SAMPLING DISTRIBUTIONS

1. What are the relationships between the chi-square, the normal, the t and the F distributions?

The horizontal and vertical components of a marksman's error are independently and identically distribted normal variates with a zero mean and a standard deriation of 2cm. Find approximately the probability that the radial distance of a shot from the centre of the target will exceed 4cm.

2. What are the relationships between the chi-square, the normal the t and the F distributions?

The horizontal and vertical components of a marksman's error in shooting at a concentric target are independently and identically distributed normal variates with a mean of zero and a standard deviation of 2 cms, Find approximately the probability that the radial distance of a shot from the centre of the target will exceed 4 cms.

3. Describe the relationships amongst the normal, the chi-square, the t and the F distributions.

On a windless day, the coordinates of a parachutist's landing point, measured relative to two axes passing at right angles through the target centre, are distributed as independent normal variates with a mean of zero and a standard deviation of 9 metres. The target area is surrounded by a circular pebble pit of radius 15 metres. Calculate the probability that the parachutist will land on the pebbles. What is the least amount of rope required to enclose an area into which he can be 90% confident of falling?

Answer. Let x and y be the coordinates. Then

$$x, y \sim N(0, \sigma = 9) \quad \text{implies} \quad \frac{x^2 + y^2}{9^2} \sim \chi^2(2).$$

We must evaluate $P(\sqrt{x^2 + y^2} < 15)$ which is equivalent to

$$\frac{x^2 + y^2}{9^2} < \frac{15^2}{9^2} = \frac{25}{9} = 2.\bar{7}.$$

From tables, $P(\chi^2(2) \geq 2.773) = 0.25$, so

$$P(\sqrt{x^2 + y^2} < 15) \simeq 0.75.$$

Next

$$P(\chi^2(2) < q) = 0.9 \quad \text{implies} \quad \frac{x^2 + y^2}{9^2} = q = 4.605.$$

Therefore $r = \sqrt{x^2 + y^2} = \sqrt{4.605 + 81} = 19.3$ is the radius of a circle into which the parachutist can be 90% confident of landing. To enclose this circle, we need a rope of length

$$2\pi r = 2 \times 3.1416 \times 19.3 = 121.27.$$

4. Describe fully the relationships amongst the normal the chi-square, the t and the F distributions.

Let x_1 and x_2 be independent standard normal variates

(i) Find the radius of a circle C centred at zero, such that the probability that the point $[x_1, x_2]$ falls in the circle is $P([x_1, x_2] \in C) = 0.9$

(ii) Find the lengths of the sides of a square S such that $P([x_1, x_2] \in S) = 0.9$

(iii) Find the relative areas of the circle and the square.

Hint: Use $P([x_1, x_2] \in S) = P(-a \leq x_1 \leq a) \times P(-a \leq x_1 \leq a)$, where a is half the length of a side of the square.

Answer. Let q be the radial distance of the point (x_1, x_2) from the centre of the circle. Then $q^2 = x_1^2 + x_2^2 \sim \chi^2(2)$ and

$$P(q^2 \leq r^2) = 0.9 \Longrightarrow \begin{cases} r^2 = 4.605; \\ r = 2.146. \end{cases}$$

Now let us find S such that $P([x_1, x_2] \in S) = 0.9$. From tables of the standard normal, we find that

$$P(-a \leq x_i \leq a) = \sqrt{0.9} = 0.9487 \quad \text{implies that} a = 1.95$$

Hence the length of the side of S is $2a = 3.90$. We deduce that

$$\begin{aligned} &\text{The area of } S \text{ is} \quad (3.9)^2 = 15.21, \\ &\text{The area of } C \text{ is} \quad \pi r^2 = 2.1416 \times 4.605 = 14.487. \end{aligned}$$