QUESTIONS IN PREPRATION FOR THE EXAMINATION

The examination asks for THREE answers

The time allowed is TWO hours

1. Give an account of the axioms of Boolean algebra and, in the process, compare them with the axioms of arithmetic.

Let A and B be two events within the sample space S, for which $P(S) = 1$. Let A^c and B^c be the complements of A and B respectively. Given that $P(A \cup B^c) = 0.3$ and $P(A \cap B) = 0.1$, find $P(B)$.

2. Show how Bayes' theorem is used to obtain the posterior likelihood of an hypothesis from the prior likelihood when an event has occured which throws some light on the hypothesis.

Usually I wash the breakfast dishes before leaving for the office. If I do not do so, then my wife will do so nine times out of ten if she returns from work first; but there is only a one-in-ten chance that my children will do so if they return from school before I get home.

This morning I failed to wash the dishes. Also my wife told me that there is a fifty-fifty chance that she would return later than me. When I returned home, I discovered that the dishes had been washed, and I could hear that my children were at home. What were the chances that my wife was also at home?

3. Derive, from first principles, the function expressing the probability of obtaining x successes in n independent trials when the probability of a success in any trial is p.

On my journey home, I encounter six sets of traffic lights at widely spaced intervals. They are all timed to give 60 seconds of green and 40 seconds in total of red and amber; and, unless they show green, I am obliged to come to a halt. Assuming that my arrival time at the lights is uniformly random over a wide interval, find the probability that I will not be delayed by more than 3 sets of lights. What is the expected value per journey of the overall delay at traffic lights?

4. Let x, y be linearly related random variables such that $E(y|x) = \alpha + \beta x$. Show that $\beta = C(x, y)/V(x)$ and that $\alpha = E(y) - \beta E(x)$

Let the expected yields in pounds of three investments be $E(x, y, z) =$ (200, 150, 350) and let

$$\begin{bmatrix} V(x) & C(x,y) & C(x,z) \\ C(y,x) & V(y) & C(y,z) \\ C(z,x) & C(z,y) & V(z) \end{bmatrix} = \begin{bmatrix} 12 & -2 & 6 \\ -2 & 11 & -9 \\ 6 & -9 & 9 \end{bmatrix}$$

What is the variance of the total earnings? Assuming that $E(y|x) = \alpha + \beta x$ and that $E(z|x) = \gamma + \delta x$, what are the expected earnings given that $x = 110$ has already been received?

5. Describe (a) the difference between a one-tailed test of a statistical hypothesis and a two-tailed test, and (b) the difference between a Type I error and a Type II error.

Imagine that a random sample of size 160 is drawn from a normal population where the standard deviation is 25 in order to test the null hypothesis that the mean is 104 against the alternative that it is 100. Calculate the probability of a Type II error in a one-tailed test with a 5% significance level. What sample size is required for the probability of the Type II error to be 0.05?

6. Demonstrate how the moments of a random variable may be obtained from the derivatives in respect of t of the function $M(t) = E\{\exp(xt)\}$.

If $x = 1, 2, 3, \ldots$ has the geometric distribution $f(x) = pq^{x-1}$, where $q = 1 - p$, show that the moment generating function is

$$M(t) = \frac{pe^t}{1 - qe^t}.$$

Find $E(x)$.