INTRODUCTORY ECONOMETRICS : Exercise 6 (Assessed)



Modelling a Series with Seasonal Fluctuations

Figure 1. The graph of 75 observations on quarterly consumption of coal (millions of tonnes) in the United Kingdom from 1971/1 to 1989/3.

| Year | Winter | Spring | Summer | Autumn |
|------|--------|--------|--------|--------|
| 1971 | 42.1 | 33.1 | 27.7 | 36.4 |
| 1972 | 30.7 | 28.6 | 27.1 | 36.0 |
| 1973 | 37.4 | 31.7 | 27.6 | 36.8 |
| 1974 | 25.8 | 30.2 | 27.8 | 34.0 |
| 1975 | 35.2 | 29.3 | 24.8 | 33.1 |
| 1976 | 35.2 | 28.5 | 25.2 | 34.7 |
| 1977 | 36.2 | 29.7 | 25.9 | 32.2 |
| 1978 | 34.1 | 28.3 | 25.3 | 32.8 |
| 1979 | 36.6 | 31.1 | 27.7 | 33.9 |
| 1980 | 35.6 | 29.2 | 25.9 | 30.2 |
| 1981 | 32.5 | 27.8 | 25.6 | 32.5 |
| 1982 | 31.0 | 27.0 | 23.6 | 29.3 |
| 1983 | 31.2 | 26.3 | 24.5 | 29.8 |
| 1984 | 31.6 | 16.4 | 13.0 | 16.4 |
| 1985 | 21.0 | 27.4 | 25.3 | 31.6 |
| 1986 | 34.2 | 26.7 | 24.2 | 29.1 |
| 1987 | 32.6 | 27.0 | 24.9 | 31.3 |
| 1988 | 32.3 | 25.7 | 23.9 | 29.7 |
| 1989 | 30.1 | 25.4 | 23.1 | |



Figure 2. Trigonometrical functions, of frequencies $\omega_1 = \pi/2$ and $\omega_2 = \pi$, associated with a quarterly model of a seasonal fluctuation.

The table overleaf contains 75 observations on the consumption y_t of coal in each season of 19 successive years.

As an exercise, you are asked to build a model in the form of

$$y_t = \alpha_0 + \beta_0 t + \alpha_1 \cos\left(\frac{\pi t}{2}\right) + \beta_1 \sin\left(\frac{\pi t}{2}\right) + \alpha_2 (-1)^t + e_t,$$

which could be used in assessing the secular decline in coal consumption and for predicting the series. There are three steps to be taken initially:

- (1) Remove a linear trend from the data,
- (2) Find the values of the coefficients in the model of the seasonal fluctuations, and determine whether they are all statistically significant.
- (3) Find the coefficients associated with an alternative system of seasonal dummy variables which attributes a specific effect to each season.

There two ways in which you might proceed:

- (i) You might exploit the fact that the vectors of dummy variables are orthogonal, which should enable you to estimate the parameters α_1 , β_1 and α_2 via simple regressions applied to the transformed data—obtained from the original data by extracting a linear trend.
- (ii) Alternatively, you could exploit the facilities for multiple regression which are available in *MicroFit* and in *Excel 4.0* to estimate the parameters jointly.

As a final exercise, you are asked to find some way of accommodating the effect of the anomalous year 1984. For example, you might include some additional dummy variables, or you might devise a systematic way of "correcting" the data.