INTRODUCTORY ECONOMETRICS : Exercise 5 (Supplementary)

Matrices of Zeros and Units for Modelling Seasonal Variations

1. The values assumed by the variable y_t in the four quarters of the year τ are given by the equation

$$(1) \qquad \begin{bmatrix} y_{\tau 0} \\ y_{\tau 1} \\ y_{\tau 2} \\ y_{\tau 3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_0 \\ \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix} + \begin{bmatrix} e_{\tau 0} \\ e_{\tau 1} \\ e_{\tau 2} \\ e_{\tau 3} \end{bmatrix},$$

or equally by the equivalent equation

$$(2) \qquad \begin{bmatrix} y_{\tau 0} \\ y_{\tau 1} \\ y_{\tau 2} \\ y_{\tau 3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \gamma_0 \\ \gamma_1 \\ \gamma_2 \end{bmatrix} + \begin{bmatrix} e_{\tau 0} \\ e_{\tau 1} \\ e_{\tau 2} \\ e_{\tau 3} \end{bmatrix}.$$

Find the matrix of the transformation which maps the vector $[\delta_0, \delta_1, \delta_2, \delta_3]'$ into the vector $[\phi, \gamma_0, \gamma_1, \gamma_2]'$ and confirm, by matrix multiplication, that this is the inverse of the matrix of (2).

2. Let the seasonal variation in y_t be represented by the equation

$$(3) \qquad y_t = \alpha_0 + \alpha_1 \cos\left(\frac{\pi t}{2}\right) + \beta_1 \sin\left(\frac{\pi t}{2}\right) + \alpha_2(-1)^t + e_t.$$

Show that, for the four quarters of the year τ, we have the following matrix equation:

$$(4) \qquad \begin{bmatrix} y_{\tau 0} \\ y_{\tau 1} \\ y_{\tau 2} \\ y_{\tau 3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 \\ 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \beta_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} e_{\tau 0} \\ e_{\tau 1} \\ e_{\tau 2} \\ e_{\tau 3} \end{bmatrix}.$$

3. Find explicit expressions for the simple least-squares estimates of the coefficients α_0, α_1, β_1, and α_2 of equation (4) when the data consist of $T = 4p$ observations spanning p years. For example, $\alpha_0 = T^{-1}\sum_{t=1}^{T} y_t$.

4. Confirm the following identity:

$$(5) \qquad \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \beta_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} \delta_0 \\ \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}.$$

5. By using the results from question 1, or by any other means, find the matrix transformation which maps from $[\alpha_0, \alpha_1, \beta_1, \alpha_2]'$ to $[\phi, \gamma_0, \gamma_1, \gamma_2]'$ together with its inverse.