AN ERROR-CORRECTION FORMULATION OF THE CONSUMPTION FUNCTION: THE DHSY MODEL

In many models, the natural logarithms of the economic variables are used in the regression equation. Consider the case where the dependent variable Y maintains a long-term proportionality with the explanatory variable X. For example, Y and X might be consumption and income respectively. In that case, the equilibrium condition which characterises a stationary state or an equilibrium growth path is $Y = KX$. Letting $x = \ln X$, $y = \ln Y$ and $k = \ln K$ gives $y = k + x$; and, on an equilibrium path, the proportional rate of growth of the two variables will be $\nabla y = \nabla x = r$.

Our purpose is to reconcile these equilibrium conditions with a dynamic regression equation in the form of

$$y(t) = \mu + \phi_1 y(t-1) + \beta_0 x(t) + \beta_1 x(t-1) + \varepsilon(t). \tag{1}$$

This is rendered in error-correction form by taking $y(t-1)$ from both sides of the equation and supplementing the RHS by $\pm\beta_0 x(t-1)$. The result is

$$\begin{aligned}\nabla y(t) &= \mu + (\phi_1 - 1)y(t-1) + \beta_0 \nabla x(t) + (\beta_1 + \beta_0)x(t-1) + \varepsilon(t)\\ &= \mu + (1-\phi_1)\left\{\frac{\beta_1+\beta_0}{1-\phi_1}x(t-1) - y(t-1)\right\} + \beta_0\nabla x(t) + \varepsilon(t).\end{aligned} \tag{2}$$

Let the rate of growth be r so that $Y_t = Y_{t-1}e^r$ and $X_t = X_{t-1}e^r$, which give $y_t = y_{t-1} + r$ and $x_t = x_{t-1} + r$, respectively. Putting these conditions in (1), eliminating the disturbance term and suppressing the temporal indices gives

$$y + r = \mu + \phi_1 y + \beta_0(x + r) + \beta_1 x, \tag{3}$$

from which

$$y = \frac{\mu + (\beta_0 - 1)r}{1-\phi_1} + \frac{\beta_0 + \beta_1}{1-\phi_1}x. \tag{4}$$

To reconcile this with the equation $y = k + x$ which characterises the growth path, we must impose the condition that $(\beta_0 + \beta_1)/(1-\phi_1) = 1$ or, equivalently, that $\beta_0 + \beta_1 + \phi_1 = 1$. Notice that $k = \{\mu + (\beta_0 - 1)r\}/(1-\phi_1)$ is dependent on the growth rate r.

One might doubt whether it is reasonable to postulate an equilibrium growth rate that prevails over the entire sample period. However, if such a postulate is accepted, then it becomes appropriate to fit an equation the form of

$$\nabla y(t) = \mu + \lambda\{x(t-1) - y(t-1)\} + \beta_0\nabla x(t) + \varepsilon(t), \tag{5}$$

which is the resulting specialisation of equation (2). This equation and the foregoing analysis were the basis of an influential article on consumer's expenditure in the U.K. by Davidson *et al.*

To understand the dynamic implications of the equation, let us set $\mu = 0$. Then, in a steady state, with a growth rate of r and with $\varepsilon(t) = 0$ for all t, we should have $\nabla y(t) = r$, $\beta_0 \nabla x(t) = \beta_0 r$ and $\lambda \{x(t-1) - y(t-1)\} = -\lambda\kappa = (1 - \beta_0)r$. It follows that, the faster the growth rate, the wider is the gap between income and consumption. In the absense of an intercept term μ, the gap would disappear altogether at a zero rate of growth. It seems that, in order to avoid this implication, an intercept term should be present in the equation.

Reference

Davidson, J.E.H., D.F. Hendry, F. Srba and S. Yeo, (1978), "Econometric Modelling of the Aggregate Time-Series Relationship between Consumer's Expenditure and Income in the United Kingdom," *The Economic Journal,* **88,** 661–692.