EXERCISES IN STATISTICS

Series A, No. 1

1. The number of children recorded to each of 25 families were

$$2, 4, 1, 0, 1, 3, 0, 4, 2, 6, 0, 0, 2, 3, 1, 5, 4, 0, 3, 1, 2, 5, 3, 4, 1.$$

   (a) Construct a frequency table and a graph for the distribution of family size in this sample.

   (b) Find the sample mean and the standard deviation.

Answer:

x_i		f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0	IIIII	5	0	0	0
1	IIIII	5	5	1	5
2	IIII	4	8	4	16
3	IIII	4	12	9	36
4	IIII	4	16	16	64
5	II	2	10	25	50
6	I	1	6	36	36
		25	57		207
		$\sum f_i$	$\sum f_i x_i$		$\sum f_i x_i^2$

We find that

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f i x_i}{N} = \frac{57}{25} = 2.28$$

We can say that the average number of children per family is 2.28. One should take care to state this correctly. For example, it is absurd to say that "The average family has 2.28 children". We also find that

$$\begin{aligned} \text{Variance} &= \frac{\sum f_i (x_i - \bar{x})^2}{N} = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N} \right)^2 \\ &= \frac{207}{25} - \left(\frac{57}{25} \right)^2 = 3.082 \end{aligned}$$

2. Find expressions in terms of the mean and variance of x for
   (a) the mean and variance of $y = ax$,
   (b) the mean and variance of $y = ax + b$.

Answer:

(a)
$$\bar{y} = \frac{\sum y_i}{N} = \frac{\sum ax_i}{N} = a\bar{x}$$
$$s_y^2 = \frac{\sum (y_i - \bar{y})^2}{N} = \frac{\sum (ax_i - a\bar{x})^2}{N}$$
$$= \frac{\sum a^2 (x_i - \bar{x})^2}{N} = a^2 s_x^2.$$

(b)
$$\bar{y} = \frac{\sum y_i}{N} = \frac{\sum (ax_i + b)}{N} = a\bar{x} + b$$
$$s_y^2 = \frac{\sum (y_i - \bar{y})^2}{N} = \frac{\sum \{(ax_i + b) - (a\bar{x} + b)\}^2}{N}$$
$$= \frac{\sum a^2 (x_i - \bar{x})^2}{N} = a^2 s_x^2.$$

3. The Centigrade temperatures of ten capital cities on Monday 6th October were
$$24, 27, 26, 16, 25, 28, 22, 15, 16, 26.$$
The Fahrenheit equivalents of these are
$$75, 81, 79, 61, 77, 82, 72, 59, 61, 79.$$
Find the mean and the variance of these temperatures in both Centigrade and Fahrenheit.

Answer: Let x be a Centigrade Temperature. Then we have

$$\text{Mean of } x = \frac{\sum f_i x_i}{\sum f_i} = \frac{\sum f i x_i}{N} = \frac{225}{10} = 22.5$$

$$\text{Variance of } x = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2$$
$$= 528.7 - (22.5)^2 = 22.4$$

Now consider the conversion from Centigrade to Fahrenheit. We know that $212°F = 100°C$ and that $32°F = 0°C$. (In fact, $100°F$ was supposed to represent blood temperature and $0°F$ was the freezing point of a saturated solution of salt water, which was the lowest temperature which could be obtained in the laboratory at the time of 1714). Therefore, if $y = ax+b$ is the linear conversion from Centigrade to Fahrenheit, we have

$$\begin{array}{rl} 212 = & b + a100 \\ 32 = & b + a0 \\ \hline 180 = & \quad\; a100 \end{array}$$

which implies that $a = 9/5$ and $b = 32$ or $F° = 32 + (9/5)C°$. Therefore

$$\text{Mean } F° = \frac{9}{5}(22.5) + 32 = 72.5$$

and

$$\text{Variance } F° = \left(\frac{9}{5}\right)^2 (22.4) = 72.58$$

4. If the sample space is $S = A_1 \cup A_2$, and if $P(A_1) = 0.8$, $P(A_2) = 0.5$, what is $P(A_1 \cap A_2)$?

Answer: We have $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$. Therefore, since $P(A_1 \cup A_2) = 1$, we get

$$\begin{aligned} P(A_1 \cap A_2) &= P(A_1) + P(A_2) - P(A_1 \cup A_2) \\ &= 0.8 + 0.5 - 1 = 0.3. \end{aligned}$$

5. A man forgets his banker's card 10% of the time, he forgets his cheque book 5% of the time and he forgets both 2% of the time.
   (a) What is the probability that, on any one day, he will have both his banker's card and his cheque book?
   (b) What is the probability that he will find his banker's card in his pockets given that he has already found his cheque book?

Answer:

The probability of forgetting the banker's Card is $P(A) = 0.10$,

The probability of forgetting the Cheque Book is $P(B) = 0.05$,

The probability of forgetting both Book and Card is $P(A \cap B) = 0.02$.

We know that

$$P(A^c \cap B^c) = 1 - P(A^c \cap B^c)^c = 1 - P(A \cup B).$$

Also

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.10 + 0.05 - 0.02 = 0.13. \end{aligned}$$

Therefore

$$P(A^c \cap B^c) = 1.0 - 0.13 = 0.87$$

and

$$\begin{aligned} P(A^c \cap B^c | B^c) &= \frac{P(A^c \cap B^c)}{P(B^c)} \\ &= \frac{0.87}{1 - 0.05} = 0.916. \end{aligned}$$

6. A motor car is placed in the luxury class for the purposes of taxation if its engine has no fewer than six cylinders or a capacity of no less than 3 litres. 15% of all cars have no fewer than six cylinders, and 10% have no less than 3 litres. We know that 80% of all cars of 3 litres and more have at least six cylinders. What proportion of all cars fall in the luxury class?

Answer:

The probability of no fewer than 6 cylinders in $P(C) = 0.15$

The probability of no less than 3 litres is $P(L) = 0.10$

The probability of no fewer than 6 cylinders given 3 litres or more is $P(C|L) = 0.80$

It follows that the probability of a car being in the luxury class is

$$\begin{aligned} P(L \cup C) &= P(L) + P(C) - P(L \cap C) \\ &= P(L) + P(C) - P(C|L)P(L) \\ &= 0.10 + 0.25 - (0.80 \times 0.10) = 0.17 \end{aligned}$$