# EXERCISES IN STATISTICS

## Series A, No. 5

1. Find the moment generating function of $x \sim f(x) = 1$, where $0 < x < 1$, and thereby confirm that $E(x) = \frac{1}{2}$ and $V(x) = \frac{1}{12}$.

2. Find the moment generating function of $x \sim f(x) = ae^{-ax}; x \geq 0$.

3. Prove that $x \sim f(x) = xe^{-x}; x \geq 0$ has a moment generating function of $1/(1-t)^2$. Hint: Use the change of variable technique to integrate with respect to $w = x(1-t)$ instead of $x$.

4. Using the theorem that the moment generating function of a sum of independent variables is the product of their individual moment generating functions, find the m.g.f. of $x_1 \sim e^{-x_1}$ and $x_2 \sim e^{-x_2}$ when $x_1, x_2 \geq 0$ are independent. Can you identify the p.d.f. of $f(x_1 + x_2)$ from this m.g.f.?

5. Find the moment generating function of the point binomial

$$f(x) = p^x(1-p)^{1-x}$$

   where $x = 0, 1$. What is the relationship between this and the m.g.f. of the binomial distribution ?