TIME-SERIES ANALYSIS: PASCAL PROGRAM

This program demonstrates alternative ways of characterising a stationary stochastic process. On the one hand are the time-domain characterisations, which depend upon the autocorrelation and partial autocorrelation functions. These functions convey the same information in different forms. On the other hand is the frequency-domain characterisation, which is provided by the spectral density function. This conveys the same information as the autocorrelation functions.

The program first asks the user to specify the parameters of a stationary autoregressive moving-average model. It proceeds to generate the corresponding autocorrelation functions and the spectral density function. The program also generates a pseudo random sequence from which the empirical autocorrelation functions can be calculated and compared to their theoretical counterparts.

The Menu

The *TSERIES* program displays the following menu:

TIME-SERIES ANALYSIS

1. *Get Page Parameters*
2. *Get Model Parameters*
3. *Get the Data and Plot it*
4. *SACF: Sample Autocorrelation Function*
5. *SPACF: Sample Partial Autocorrelation Function*
6. *TACF: Theoretical Autocorrelation Function*
7. *TPACF: Theoretical Partial Autocorrelation Function*
8. *Spectral Density Function*
9. *Periodogram*
10. *FOR EXIT*

The manner in which the program operates is largely self-explanatory. It poses questions that are typically answered by typing Y for yes or N for no or by supplying the numerical values corresponding to model parameters or the graphical specifications.

Specifying Graphics Parameters

The program plots a variety of diagrams on the computer screen and it also generates the corresponding *PostScript* code, which may be viewed and printed via the *GhostScript* program or incorporated into a TEX or a LaTEX document.

The graphics parameters are specified by selecting the item *1. Get Page Parameters* and by issuing the relevant commands. First, the user is asked whether they require *PostScript graphics.* If so, then the choice is between the

Textures format and the *Encapsulated PostScript* (EPS) format, which is the default option.

Next, the user is asked to specify the dimensions of the frame surrounding the graph. This frame is the bounding box of the graphic. If the only purpose is to view the graphics on the screen, then the maximum dimensions should be chosen. This can be achieved by typing large values in excess of the maximum sizes. Thus, an adequate response to the requests to *Specify the width* and *Specify the height* would be to type 99 in both cases.

Data Acquisition

Data are usually created within the program using a pseudo-random number generator in conjunction with the specification of an autoregressive moving-average model. Data can also be imported into the program via *3. Get the Data and Plot it.* If the response to the question *Do you want to generate pseudo random data?* is no and if it is decided to read a data file, then the exact name of a data file, located within the same directory as the program, must be provided. The procedure for importing data is fragile. Tying an incorrect name or the name of a non-existent data file will cause the program to crash.

Model Specification

An autoregressive moving-average model is specified either by listing the coefficients of the autoregressive and the moving-average operators or by specifying the roots of the corresponding auxiliary polynomials. The roots may be specified in Cartesian form or in polar form.

It is preferable, in most circumstances, to specify the roots in polar form, since this facilitates the choice of values that fulfil the conditions of stationarity or inversibility. To illustrate the matter, consider an ARMA(2, 1) model that might be represented by the notation

$$(\alpha_0 + \alpha_1 L + \alpha_2 L^2)y(t) = (1 + \mu L)\varepsilon(t),$$

where L denotes the lag operator. The auxiliary polynomial corresponding to the autoregressive operator is

$$\begin{aligned} z^2\alpha_0 + z\alpha_1 + \alpha_2 &= (z - \lambda_1)(z - \lambda_2) \\ &= z^2 - z(\lambda_1 + \lambda_2) + \lambda_1\lambda_2. \end{aligned}$$

Since the autoregressive order is 2, the question *Are there any poles?* would be answered in the affirmative and the order of 2 would be specified for the polynomial operator. The corresponding question in respect of the moving-average operator is *Are there any zeros?* In the case of the example, the order of 1 would be supplied for the moving-averge operator.

If the roots are real-valued, then the Cartesian and the polar representations coincide, and the real values of λ_1, λ_2 should be specified. Complex-valued roots come in conjugate pairs represented by

$$\lambda_1 = \alpha + i\beta = \rho(\cos\theta + \mathrm{i}\sin\theta) = \rho r^{\mathrm{i}\theta},$$

$$\lambda_2 = \alpha - i\beta = \rho(\cos\theta - \mathrm{i}\sin\theta) = \rho r^{-\mathrm{i}\theta}.$$

Here, ρ is described as the modulus of the roots, whereas θ is their argument. To ensure the stationarity of the process, the parameters must fulfil the condition that $\alpha^2 + \beta^2 = \rho^2 < 1$. Also, θ lies in the interval $[-\pi, \pi]$, which is $[-180, 180]$ in degrees.

The procedure for specifying the parameters of the autoregressive or the moving-average polynomial begins by asking *Do you wish to specify the auxiliary polynomial in polar form?* If the answer is no, then it proceeds to ask *Do you wish to specify in Cartesian form?* If the answer is no again, then the program infers that *You have chosen to specify the polynomial coefficients.*

Data that have been generated within the program can be saved via the item *10. FOR EXIT.* Having saved the data, the user can choose either to exit the progam or to return to the main menu.