ENHANCED METHODS OF SEASONAL ADJUSTMENT

1. Introduction

The effect of the conventional model-based methods of seasonal adjustment is to nullify the elements of the data that reside at the seasonal frequencies and to attenuate the elements at the adjacent frequencies.

It may be desirable to nullify some of the adjacent elements instead of merely attenuating them. For this purpose, two alternative procedures are presented that have been implemented in a computer program.

In the first procedure, the basic seasonal-adjustment filter is applied in series with additional filters that that are targeted at the adjacent frequencies.

In the second procedure, a Fourier transform is deployed to reveal the elements of the data at all the frequencies. This allows the elements in the vicinities of the seasonal frequencies to be eliminated or attenuated at will.

2. Comb Filters

Any time-domain procedure for seasonal adjustment must contain a component that acts like a comb filter. This can be represented as a ratio of two polynomials of which the argument is a complex number z:

$$\frac{\Sigma(z)}{P(z)} = \frac{1 + z + z^2 + \cdots + z^{s-1}}{1 + \rho z + (\rho z)^2 + \cdots + (\rho z)^{s-1}} = \frac{(1 - z^s)(1 - \rho z)}{(1 - z)(1 - \rho^s z^s)}. \tag{1}$$

Here, $\rho \in (0, 1)$ and $s = 4, 12$ denotes either a quarterly or a monthly frequency of observation.

The numerator polynomial contains zeros at the seasonal frequencies $\omega_j = 2\pi j/s; j = 1, 2, \ldots, s-1.$ which are amongst the roots of the equation $1 - z^s = 0$,

The denominator polynomial contains the poles $\rho \exp(\mathrm{i}2\pi j/s); j = 1, 2, \ldots, s-1$, which lie on a circle in the complex plane of radius ρ^{-1}.

The seasonal elements are nullified by the zeros of the filter. The effects of these zeros at other frequencies is limited by the poles. At frequencies remote from the seasonal frequencies, the effects of the poles and the zeros are largely cancelled.

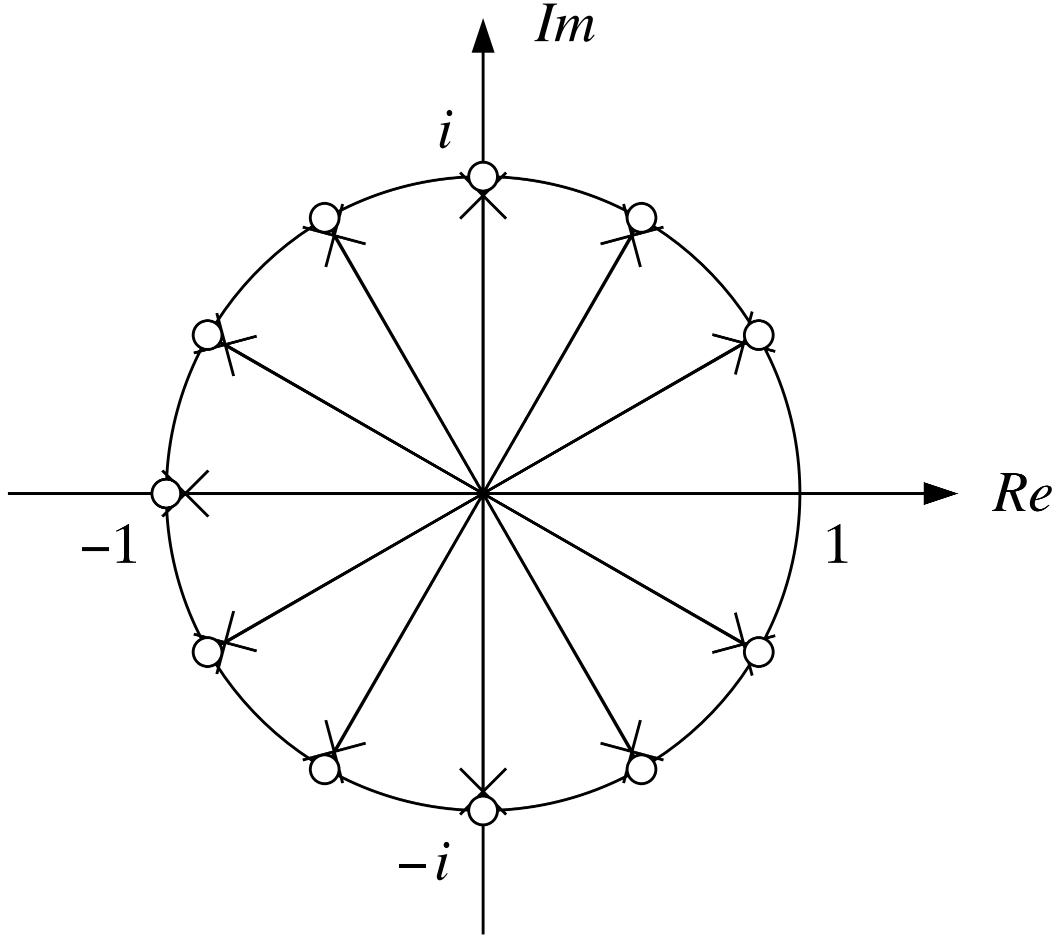

Figure 1. The pole-zero diagram of the unidirectional comb filter for monthly data. The poles are marked by crosses and the zeros are marked by circles.

3. The Bidirectional Comb Filter

The filter of equation (1) is unidirectional and backward looking. The filter will induce a phase shift or time lag in the filtered data.

To avoid this effect, the filter must reach equally forwards and backwards in time, and it is appropriate adopt a bidirectional filter:

$$B(z) = \mu \frac{\Sigma(z^{-1})\Sigma(z)}{P(z^{-1})P(z)}. \tag{2}$$

The filter is applied in two pass running forwards an backwards through the data:

$$P(z)q(z) = \Sigma(z)y(z) \qquad \text{and} \qquad P(z^{-1})x(z) = \Sigma(z^{-1})q(z), \tag{3}$$

The frequency response function shows how the filter modifies the amplitudes of the sinusoidal elements of which a stationary data sequence is composed. It is obtained by setting $z = \exp\{i\omega\}$ and by running ω from zero to the limiting frequency of π.

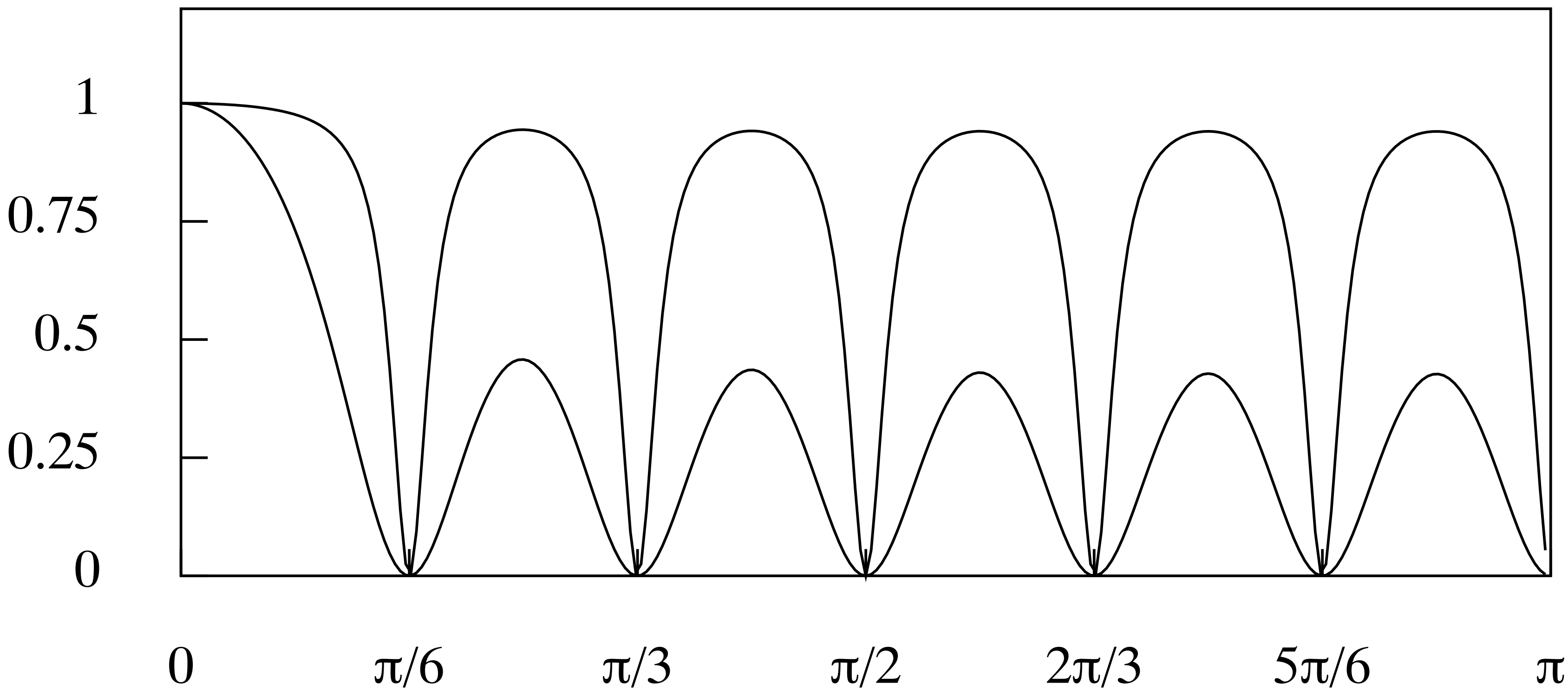

Figure 2. The frequency response function of the bidirectional comb filter for monthly data with $\rho = 0.8$, giving the lesser peaks, and $\rho = 0.9$, giving the higher peaks.

4. The Wiener–Kolmogorov Filter

The comb filter does not fully preserve non-seasonal elements of the data. It offers little control over the width of the clefts that surround the seasonal frequencies.

A Wiener–Kolmogorov filter offers a better solution:

$$\Psi(z) = \mu \frac{\Sigma(z^{-1})\Sigma(z)}{\Sigma(z^{-1})\Sigma(z) + \lambda P(z^{-1})P(z)}. \tag{4}$$

The filter can derived from the following statistical model:

$$\begin{aligned} y(z) &= \frac{P(z)}{\Sigma(z)}\nu(z) + \eta(z) \\ &= \xi(z) + \eta(z), \end{aligned} \tag{5}$$

where $y(z)$ is the z-transform of the data sequence and where $\xi(z)$ represents the seasonal fluctuations. Also, $\eta(z)$ and $\nu(z)$ represent mutually independent white-noise processes with variances of σ_η^2 and σ_ν^2, respectively.

5. The Wiener–Kolmogorov Filter

The model does not provide a realistic description of the data process. Instead, it is to be regarded solely as a means of deriving an appropriate filter. Multiplying (5) by $\Sigma(z)$ achieves stationarity:

$$g(z) = \Sigma(z)y(z) = P(z)\nu(z) + \Sigma(z)\eta(z). \tag{6}$$

The conditional expectation $\eta(z)$ given $g(z)$ is

$$E\{\eta(z)|g(z)\} = E\{\eta(z)\} + \frac{C\{\eta(z), g(z)\}}{V\{g(z)\}}[g(z) - E\{g(z)\}]. \tag{7}$$

Given that

$$V\{g(z)\} = \sigma_\nu^2 P(z^{-1})P(z) + \sigma_\eta^2 \Sigma(z^{-1})\Sigma(z) \quad \text{and} \quad C\{g(z), \eta(z)\} = \sigma_\eta^2 \Sigma(z), \tag{8}$$

and given that $E\{\eta(z)\} = E\{g(z)\} = 0$, it follows that

$$E\{\eta(z)|g(z)\} = \frac{\Sigma(z^{-1})\Sigma(z)}{\Sigma(z^{-1})\Sigma(z) + \lambda P(z^{-1})P(z)} y(z), \tag{9}$$

where $\lambda = \sigma_\nu^2/\sigma_\eta^2$.

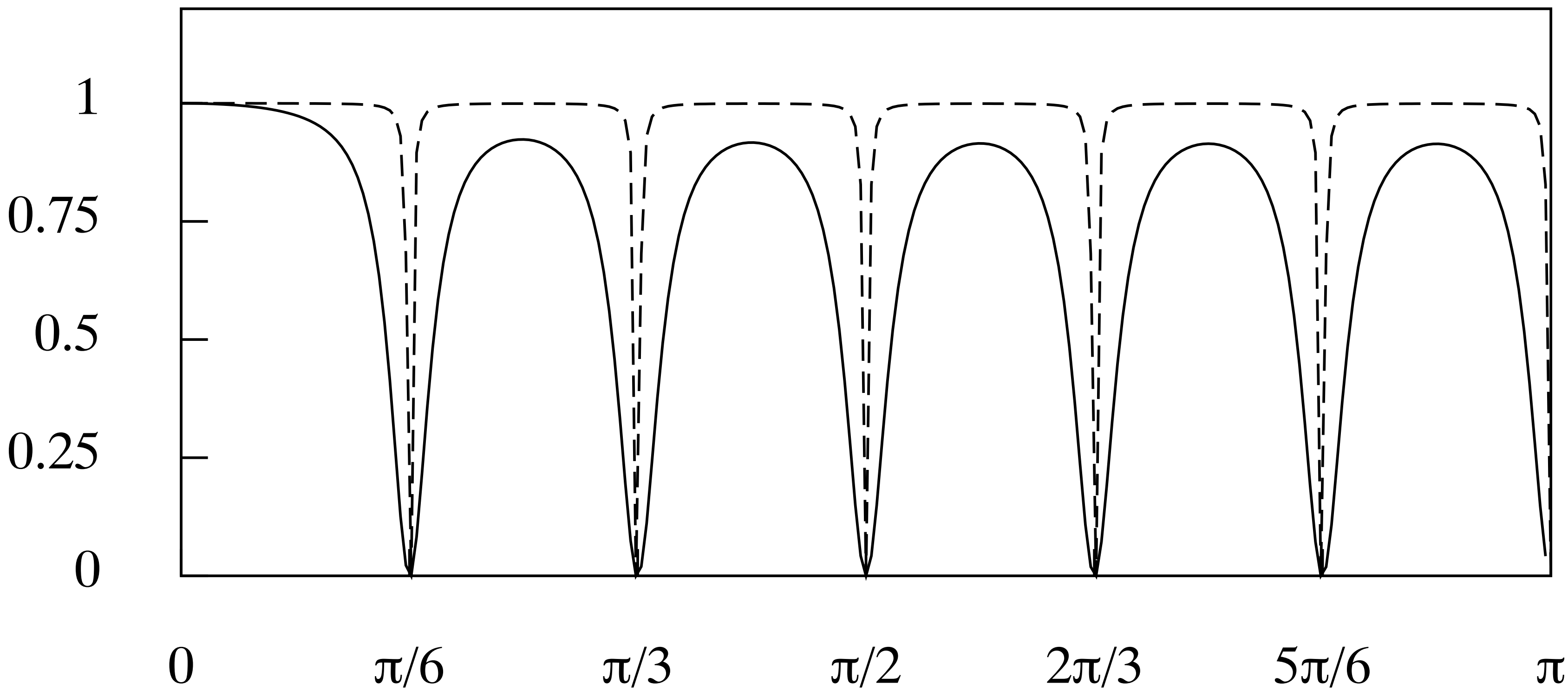

Figure 3. The frequency response functions of the ordinary seasonal adjustment filter for monthly data with $\lambda = 0.5$. and $\rho = 0.8$ (the solid line) and with $\lambda = 0.5$. and $\rho = 0.99$ (the dashed line).

6. The Finite-Sample Filter and the Matrix Lag Operator

To create a genuine finite-sample filter, appropriate to a vector of T observations, we may replace z by the matrix lag operator $L_T = [e_1, e_2, \ldots, e_{T-1}, 0]$ or by the circulant matrix $K_T = [e_1, e_2, \ldots, e_{T-1}, e_0]$, which are derived from the identity matrix $I_T = [e_0, e_1, e_2, \ldots, e_{T-1}]$.

In the case of the matrix lag operator, the necessary presample elements are provided by zeros. In the case of the circulant matrix, they come from the end of the sample.

The presample problem can be avoided by deleteing the initial rows of $\Sigma(L_T)$ and $P(L_T)$. Consider

$$\Sigma(L_4) = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \hline 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array}\right] = \begin{bmatrix} S_* \\ S' \end{bmatrix}, \quad P(L_4) = \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ \rho & 1 & 0 & 0 \\ \hline \rho^2 & \rho & 1 & 0 \\ 0 & \rho^2 & \rho & 1 \end{array}\right] = \begin{bmatrix} P_* \\ P' \end{bmatrix}.$$

Using S' and P' in place of $\Sigma(z)$ and $P(z)$ in equations (4) and (9) gives

$$h = \Psi y = S(S'S + \lambda R'R)^{-1} S'y. \tag{13}$$

7. The Finite-Sample Wiener–Kolmogorov Filter

This equation can also be derived from a conditional expectation. Applying S' to the equation $y = \xi + \eta$, representing the seasonally fluctuating data, gives

$$S'y = R'\nu + S'\eta = g. \tag{14}$$

The relevant expectations and the dispersion matrices are

$$E(g) = E(\eta) = 0, \quad D(g) = \sigma_\nu^2 R'R, +\sigma_\eta^2 S'S. \tag{15}$$

The conditional expectation of η, given the transformed data $g = S'y$, is provided by the formula

$$\begin{aligned} h = E(\eta|g) &= E(\eta) + C(\eta, g)D^{-1}(g)\{g - E(g)\} \\ &= S(S'S + \lambda R'R)^{-1}S'y. \end{aligned} \tag{16}$$

A simple procedure for calculating h begins by solving the equation

$$(S'S + \lambda R'R)b = S'y = g \tag{19}$$

for the value of b. Thereafter, one can generate $h = Sb$.

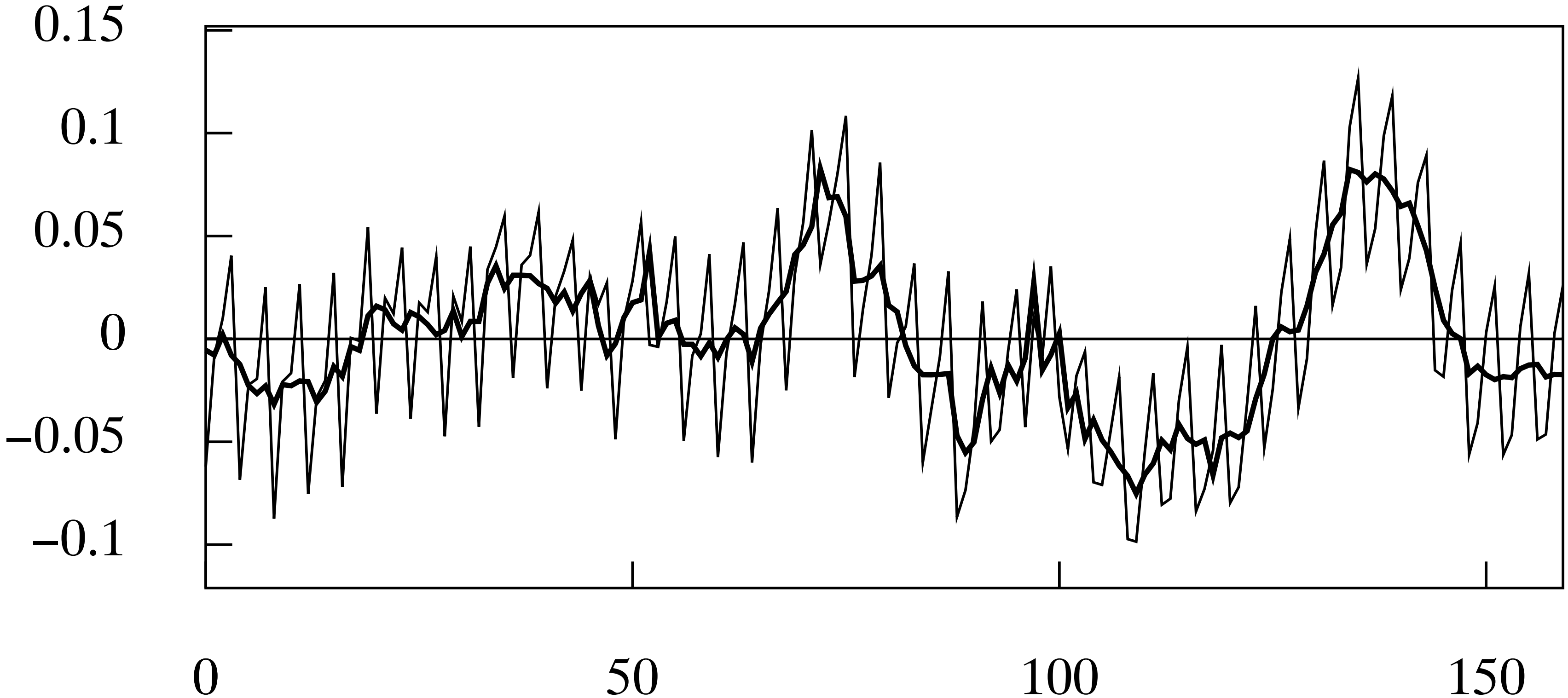

Figure 4. The residuals from a linear detrending of the logarithms of an index of quarterly U.K. Consumption for the years 1955 to 1994, with a superimposed seasonally adjusted sequence.

8. Widening the Seasonal Stopbands

To increase the attenuation of the elements of the data adjacent to the seasonal frequencies, the poles can be retracted from the unit circle by reducing value of ρ within $P(z)$.

It may be required to impose a greater attenuation on the adjacent elements than can be achieved by reductions in the value of ρ.

Then, it might be appropriate to apply the seasonal-adjustment filter twice or more in succession and with poles and zeros that are displaced from the seasonal frequencies by small angles.

A twofold filter with equal displacements on either side of the seasonal frequencies could take the form of

$$\Psi_\zeta(\omega) = \Psi(\omega - \zeta)\Psi(\omega + \zeta), \tag{21}$$

where ζ is the angle of the displacement.

The lack of a central filter aimed at eliminating the elements at the seasonal frequencies may the cause an unacceptable leakage. Therefore, it may be desirable to apply a central filter in series with two offset filters.

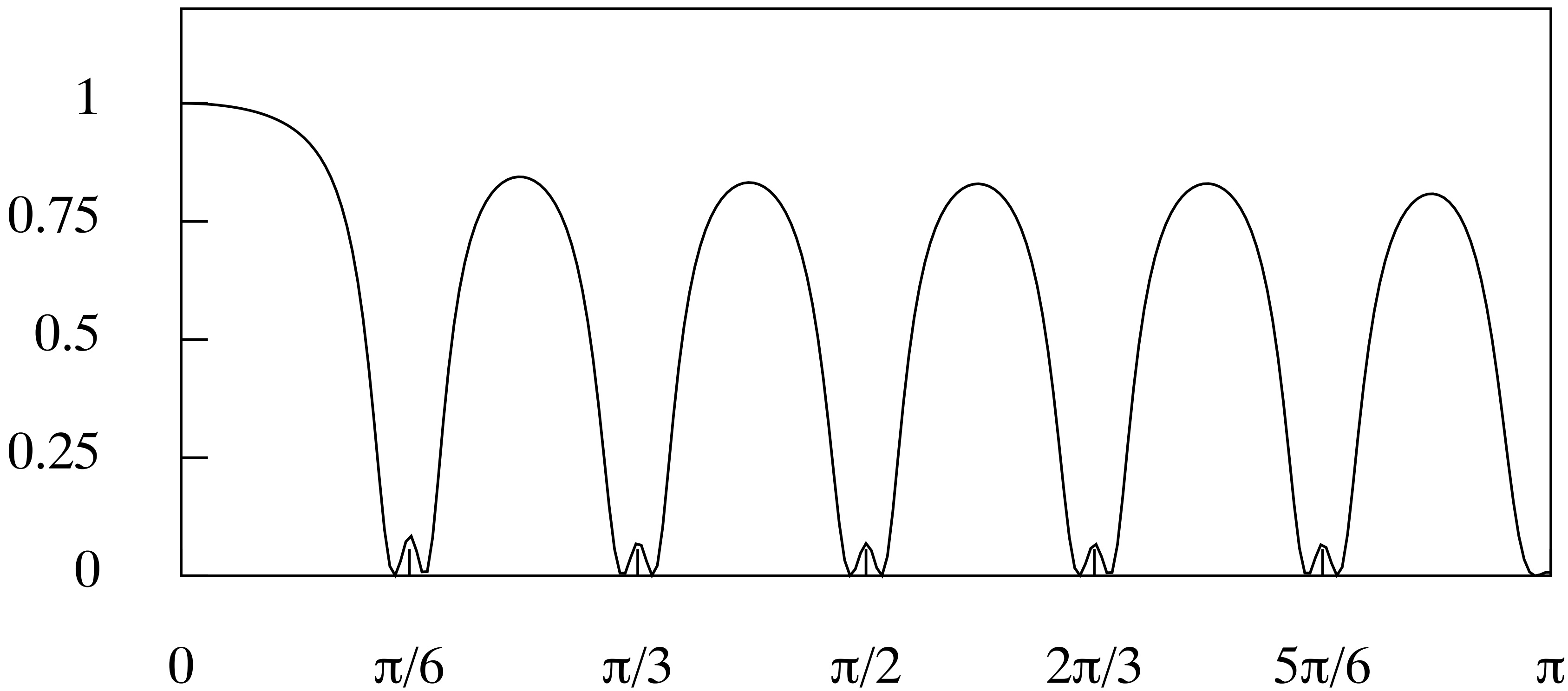

Figure 5. The frequency response function of the double seasonal adjustment filter for monthly data with offsets of 2 degrees.

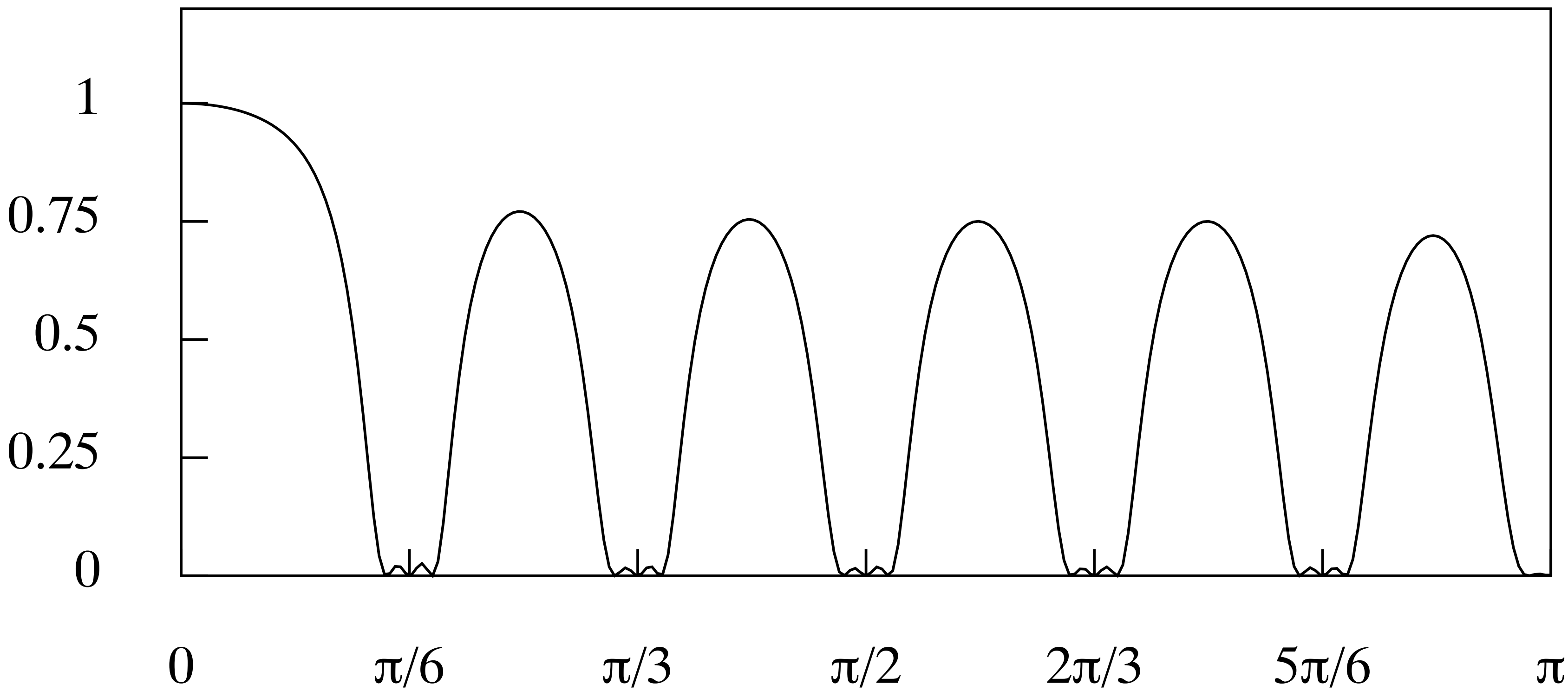

Figure 6. The frequency response function of the triple seasonal adjustment filter for monthly data with offsets of 3 degrees.

9. SEATS-TRAMO and the Trend-Extraction Filter

In the conventional models, the trend is represented by an integrated random walk. We use a polynomial function to extract the trend. When the residuals have been adjusted, they are added back to the trend to create the seasonally-adjusted data.

The SEATS-TRAMO procedures are based on the airline passenger model of Box and Jenkins:

$$y(z) = \frac{(1-\phi z)(1-\Theta z^s)}{(1-z)(1- \quad z^s)}\varepsilon(z) = \frac{(1-\phi z)(1-\Theta z^s)}{\nabla^2(z)\Sigma(z)}\varepsilon(z). \tag{23}$$

The model is decomposed via a princicpal of canonical decomposition to create a *meta model* comprising a trend component, a seasonal component and a residual component to which all of the white noise, potentially residing in the other components, is assigned:

$$y(z) = \frac{U(z)}{\nabla^2(z)}\epsilon(z) + \frac{V(z)}{\Sigma(z)}\zeta(z) + \eta(z). \tag{24}$$

Here, $\epsilon(z)$, $\zeta(z)$ and $\eta(z)$ represent mutually independent white-noise processes.

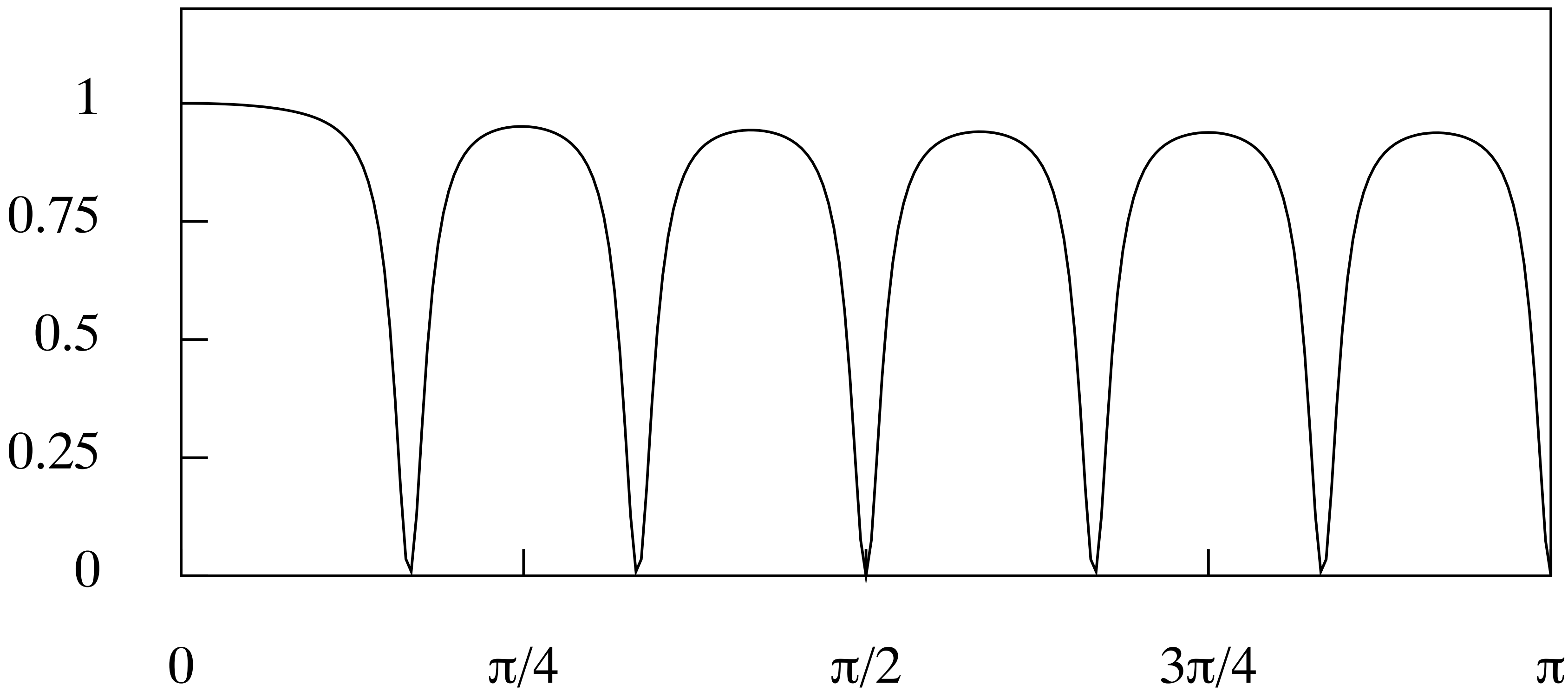

Figure 7. The gain of the seasonal-adjustment filter associated with the monthly airline passenger model.

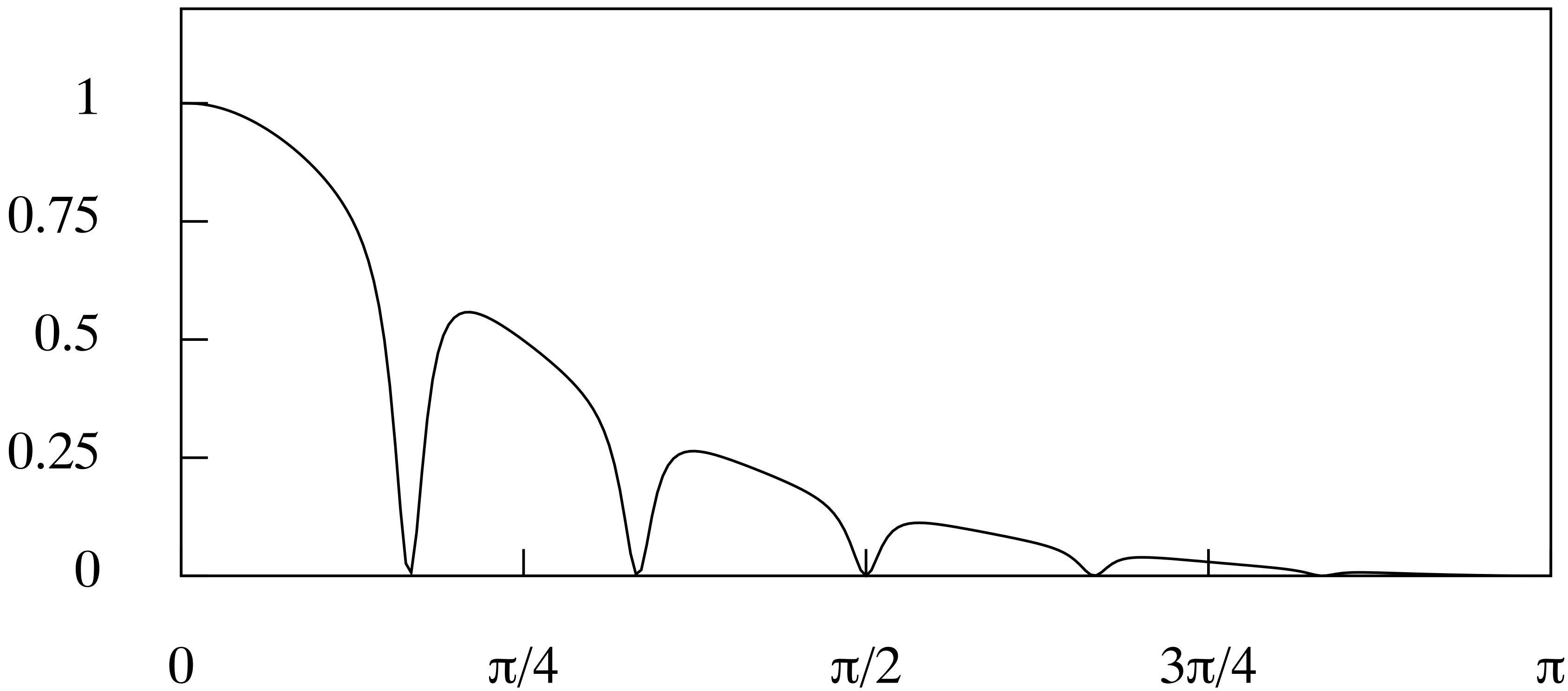

Figure 8. The gain of the trend extraction filter associated with the monthly airline passenger model.

10. Mimicking the Trend-Extraction Filter

In order to mimic the trend extraction filter of Figure 8, a smoothing filter based on a second-order moving average $\mu(z) = 1+\mu_1 z+\mu_2 z^2$ can be applied in series with the seasonal adjustment filter of (13).

The smoothing filter is represented by

$$M(z^{-1})M(z) = \frac{(1+\mu_1 z^{-1}+\mu_2 z^{-2})(1+\mu_1 z+\mu_2 z^2)}{(1+\mu_1+\mu_2)^2}. \tag{26}$$

The purpose of the denominator is to ensure that the filter has a unit gain at the zero frequency.

A parsimonious parametrisation of the smoothing filter is achieved by adopting a second-order moving average of the form $1+(1+\kappa)z+\kappa z^2$, with $\kappa \in [0,1]$.

Figure 9 shows the frequency response function of a filter that compounds the smoothing filter with the filter of which the frequency response is depicted in Figure 3 by the unbroken line. In this case, The smoothing parameter is $\kappa = 0.6$.

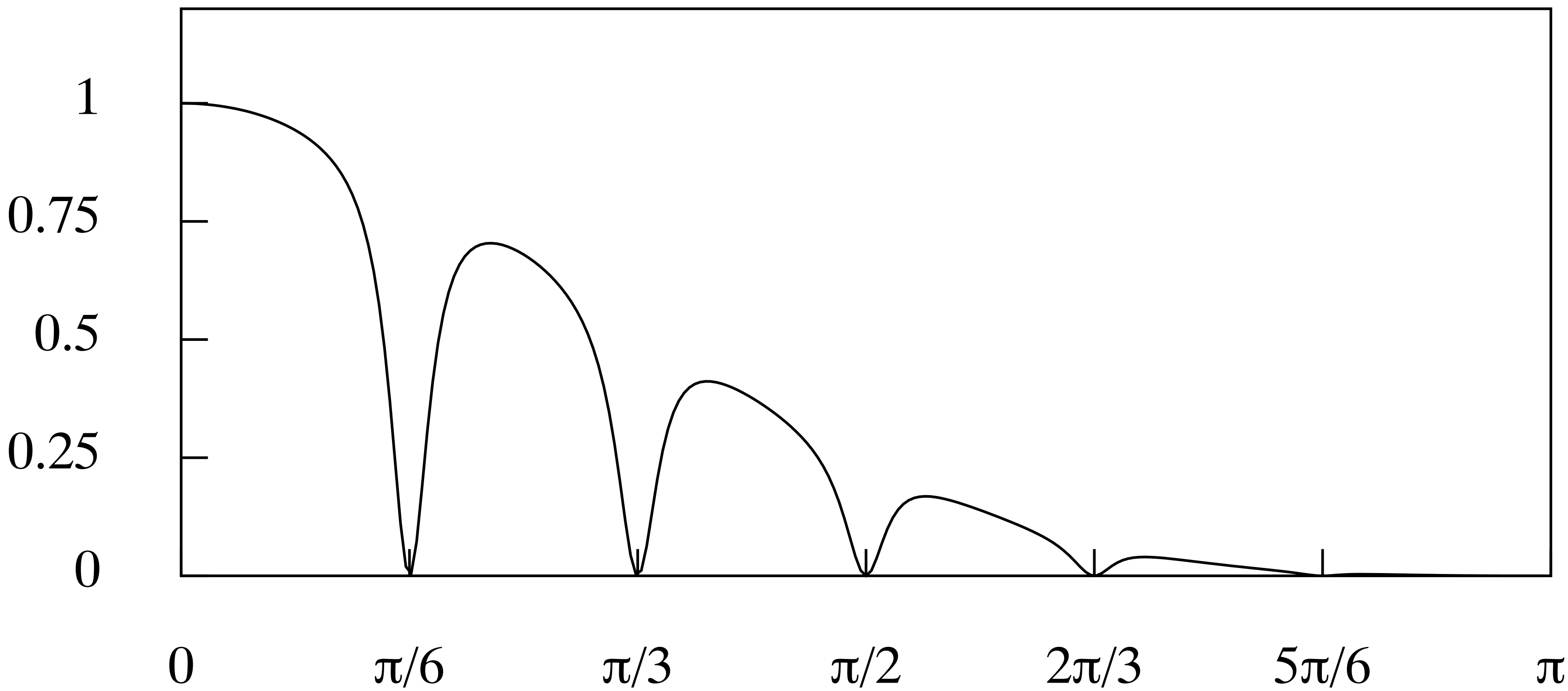

Figure 9. The frequency response of a trend extraction filter that mimics that of the monthly airline passenger model.

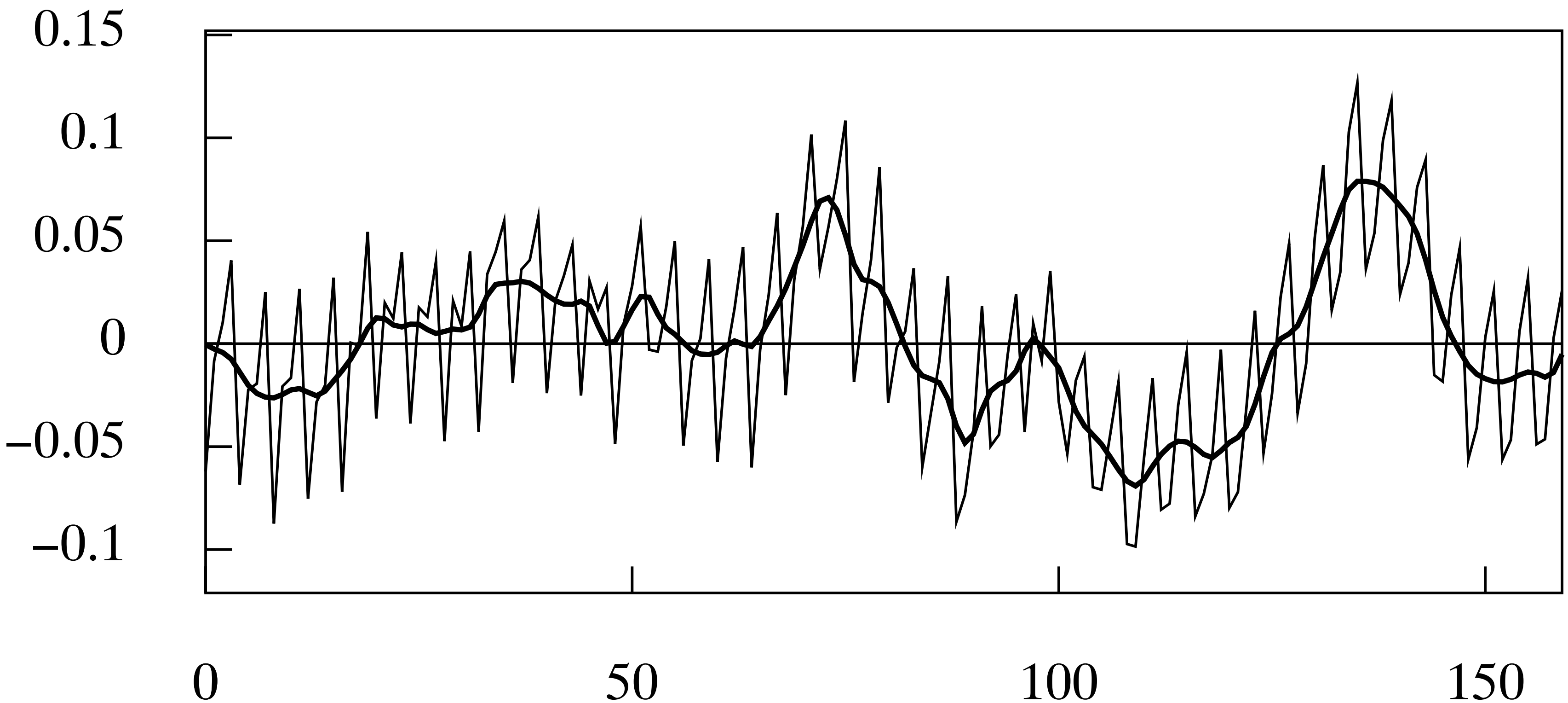

Figure 10. The effect of applying the trend extraction filter to the sequence depicted in Figure 4.

11. Fourier Methods of Seasonal Adjustment and Trend-Extraction

The Fourier ordinates of a detrended data sequence can be rescaled in any way that is deemed to be appropriate, thereby altering the amplitude of the sinusoidal elements of which the data are composed.

An inspection of the periodogam of the data will indicate which of the elements need to removed or to be attenuated in pursuit of the seasonal adjustment.

The seasonal fluctuations are liable to comprise elements at frequencies that are adjacent to the seasonal frequency and its harmonics. These can be removed easily by operating in the frequency domain.

Such elements may be due to strikes, holidays and calender irregularities. Data irregularities are usually addressed directly by adjusting the data. This can be avoided by operating on the Fourier ordinates in the frequency domain.

The periodogram of Figure 11 indicates, via the highlighted bands, the elements that should be removed in pursuit of seasonal adjustment.

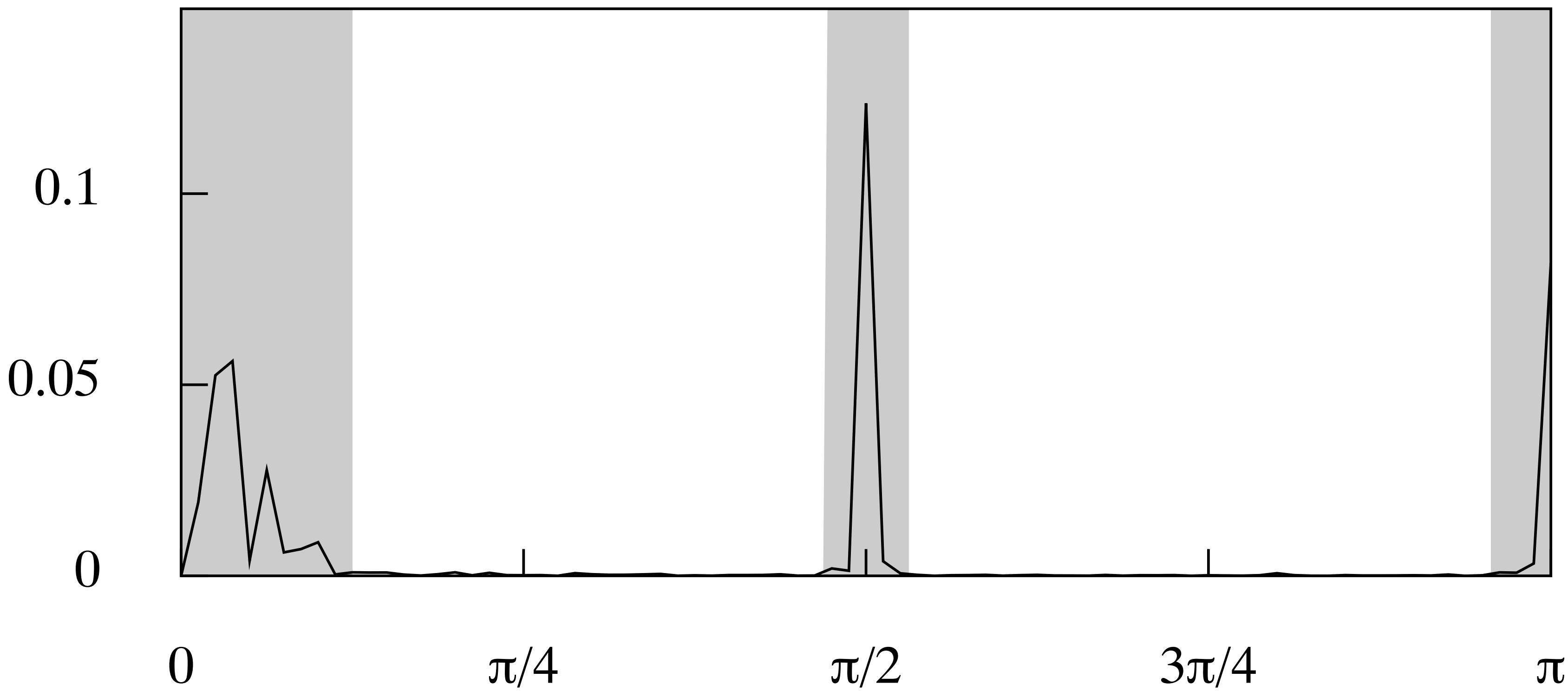

Figure 11. The periodogram of the residual sequence from the linear detrending of the logarithmic consumption data.

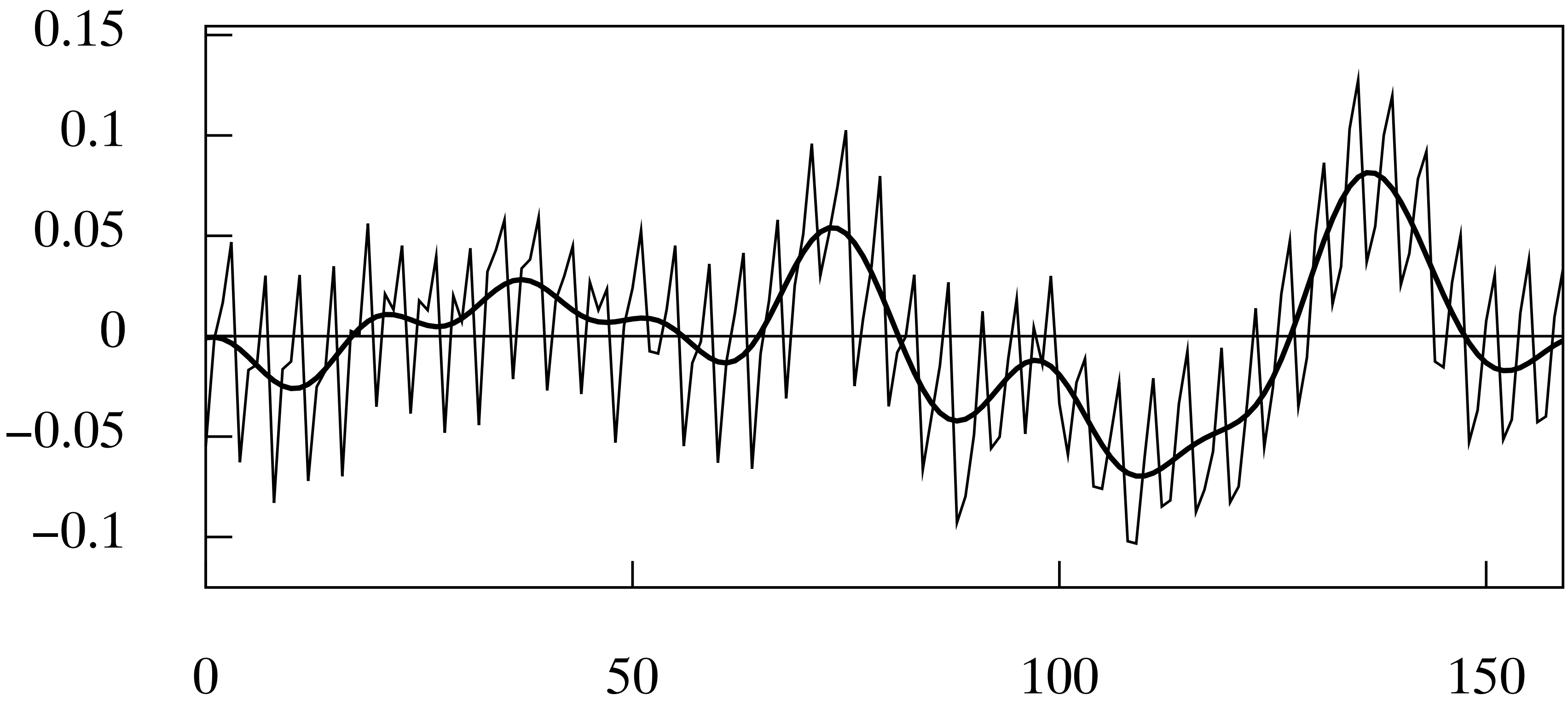

Figure 12. The residual sequence from fitting a linear trend to the logarithmic consumption data with an interpolated line representing the business cycle, obtained by the frequency-domain method.

12. The Trend-Cycle Trajectory and the Seasonally-Adjusted Data

Figure 12 shows the residual sequence from fitting a linear trend to the logarithmic consumption data together with an interpolated line representing the business cycle, synthesised from the Fourier ordinates in the interval $[0, \pi/8]$.

This representation of the business cycle is liable to be preferred to that of Figure 10, which has been produced by a time-domain method and which is affected by some of the noise from within the deadspaces and by some proportion of the elements that are adjacent to the seasonal frequencies.

The difference between the trajectory of Figure 12 and the seasonally adjusted sequence of Figure 4 is noise that is devoid of any economic information. Therefore, it can be argued that the estimated trend-cycle sequence of Figure 12 should serve as the seasonally adjusted data.

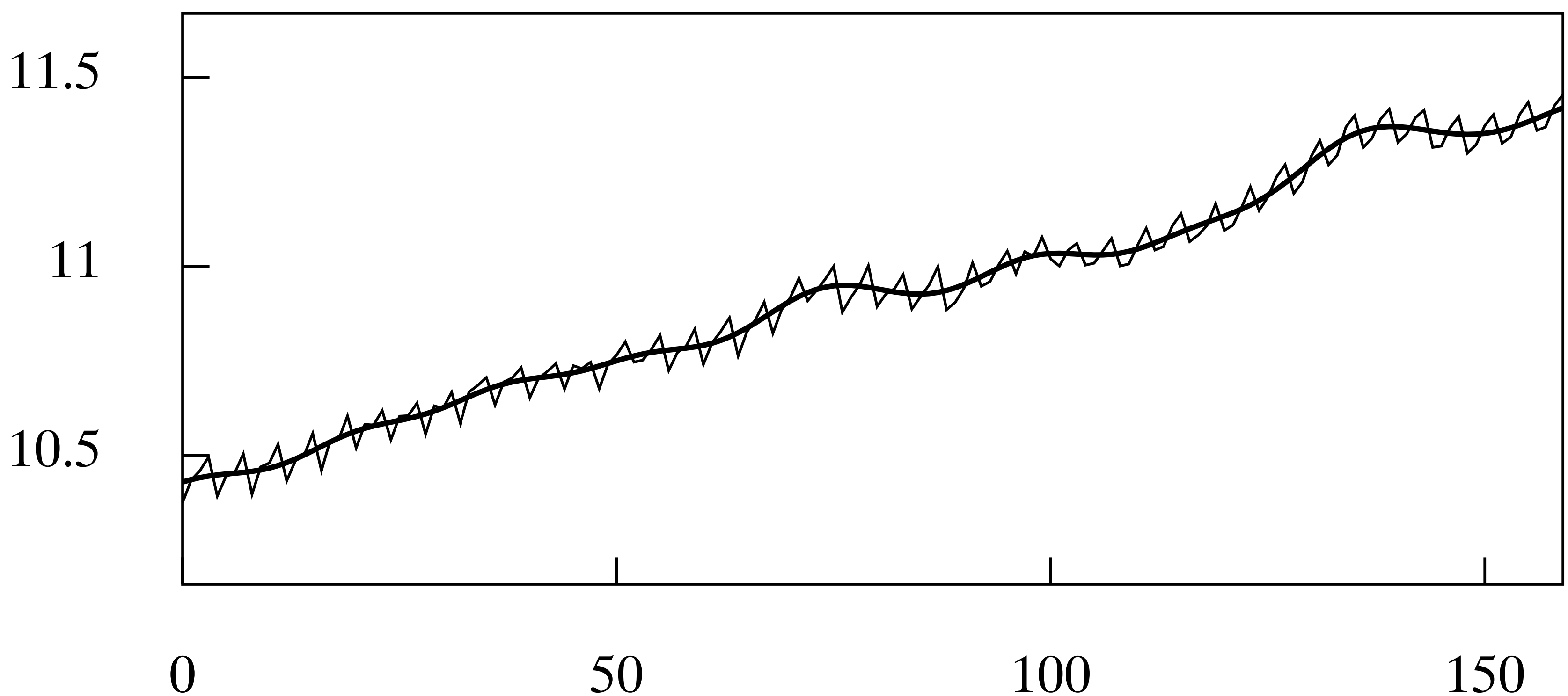

Figure 14. The trend-cycle component derived by adding the interpolated polynomial to the low-frequency components of the residual sequence

13. Defining the Frequency-Response Function

Any frequency response function that can be plotted as a graph can be imposed on the Fourier ordinates of the data.

In Figure 11, the clefts that surround the seasonal frequencies are vertical shafts. More generally, the clefts can be defined on a frequency interval

$$[M - a_1, M - b_1, M + b_2, M + a_2],$$

where M is a seasonal frequency, $[M - a_1, M - b_1]$ and $[M + b_2, M + a_2]$ are transition bands and $[M - b_1, M + b_2]$ is the stop band.

The transition function defined on the lower transition band $[M - a, M - b]$ might take the form of

$$\frac{1}{1-r}\left[\cos\left(x\frac{\pi}{2q}\right) - r\right] \text{ where } x = \frac{\omega - [M-a]}{a-b} \quad \text{and} \quad r = \cos(\pi/2q).$$

With $q = 1$ and $r = 0$, this function declines from unity to zero over the interval $[M - a, M - b]$. If $q < 1$ it does likewise, but with a slower initial rate of descent.

An alternative transition function based on $1 - \sin(x)$ will begin with a rapid descent that slows as zero is approached as $x \longrightarrow \pi/2$.

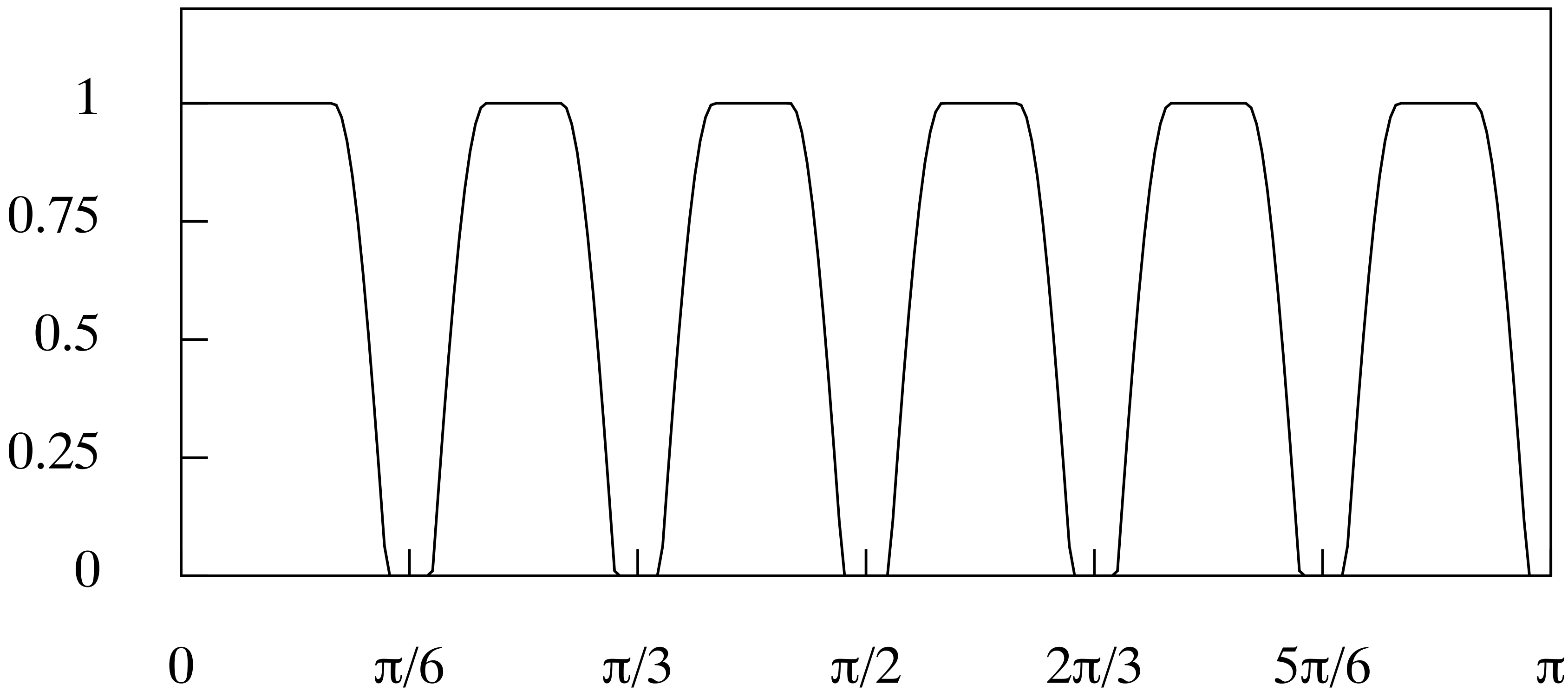

Figure 15. A frequency response function with a cosine transition that might be applied to the Fourier ordinates of a de-trended monthly data sequence.