**INTRODUCTORY ECONOMETRICS : Exercise 4 (Tutorial)**
Multiple Regression and Matrix Algebra

The following is a table of figures relating to the consumption of textiles in the Netherlands in the period 1923—1939:

| YEAR | TEX | INC | PRICE | LOGTEX | LOGINC | LOGPRICE |
|---|---|---|---|---|---|---|
| 1923 | 99.2 | 96.7 | 101.1 | 1.99651 | 1.98543 | 2.00475 |
| 1924 | 99.0 | 98.1 | 100.1 | 1.99564 | 1.99167 | 2.00043 |
| 1925 | 100.0 | 100.0 | 100.0 | 2.00000 | 2.00000 | 2.00000 |
| 1926 | 111.6 | 104.9 | 90.6 | 2.04766 | 2.02078 | 1.95713 |
| 1927 | 122.2 | 104.9 | 86.5 | 2.08707 | 2.02078 | 1.93702 |
| 1928 | 117.6 | 109.5 | 89.7 | 2.07041 | 2.03941 | 1.95279 |
| 1929 | 121.1 | 110.8 | 90.6 | 2.08314 | 2.04454 | 1.95713 |
| 1930 | 136.0 | 112.3 | 82.8 | 2.13354 | 2.05038 | 1.91803 |
| 1931 | 154.2 | 109.3 | 70.1 | 2.18808 | 2.03862 | 1.84572 |
| 1932 | 153.6 | 105.3 | 65.4 | 2.18639 | 2.02243 | 1.81558 |
| 1933 | 158.5 | 101.7 | 61.3 | 2.20003 | 2.00732 | 1.78746 |
| 1934 | 140.6 | 95.4 | 62.5 | 2.14799 | 1.97955 | 1.79588 |
| 1935 | 136.2 | 96.4 | 63.6 | 2.13418 | 1.98408 | 1.80346 |
| 1936 | 168.0 | 97.6 | 52.6 | 2.22531 | 1.98945 | 1.72099 |
| 1937 | 154.3 | 102.4 | 59.7 | 2.18837 | 2.01030 | 1.77597 |
| 1938 | 149.0 | 101.6 | 59.5 | 2.17319 | 2.00689 | 1.77452 |
| 1939 | 165.5 | 103.8 | 61.3 | 2.21880 | 2.01620 | 1.78746 |

The variables are defined as follows:

(1) TEX: Volume of Textile Consumption per Capita,
(2) INC: Real Income per Capita,
(3) PRICE: Relative Price of Textiles,
LOGTEX, LOGINC, LOGPRICE: Logarithms (base 10) of (1), (2), and (3).

The object is to estimate the parameters of the regression equation

$$y = i\beta_0 + x_1\beta_1 + x_2\beta_2 + \varepsilon,$$

where $y$ = LOGTEX, $x_1$ = LOGINC, $x_2$ = LOGPRICE and $i$ is a vector of units. This is to be done in *Lotus 1-2-3* without the use of the *Data-Regression* command. Then the results should be compared with those from *Microfit.*

To express the problem in terms of matrix algebra, let us define the matrix

$$[\,y \quad X\,] = [\,y \quad i \quad x_1 \quad x_2\,].$$

Using the matrix facilities of *Lotus 1-2-3*, one can proceed to calculate

$$\begin{bmatrix} y' \\ X' \end{bmatrix} [\,y \quad X\,] = \begin{bmatrix} y'y & y'X \\ X'y & X'X \end{bmatrix}.$$

In order to form the transposed matrix on the LHS, there is no need to input the data a second time in a row-wise manner. Instead, we may use the *Range-Transpose* command which copies a range of columns into a workspace defined by a range of rows.The matrix multiplication is accomplished by the *Data-Matrix-Multiply* command.

The matrix on the RHS takes the form of

$$\begin{bmatrix} y'y & y'i & y'x_1 & y'x_2 \\ i'y & i'i & i'x_1 & i'x_2 \\ x_1'y & x_1'i & x_1'x_1 & x_1'x_2 \\ x_2'y & x_2'i & x_2'x_1 & x_2'x_2 \end{bmatrix} = \begin{bmatrix} \sum_t y_t^2 & \sum_t y_t & \sum_t y_t x_{t1} & \sum_t y_t x_{t2} \\ \sum_t y_t & T & \sum_t x_{t1} & \sum_t x_{t2} \\ \sum_t x_{t1} y_t & \sum_t x_{t1} & \sum_t x_{t1}^2 & \sum_t x_{t1} x_{t2} \\ \sum_t x_{t2} y_t & \sum_t x_{t2} & \sum_t x_{t2} x_{t1} & \sum_t x_{t2}^2 \end{bmatrix}$$

$$= \begin{bmatrix} 76.65898 & 36.07631 & 72.59642 & 67.44192 \\ * & 17 & 34.20783 & 31.83389 \\ * & * & 68.84179 & 64.06457 \\ * & * & * & 59.75950 \end{bmatrix}.$$

Of course, for a symmetric matrix of order $n$, there are only $q(n) = (n^2 + n)/2$ distinct elements to be calculated; and, when $n = 4$, we have $q(n) = 10$. It is possible to form each of the inner products within the matrix on the RHS via separate operations.

We also need the inverse of the matrix $X'X$:

$$(X'X)^{-1} = \begin{bmatrix} 510.8912 & -254.2522 & 0.4167 \\ * & 132.7044 & -6.8242 \\ * & * & 7.1106 \end{bmatrix}.$$

This is obtained via the*Data-Matrix-Invert* command. It remains to form the product

$$b = (X'X)^{-1}X'y = \begin{bmatrix} 1.3742 \\ 1.1430 \\ -0.8289 \end{bmatrix}.$$

This gives us the estimated regression coefficients including the intercept term.

We could have calculated this regression rather easily by taking the data in deviation form. We could calculate the elements of the cross-product matrix

$$\begin{bmatrix} (y-\bar{y})'(y-\bar{y}) & (y-\bar{y})'(x_1-\bar{x}_1) & (y-\bar{y})'(x_2-\bar{x}_2) \\ (x_1-\bar{x}_1)'(y-\bar{y}) & (x_1-\bar{x}_1)'(x_1-\bar{x}_1) & (x_1-\bar{x}_1)'(x_2-\bar{x}_2) \\ (x_2-\bar{x}_2)'(y-\bar{y}) & (x_2-\bar{x}_2)'(x_1-\bar{x}_1) & (x_2-\bar{x}_2)'(x_2-\bar{x}_2) \end{bmatrix}$$

and, using the known formula for the elements of the inverse of a $2 \times 2$ matrix, we could proceed to calculate

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} (x_1-\bar{x}_1)'(x_1-\bar{x}_1) & (x_1-\bar{x}_1)'(x_2-\bar{x}_2) \\ (x_2-\bar{x}_2)'(x_1-\bar{x}_1) & (x_2-\bar{x}_2)'(x_2-\bar{x}_2) \end{bmatrix}^{-1} \begin{bmatrix} (x_1-\bar{x}_1)'(y-\bar{y}) \\ (x_2-\bar{x}_2)'(y-\bar{y}) \end{bmatrix}.$$

Then we could calculate

$$b_0 = \bar{y} - b_1\bar{x}_1 - b_2\bar{x}_2,$$

where $\bar{y}$, $\bar{x}_1$, $\bar{x}_2$ are the means of the appropriate columns.