EXERCISES IN STATISTICS

Series A, No. 8

1. The value of the mean of a random sample of size 20 from a normal population is $\bar{x} = 81.2$ Find the 95% confidence interval for the mean of the population on the assumption that the variance is $V(x) = 80$.

Answer. We have $\bar{x} = 81.2$, $V(x) = 80$, $n = 20$ and a confidence level of $Q = 0.95$. The confidence interval is derived from the following probability statement:

$$P\left(\bar{x} - \beta\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \beta\frac{\sigma}{\sqrt{n}}\right) = Q.$$

The values of β corresponding to $Q = 0.95$ is $\beta = 1.960$. Therfore the confindence interval is

$$\bar{x} \pm \beta\frac{\sigma}{\sqrt{n}} = 81.2 \pm 1.96\frac{\sqrt{80}}{\sqrt{20}} = [77.28, 85.12].$$

2. Let $\bar{x}$ be the mean of a random sample of size n from an $N(\mu, \sigma^2)$ population. What is the probability that the interval $(\bar{x} - 2\sigma/\sqrt{n}, \bar{x} + 2\sigma/\sqrt{n})$ includes the point μ?

Answer. We have

$$P\left(\bar{x} - 2\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + 2\frac{\sigma}{\sqrt{n}}\right) = P\left(-2 \leq z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \leq 2\right),$$

where $z \sim N(0, 1)$ is a standard normal variate. From tables, we find that $P(z \in [0, 2]) = 0.4772$. Therefore $P(-2 \leq z \leq 2) = 0.9544$.

3. The mean of a random sample of size 17 from a normal population is $\bar{x} = 4.17$. Determine the 90 % confidence interval for the population mean when the estimate variance of the population is 5.76.

Answer. We are given $\bar{x} = 4.17$, $\hat{\sigma}^2 = 5.76$ and $n = 17$. Also the level of confidence is $Q = 0.90$. We infer that

$$\frac{\bar{x} - \mu}{\hat{\sigma}/\sqrt{n}} \sim t(16)$$

has a t dustribution of 16 degress of freedom; and, from tables, we find that $P(t \in [-b, b]) = 0.90$ implies that $b = 1.746$. From the probability statement

$$P\left(\bar{x} - b\frac{\hat{\sigma}}{\sqrt{n}} \leq \mu \leq \bar{x} + b\frac{\hat{\sigma}}{\sqrt{n}}\right) = Q$$

we derive a confidence interval of the form

$$\bar{x} \pm b\frac{\hat{\sigma}}{\sqrt{n}} = 4.17 \pm 1.746\frac{\sqrt{5.76}}{\sqrt{17}} = [3.154, 5.186]$$

4. Let $\bar{x}$ be the mean of a random sample of size n from a distribution which is $N(\mu, \sigma^2)$ where $\sigma^2 = 90$. Find n such that $P(\bar{x} - 1 \leq \mu \leq \bar{x} + 1) = 0.9$ approximately.

Answer. If $x \sim N(\mu, \sigma^2)$, then

$$P\left(\bar{x} - \beta\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \beta\frac{\sigma}{\sqrt{n}}\right) = 0.90$$

implies $\beta = 1.645$. With $\sigma^2 = 90$, we have

$$\beta\frac{\sigma}{\sqrt{n}} = 1 = 1.645\frac{\sqrt{90}}{\sqrt{n}},$$

which implies that

$$\sqrt{n} = 1.645\sqrt{90} \quad \text{and therefore} \quad n \simeq 244.$$