CYCLES, SYLLOGISMS AND SEMANTICS: EXAMINING THE IDEA OF SPURIOUS CYCLES IN MACROECONOMIC DATA

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The claim that linear filters are liable to induce spurious fluctuations has been repeated many times of late. However, there are good reasons for asserting that this cannot be the case for the filters that, nowadays, are commonly employed by econometricians.

If these filters cannot have the effects that have been attributed to them, then one must ask what effects the filters do have that could have led to the aspersions that have been made against them.

1. The History of an Idea

Slutsky (1927, 1937) applied a moving-average filter to random numbers drawn from a public lottery to produce a sequence that had the characteristics of a macroeconomic business cycle.

Yule (1927) demonstrated the manner in which a second-order autoregressive model, driven by a white noise sequence of independently and identically distributed random variables, can give rise to an output that contains cycles of such regularity that one might imagine that they have a mechanical origin.

The danger of being misled by an inappropriate use of filters was emphasised by Howrey (1968), who discovered that the long-run economic cycles that Kuznets (1961) claimed to have detected were, in fact, the artefacts of his data processing. It seemed appropriate to describe these cycles as spurious.

FIGURE 3.————An index of English business cycles from 1855 to 1877; scale on the left side. ---------Terms 20 to 145 of Model I; scale on the right side.

1. The Effects of a Linear Filter

A linear filter combines the values of an input sequence $x(t)$ to create an output sequence

$$y(t) = \sum_j \psi_j x(t-j).$$

The effects of the filter can be shown by considering a complex exponential input sequence of the form $x(t) = \cos(\omega t) + \mathrm{i}\sin(\omega t) = \exp\{\mathrm{i}\omega t\}$. The corresponding output is

$$y(t) = \sum_j \psi_j e^{\mathrm{i}\omega(t-j)} = \left\{\sum_j \psi_j e^{-\mathrm{i}\omega j}\right\} e^{\mathrm{i}\omega t} = \psi(\omega)e^{\mathrm{i}\omega t}.$$

The effects are summarised by the complex function

$$\psi(\omega) = |\psi(\omega)|e^{-\theta(\omega)}.$$

The modulus $|\psi(\omega)|$ alters the amplitudes of the cyclical elements of the data, which is the *gain effect.* The argument $\theta(\omega)$ displaces the elements in time, which is the *phase effect.* A phase effect can be avoided if the filter coefficients are disposed symmetrically about a central point, such that the filter reaches equally forward and backwards in time.

Slutsky's Filter

Slutsky's filter was a ten-point moving average

$$y(t) = \frac{1}{10}\{x(t) + x(t-1) + \cdots + \ x(t-9)\}$$

The transfer function is defined by setting $z = \exp\{-\mathrm{i}\omega\}$ in the polynomial

$$\psi(z) = \frac{1}{10}(1 + z + \cdots + z^9).$$

Then $|\psi(z)|^2 = \psi(z)\psi(z^{-1})$. Setting $z = \exp\{\mathrm{i}\omega\}$ gives

$$|\psi(z)| = \frac{\sin(5\omega)}{10\sin(\omega/2)}.$$

In Slutsky's filter, the gain is unity at zero frequency and it declines rapidly with rising frequency. Therefore, the filter preserves the cycles at the lowest frequencies, which would include the trend, and it attenuates those at higher frequencies.

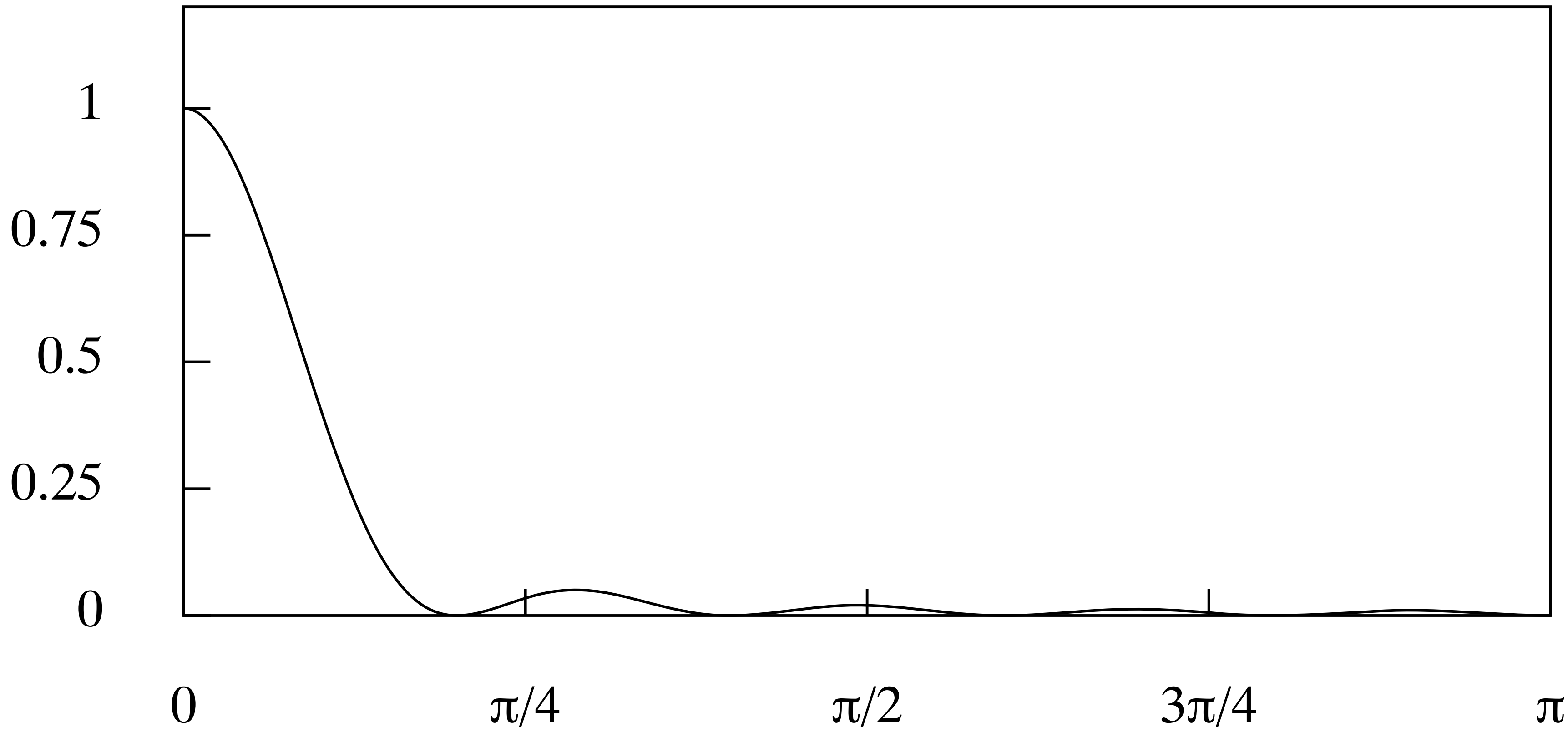

Figure 2. The squared gain of the Slutsky filter.

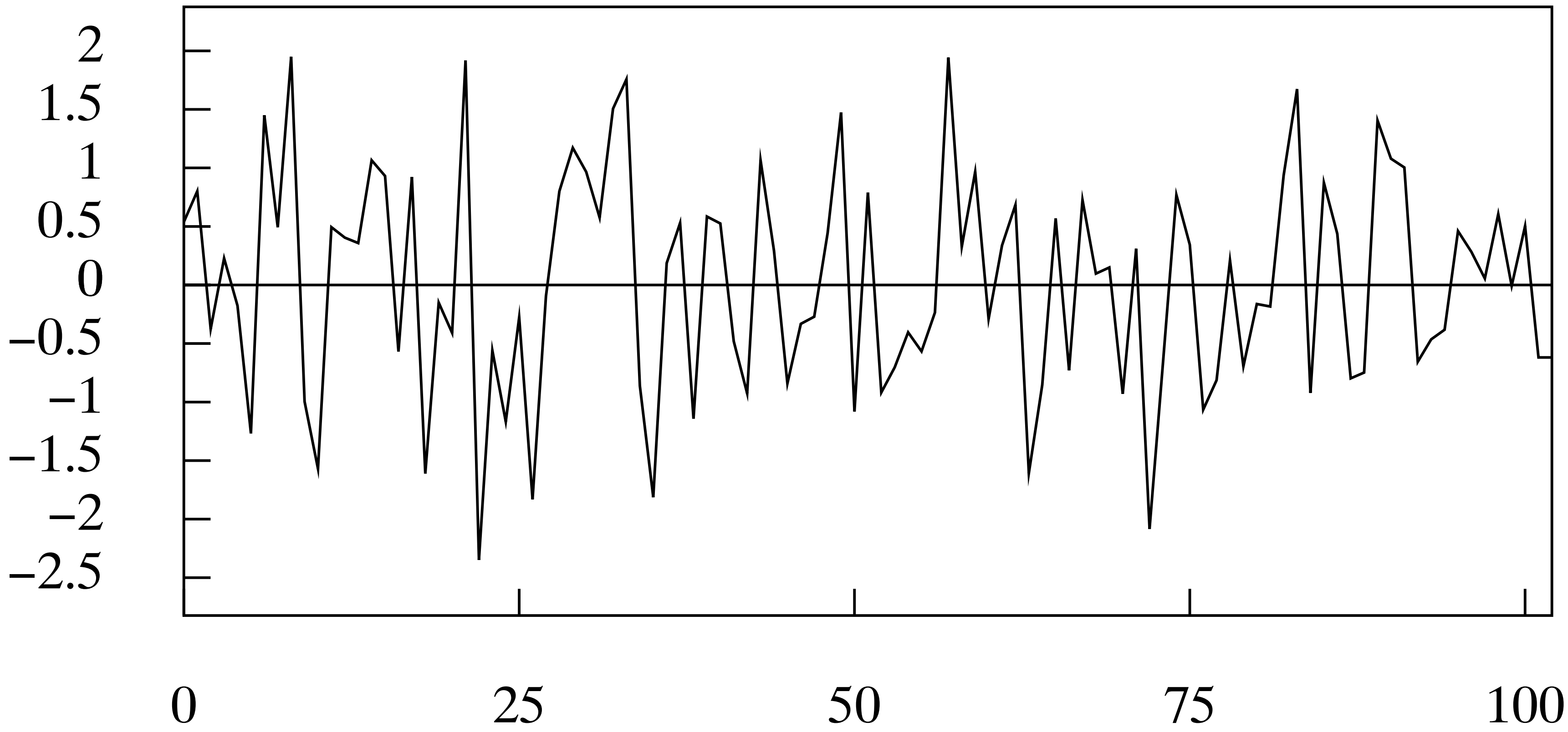

Figure 3. A simulated series of 103 points of a white-noise process.

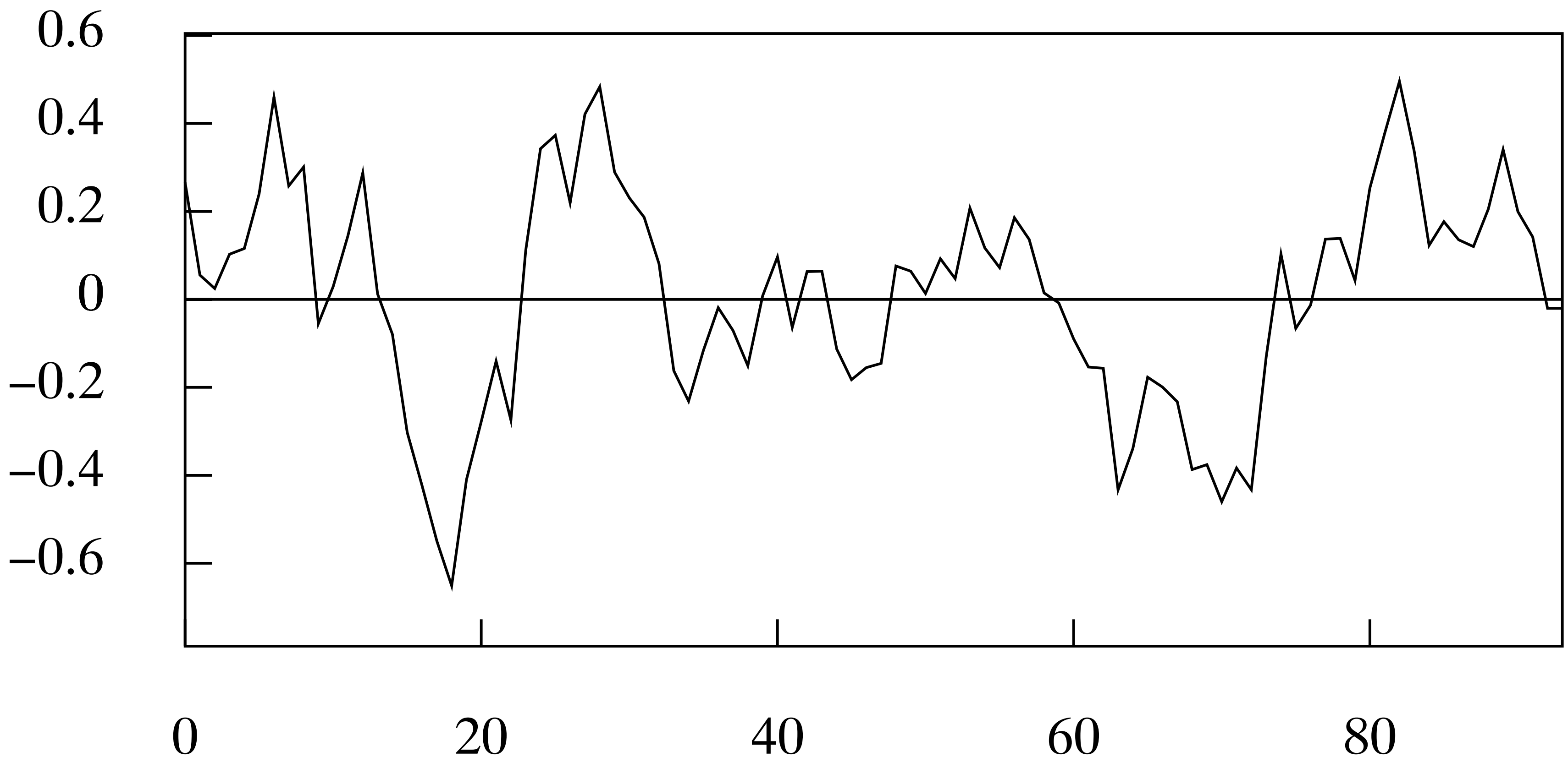

Figure 4. A sequence of 92 points obtained by applying the filter of Slutsky to a white-noise process.

The Filter of Kuznets

The filter of Kuznets compounded two operations. The first was a symmetric five-point moving average:

$$w(t) = \frac{1}{5}\{x(t+2) + x(t+1) + x(t) + x(t-1) + x(t-2)\}.$$

The second operation involved a difference across eleven points:

$$y(t) = w(t+5) - w(t-5).$$

The gain of the filter is given by

$$|\psi(\omega)| = \frac{2\sin(5\omega/2)\sin(5\omega)}{5\sin(\omega/2)}.$$

This has zero gain at zero frequency. The gain rises to a maximum of 3.30 at the frequency of $\pi/10$ and, thereafter, it declines rapidly with rising frequency. Therefore, for annually observed data, the filter amplifies more than threefold the elements of the data that have a 20 year cycle. This, according to Howrey (1968), was the provenance of the long-swings that were discovered by Kuznets.

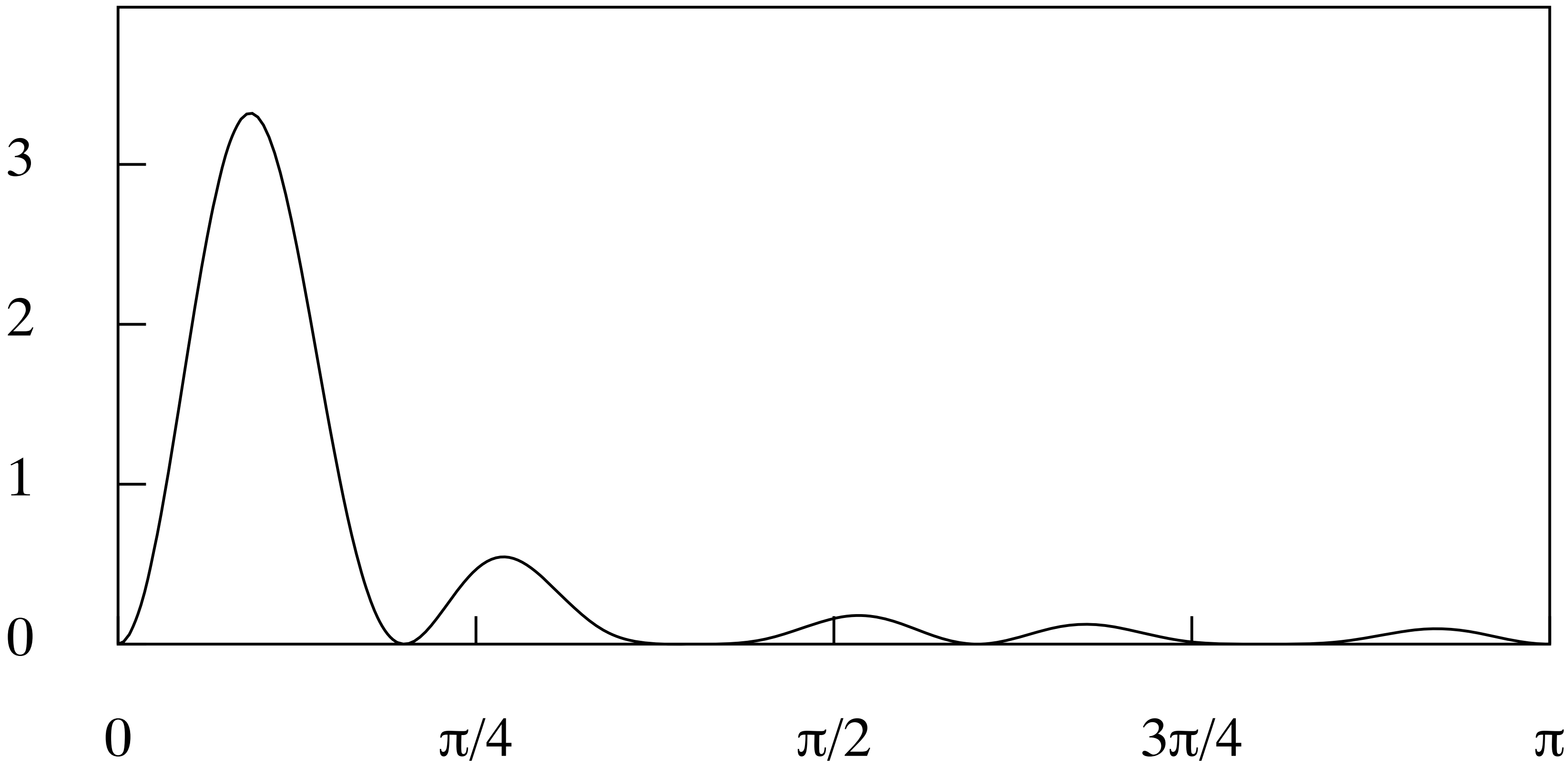

Figure 5. The squared gain of the filter of Kuznets.

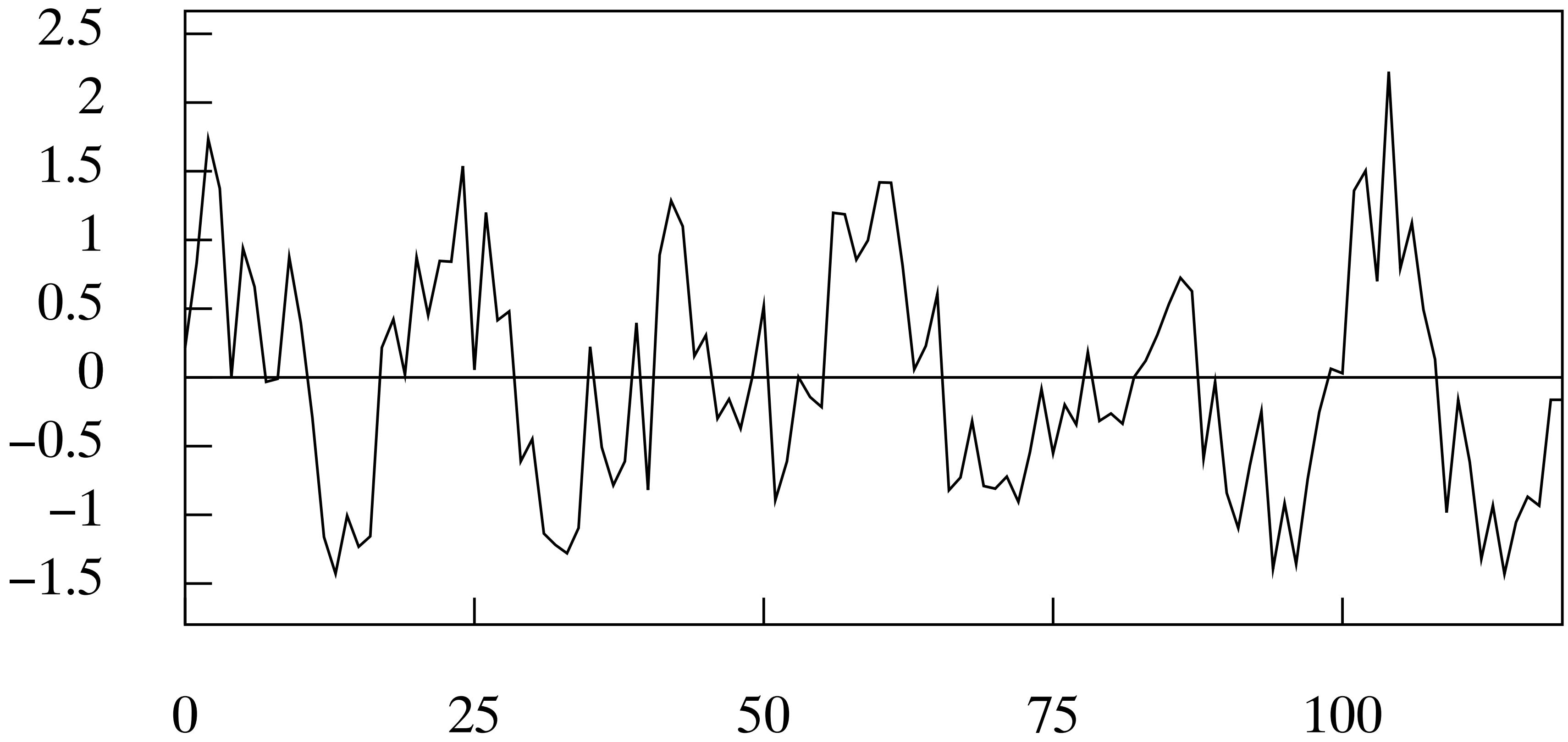

Figure 6. A sequence of 120 points obtained by applying the filter of Kuznets to a white-noise process.

The Linear Detrending of the Logarithmic Consumption Data

One may be doubtful of the meaning of a low-frequency cycle that emerges from the filtering of an undifferentiated white-noise sequence that has a uniform spectrum extending over the entire frequency range $[0, \pi]$.

A filtered sequence becomes meaningful if it represents a component of the data that resides in a frequency band that is separate from the frequency bands of other components. Such is the case of the low-frequency component of the logarithmic consumption data of the U.K. economy.

A filtering exercise might also be meaningful if it endeavours to separate a signal component from a white-noise contamination that extends evenly over frequency range. In that case, there will be no identifiable frequency value that separates the signal from the noise; and the separation of the two is bound to be tentative.

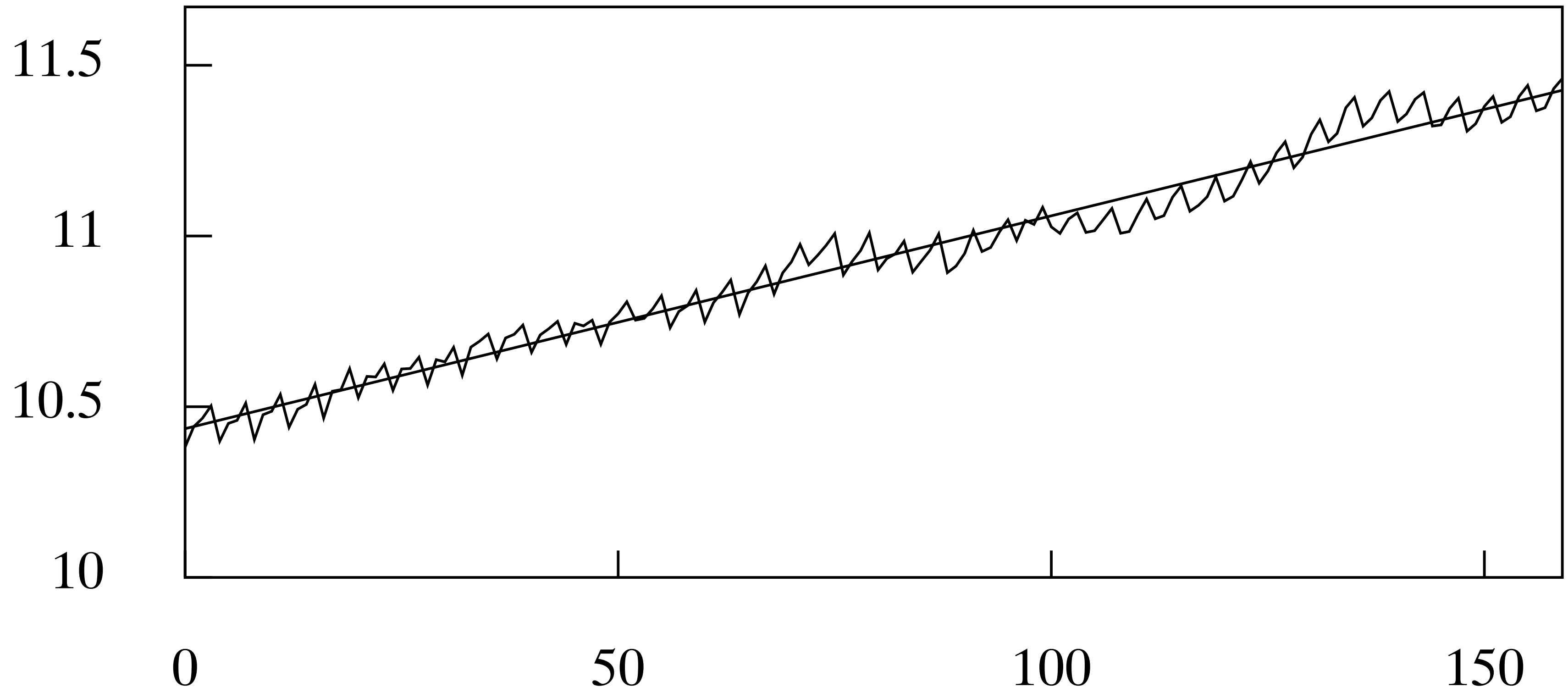

Figure 7. The quarterly series of the logarithms of consumption in the U.K., for the years 1955 to 1994, together with a linear trend interpolated by least-squares regression.

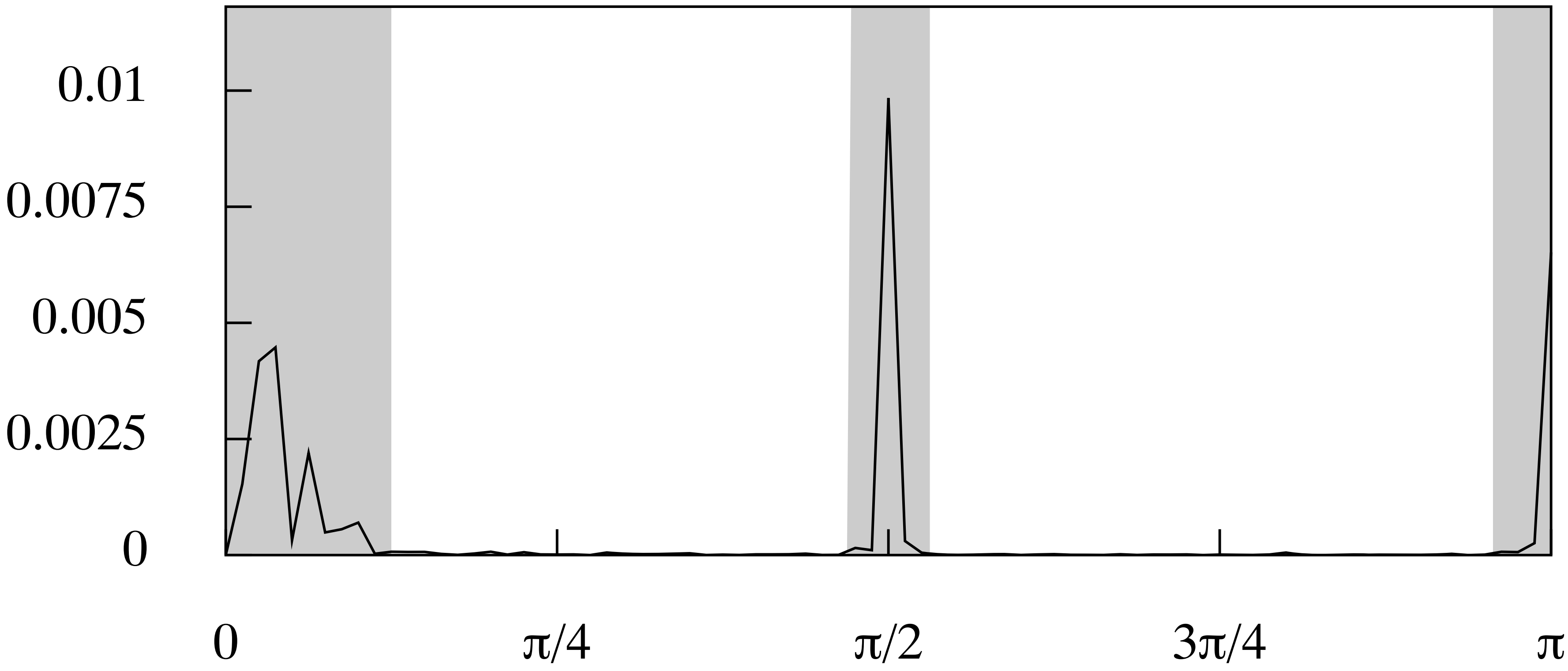

Figure 8. The periodogram of the residual sequence obtained from the linear detrending of the logarithmic consumption data. The shaded band on the interval $[0, \pi/8]$ contains the elements of the business cycle, and the bands in the vicinities of $\pi/2$ and π contain elements of the seasonal component.

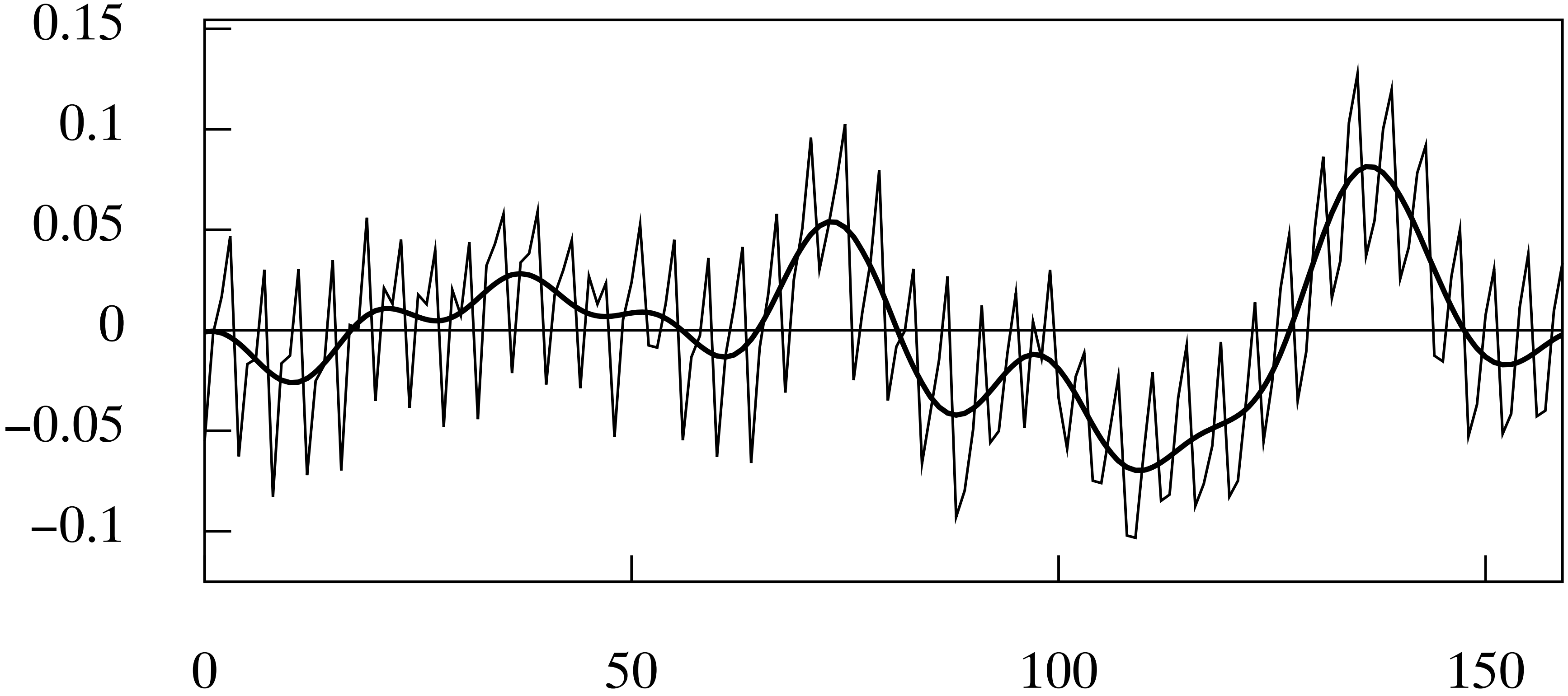

Figure 9. The residual sequence from fitting a linear trend to the logarithmic consumption data with an heavy interpolated line representing the business cycle, obtained by the frequency-domain method.

The Linear Detrending of a Random Walk

The belief that an inappropriate processing of the data can induce spurious fluctuations has been reaffirmed in more recent times in connection with the filtering of data generated by random walk processes.

Chan *et al.* (1977) and Nelson and Kang (1981) have described the effects of using linear and polynomial regressions to remove apparent trends from the data. They have observed that, regardless of the length of the data sequence, a random walk that has been subject to detrending exhibits major cycles that have a duration that is matched to the length of the sample.

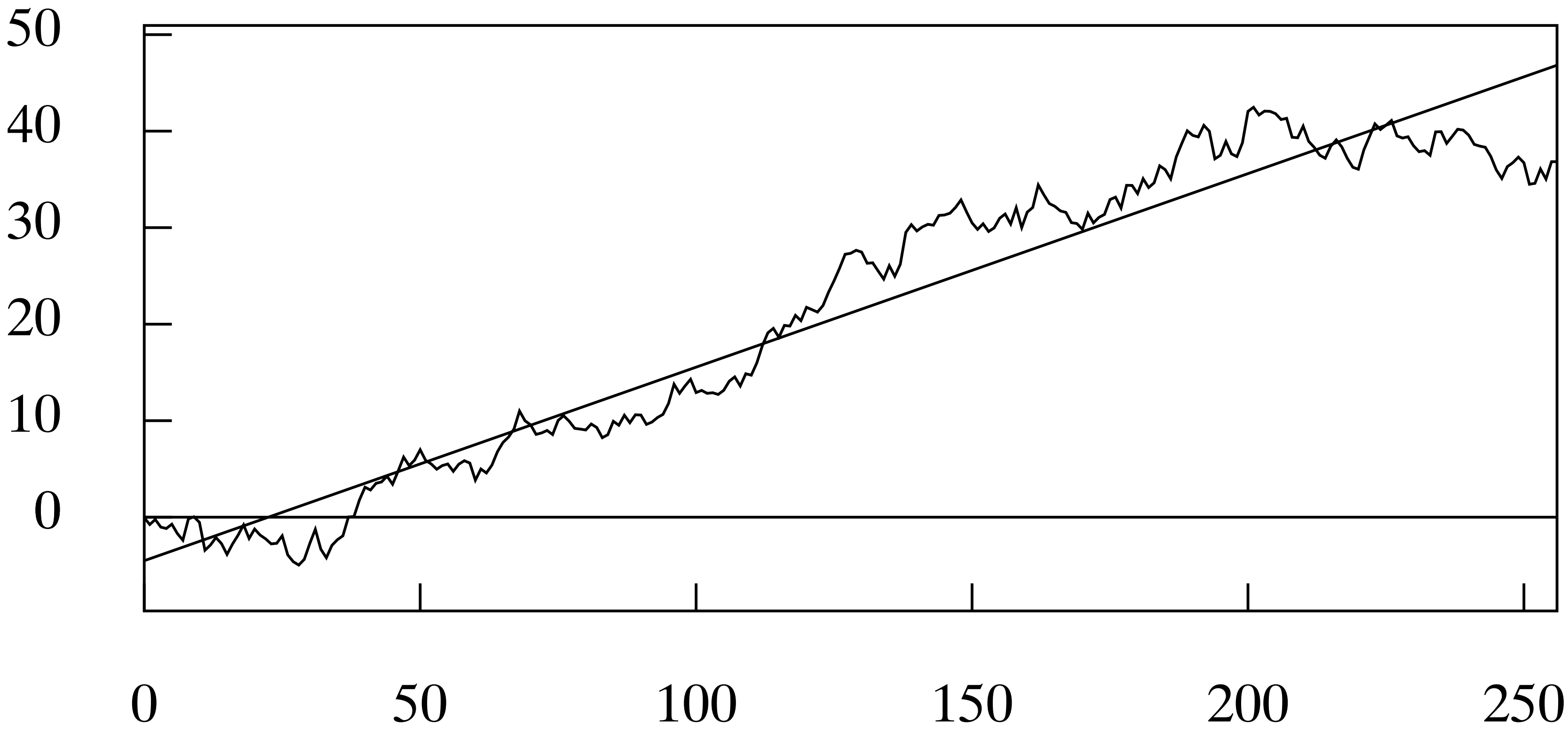

Figure 10. A random walk generated by the equation $y_t = y_{t-1} + \delta + \varepsilon_t$ together with an interpolated regression line. The variance of the white-noise disturbance is $V(\varepsilon_t) = 1$ and the drift parameter is $\delta = 0.2$.

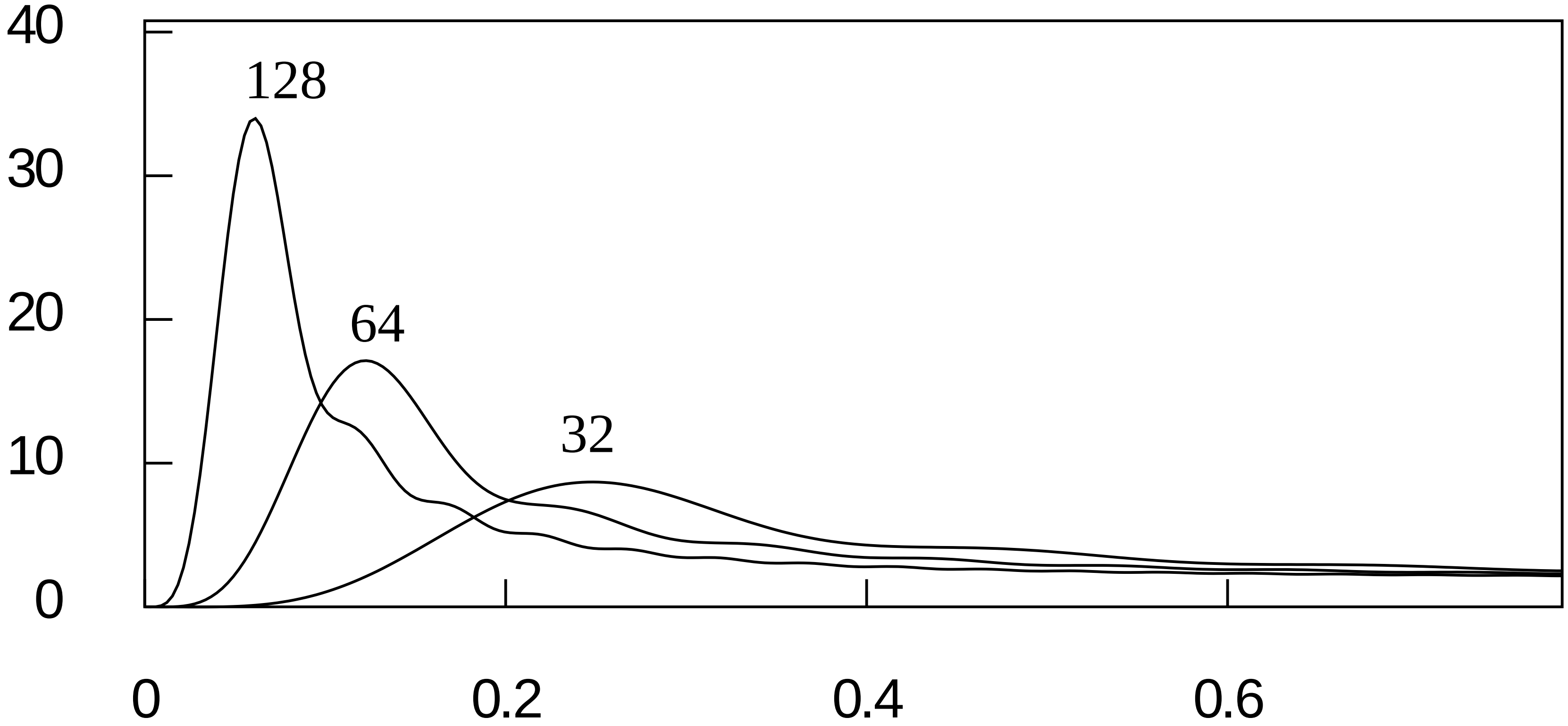

Figure 12. The spectral density function derived from the autocorrelation function of Nelson and Kang for sample sizes of 32, 64 and 128.

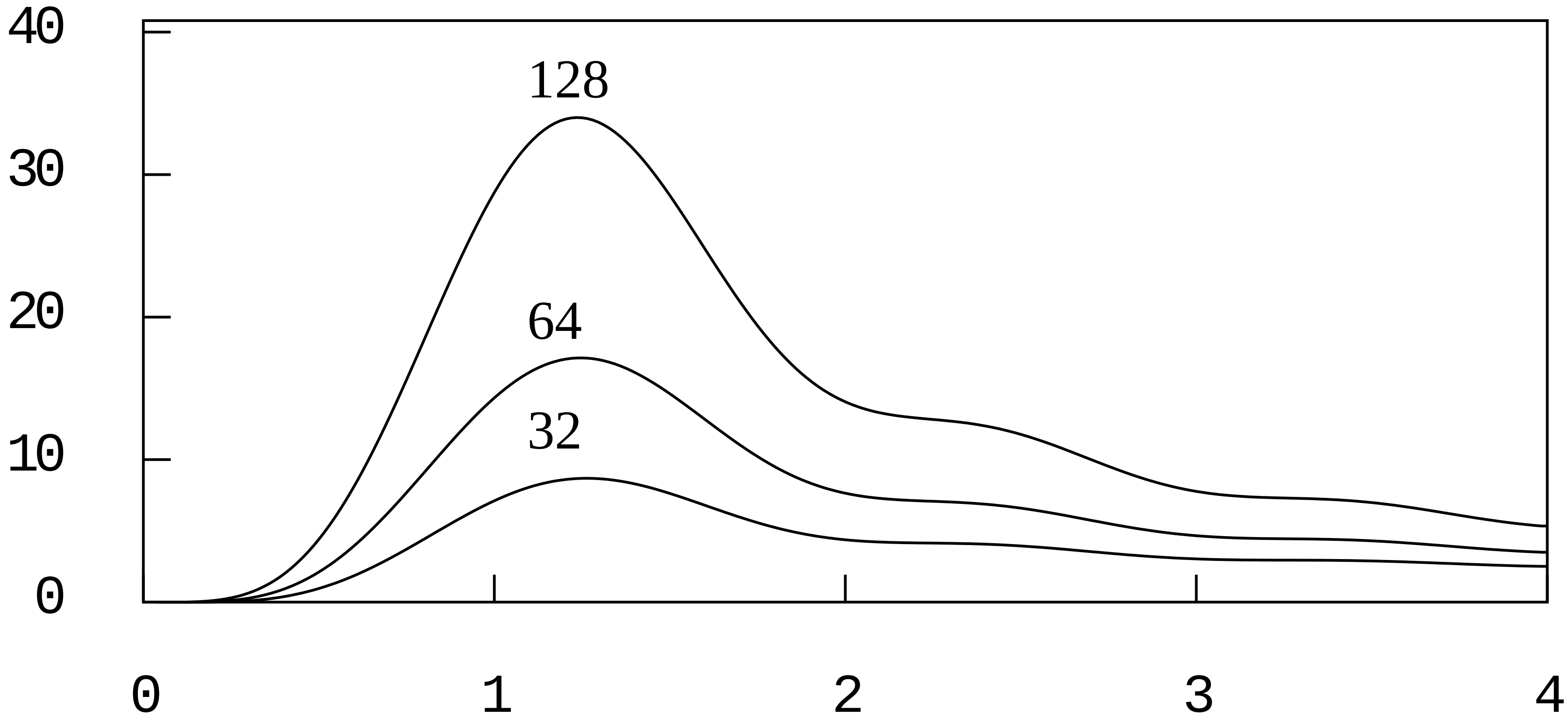

Figure 13. The spectral density functions of the regression residuals for sample sizes of 32, 64 and 128, plotted as functions of cycles per sample.

A False Conclusion from a Simple Syllogism

A random walk is generated by cumulating a white-noise sequence, which contains cycles of every frequency in the interval running from zero to the Nyquist frequency π radians per period. Therefore, the idea that there is no cyclicality in the process should be treated with caution.

It is easy to see how, via a simple syllogism, a false conclusion concerning macroeconomic data sequences can arise. The *major premise* is that the macroeconomic data can be regarded as the product of random walk. The *minor premise* is that the detrending of a random walk gives rise to spurious cycles. The *conclusion* is that the detrending of a macroeconomic sequence induces spurious cycles.

It is the complete identification of the macroeconomic process with a random walk (or with a random walk with drift) that is at fault in this argument.

In contrast to random walks, real economic processes are subject to evident constraints. They are driven by the buoyant forces of entrepreneurial endeavour and by consumer aspirations, and they are constrained by the more or less pliable limits of productive capacity and resource availability. In a thriving economy, they press alternately against the floors and the ceilings and they rebound from them in a manner that is undeniably cyclical.

The Self-Similar Nature of A Random Walk

The analysis of Nelson and Kang gives the impression that the number of times that a drifting random walk crosses an interpolated regression line is the same, on average, regardless of the length of the sample. In fact, the number of line crossings tends to infinity as the sample size increases indefinitely.

A sample n elements from a Wiener process, scaled appropriately, has a similar appearance and the same statistical properties as any other sample of n elements, regardless of its rate of sampling. Let samples be taken at intervals of one and T time units. Then

$$T^{1/2}(x_1, x_2, \ldots, x_n) \stackrel{D}{=} (x_{T1}, x_{T2}, \ldots, x_{Tn})$$

Taking the scaling into account, the effect of increasing the rate of sampling is the same as that of increasing the length of a sample taken at a fixed rate.

The following six diagrams show nested samples of a trajectory of Brownian motion taken at ever-increasing sample rates. In passing from one diagram to the next, the rate of sampling increases by a factor of 4, which implies a magnification from first to last of $4^5 = 1024$.

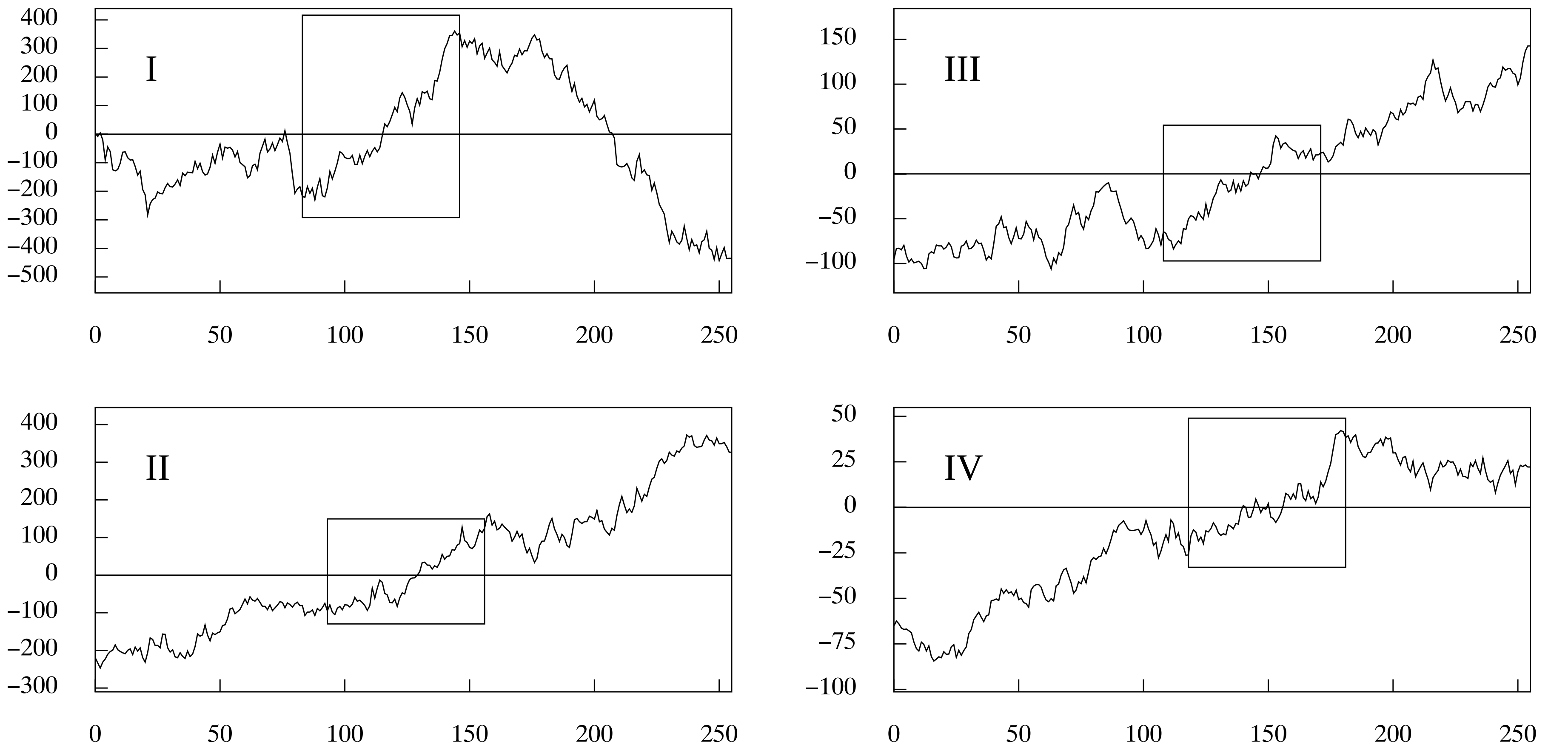

Figure 14. Nested segments of a trajectory of Brownian motion, sampled at rates that increase successively by factors of 4. The succession runs in the order top-left, bottom-left, top-right, bottom-right.

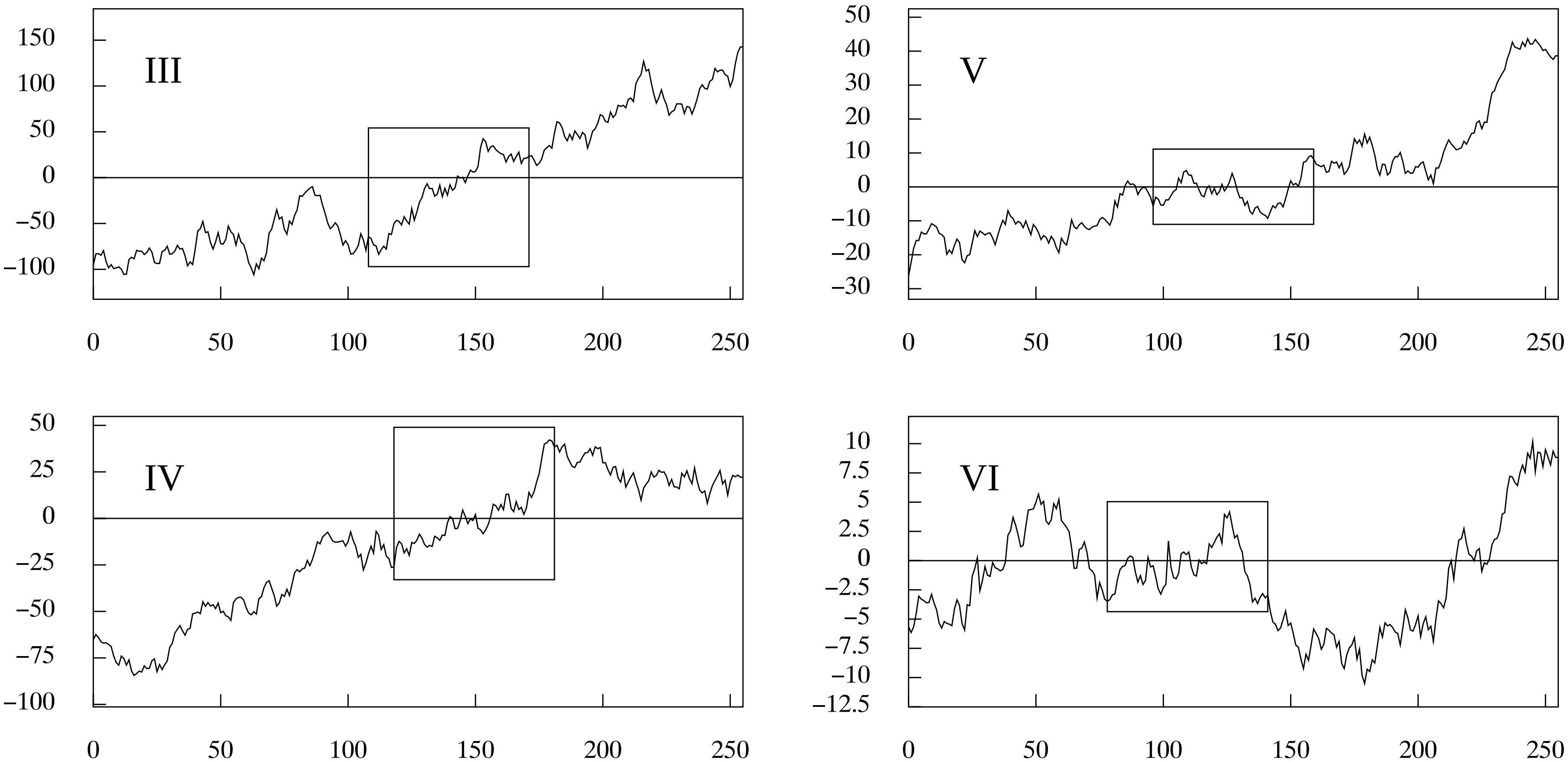

Figure 15. Nested segments of a trajectory of Brownian motion, contined from the previous diagram. The succession runs in the order top-left, bottom-left, top-right, bottom-right.

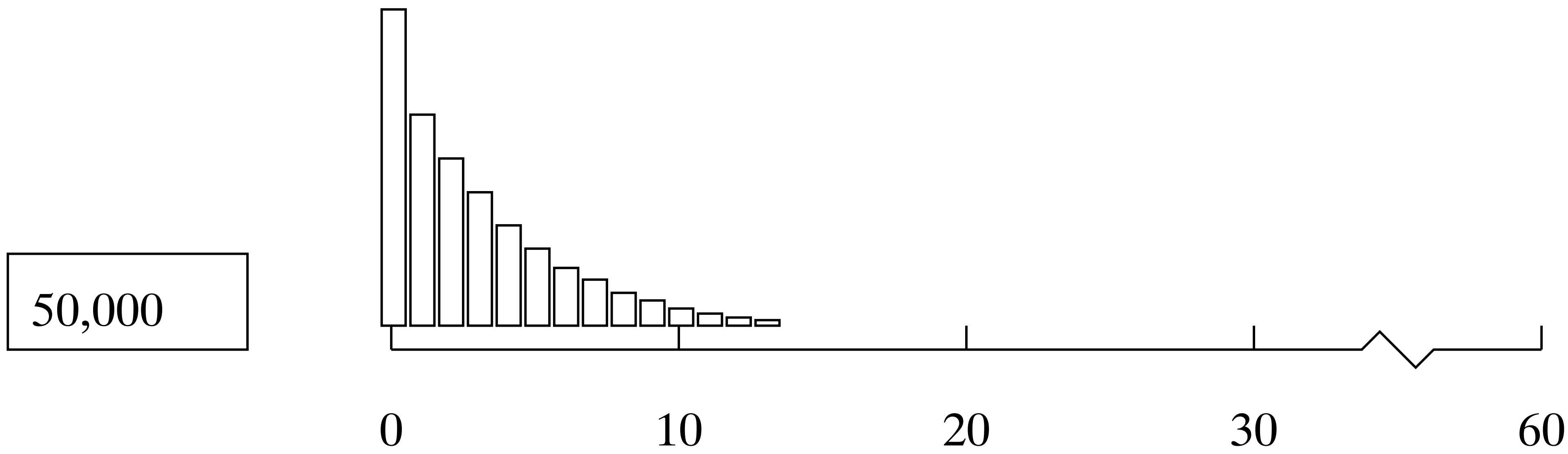

Figure 16. The histogram of the number of times a random walk of 60 steps crosses the horizontal axis.

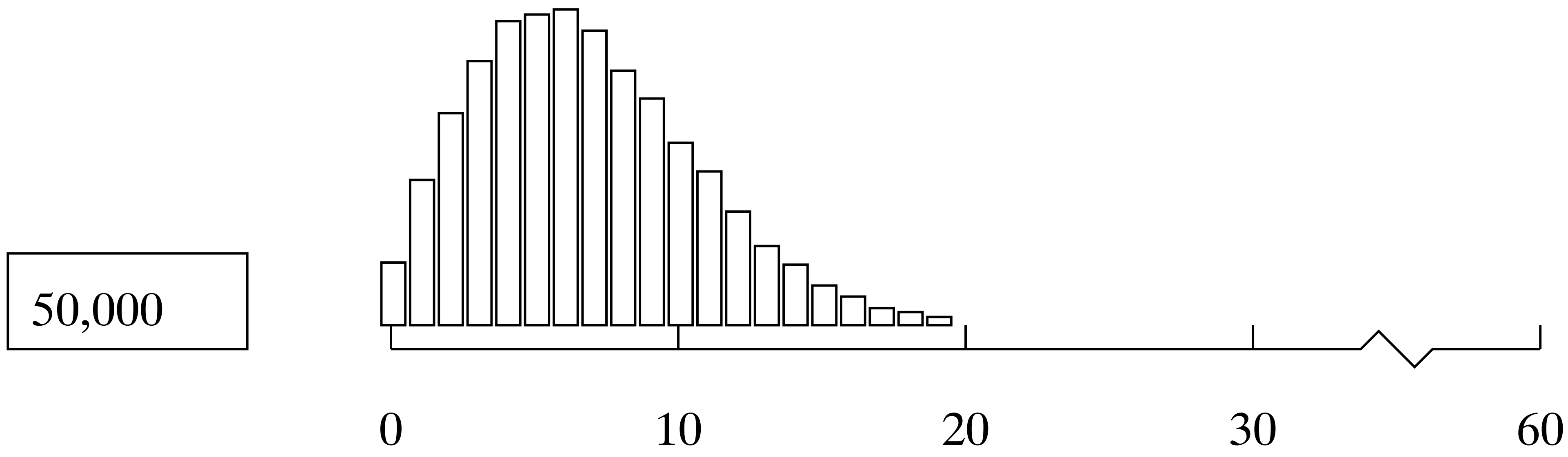

Figure 17. The histogram of the number of times a random walk of 60 steps crosses a line throught the end points of the sample.

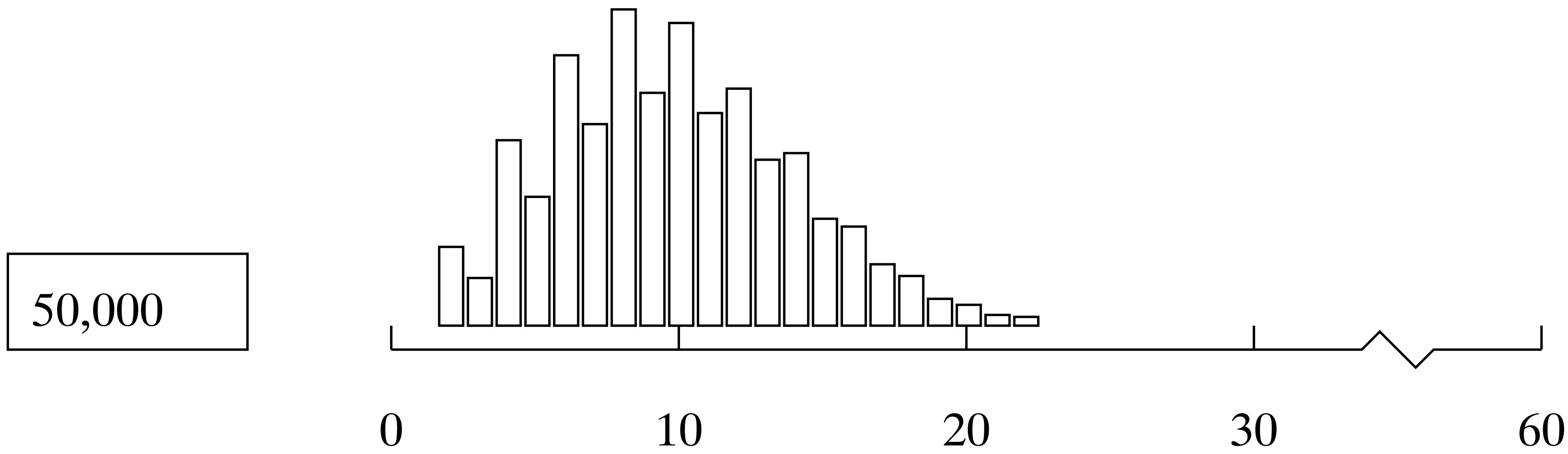

Figure 18. The histogram of the number of times a random walk of 60 steps crosses a line interpolated by least-squares regression.

The Distributions of the Numbers of Line Crossings

The asymptotic distributions can be found for the number of line crossings in the first two cases.

The number of times N_T in a sample of size T that an ordinary random walk, devoid of drift, will cross the horizontal axis increases with T at the rate of $T^{1/2}$. The limiting distribution of $T^{-1/2}N_T$ as $T \to \infty$ is $\{E|\varepsilon_t|/\sigma\}|N(0,1)|$, where $E|\varepsilon_t|$ is the mean absolute deviation of ε_t and where $|N(0,1)|$ is the distribution of $|z|$ when $z \sim N(0,1)$.

Let N_T^B denote the number of times in a sample of size T that an random walk, subject to a linear drift, will cross the line that interpolates the first and the final sample points. Then, the limiting distribution of $T^{-1/2}N_T^B$ is $\{E|\varepsilon_t|/\sigma\}|R(x)$, where $R(x)$ is a standard Raleigh distribution, which has a density function of the form

$$R(x) = xe^{-x^2/2}.$$

The distribution of the number of times that a drifting random walk crosses an line interpolated by least-squares regression does not seem to possess a simple analytic form. For any sample size, even numbers of crossing are, on average, more numerous than odd numbers of crossings.

Aspersions against the Hodrick–Prescott Filter

Numerous authors have inveighed against the use of the Hodrick–Prescott filter as a device for extracting trends from economic data. The typical analysis concerns the interaction of the frequency response of the filter with the pseudo spectrum of a random-walk process defined on a doubly infinite set of indices.

Such a random walk is truly an unimaginable process; and its values have a zero probability of being found within a finite distance of the origin. Also, the pseudo spectrum is unbounded in the vicinity of the zero frequency.

When the pseudo spectrum is modulated by the frequency response function of the H–P filter, a spectral density function is produced that has a prominent peak in the low-frequency region. This spectral density function, which corresponds to the output of the filter, is identified with a cyclical process. It is asserted, on this basis, that the filter is liable to induce spurious cycles.

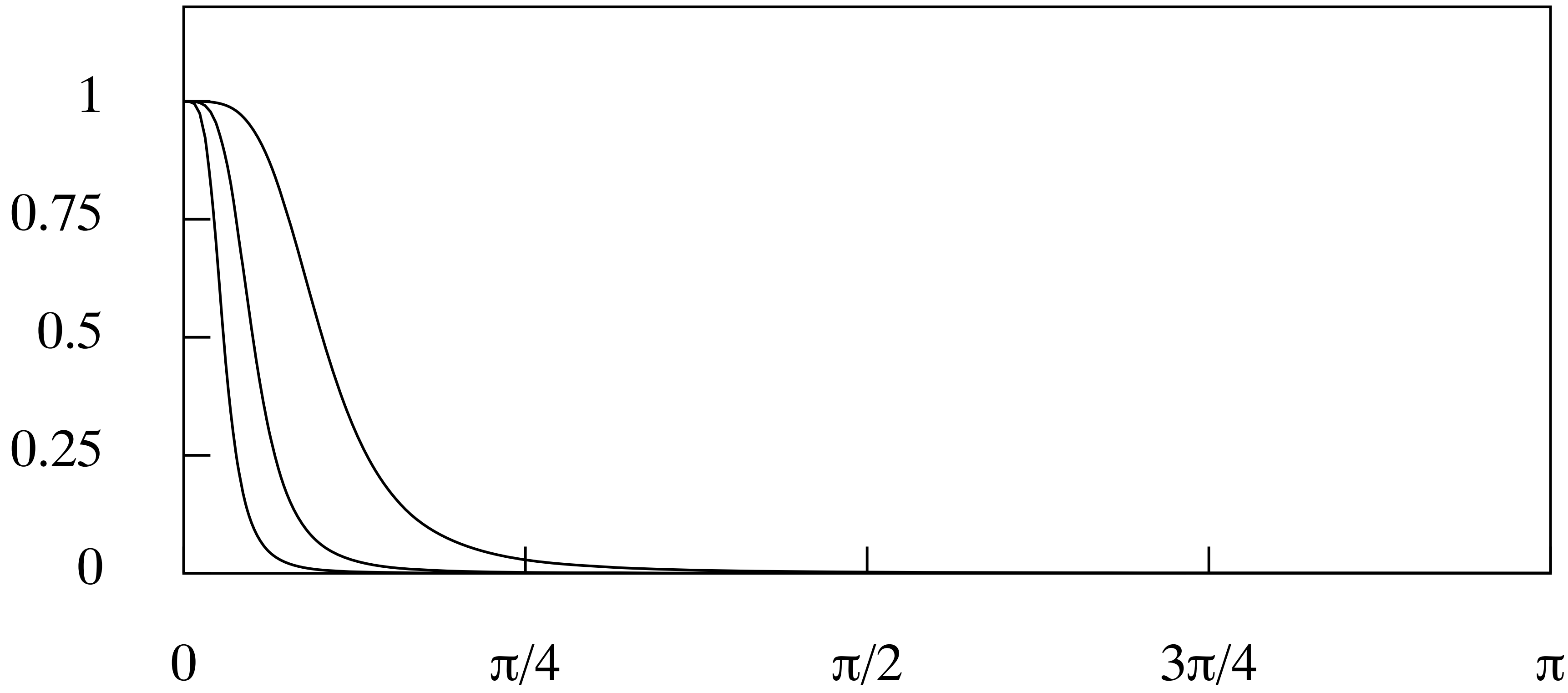

Figure 19. The gain of the Hodrick–Prescott lowpass filter with a smoothing parameter set to 100, 1,600 and 14,400.

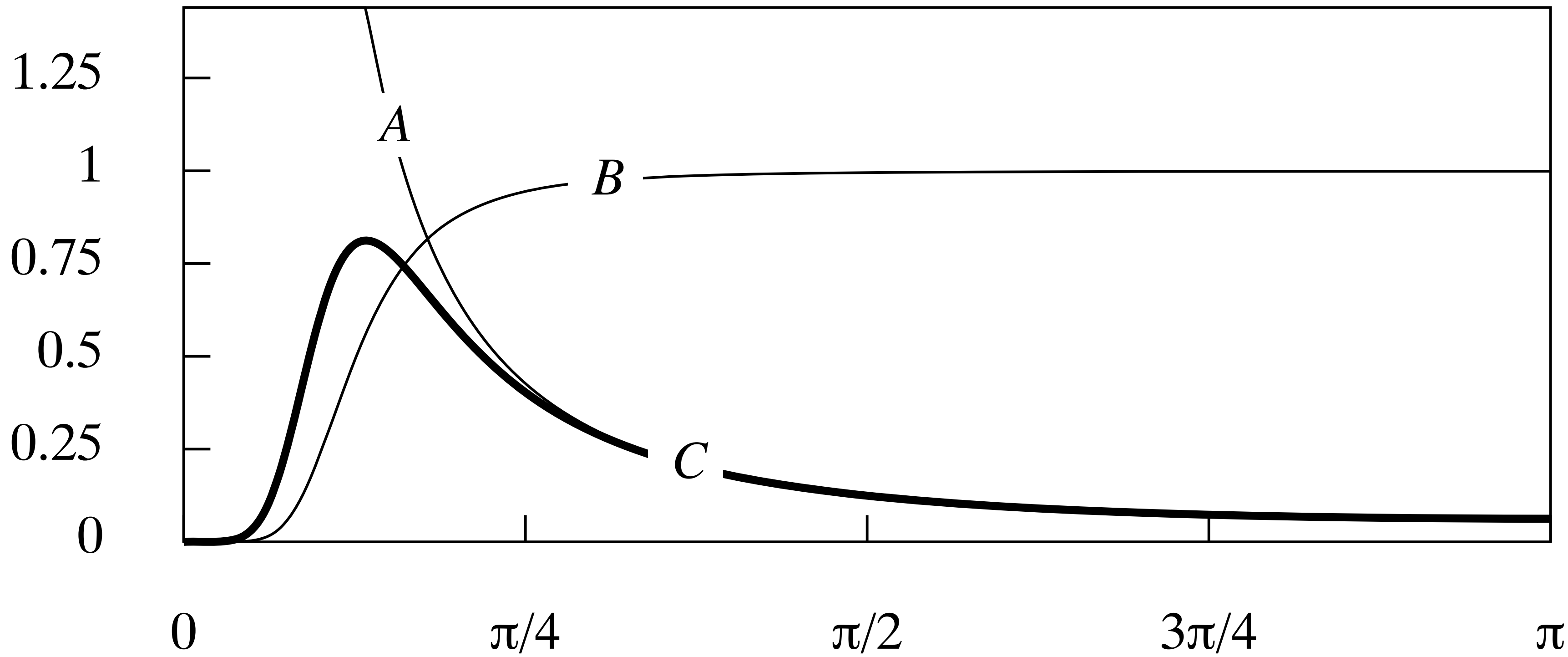

Figure 20. The pseudo-spectrum of a random walk, labelled A, together with the squared gain of the high pass Hodrick–Prescott filter with a smoothing parameter of $\lambda = 100$, labelled B. The curve labelled C represents the spectrum of the filtered process.

The Finite Sample Form of the Hodrick–Prescott Filter

The Hodrick–Prescott filter depends on the matrix version of the second-order difference operator. We may define, for example,

$$\nabla_5^2 = \begin{bmatrix} Q'_* \\ Q' \end{bmatrix} = \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 \\ \hline 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \end{array}\right].$$

The lowpass filter applied to the data vector $y = [y_0, y_1, \ldots, y_{T-1}]'$ gives rise to the trend estimate

$$x = y - Q(\lambda^{-1}I + Q'Q)^{-1}Q'y,$$

where λ is the so-called smoothing parameter. By letting $\lambda \to \infty$, we derive the interpolated regression line

$$x = y - Q(Q'Q)^{-1}Q'y = y - e,$$

where $e = Q(Q'Q)^{-1}Q'y$ is the residual vector. Observe that

$$Q(\lambda^{-1}I + Q'Q)^{-1}Q'y = Q(\lambda^{-1}I + Q'Q)^{-1}Q'e.$$

Thus, the highpass filter gives the same result whether it is applied to the data vector y or to the residual vector e

The Linear Trend and the Hodrick–Prescott Trend

Let $P = Q(Q'Q)^{-1}Q'$ be the matrix transformation that generates the residuals from fitting a linear trend, and let $H = Q(\lambda^{-1}I + Q'Q)^{-1}Q'$ be the matrix of the highpass version of the Hodrick–Prescott filter. Observe that $HP = H$.

It follows that

$$(I - P)y + (I - H)Py = (I - H)y.$$

Here, $(I - P)y$ is a linear trend estimated by ordinary least-squares regression, and $Py = e$ is the residual sequence. Also,

$(I - H)y$ is the trend estimated by the Hodrick–Prescott lowpass filter, and

$(I - H)Py = (I - H)e$ represents the effect of applying the filter to the residual sequence.

Therefore, the equation above indicates that the Hodrick–Prescott trend can be calculated by adding the filtered residuals to the linear trend.

Estimating the Business Cycle

The estimation of the business cycle requires two steps. First, the trend must be removed from the data. Next, the extraneous elements must be removed from the detrended (residual) sequence. These will include any seasonal fluctuations that are present in the data as well as any elements of noise that lie beyond the range of the business-cycle frequencies.

A straight line fitted by least-squares regression to the logarithms of a macroeconomic data sequence provides a benchmark of constant exponential growth, relative to which one can measure the fluctuations of the business cycle. A polynomial of degree not less than 2 can be used to create a trajectory with a changing rate of growth.

A Hodrick–Prescott filter with the smoothing parameter set to the conventional value will generate a more flexible trend that will partially absorb the business cycle fluctuations.

To cleanse the detrended sequence of the extraneous elements, we use a frequency domain filter that nullifies all components that fall beyond the range of the business cycle frequencies. The limiting frequency of the business cycle can be perceived via the periodogram of the linearly detrended data.

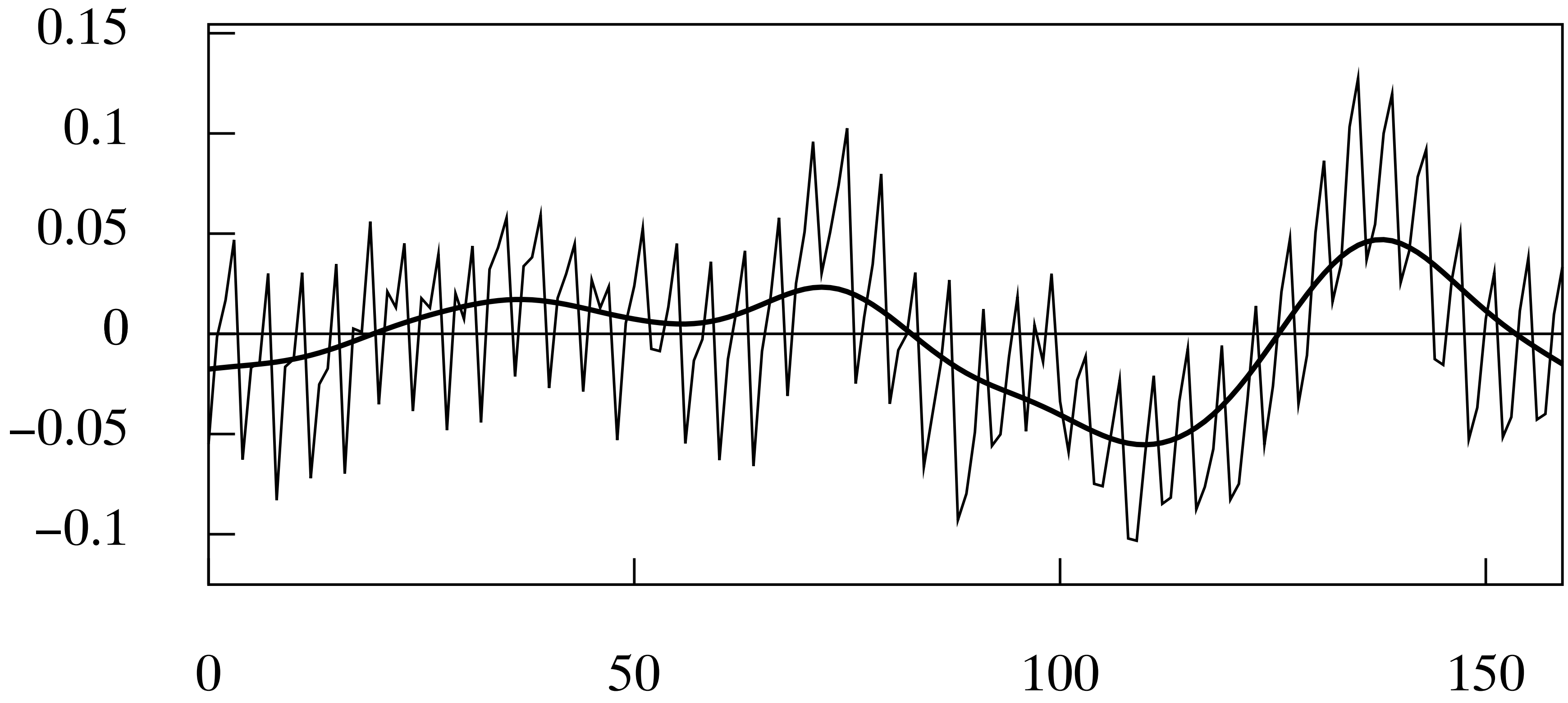

Figure 21. The residual sequence $e = Py$ obtained by extracting a linear trend from the logarithmic consumption data, together with a low-frequency trajectory $(I - H)Py$ that has been obtained via the lowpass Hodrick–Prescott filter.

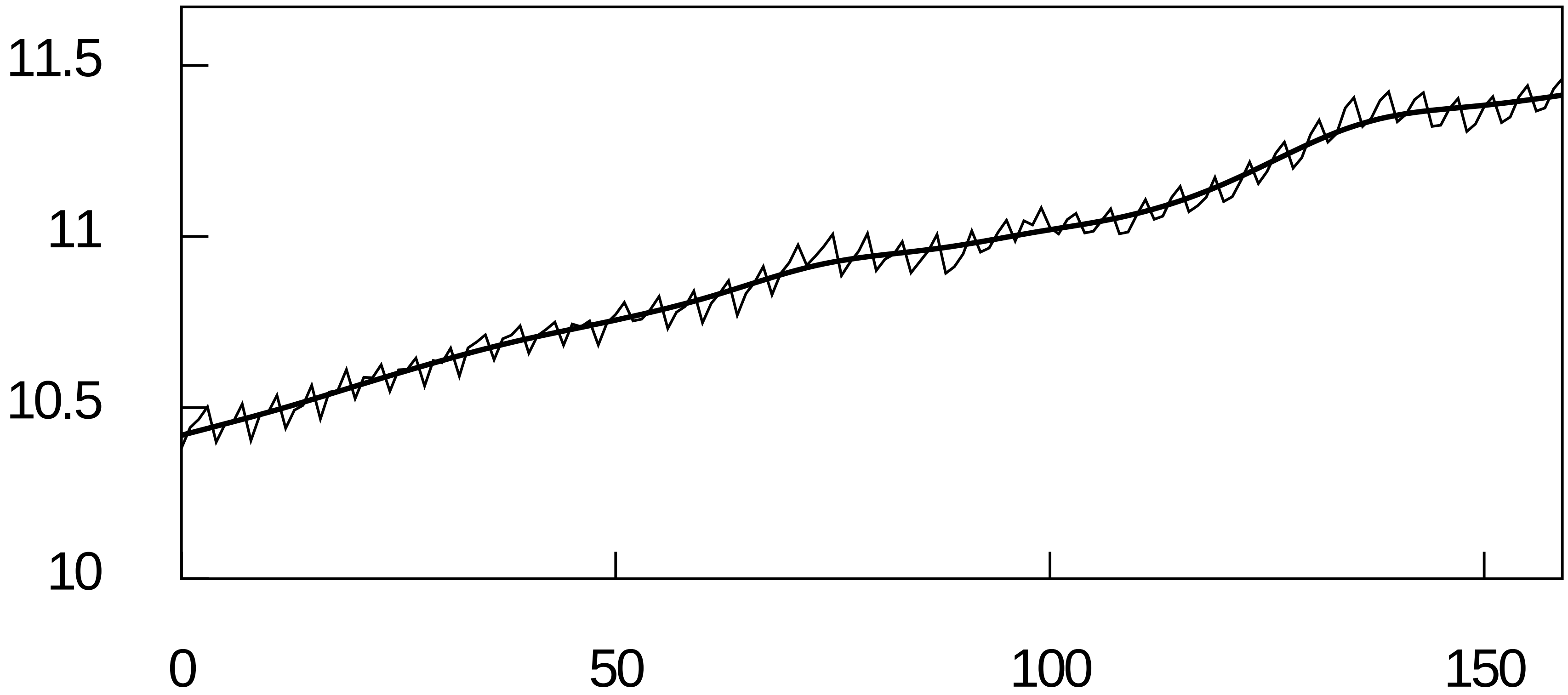

Figure 22. The quarterly logarithmic consumption data together with a trend $(I-H)y = (I-P)y + (I-H)Py$ interpolated by the lowpass Hodrick–Prescott filter with the smoothing parameter set to $\lambda = 1,600$.

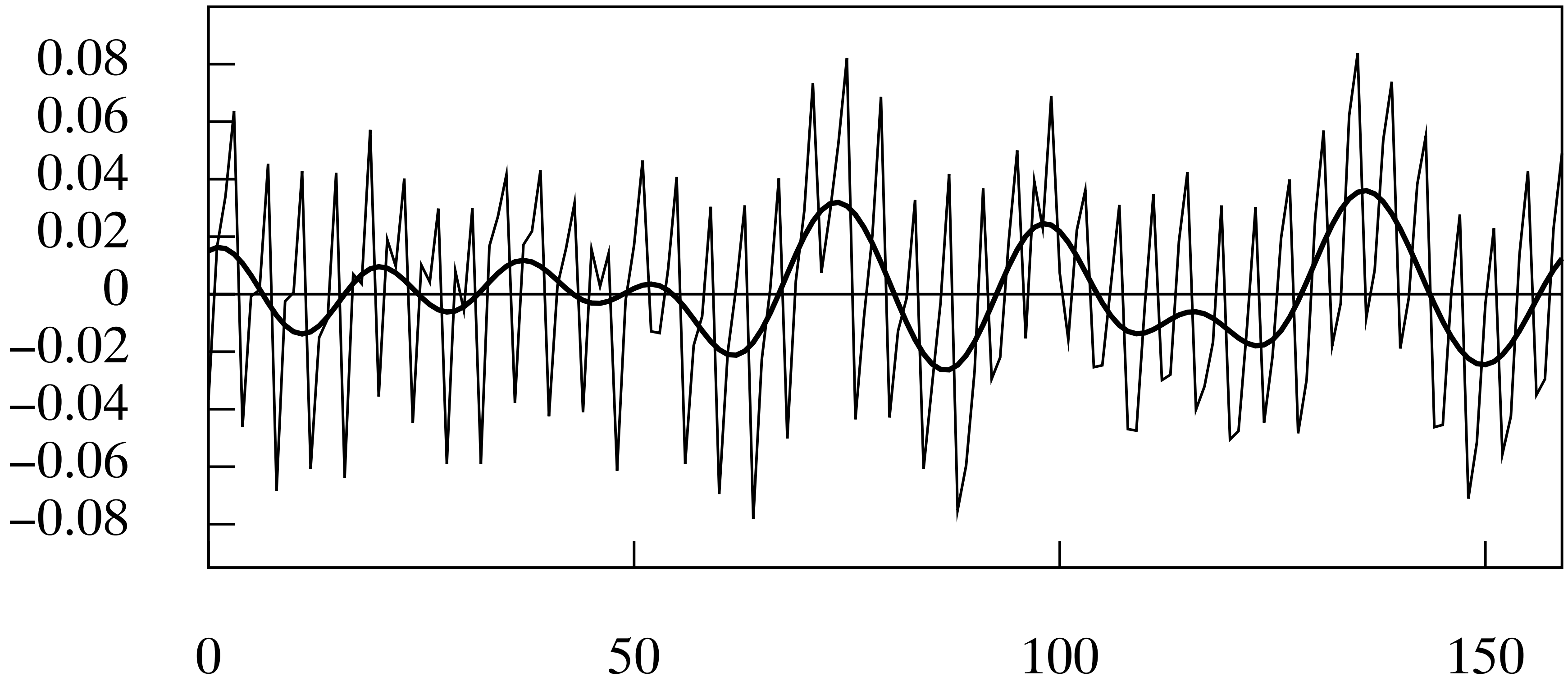

Figure 23. The residual sequence $Hy = He$ obtained by using the lowpass Hodrick–Prescott filter to extract the trend, together with a fluctuating component $DHy = DHe$ obtained by subjecting the sequence to a lowpass frequency-domain filter with a cut-off point at $\pi/8$ radians.

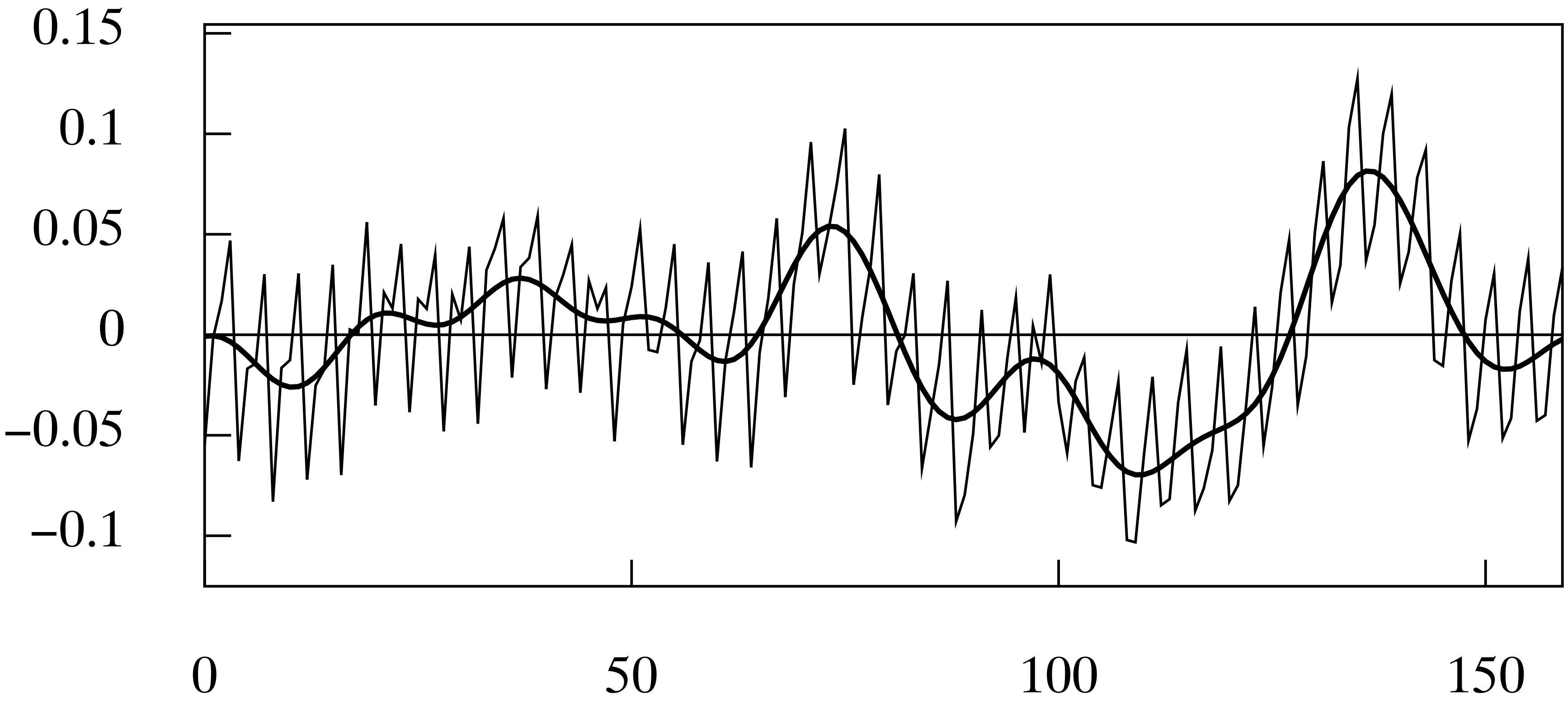

Figure 24. The residual sequence $e = Py$ from fitting a linear trend to the logarithmic consumption data with an heavy interpolated line $De = DPy$ representing the business cycle, obtained by the frequency-domain method.

The Effect of the Hodrick–Prescott Filter

In detrending the data, the Hodrick–Prescott filter transfers part of the fluctuations that belong to the business cycle into the trend. The amplitudes of the business cycle fluctuations are diminished and regularised. These fluctuations are present in the data obtained via a linear detrending. Therefore, it cannot be said that the H–P filter induces spurious fluctuations. The most that can be said is that it gives the fluctuations a spurious regularity.

The trend should be maximally stiff, unless it is required to accommodate a structural break. In times of normal economic activity, a log linear trend, which represents a trajectory of constant exponential growth, may be appropriate. At other times, the trend should be allowed to adapt to reflect untoward events.

A device that achieves this is available in the form of a version of the H–P filter that has a smoothing parameter that is variable over the sample. When the trajectory of the trend is required to accommodate a structural break, the smoothing parameter λ can be set to a value close to zero within the appropriate locality. Elsewhere, it can be given a high value to ensure that a stiff curve is created. Such a filter is available in the IDEOLOG computer program.

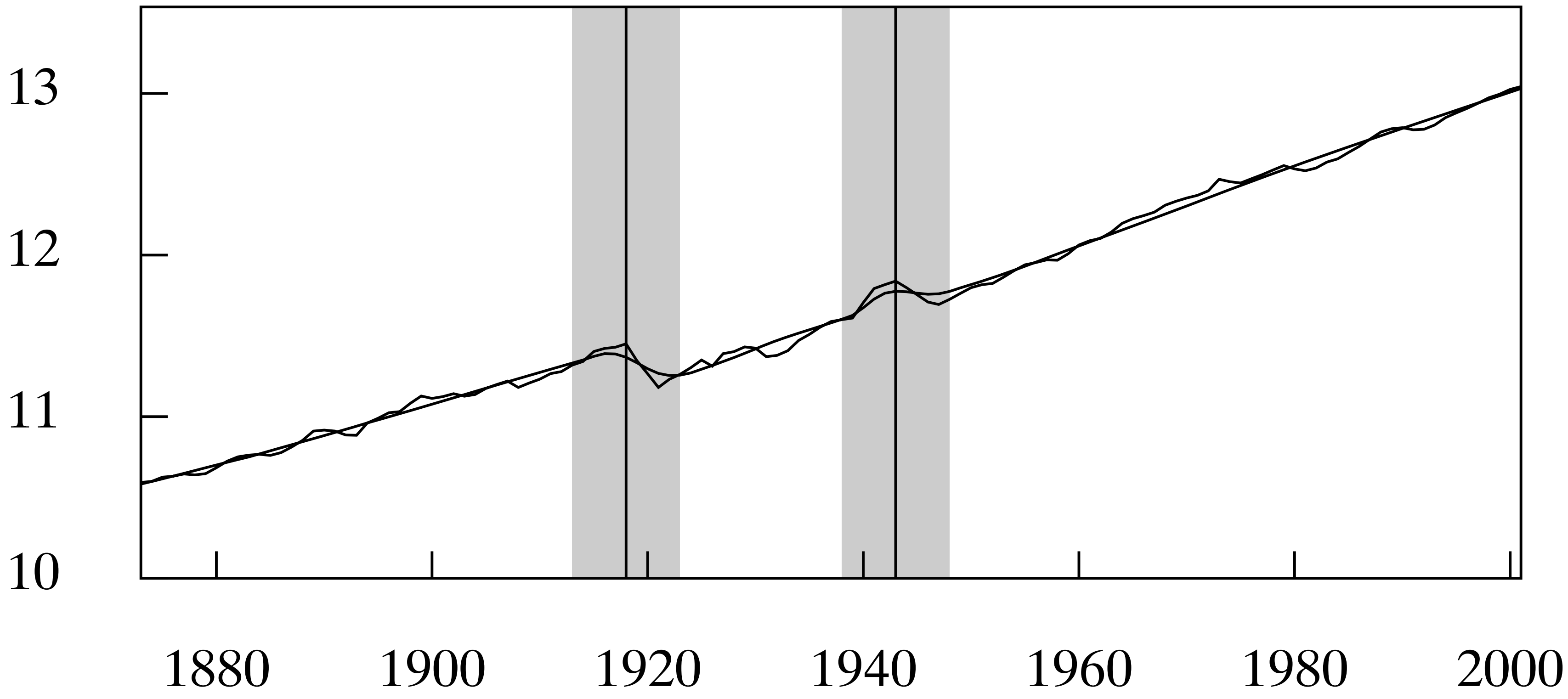

Figure 25. The logarithms of annual U.K. real GDP from 1873 to 2001 with an interpolated trend. The trend is estimated via a filter with a variable smoothing parameter.