

OPTIMAL HEDGE RATIO

If an asset has been hedged, then the movements in its spot price and in the accompanying short hedge should constitute compensating variations. The optimum size of the hedge will be a function of the variances of the spot price and the futures price and of the covariance of the two.

Let $\nabla S = S_{t_2} - S_{t_1}$ denote the change in the spot price between times t_1 and t_2 , and let $\nabla F = F_{\tau|t_2} - F_{\tau|t_1}$ be the change in the futures price. Also, let h denote the hedge ratio, which is the ratio of the value of the futures contract at time t to the value of the asset in question. The change in the value of the hedger's position between time t_1 and t_2 is

$$\nabla S - h\nabla F.$$

We may denote the variance of ∇S by σ_S^2 and the variance of ∇F by σ_F^2 . Then, the variance of the hedger's position is

$$\nu = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F,$$

where ρ is the correlation between ∇S and ∇F and where, consequently, $\rho\sigma_S^2\sigma_F^2$ is the covariance of ∇S and ∇F .

The value of h which minimises ν is

$$h = \rho \frac{\sigma_S}{\sigma_F}.$$