

EXERCISES IN STATISTICS

Series A, No. 7

1. Let x and y be jointly distributed random variables with conditional expectations which can be written as $E(y|x) = \alpha + \beta x$ and $E(x|y) = \gamma + \delta y$. Express β and δ in terms of the moments of the joint distributions and show that $\beta \leq 1/\delta$.
2. A marksman's scores are a sequence of random variables y_i with $E(y_i) = 90$ and $V(y_i) = 16$ for all i . The correlation between successive scores is 0.9, and the expectation of a score conditional upon the previous score is given by $E(y_i|y_{i-1}) = a + by_{i-1}$ where $a = (1 - b)E(y_i)$. Find the expected score given that the previous score was 80.
3. The expected rainfalls in September, October and November are 10 ins, 8 ins 6 ins respectively, with a variance-covariance matrix of

$$\begin{bmatrix} 6 & -3 & 1.5 \\ -3 & 6 & -3 \\ 1.5 & -3 & 6 \end{bmatrix}.$$

Calculate the expected rainfall throughout these three months and find its variance.

If the September rain was unusually high, in what direction would ones estimates of rainfall be revised

- (a) in the two months following, and
 - (b) for the three month period?
4. A man runs on Hampstead Heath twice a week. The average duration of one of his outings is 40 minutes with a standard deviation of 5 minutes. His average running time per week is 1 hour 20 minutes with a standard deviation of 4 minutes. Given that he ran for only 15 minutes on Monday, what is the expected duration of his Friday outing?
 5. An investor has a choice of three financial assets. The expected yields of these assets are given in the vector $[0.04 \quad 0.03 \quad 0.05]$ and the variances and covariances of the yields are given in the matrix

$$10^{-4} \begin{bmatrix} 3.29 & -0.83 & 0 \\ -0.83 & 3.41 & 0 \\ 0 & 0 & 2.01 \end{bmatrix}.$$

Derive expressions for the expected yield and variance of a portfolio containing λQ of the first asset and $(1 - \lambda)Q$ of the second asset, and ascertain whether there is any value of λ such that the variance of this portfolio is less than the variance of an investment Q in the third asset.