Methods of TimeSeries Analysis I

Course Summary 1990

1. Since we are dealing with dynamic phenomena, we ought to know something about non-stochastic dynamic theory. We ought to know something about differential equations, but since we are dealing with discrete-time models we concentrate on difference equations. The second-order difference equation exemplifies most of the relevant theory. We should know how to obtain a general solution—at least in the homogeneous case, and we should know how the nature of this solution is affected by the nature of the roots of the auxiliary equation i.e. are they real or complex or do they coincide? Do they have a modulus greater or less than unity?

2. The AR model $\alpha(L)y(t) = \varepsilon(t)$ is just a difference equation with a forcing function which is a white-noise process. Once more, the second-order case contains most of the interesting features. There are two ways of parametrising an AR process: (1) in terms of the parameters of $\alpha(L)$ and the variance of $\varepsilon(t)$ and (2) in terms of the autocovariances of the process. The Yule–Walker equations, which you must be able to derive, provide a link between (1) and (2). Usually they are employed for the purposes of going from (2) $\{\gamma_0, \gamma_1, \ldots, \gamma_p\}$ to (1) $\{\alpha_1, \ldots \alpha_p, \sigma_\varepsilon^2\}$, but we have also shown how to go in the reverse direction.

3. The other basic model of time-series analysis is the MA model. We should be able to establish the same sort of relationship between the autocovariances of the MA operator $\mu(L)$ as we have in the case of the AR model. But now we find a strange reversal. In the case of the MA model, it is almost trivial to find the autocovariances given the parameters. But to find the parameters given the autocovariances is a difficult problem. For this, we need special techniques for solving non-linear equations, e.g. The Newton–Raphson procedure.

4. The AR and the MA models are mirror images of each other. If the conditions of stationarity and invertibility are satisfied, then the AR(p) model is equivalent to an MA(∞) model, and if the conditions of invertibility are satisfied, then the MA(q) model is equivalent to an AR(∞) model. It hardly makes any sense to consider an AR model which fails to satisfy the conditions of stationarity, for then it corresponds to an explosive process. On the other hand, some people would argue that there is no reason why we should not consider a non-invertible MA process. However, it is easy to avoid doing so provided that none of the roots of the MA operator are on the unit circle. For, if we have a non-invertible MA process, then we

can easily find an equivalent invertible process by inverting some of the roots of the MA operator and by adjusting the variance of $\varepsilon(t)$. Both the invertible and the non-invertible processes will have the same autocovariances. Given a non-invertible MA(2) process, you should be able to find the corresponding invertible process as well as the autocovariances which are shared by the two processes.

5. So far , we have talked only of the time domain. We are just as interested in the frequency domain. We know, for example, that any stationary stochastic process $y(t)$ can be represented as an MA process of finite or infinite order; and, to demonstrate this result, we had to use arguments in the frequency domain. Essentially, we had to demonstrate that, if $y(t)$ has a spectral density function $f_y(\omega)$, then there exits a factorisation of the form $f_y(\omega) = \mu^*(\omega)\mu(\omega)/2\pi$, where $\mu(\omega) = \mu_0 + \mu_1 z + \cdots + \mu_q z^q$ with $z = e^{-i\omega}$ and where $\mu^*(\omega)$ is the complex conjugate of $\mu(\omega)$. We saw that, in view of this factorisation, we were able to write $y(t) = \mu(L)\varepsilon(t)$, which is the MA representation of the process.

6. We also saw how to derive the spectral density functions which correspond to simple AR and MA processes—e.g. the AR(1) and the MA(1) processes—and we saw how to infer the nature of the time series from the shape of the spectrum and vice versa. We regarded both the MA process $y(t) = \mu(L)\varepsilon(t)$ and the AR process $y(t) = \alpha^{-1}(L)\varepsilon(t)$ as the result of passing the white-noise process $\varepsilon(t)$ through a linear filter.