EXERCISE: The Identification of ARMA Models

An appropriate ARMA model for fitting to a stationary series may be identified by inspecting the following three functions:

(1) The Empirical Autocorrelation Function,
(2) The Empirical Partial Autocorrelation Function,
(3) The Nonparametric Estimate of the Spectral Density Function.

The procedures for determining the orders of an ARMA process from the first two of these functions described in the appendix to Lecture 4 which is titled *Identification of ARIMA Models*. Some practice in identifying the orders can be gained by using the *TSERIES* program and the *MESOSAUR* program.

The Pseudo-Random Data Series

Within the *TSERIES* program, there are facilities for generating pseudo-random data from specified models. You are invited to generate data from each of the models specified in the following list under the numerals (i) to (xiv). In the process, you may consider the effects of varying the sample size. Alternatively, data that have been generated in this manner by *TSERIES* are available on website; and they can be read into the program or into *MESOSAUR*. All of these data files comprise 160 observations.

Using either *TSERIES* or *MESOSAUR*, you can plot the data sequence corresponding to each of the models, before examining the autocorrelation function, the partial autocorrelation function and the periodogram. You should understand how the features of these functions reflect those of the corresponding theoretical functions, which would be generated by the true parameters of the processes.

The *TSERIES* program has facilities for plotting both the theoretical functions and their empirical counterparts. In the case of the spectral density function, *TSERIES* provides only the periodogram as an empirical counterpart. However, *MESOSAUR* provides an estimate of the spectral density function via a smoothed version of the periodogram. Whereas this program lacks the facility for generating pseudo-random data, it does provide the means for fitting an ARMA model to the imported data.

(i)
$$\begin{aligned} \text{1ARMA01: } y(t) &= (1-\theta L)\varepsilon(t) \\ y(t) &= (1+0.75L)\varepsilon(t) \end{aligned}$$

(ii)
$$\begin{aligned} \text{2ARMA01: } y(t) &= (1-\theta L)\varepsilon(t) \\ y(t) &= (1-0.75L)\varepsilon(t) \end{aligned}$$

(iii)
$$\begin{aligned} \text{1ARMA10: } (1-\phi L)y(t) &= \varepsilon(t) \\ (1-0.75L)y(t) &= \varepsilon(t) \end{aligned}$$

(iv)
$$\begin{aligned} \text{2ARMA10: } (1-\phi L)y(t) &= \varepsilon(t) \\ (1+0.75L)y(t) &= \varepsilon(t) \end{aligned}$$

(v)
$$\begin{aligned} \text{1ARMA11: } (1-\phi L)y(t) &= (1-\theta L)\varepsilon(t) \\ (1-0.9L)y(t) &= (1+0.9L)\varepsilon(t) \end{aligned}$$

(vi)
$$\begin{aligned} \text{2ARMA11: } (1-\phi L)y(t) &= (1-\theta L)\varepsilon(t) \\ (1+0.9L)y(t) &= (1-0.9L)\varepsilon(t) \end{aligned}$$

(vii)
$$\begin{aligned}\text{1ARMA20: } &(1+\alpha_1 L+\alpha_2 L^2)y(t)=\varepsilon(t)\\ &(1-1.273L+0.81L^2)y(t)=\varepsilon(t)\end{aligned}$$

(viii)
$$\begin{aligned}\text{2ARMA20: } &(1+\alpha_1 L+\alpha_2 L^2)y(t)=\varepsilon(t)\\ &(1+1.85L+0.855L^2)y(t)=\varepsilon(t)\end{aligned}$$

(ix)
$$\begin{aligned}\text{1ARMA02: } &y(t)=(1+\mu_1 L+\mu_2 L^2)\varepsilon(t)\\ &y(t)=(1-1.273L+0.81L^2)\varepsilon(t)\end{aligned}$$

(x)
$$\begin{aligned}\text{2ARMA02: } &y(t)=(1+\mu_1 L+\mu_2 L^2)\varepsilon(t)\\ &y(t)=(1+1.85L+0.855L^2)\varepsilon(t)\end{aligned}$$

(xi)
$$\begin{aligned}\text{1ARMA21: } &(1+\alpha L+\alpha_2 L^2)y(t)=(1+\mu L)\varepsilon(t)\\ &(1-1.785L+0.9025L^2)y(t)=(1+0.95L)\varepsilon(t)\end{aligned}$$

(xii)
$$\begin{aligned}\text{2ARMA21: } &(1+\alpha L+\alpha_2 L^2)y(t)=(1+\mu L)\varepsilon(t)\\ &(1+1.691L+0.81L^2)y(t)=(1-0.95L)\varepsilon(t)\end{aligned}$$

(xiii)
$$\begin{aligned}\text{1ARMA22: } &(1+\alpha L+\alpha_2 L^2)y(t)=(1+\mu_1 L+\mu_2 L^2)\varepsilon(t)\\ &(1-1.4745L+0.51L^2)y(t)=(1-1.157L+0.81L^2)\varepsilon(t)\end{aligned}$$

(xiv)
$$\begin{aligned}\text{2ARMA22: } &(1+\alpha L+\alpha_2 L^2)y(t)=(1+\mu_1 L+\mu_2 L^2)\\ &(1-1.275L+0.81L^2)y(t)=(1+1.273L+0.81L^2)\varepsilon(t)\end{aligned}$$

Pseudo Random Data from Unspecified Processes

Model identification is essentially a matter of practice. Therefore, a large collection of pseudo-random computer-generated data is provided in the files `XYARMA` which you may examine in detail. The processes that have generated the data series in `XYARMA` will be revealed to you only after you have attempted to guess the orders of the processes: