## ECONOMETRIC THEORY: Exercise 2

Matrix Multiplication: Orthonormal Matrices and Rotations.

1. Find following matrix product:

$$
\begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}
\begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}
\begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix}.
$$

2. Confirm that, if $\theta = 45°$, then

$$
\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}
=
\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.
$$

3. Use the definitions of a sine and a cosine together with Pythagorus theorem to prove that $\sin^2 + \cos^2 = 1$.

4. Confirm that

$$
\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}
\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}
=
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

and that

$$
\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}
\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}
=
\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
$$

5. Calculate the product

$$
\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}
\begin{bmatrix} 1-\rho & 0 \\ 0 & 1+\rho \end{bmatrix}
\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.
$$

6. Calculate the product

$$
\frac{1}{1-\rho^2}
\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}
\begin{bmatrix} 1+\rho & 0 \\ 0 & 1-\rho \end{bmatrix}
\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.
$$

7. Show that the product under 6. is the inverse of the product under 5.

8. Use the method of Gaussian elimination to solve the following set of equations:

$$
\begin{bmatrix} 2 & 4 & 3 \\ 2 & 2 & 1 \\ 1 & 2 & -3 \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
=
\begin{bmatrix} 14 \\ 8 \\ -2 \end{bmatrix}.
$$