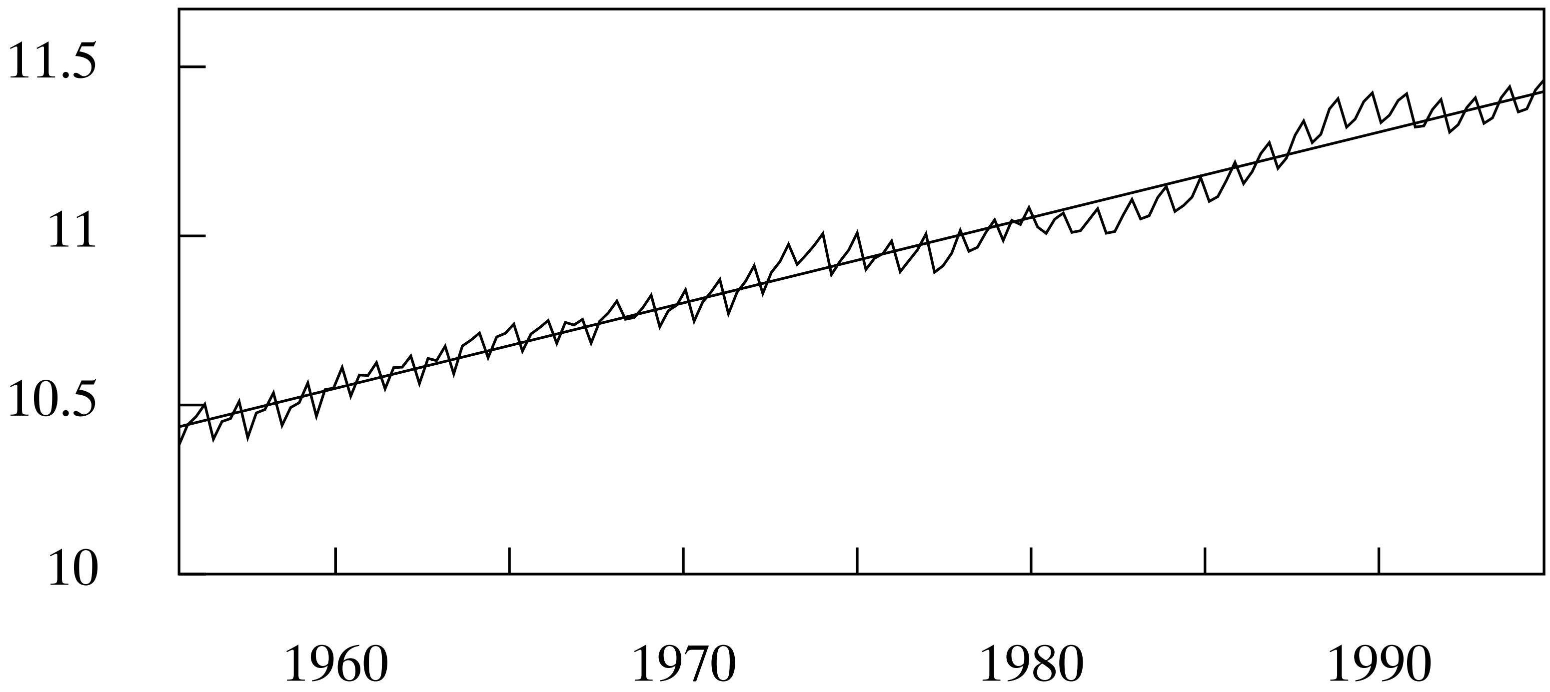

Figure 4. The quarterly sequence of the logarithms of consumption in the U.K., for the years 1955 to 1994, together with a linear trend interpolated by least-squares regression.

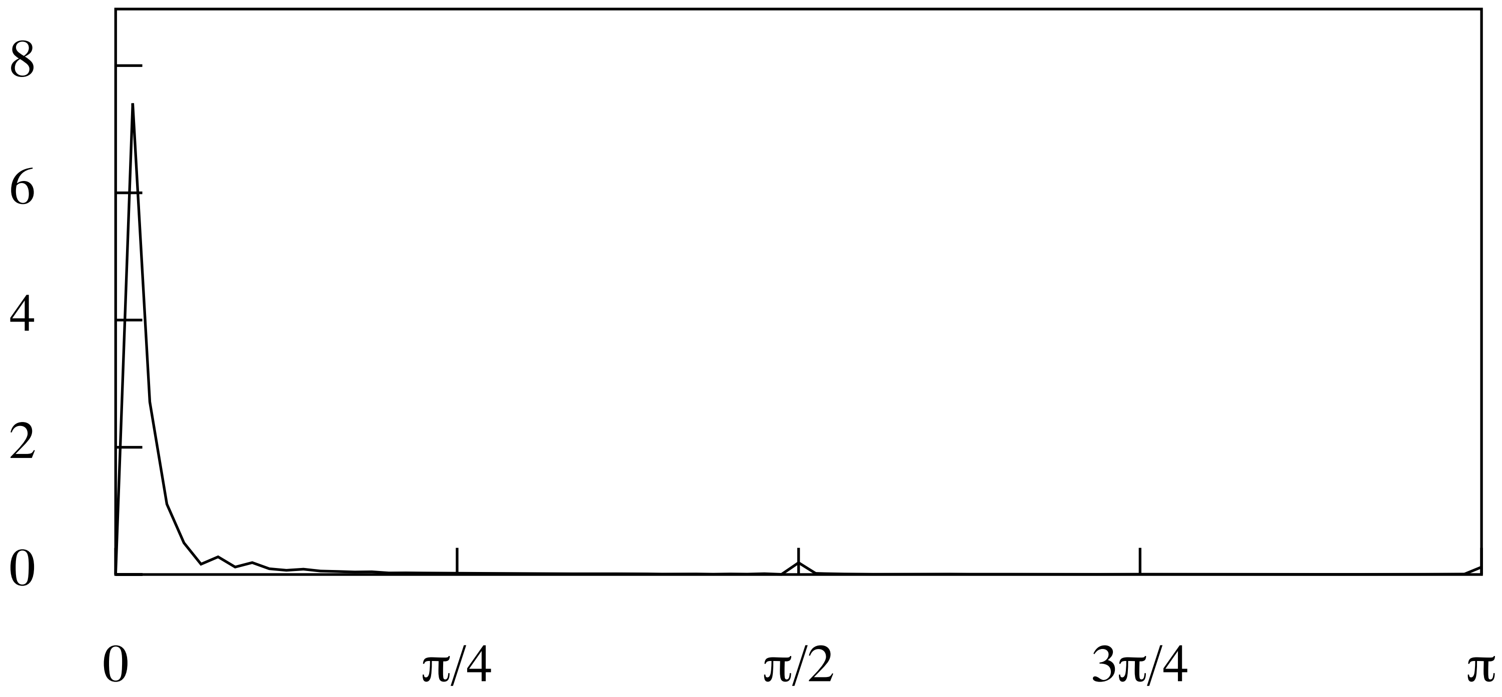

Figure 5. The periodogram of the trended U.K. logarithmic consumption data.

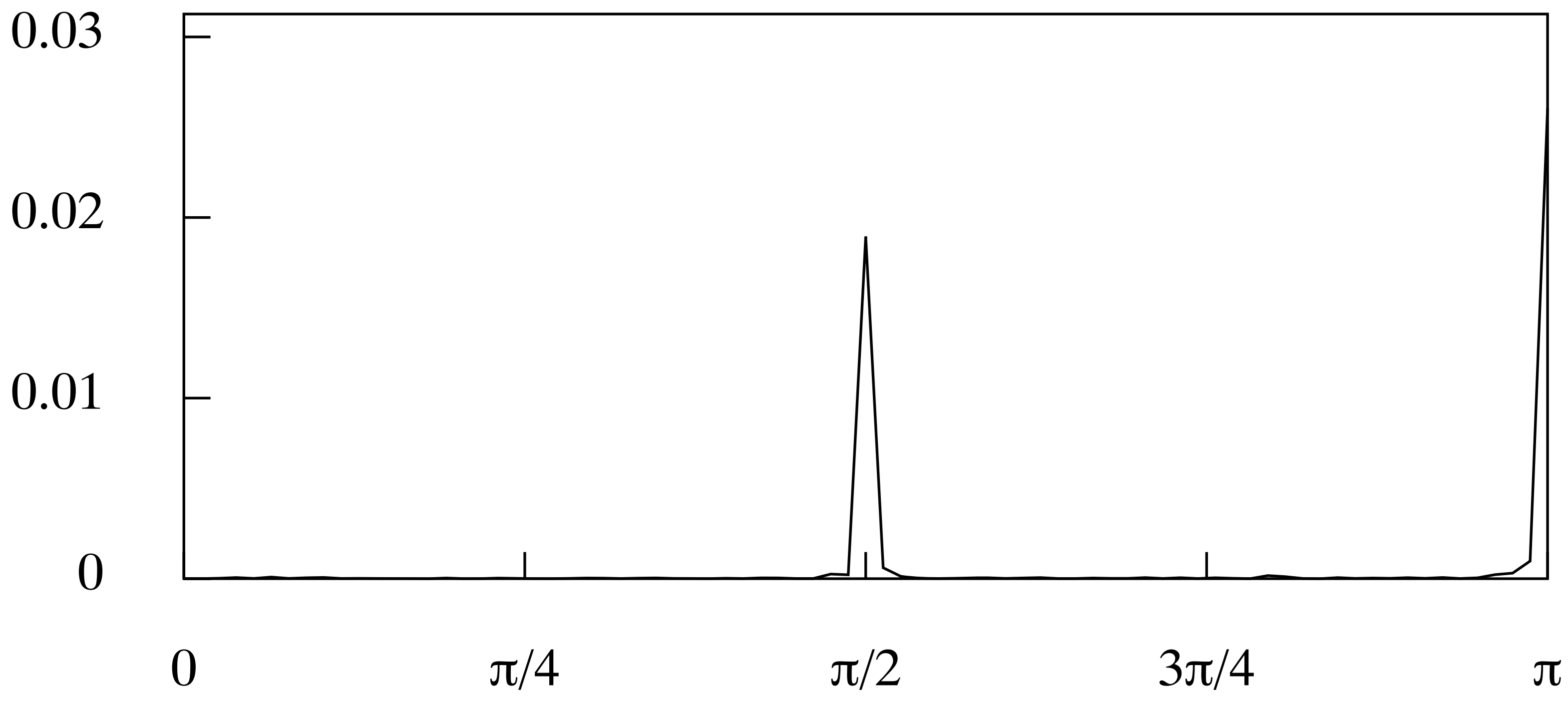

Figure 6. The periodogram of the first differences of the U.K. logarithmic consumption data.

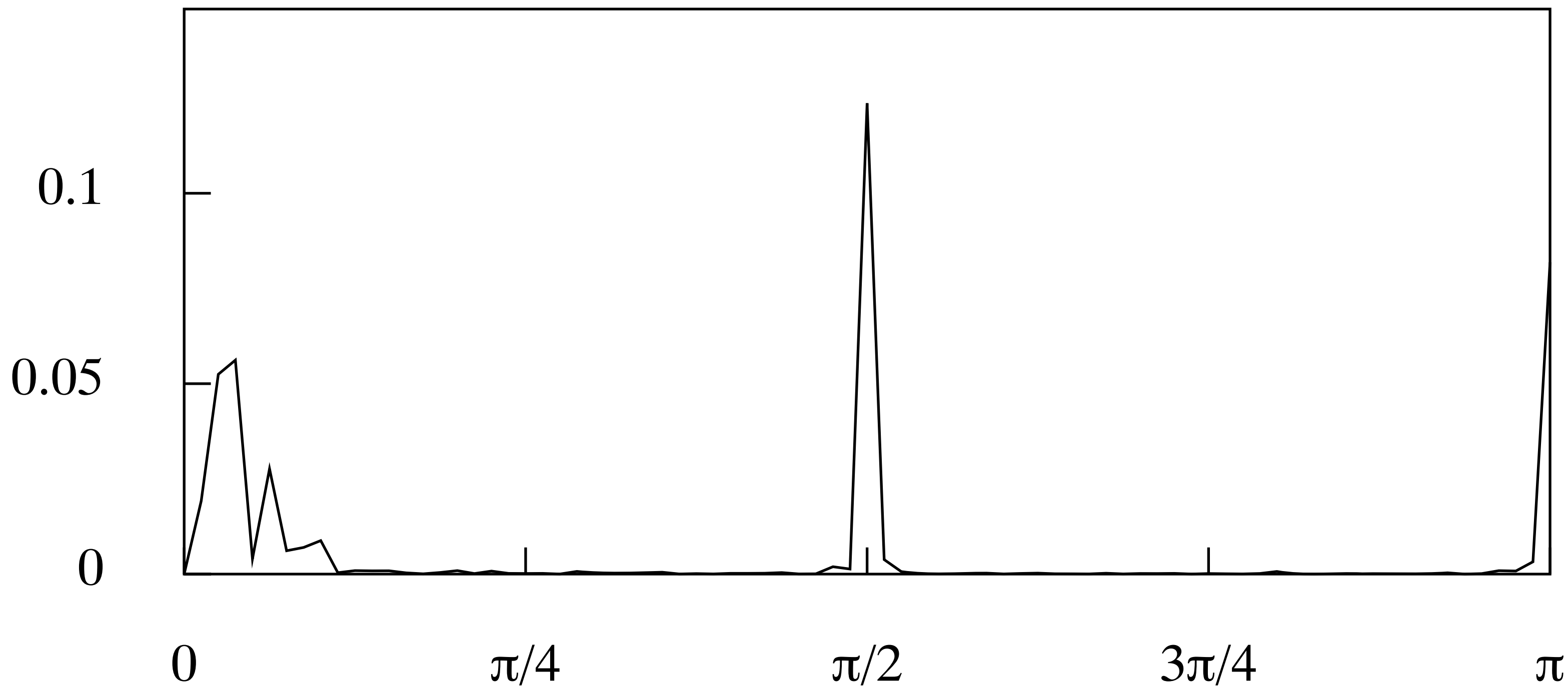

Figure 7. The periodogram of the residual deviations of the logarithmic consumption data from an interpolated line.

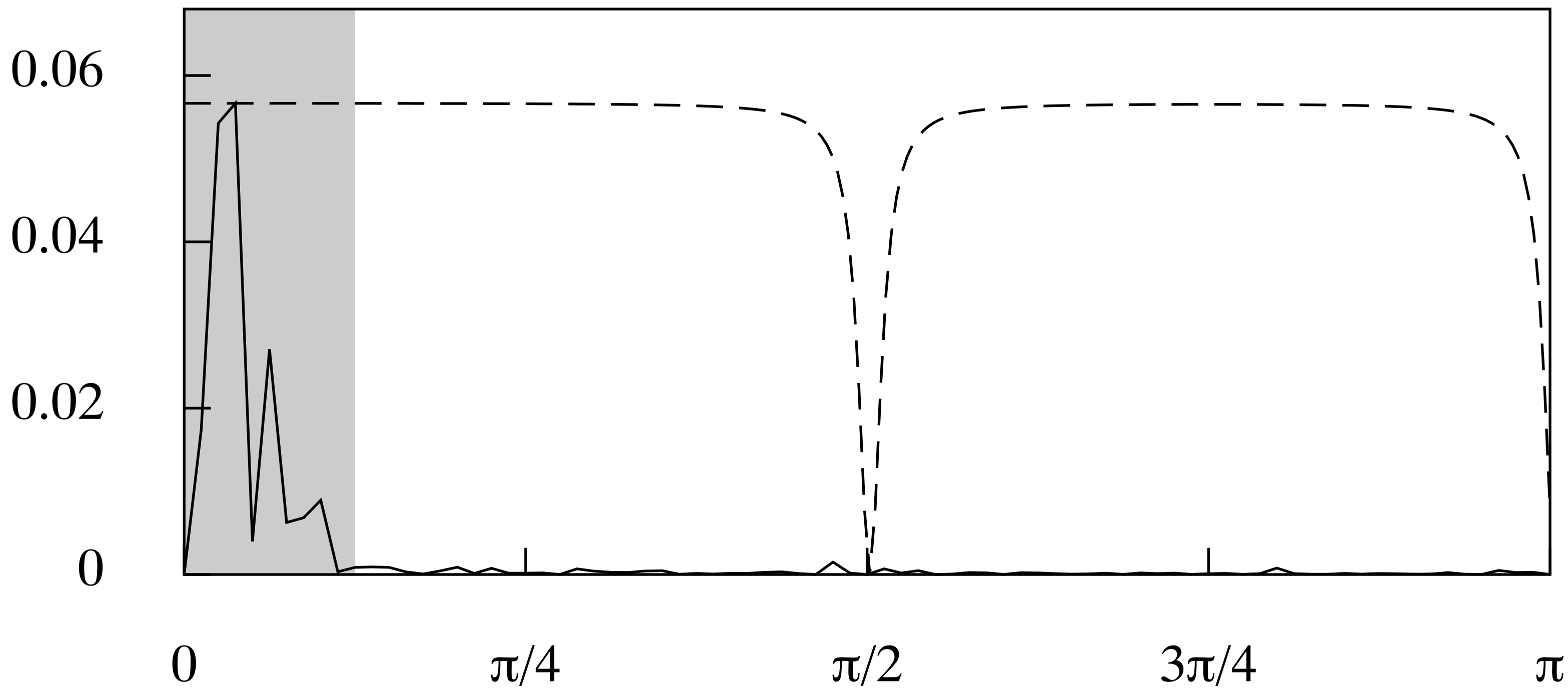

Figure 8. The periodogram of the residuals from a linear detrending of the seasonally-adjusted logarithmic consumption data, with the frequency-response function of the filter superimposed. The shaded band contains the elements of the business cycle.

4. FILTERING TRENDED DATA

There are three methods that may be used in filtering trended data:

1. The filter can be applied directly to the trended data. The forwards pass of the filter requires some pre-sample starting values and the reverse pass requires some post-sample extrapolations.

2. The filter can be applied to the residuals from fitting a linear or polynomial trend. Then, the filtered sequence can be added back to the trend.

3. The trended sequence can be reduced to stationarity by a twofold differencing. After the differenced sequence has been filtered, it can be reinflated via a twofold summation.

In the case of the lowpass filtering of a differenced sequence, some initial conditions will be required to begin a process of summation that recovers the trend. In the case of a high pass or bandpass filtering of the differenced sequence, the summation will generate a stationary mean-zero sequence, and the requisite initial conditions will be zero-valued.

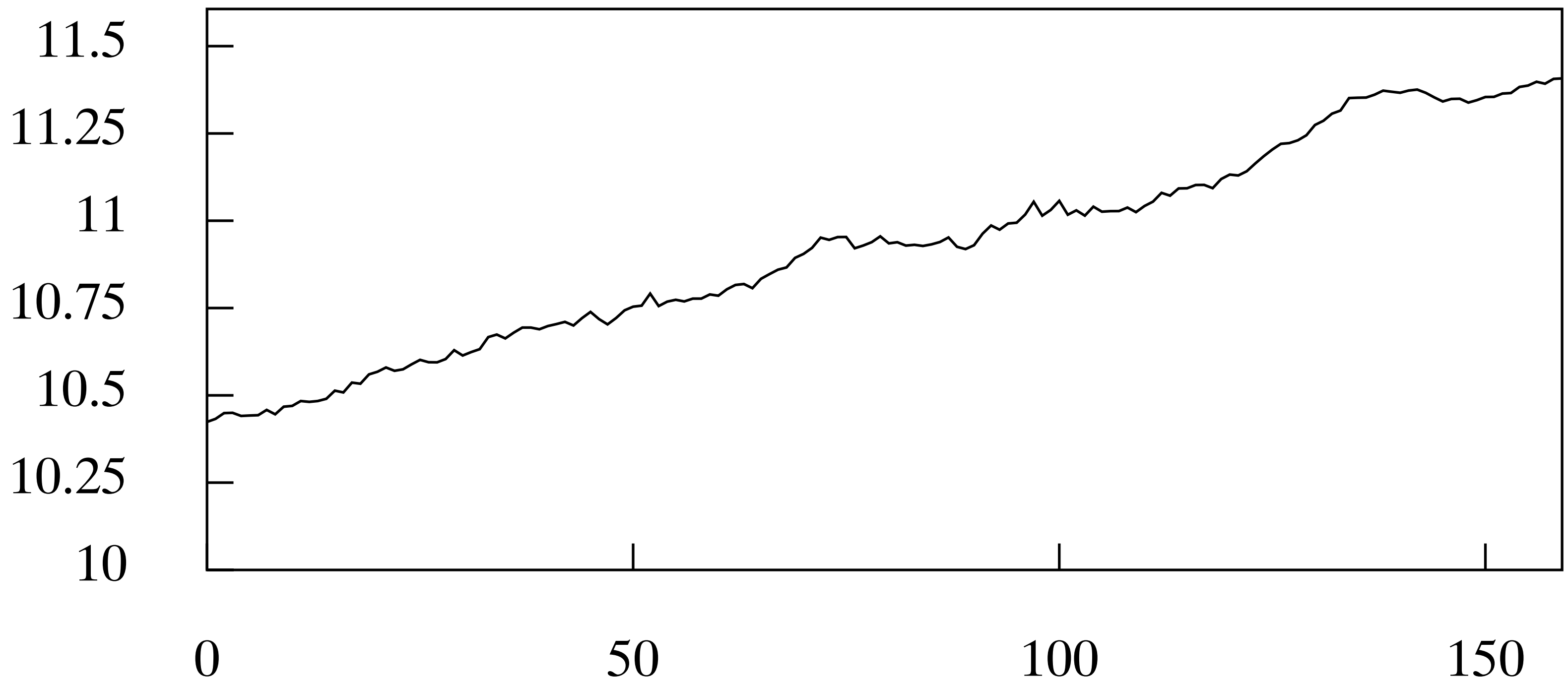

Figure 9. The plot of a seasonally-adjusted version of the logarithmic consumption data of Figure 4, generated by the simplified Wiener–Kolmogorov filter.

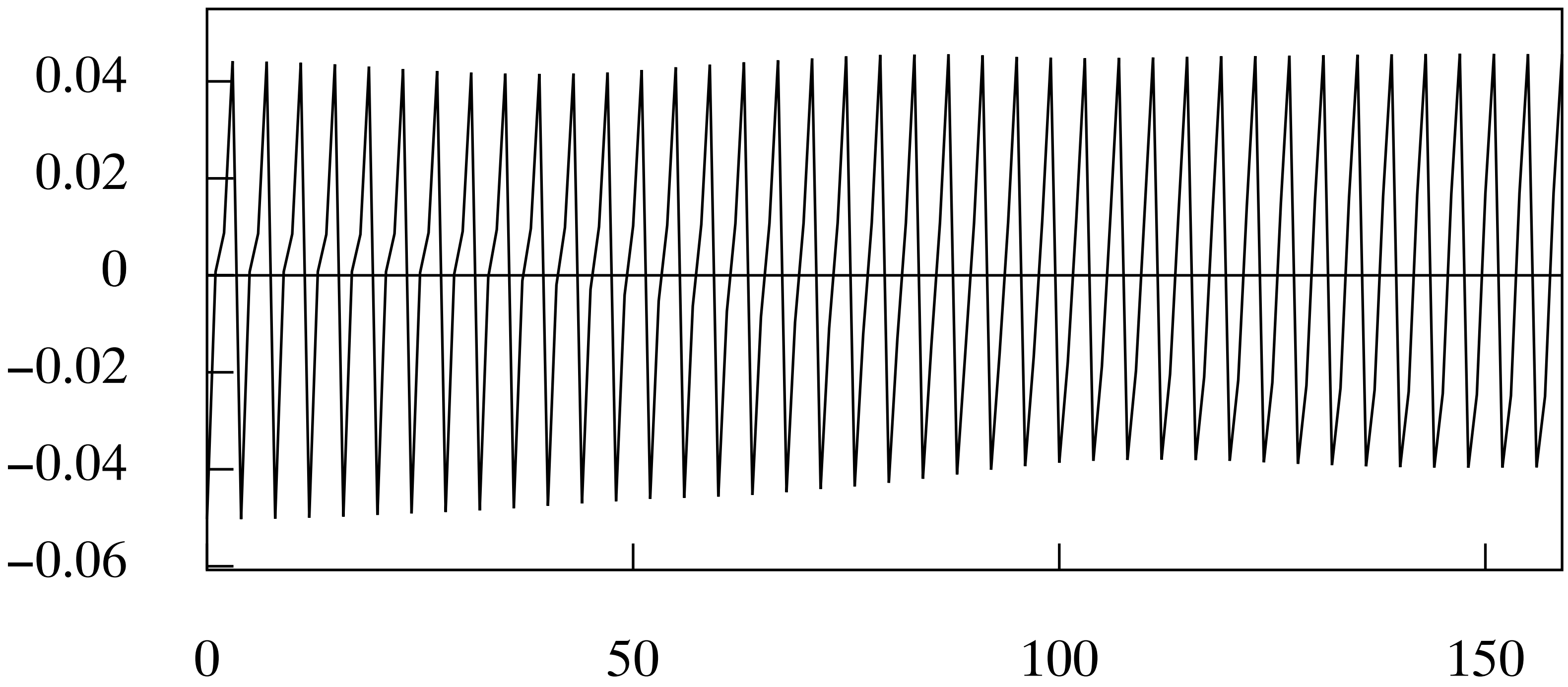

Figure 10. The seasonal component extracted from the logarithmic consumption data by the simplified Wiener–Kolmogorov filter.

5. FREQUENCY-DOMAIN SEASONAL ADJUSTMENT

The seasonal component generated by the Wiener–Kolmogorov method is synthesised from the sinusoidal elements at the fundamental seasonal frequency and at the harmonic frequencies, and from little else besides. Therefore, it has a very regular appearance.

The frequency-domain method of seasonal adjustment permits the selection of a wider range of elements adjacent to the seasonal frequencies. Therefore, it is liable to generate a more variable pattern of seasonal fluctuations.

Observe that the combination of two elements at Fourier frequencies that are adjacent will generate a pattern that evolves throughout the entire sample period, moving from constructive interference to destructive interference and back.

(Over several sample periods, this would give rise to a phenomenon of low-frequency beats.)

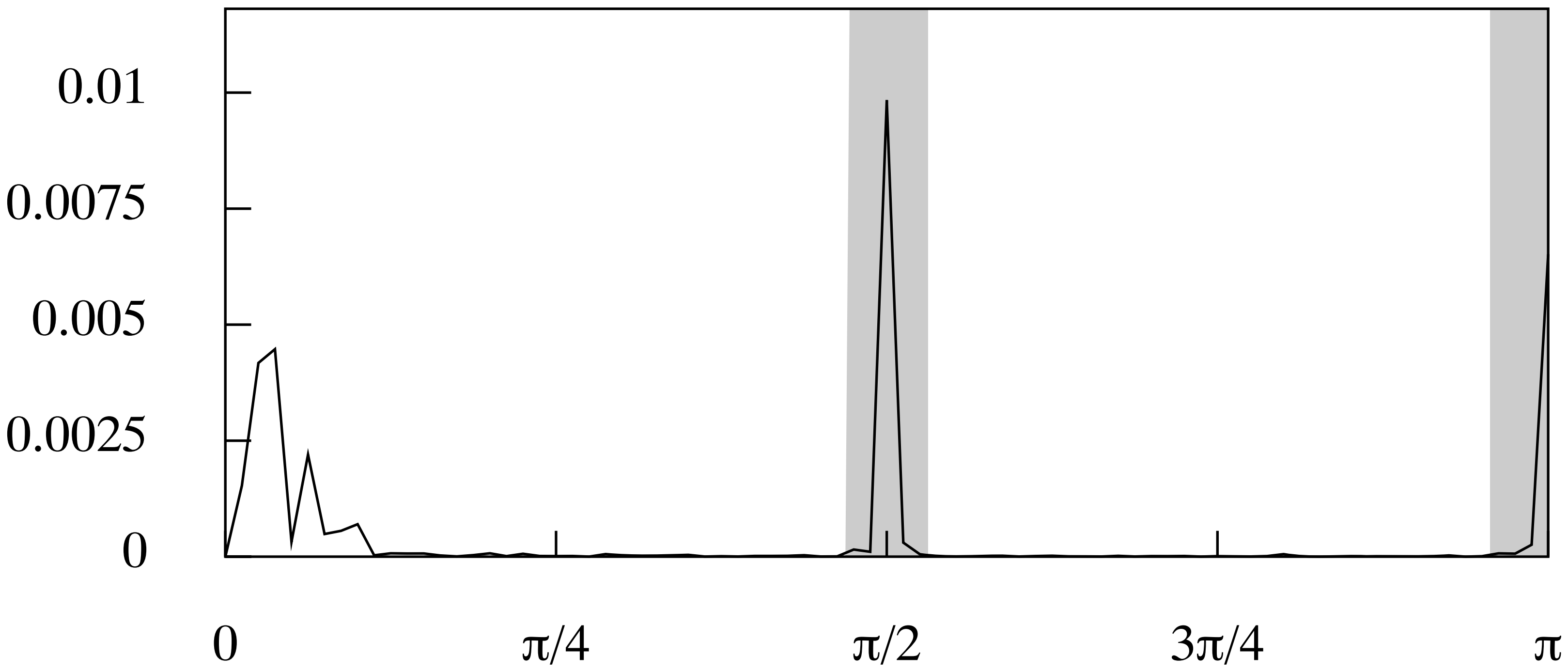

Figure 11. The periodogram of the residual sequence obtained from the linear detrending of the logarithmic consumption data. The shaded bands contain the elements of the seasonal component.

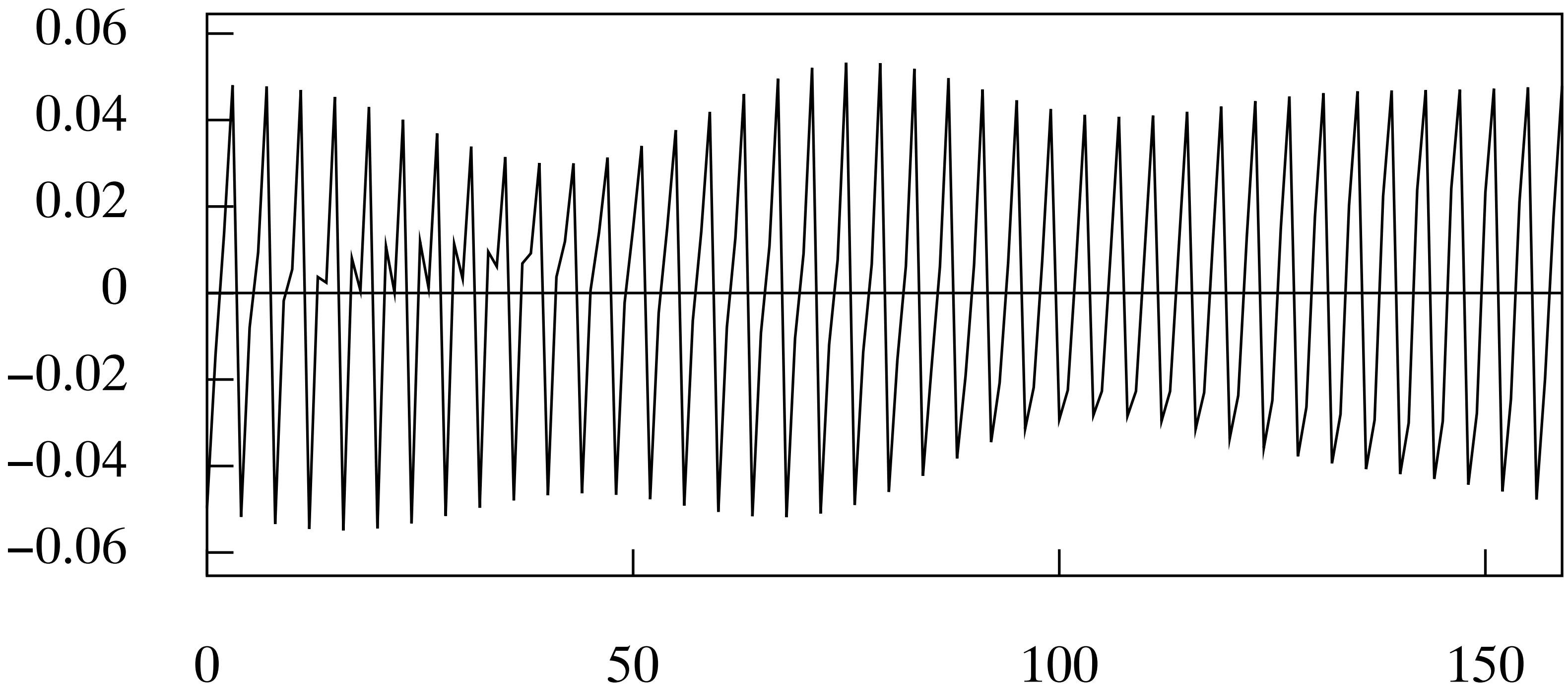

Figure 12. The seasonal component extracted from the logarithmic consumption data by the frequency-domain method.

6. THE LOW-FREQUENCY COMPONENT OF U.K CONSUMPTION

The profile of the seasonally-adjusted U.K. data has a roughness that is attributable to minor elements of noise, of which the spectral traces are visible in the periodogram of Figure 7. It it proposed that these elements have no economic significance and that they can be discarded.

A smooth continuous trajectory can be synthesised from the Fourier ordinates of the detrended data that reside in the shaded frequency band covering the interval $[0, \pi/8]$.

The smooth trajectory can be added to the linear trend to create a trend–cycle component. This constitutes the seasonally-adjusted data.

Observe that all of the information of the smooth trajectory can be captured in a sample taken at the rate of one observation in every two years, which is at 1/8 of the original quarterly sampling rate.

The sample has 20 data points, compared to 160 data points in the original sample of quarterly data.

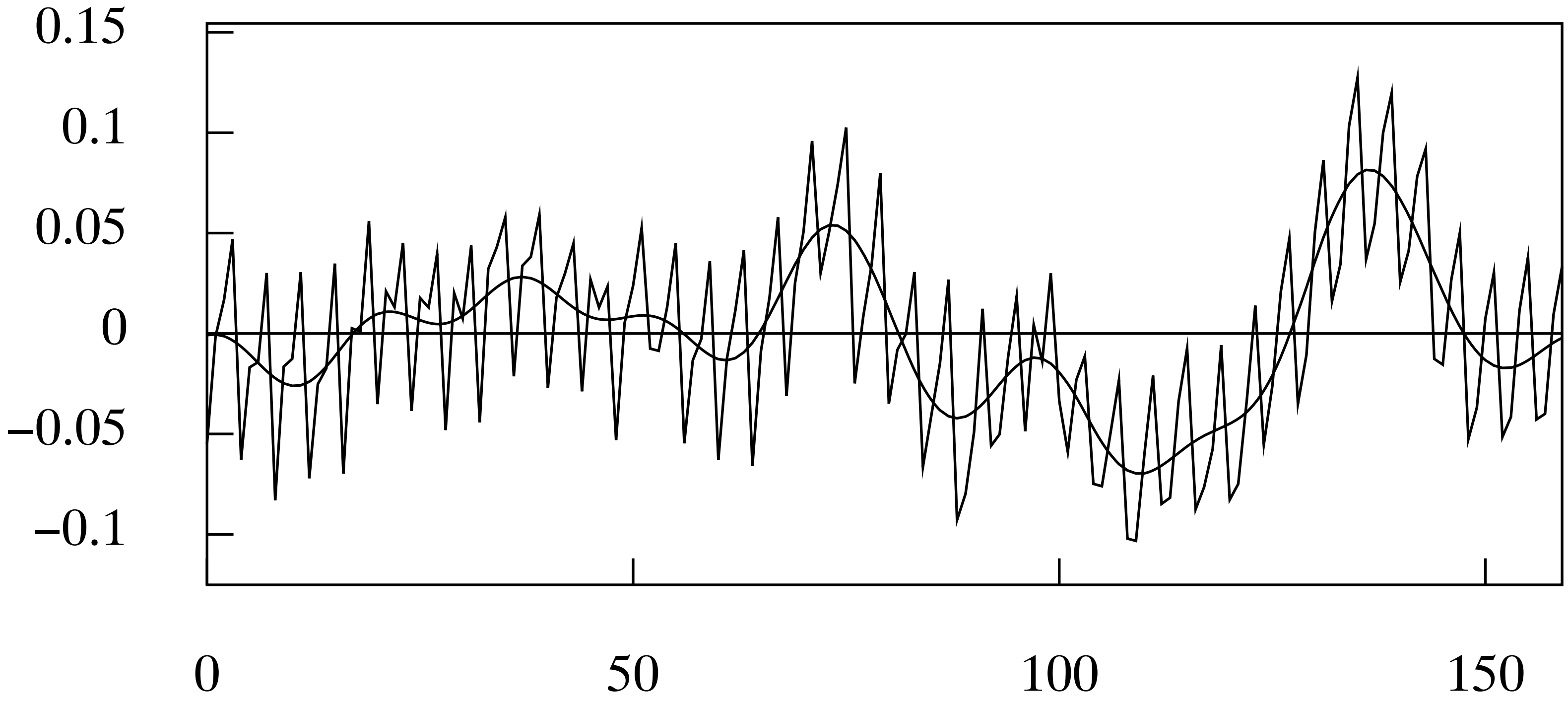

Figure 13. The residual sequence from fitting a linear trend to the logarithmic consumption data with an interpolated function representing the business cycle.

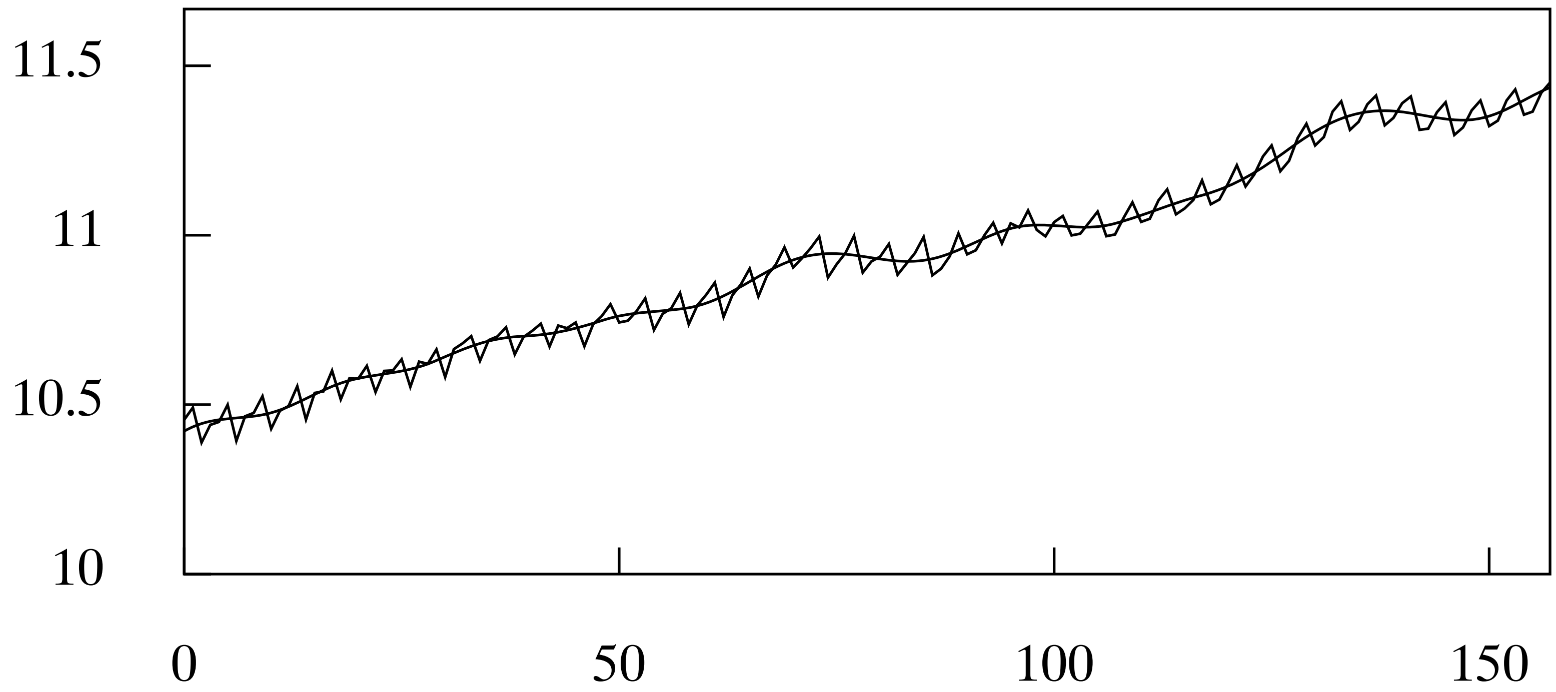

Figure 14. The trend–cycle component of U.K. consumption determined by the frequency-domain method, superimposed on the logarithmic data.

7. ANALYSIS OF MONTHLY U.S. SALES DATA

For the U.S. sales data, there is barely a distinction between the sequence obtained by eliminating the seasonal elements and the sequence that is synthesised from the Fourier ordinates that fall in the interval $[0, \pi/8]$.

The frequency value of $\pi/8$ is higher in monthly data (one cycle in 9 months) than in quarterly data (one cycle in 2 years). It is less than the seasonal frequency of one cycle in 12 months. Therefore, the trajectory synthesised from the ordinates in the interval $[0, \pi/8]$ is free of seasonal effects.

The residual deviations from an interpolated linear trend have been mapped onto the circumference of a circle. A synthetic segment has been inserted between the end and the beginning the residual sequence. This has been generated by morphing the final-year residuals into the first-year residuals.

A trajectory synthesised from the Fourier ordinates in the interval $[0, \pi/8]$ of the augmented data has been added to the linear trend to form the tend/cycle function that represents the deseasonalised trajectory of the original trended logarithmic data.

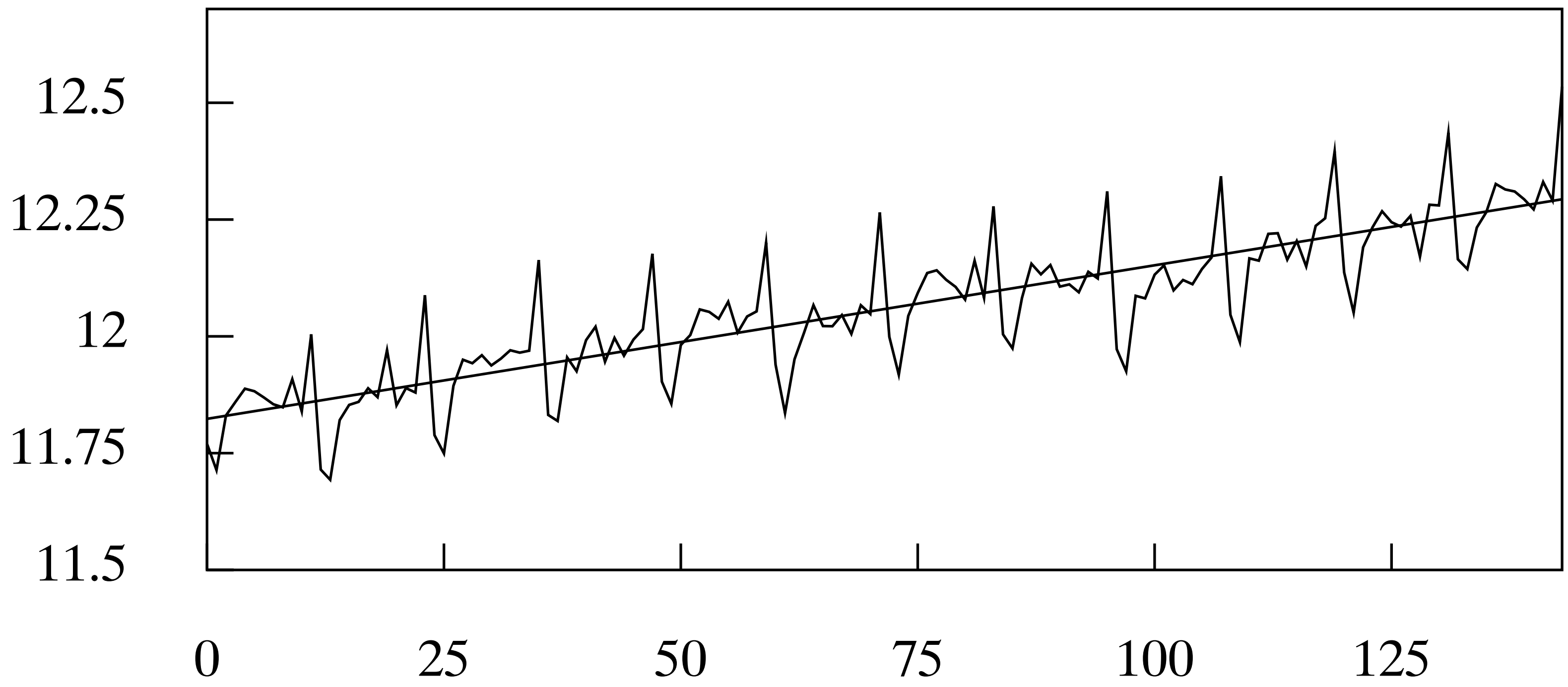

Figure 15. The logarithms of U.S. total retail sales from January 1953 to December 1964 with an interpolated linear function.

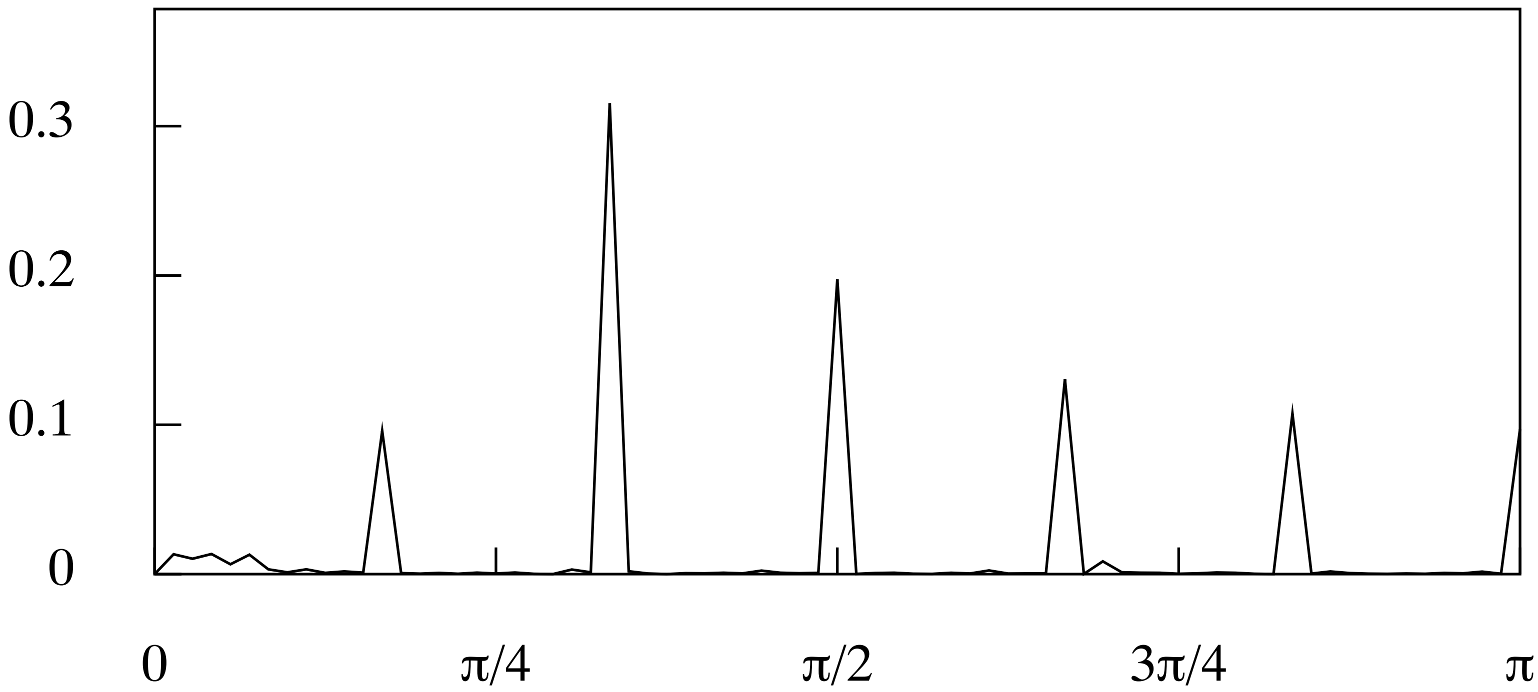

Figure 16. The periodogram of residuals from fitting a linear trend to the logarithmic sales data.

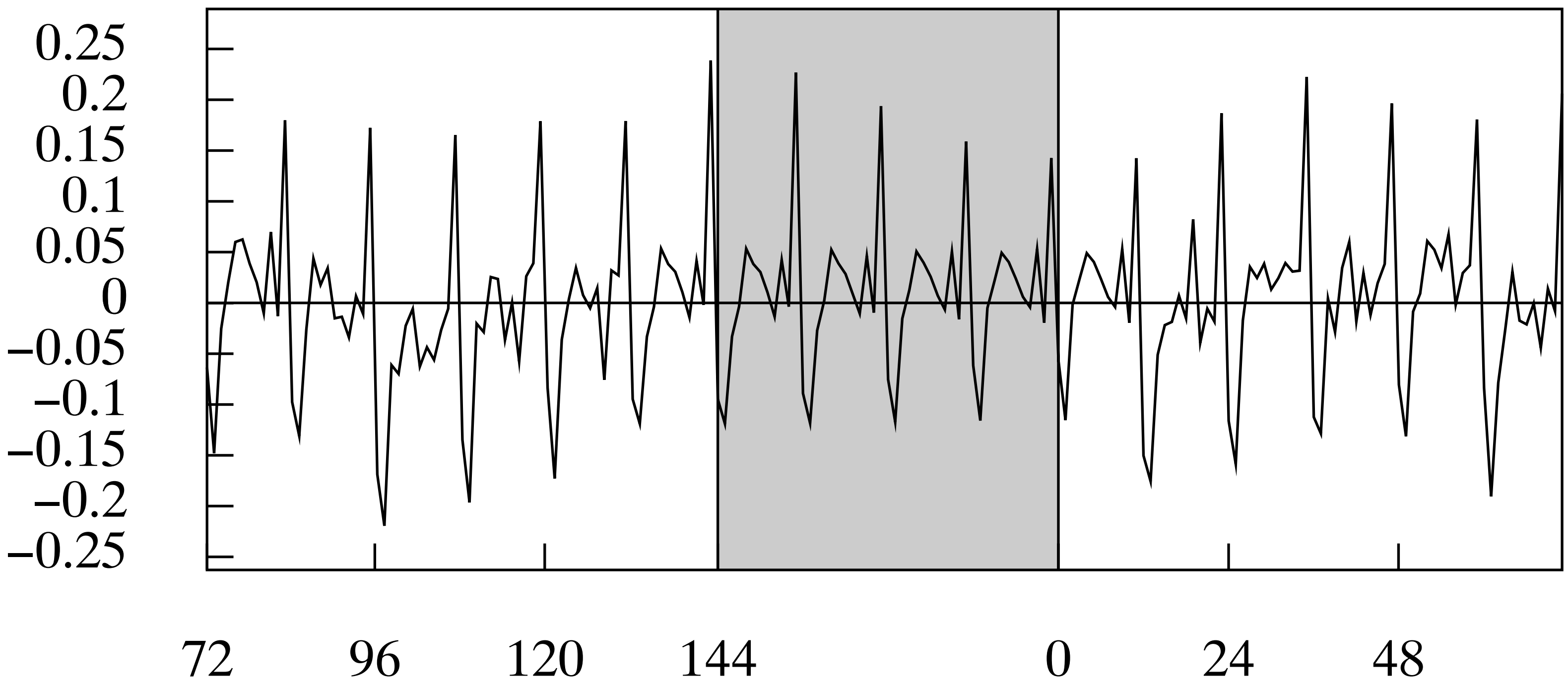

Figure 17. The residuals from a linear detrending of the sales data, with an interpolation of four years length, marked by the shaded band, inserted between the end and the beginning of the circularised sequence.

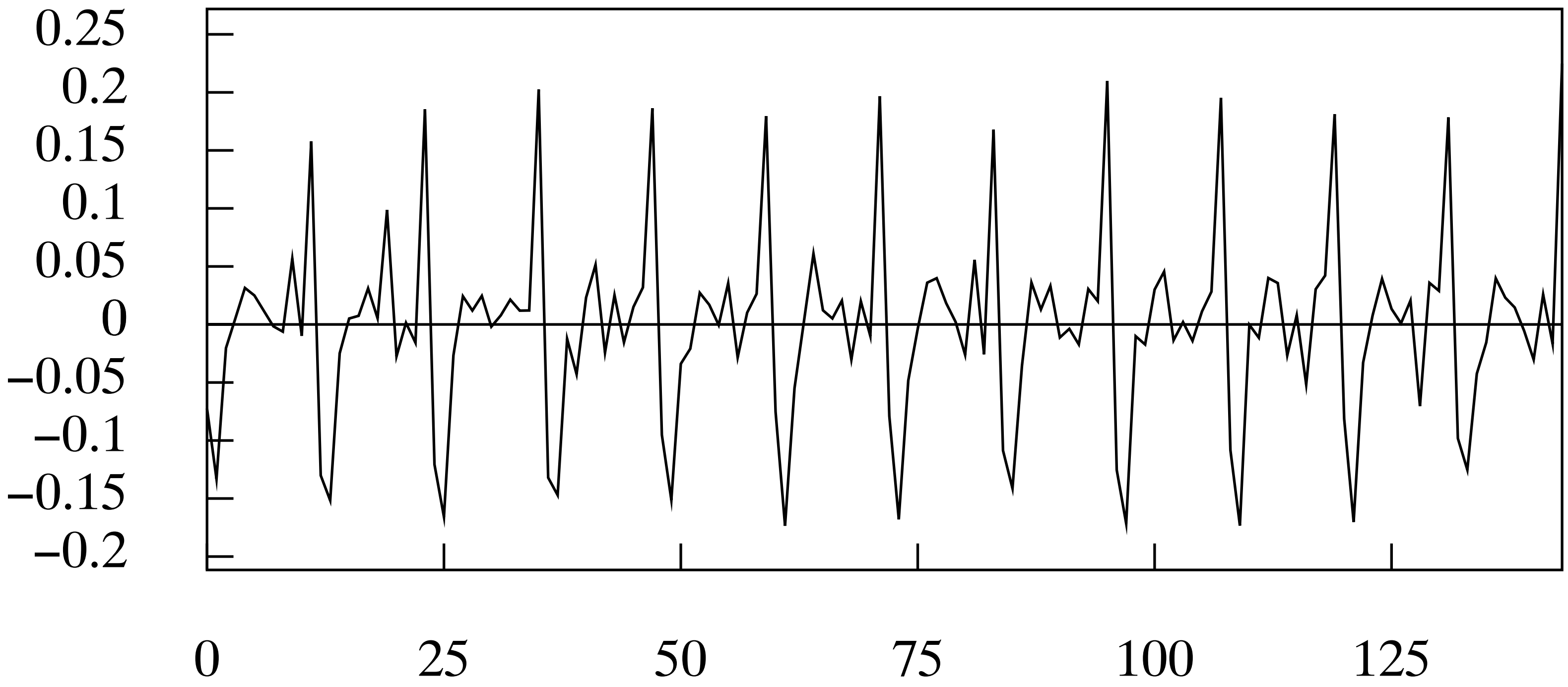

Figure 18. The sequence of residual deviations of the sales data from their trend, which may be regarded as the seasonal component.

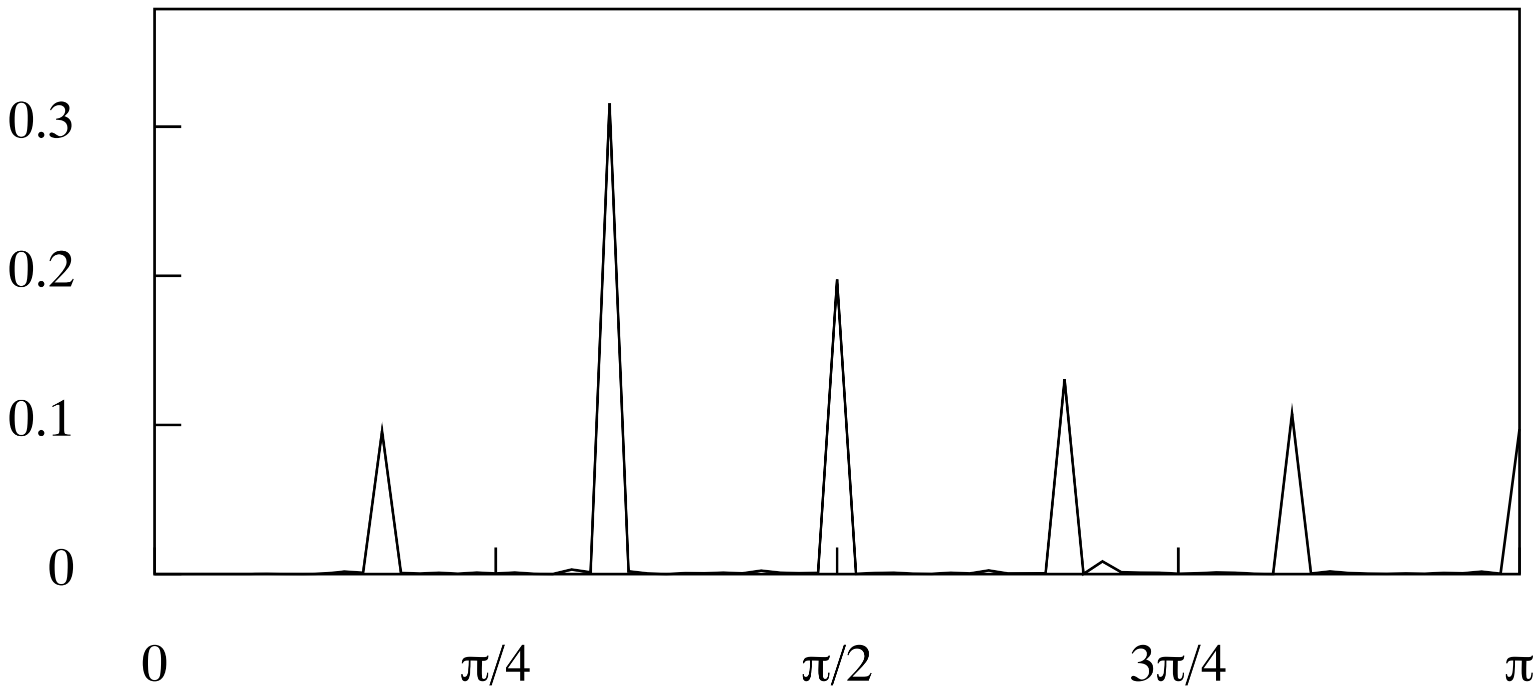

Figure 19. The periodogram of the residual deviations of the sales data from their trend.

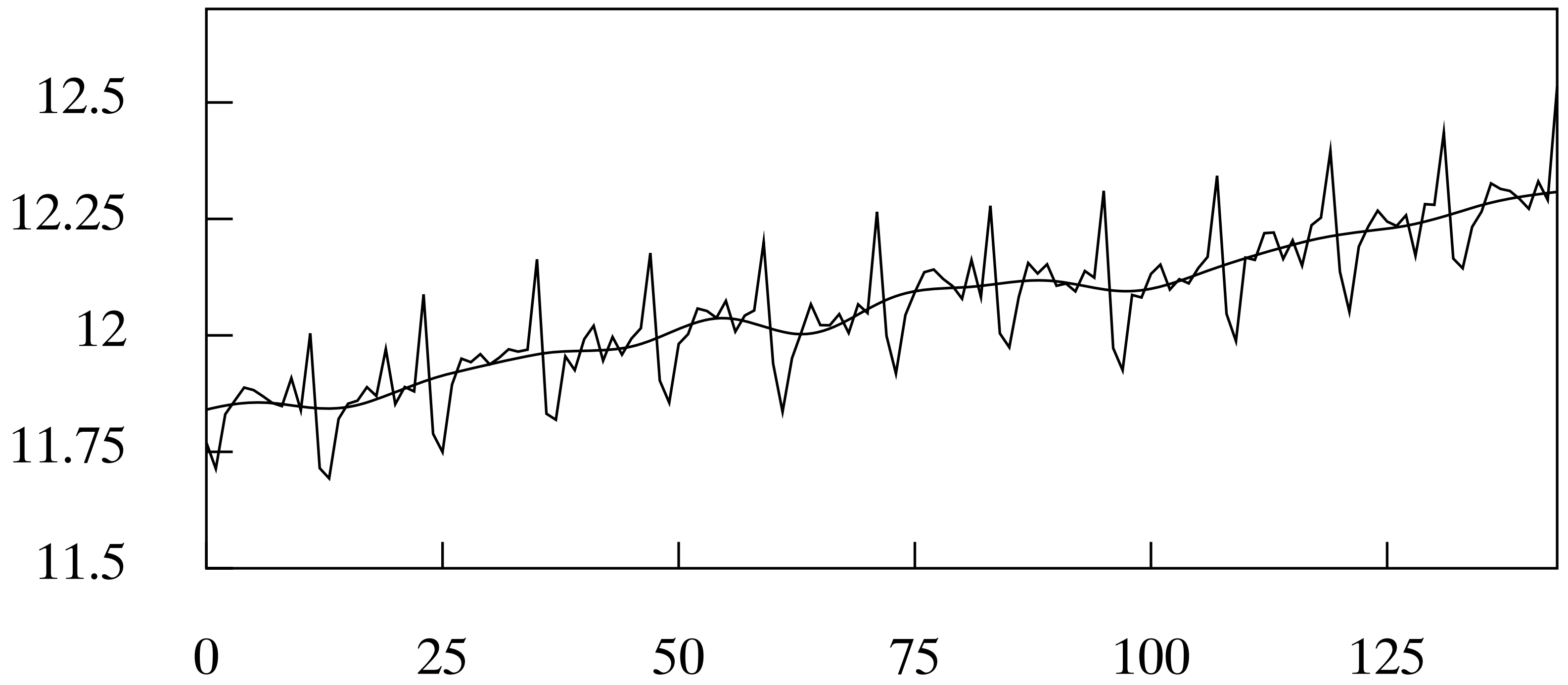

Figure 20. The logarithms of U.S. total retail sales from January 1953 to December 1964 with an interpolated trend function.

8. BAND-LIMITED ANALYTIC STOCHASTIC FUNCTIONS

A continuous function synthesised from a finite number of the sinusoidal elements within a limited frequency band is analytic and differentiable.

The set of continuous functions defined over the real line $\mathcal{R} = \{t \in (-\infty, \infty)\}$ and band-limited in frequency to the interval $\mathcal{F} = \{\omega \in [0, \pi]\}$, has an orthononormal basis consisting of the sinc functions at unit displacements.

Thus, if $\{y_t, t = 0, \pm 1, \pm 2, \ldots\}$ are the sampled ordinates of a continuous function $y(t)$, band-limited to $[0, \pi]$, then

$$y(t) = \sum_{k=-\infty}^{\infty} y_t \,\mathrm{sinc}(t - k), \quad \text{where} \quad \mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}.$$

On the assumption that our data have been sampled from continuous band-limited processes, we can find the derivatives of their underlying trajectories. This can assist us, for example, in finding the turning points of the trajectories.

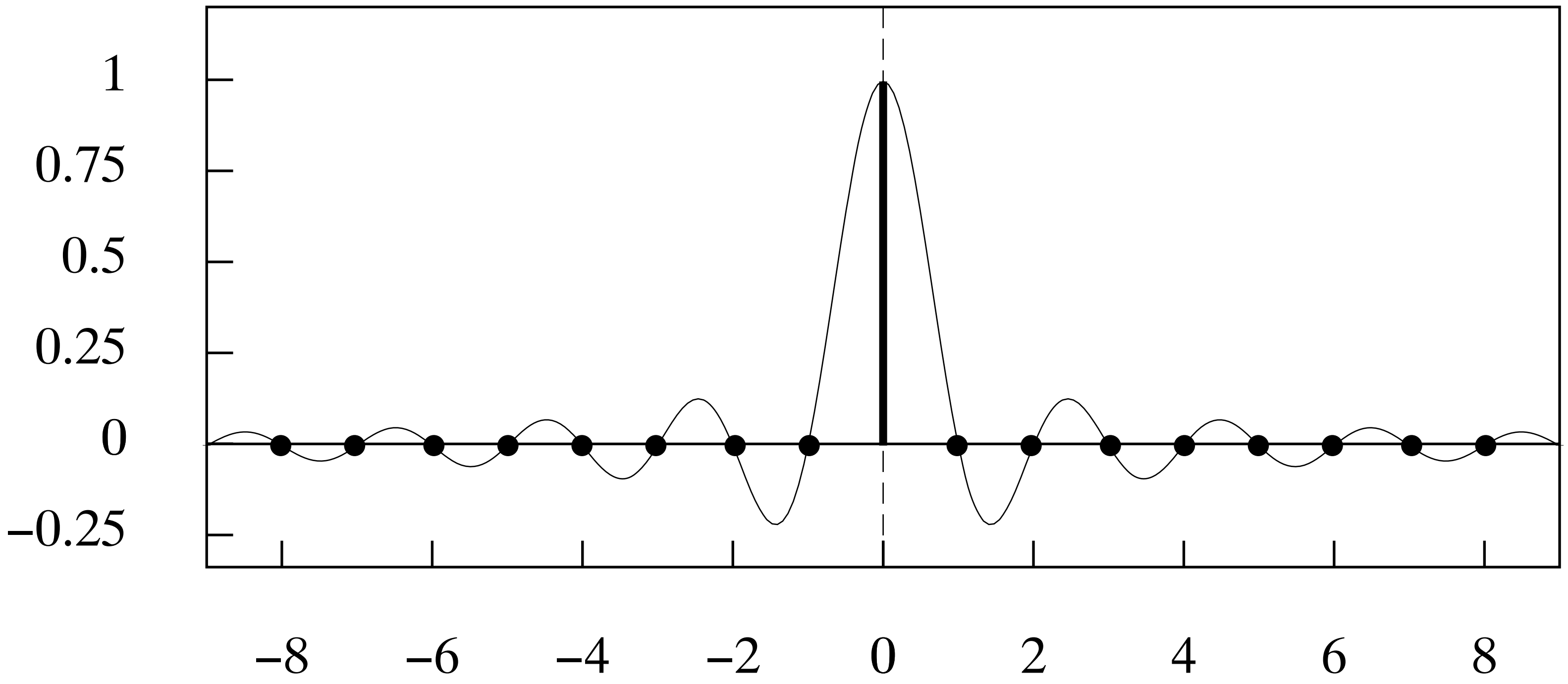

Figure 21. The sinc function $\psi(t) = \sin(\pi t)/\pi t$.

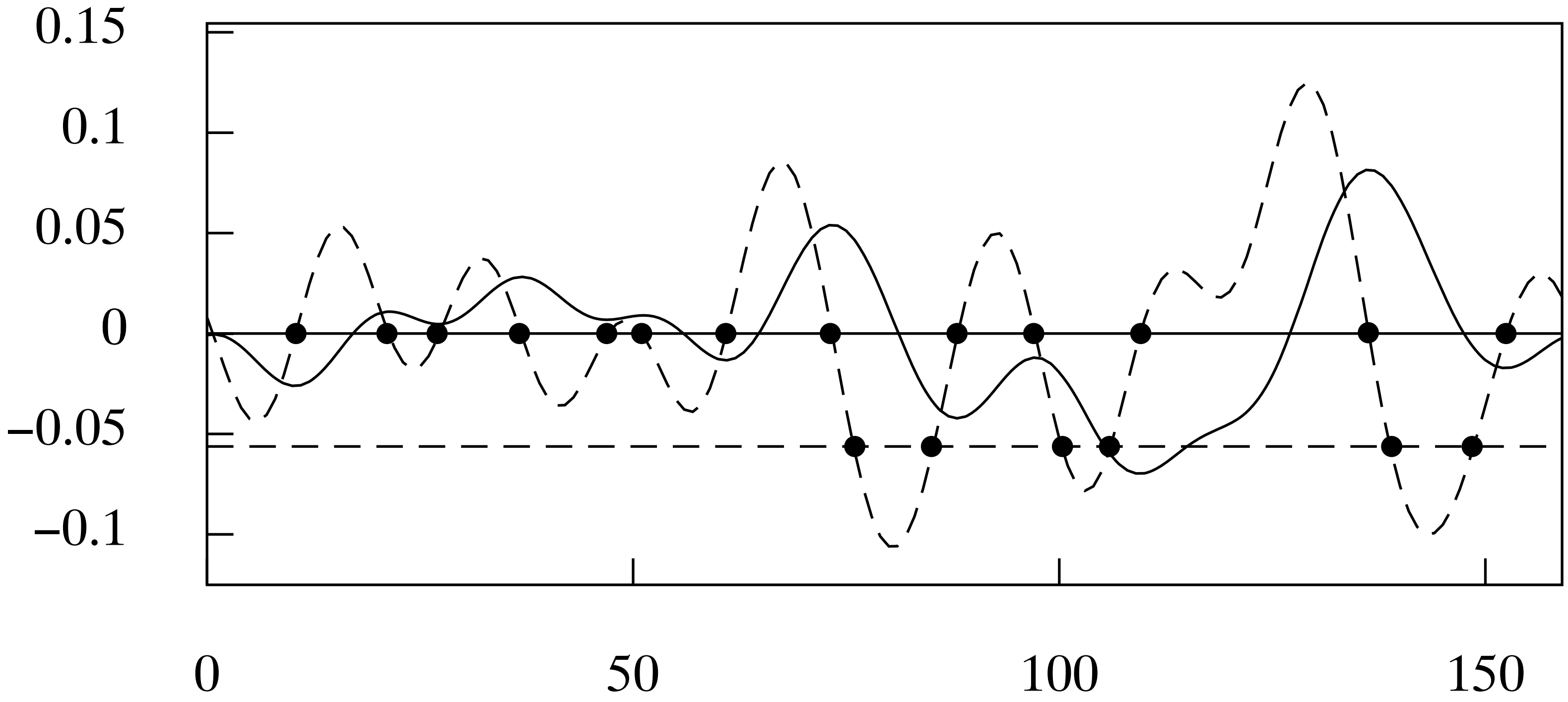

Figure 22. The turning points of the business cycle marked on the horizontal axis by black dots. The solid line is the business cycle of Figure 16. The broken line is the derivative function.

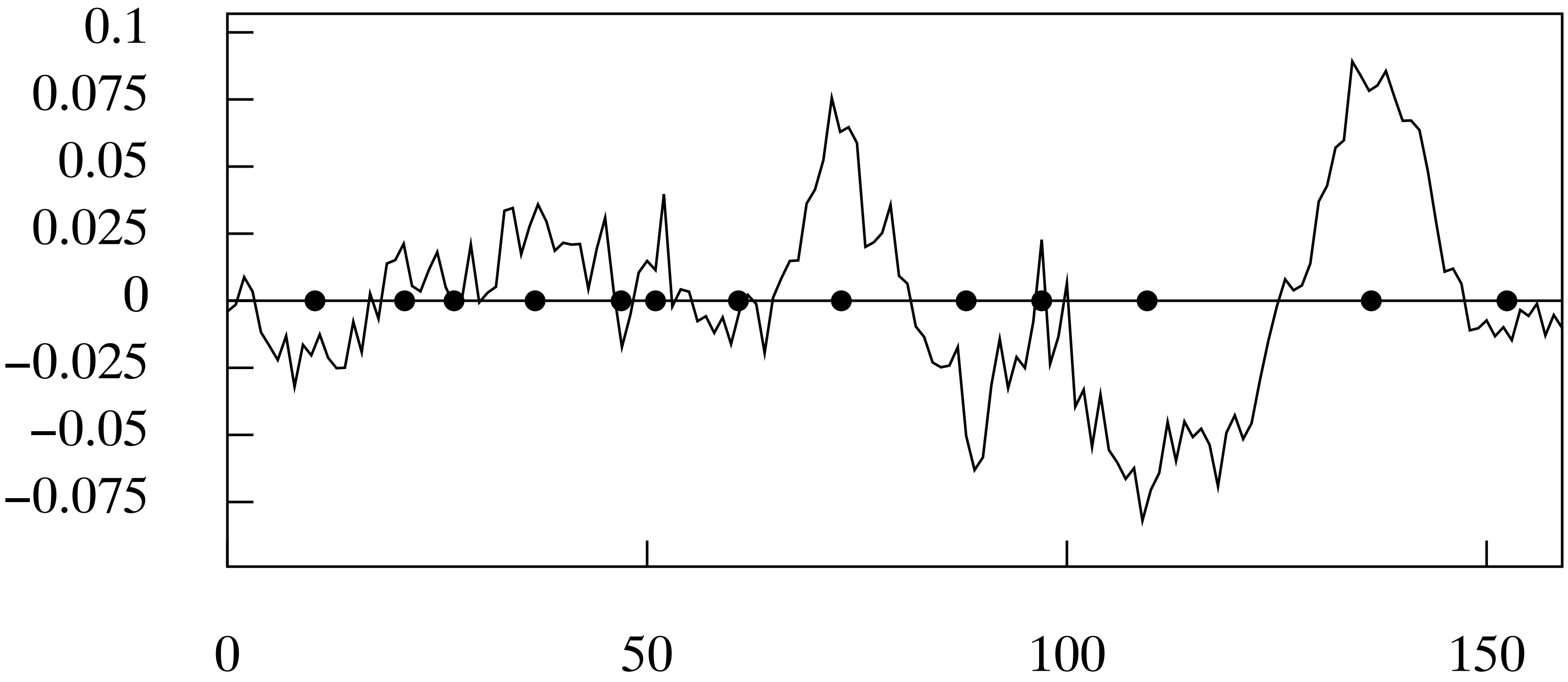

Figure 23. A sequence derived from the detrended data of Figure 16 via the time-domain method of seasonal adjustment.

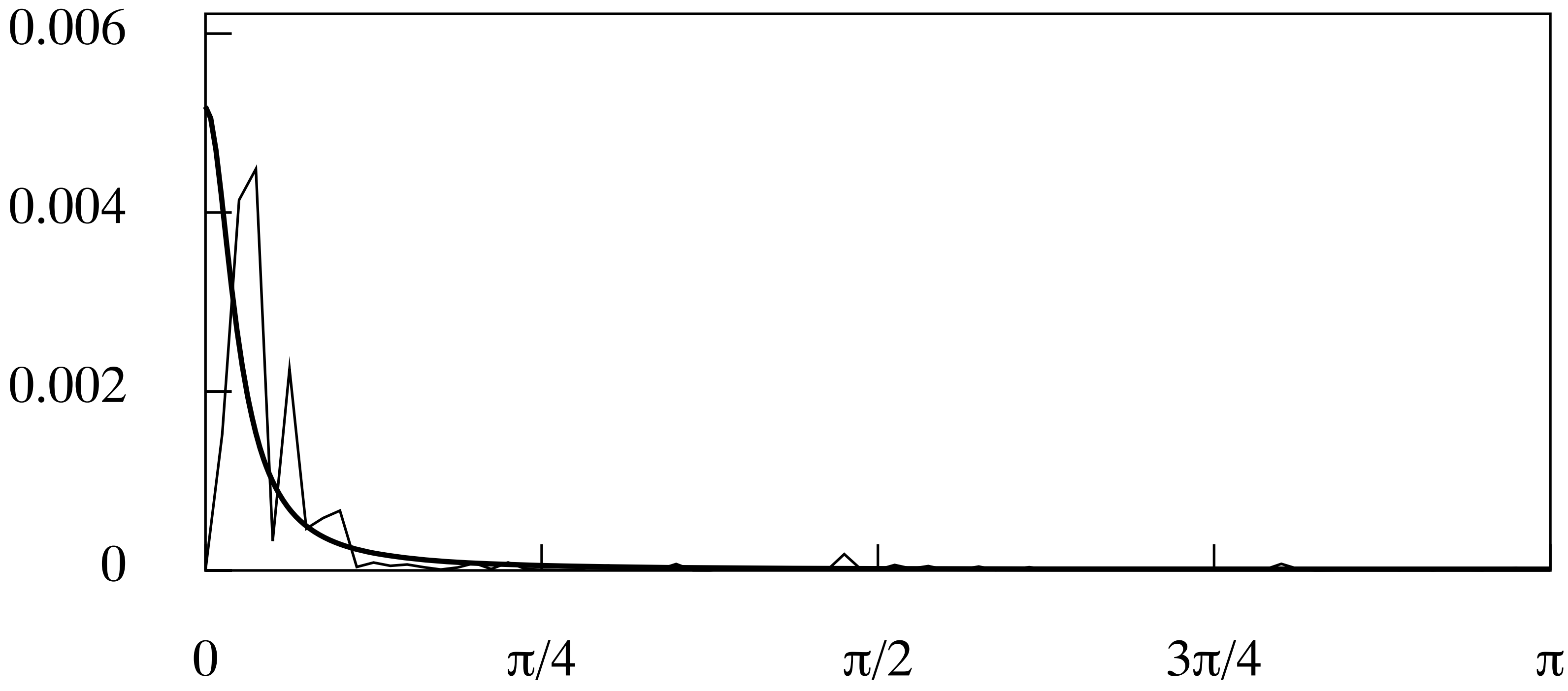

Figure 24. The spectrum of an AR(2) model fitted to the detrended, seasonally adjusted logarithmic consumption data, superimposed on the periodogram.

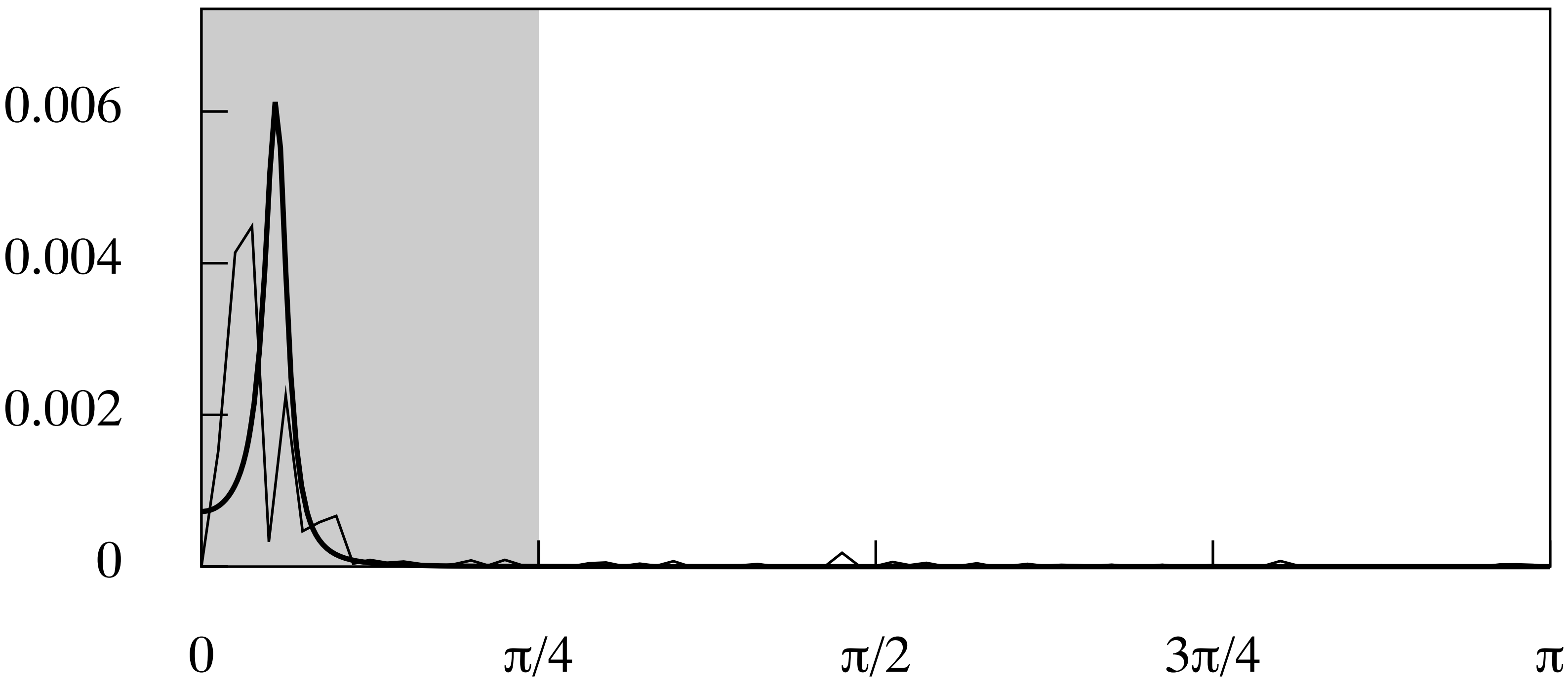

Figure 25. The spectrum of an AR(2) model fitted to the detrended, deseasonalised logarithmic income data via a band-limited autoregressive estimation, superimposed on the periodogram.

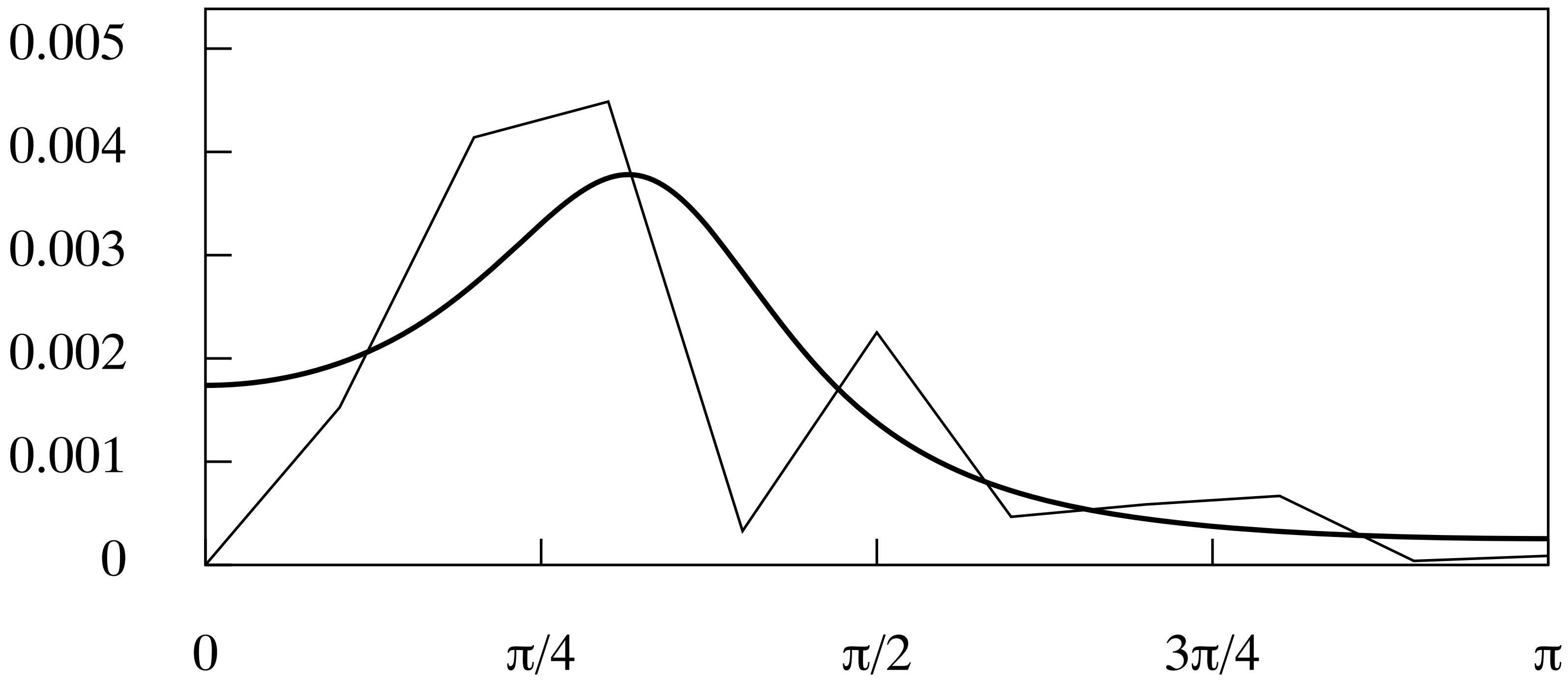

Figure 26. The periodogram of the sub sampled anti-aliased data with the parametric spectrum of an estimated AR(2) model superimposed.