FORWARD CONTRACTS

In a forward contract, a party agrees to buy or sell an asset at a given price at a future date τ. The party that agrees to buy the asset, is taking a *long* position. The party that is selling is taking a *short* position. The *spot price* S_τ is the price in the open market of the asset of time τ. The delivery price K_τ is the price agree in the forward contract for the transaction that is to be enacted at time τ, when the money is paid and the delivery is taken.

The forward price $F_{\tau|t}$ is the price prevailing at time t for a delivery that is scheduled for time τ. When the contract is written, the delivery price is the prevailing forward. We denote this by writing $K_\tau = F_{\tau|t}$. After time t, the forward price and the delivery price may diverge.

The returns to the party taking the short position in the forward contract is

$$K_\tau - S_\tau.$$

The party taking the short position is paid a sum of K_τ at time τ and they relinquish an asset that is valued at S_τ on the open market at that time. The returns to the party taking the long position in the forward contract is

$$S_\tau - K_\tau.$$

They must pay K_τ for the delivery of an asset that is valued at S_τ at the time.

The party taking the short position hopes that the returns will be positive and that they will be sufficient to compensate for the opportunity cost of tying up their capital by holding the asset until time τ. This opportunity cost is the return R that they would obtain from the best riskless alternative investment of their capital. They will gain from the transition only if

$$K_\tau - S_\tau > R.$$

In that case, what they gain from the contract will be is greater than what they could have gained from an alternative investment.

The gains of the short party are the losses of the long party. The latter will gain from the contract only if

$$R > K_\tau - S_\tau,$$

which is when the returns from a safe investment, which is what we presume that they have received up to time τ, exceeds the discrepancy between the spot price of the asset and what they have contracted to pay for it.

The delivery price K_τ will be the sum of two components. The first is the value $E(S_\tau)$ of the spot price that is expected to prevail at time τ, the expectation being formed at the time $t = 0$ when the the contract is written. The second component is to compensate the party taking the short position, i.e. the seller of the asset, for the loss of investment income R occasioned by holding the asset. In this case, we assume that is no return from holding the asset and that there are no associated costs of storage. Then

$$K_\tau = E(S_\tau) + R.$$

Assume that the contract is to mature in one year's time. If the current price of the asset is S_0 and if the annual rate of return on a riskless financial investment is r, then $R = rS_0$. Also, the current price provides the best estimate of the spot price in one year's time, so there is $E(S_\tau) = S_0$. Substituting these into the previous equation gives

$$K\tau = (1 + r)S_0.$$

Example. Suppose that the price of gold today is $S_0 = \$300$ and that the annual rate of return of a riskless bond is 5%. Imagine that the one-year-foreward price of gold is $K_\tau = \$340$. In that case, I could take a short position by borrowing \$300 to buy the gold.

If I contracted to sell the gold in one years time for $K_\tau = \$340$, my return would be $K_\tau - S_0 = \$40$. The interest payment for the loan over a period of one year is $R = \$300 \times 0.015 = \$15$. Therefore, I would have a guaranteed overall profit of $\$40 - \$15 = \$25$.

It is unlikely that such an opportunity would persist for long. The demand to exploit it would drive the contract price K_τ to the level of \$315, where there would be no remaining profit from this strategy. This is the case when

$$K_\tau - S_0 = R$$

This condition serves to determine the delivery price K_τ.

In these circumstances, the short position will deliver a profit if $S_\tau < S_0$. For, in that case, $K_\tau - S_\tau > R$, which is to say that by holding a short position, I should derive more that I would have done had I invested my capital in a risk-free bond.

D.S.G. Pollock: stephen_pollock@sigmapi.u-net.com