SUPPLEMENT 1

The Dynamic Consumption Function

The Keynesian Multiplier

Consider the equations

$$\begin{aligned} Y &= C + I, \\ C &= a + bY, \end{aligned} \tag{1}$$

where Y is output, C is consumption and I is investment spending. Combining the equations gives $Y = a + bY + I = bY + A$, where $A = a + I$, which may be called autonomous expenditure. Solving for Y gives

$$Y = \frac{A}{1-b} = A(1 + b + b^2 + \cdots). \tag{2}$$

To demonstrate the final equality, let $S = (1 + b + b^2 + \cdots)$ so that $bS = (b + b^2 + b^3 + \cdots)$. Then $S - bS = 1$ so $S = 1/(1-b)$. S is called the Keynesian multiplier.

Suppose that autonomous expenditure A increases by ΔA and then maintains its level. We can imagine that output increases by ΔA in the first period, by $b\Delta A$ in the next period, by $b^2\Delta A$ in the third period and so on. At length, output will have increased by $\Delta Y = \Delta A(1 + b + b^2 + \cdots) = \Delta A/(1-b)$. To represent these circumstances adequately, we need to introduce a time period into the model. We may replace the equations of (1) by the following:

$$\begin{aligned} Y(t) &= C(t) + I(t), \\ C(t) &= a + bY(t-1). \end{aligned} \tag{3}$$

Here $Y(t) = \{Y_t; t = 0, \pm 1, \pm 2, \ldots\}$, $C(t)$ and $I(t)$ are sequences indexed by time. Consumption depends upon $Y(t-1)$ which is the income or output of the previous period. On defining $A(t) = I(t) + a$, we may write

$$(4)\qquad \begin{aligned} Y(t) &= A(t) + bY(t-1) \\ &= A(t) + bA(t-1) + b^2Y(t-2) \\ &\ \vdots \\ &= \big\{A(t) + bA(t-1) + b^2A(t-2) + \cdots\big\}. \end{aligned}$$

Let L denote the lag operator whose effect is that $LY(t) = Y(t-1)$, $L^2Y(t) = LY(t-1) = LY(t-2)$, etc. Then we may write

$$(5)\qquad \begin{aligned} Y(t) &= \big\{A(t) + bA(t-1) + b^2A(t-2) + \cdots\big\} \\ &= \big\{1 + bL + b^2L^2 + \cdots\big\}A(t) \\ &= \frac{1}{1-bL}A(t). \end{aligned}$$

Imagine that, in place of $A(t)$, the input sequence is a unit impulse

$$\{0, \ldots, 0, 1, 0, 0, \ldots\}.$$

Then the output sequence would be $\{0, \ldots, 0, 1, b, b^2, \ldots\}$ which is the so-called impulse-response function. If the input sequence were a step function of the form $\{0, \ldots, 0, 1, 1, 1, \ldots\}$, then the output sequence would be the so-called step-response function $\{0, \ldots, 0, 1, b, b + b^2, \ldots\}$ which consists of the partial sums of the elements in the impulse-response function.

A Dynamic Consumption Function

Keynes supposed that, as real income increases, an increasing proportion is bound to be saved. In fact, in the U.K. and elsewhere, a more or less constant proportion of the growing GNP has been saved in postwar years. Therefore, in the long run, the consumption function has the form $C^* = bY$ rather than $C = a + bY$. Nevertheless, in the short run, a relationship similar to the one which Keynes postulated seems to prevail.

We can attempt to reconcile these features by postulating a dynamic consumption function in the form of

$$(6)\qquad C(t) = (1-\lambda)C^*(t) + \lambda C(t-1),$$

where $C^*(t) = bY(t)$. Substituting for $C^*(t)$ gives

$$(7)\qquad \begin{aligned} C(t) &= (1-\lambda)bY(t) + \lambda C(t-1) \\ &= (1-\lambda)bY(t) + \lambda LC(t), \end{aligned}$$

for which the solution is

$$
\begin{aligned}
C(t) &= \frac{(1-\lambda)b}{(1-\lambda L)}Y(t) \\
&= (1-\lambda)b\big\{Y(t) + \lambda Y(t-1) + \lambda^2 Y(t-2) + \cdots\big\}.
\end{aligned} \tag{8}
$$

The impulse response and the step response of consumption to changes in income now have the same forms as the corresponding responses of income to autonomous expenditure which are to be found in the previous model. If income increases suddenly, then consumption will respond, initially, by making only a partial adjustment. In time, the full adjustment will be accomplished.