Inequality and Redistribution When Voters Have ‘Other Regarding Preferences’

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15 November 2010

Abstract

The celebrated relation between inequality and redistribution is based on selfish voters who care solely about their own-payoffs. A growing empirical literature highlights the importance of other regarding preferences (ORP) in voting over redistribution. In a simple general equilibrium model, with endogenous labor supply, we reexamine the relation between inequality and redistribution when voters have ORP. Our main contributions are three-fold. First, greater fairness leads to greater redistribution. In particular, we demonstrate that poverty can lead to increased redistribution, which implies a countercyclical social spending to GDP ratio, as observed. Second, we introduce the concept of ‘strong median dominance’ and show that disposable income in the political equilibrium ‘strongly median-dominates’ factor income. Furthermore, we show that fair voters respond to an increase in ‘strong median-dominance’ by engaging in greater redistribution. Third, we show that our framework has the potential to clarify the vexed question of the relationship between inequality and redistribution.

Keywords: Redistribution; other regarding preferences; income inequality; difference dominance; median dominance.

JEL Classification: D64 (Altruism); D72 (Economic Models of Political Processes: Rent-Seeking, Elections, Legislatures, and Voting Behavior); D78 (Positive Analysis of Policy-Making and Implementation).

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“The Americans... are fond of explaining almost all the actions of their lives by the principle of interest... In this respect I think they frequently fail to do themselves justice; for in the United States, as well as elsewhere, people are sometimes seen to give way to those disinterested and spontaneous impulses which are natural to man; but the Americans seldom allow that they yield to emotions of this kind; they are more anxious to do honor to their philosophy than to themselves.” Alexis de Tocqueville, 1985, from “Democracy In America”, Volume 2, Chapter VIII.

1. Introduction

Traditional economic theory largely assumes self-interested behavior, i.e., individuals are interested only in their own pecuniary payoffs (selfish preferences). Most current models of income redistribution trace their lineage to a simple general equilibrium model with endogenous labour supply as in Romer (1975), Roberts (1977), and Meltzer and Richard (1981), which we will, henceforth, call the RRMR model. In the RRMR model, redistributive policy is chosen directly by a majority vote (direct democracy), a political rule that is central to public economics and is of growing practical importance; see Matsusaka (2005a,b). We shall refer to the RRMR model when voters have selfish preferences as the selfish voter model.

The assumption of selfish preferences (a key ingredient in the RRMR model) fails in a range of phenomena from many diverse areas in economics such as collective action, contract theory, the structure of incentives, political economy and the results of several experimental games. Many of these phenomena can be accounted for by models that recognize that individuals have social preferences or other regarding preferences (ORP) that incorporate both altruism and envy. These models are being rapidly employed in almost all areas within economics. For a survey of models of ORP, its growing neuroeconomic foundations, applications and the empirical results, see Fehr and Fischbacher (2002), and Camerer and Fehr (2007).

Public redistribution is hugely important in terms of its actual magnitude. Furthermore, the welfare consequences of alternative levels of public redistribution are profound. Unlike the selfish voter model, which is the main model in the literature, it is entirely plausible that the human desire for fairness, as encapsulated in a variety of models of ORP, underpins observed public redistribution. We shall refer to the RRMR model in which selfish preferences are replaced by ORP as the fair voter model.

1.1. Social Preferences and Voting

In voting contexts, the emerging empirical literature is strongly supportive of the role of ORP; see, for instance, Ackert et al. (2007), Bolton and Ockenfels (2006) and Tyran
and Sausgruber (2006). These papers establish that voters often choose policies that promote equity/fairness over purely selfish considerations. Bolton and Ockenfels (2006), for instance, examine the preference for equity versus efficiency in a voting game. Groups of three subjects were presented with two alternative policies: one that promotes equity while the other promotes efficiency. The final outcome was chosen by a majority vote. About twice as many experimental subjects preferred equity as compared to efficiency. Furthermore, even those willing to change the status-quo for efficiency are willing to pay, on average, less than half relative to those who wish to alter the status-quo for equity.

1.2. Which model of social preferences is appropriate in a voting context?

A careful consideration, that we justify below, leads us to propose the Fehr-Schmidt (1999) model of ORP (henceforth, FS) applied to voting over redistribution. In this approach, voters care about their own payoffs (as in models of selfish preferences) but also derive disutility from the payoffs of voters lower than their own payoffs (advantageous-inequity arising from, say, altruism) or greater than their own payoffs (disadvantageous-inequity arising from, say, envy).

The FS model is tractable and explains the experimental results arising from several games where the prediction of standard models with selfish preferences yields results that are not consistent with the experimental evidence. These games include the ultimatum game, the gift-exchange game, the dictator game as well as the public-good game with and without punishment. Furthermore, Neilson (2006) provides an axiomatization of FS-like preferences. A possible objection to the FS model is that it ignores the role played by intentions, as highlighted in the work of Rabin (1993) and others. However, intentions seem more important in bilateral or small group interactions. Economy-wide voting, because it is impersonal and anonymous, is unlikely to be influenced by intentions.

Tyran and Sausgruber (2006) explicitly test for the importance of the FS framework when voters directly choose redistributive policy through majority voting. They conclude that the FS model predicts much better than the standard selfish-voter model and provides, in their words, “strikingly accurate predictions for individual voting in all three income classes.” The econometric results of Ackert et al. (2007), based on their experimental data, lend further support to the FS model in the context of redistributive taxation. The estimated coefficients of altruism and envy in the FS model are statistically significant and, as expected, negative in sign. Social preferences are found to influence participant’s vote over alternative taxes. They find evidence that some participants are willing to reduce their own payoffs in order to support taxes that reduce advantageous or disadvantageous inequity. In the context of voting experiments, Bolton and Ockenfels (2006) conclude that “... while not everyone measures fairness the same way, the simple measures offered
by... the Fehr-Schmidt (1999) model provide a pretty good approximation to population behavior over a wide range of scenarios that economists care about.”

1.3. Inequality and Redistribution in the selfish voter model

An unresolved, fundamental question of great practical interest in economics is the relation between inequality and government redistribution. Do we, for instance, expect redistribution to be higher or lower in societies with more unequal incomes? Meltzer and Richard (1981) provide the leading prediction in the case of the selfish voter model. They show that the extent of redistribution varies directly with the ratio between mean and median income. The intuition is that as inequality increases, the median voter (whose income is below the average), becomes relatively more poor and, hence, chooses greater redistribution. However, the evidence is largely mixed. Positive support is found by Meltzer and Richard (1981), Easterly and Rebelo (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), and Milanovic (2000). However, Lindert (1996) and Perotti (1996) do not find any support.

Consider, for instance, the comparison between Sweden and the USA. Disposable income inequality in Sweden is about 60 percent that of the USA, while the Swedish social spending to GDP ratio is about twice that of the USA.¹ These examples abound and are clearly at variance with the predictions of the RRMR model. Some attempts have been made to address these issues in a selfish voter model in Alesina and Angeletos (2005) and Benabou (2000), among others. We revisit this issue in section 4.2 below.

1.4. Inequality and redistribution in a fair voter model

Suppose that one replaced voters with selfish preferences in the RRMR model with voters who have ORP. In this fair voter model, a first question is whether a Condorcet winner will exist? Recent research, by Dhami and al-Nowaihi (2010a), shows that the existence of a Condorcet winner can be shown under fairly plausible conditions.

One can then ask at least three further, and fundamental, questions that are the focus of this paper. (1) What is the effect of poverty on redistribution in a fair voter model? (2) Are existing inequality measures used in a selfish voter model, such as first and second order stochastic dominance, appropriate in a fair voter model? If not, then what new inequality concept do we need? (3) What are the implications of the fair voter model for the classical relation between inequality and redistribution? In the next three subsections we deal with these three questions.

¹The figures on disposable income inequality can be obtained from the ‘Luxembourg Income Study’ (LIS) or from various World Bank publications. The ratio of social spending to GDP can be obtained from various OECD publications.
1.4.1. Poverty and redistribution

In the standard selfish voter model, an increase in poverty reduces redistribution. The reason is that an increase in poverty reduces average incomes, so making it less worthwhile for the median selfish voter to increase the redistributive tax rate. Insofar as periods of increased poverty are also associated with unemployment shocks, the prediction is that the ratio of social spending to GDP is pro-cyclic. However, we show that when voters have ORP, poverty can lead to increased redistribution, which implies a countercyclical social spending to GDP ratio, as observed. The intuition is that if the inequity aversion of the fair median voter is high enough, then he/she suffers a utility loss from increased poverty. An increase in the redistributive tax rate, under these circumstances, reduces post-tax poverty and makes the median voter better off.

1.4.2. Appropriate notion of income inequality

We need a formal definition of what does it mean to say that one income distribution is more unequal than another when we have a discrete set of incomes $y_1 < y_2 < \ldots < y_n$. It will become apparent, below, that no probability distribution function over these income levels is proposed. This is a well known approach, which we call the first approach to income inequality. However, existing inequality measures in the first approach presuppose selfish preferences. In order to incorporate a preference for fairness, within the first approach, we have introduced two new measures, which we call median-dominance and strong median dominance; see Definition 1, below. Our concept is closest to the concept of difference-dominance due to Marshall and Olkin (1979), see Remark 3, below. We show that fair voters respond to an increase in strong median-dominance by engaging in greater redistribution. Median-dominance may prove to be a useful concept in other applications of ORP as well.

We briefly comment on the second approach to income inequality in subsection 4.1 that is associated with inequality concepts such as first and second order stochastic dominance (SOSD), it suffices to make two comments here. First, in the absence of a probability distribution over income levels it does not even make sense to talk of SOSD. Second, even if one were to have a probability distribution over income levels, SOSD is inadequate if individuals were to have fair preferences.

1.4.3. Implications of the fair voter model for redistribution

Failing to control for fairness might have contributed to the mixed results between income inequality and redistribution. Indeed, a prediction of our model is that, controlling for fairness, higher inequality leads to greater distribution but that not controlling for fairness could lead to mixed and possibly contradictory empirical results.
1.5. A comparison of our paper with related literature on voting and ORP

The aim of the important paper by Tyran and Sausgruber (2006) is to demonstrate that fairness considerations are important in a voting context. Their concern is not, however, with an analysis of the relation between inequality and redistribution, or with a general equilibrium framework. Furthermore, they consider a more restricted tax policy choice than us. While we consider, as in the RRMR framework, changes in a linear progressive income tax that affect all they only consider redistribution from the rich to the poor that leaves the middle income voters unaffected. They do introduce a cost of such redistribution to the middle income voters, but it is not an integral part of the redistributive fiscal package considered.

Galasso (2003) allows for fairness concerns in the RRMR model. An important insight of the FS framework, that is consistent with results from voting experiments, is to show that altruism and envy (along with self interest) are important. However, Galasso only focusses on altruism. Furthermore, Galasso’s (2003) notion of altruism is of a specific form that is inconsistent with accepted models of ORP. In particular, fair voters care about their own payoffs but suffer disutility through a term that is linear in their payoffs relative to the worse off voter in society. Within this framework, there is greater redistribution when there is a mean preserving spread in inequality. However, this leaves open the relation between inequality and redistribution in a standard model of fairness, such as the FS model. For the FS model, we find that a mean preserving spread is not even the appropriate notion of inequality. In this sense, the machinery developed in Galasso (2003) does not carry over to our paper.

While Dhami and al-Nowaihi (2010a) prove the existence of a Condorcet winner in a fair voter model, Dhami and al-Nowaihi (2010b) ask what is the effect on redistribution in this model when there are different mixtures of selfish and fair voters? They focus on a three-classes model: poor, middle-class, and rich. However, none of these papers explore the classical problem of the relation between inequality and size of the government.

1.6. Schematic outline

The plan of the paper is as follows. Section 2 describes the model. Section 3 gives the comparative static results. In Section 4 we develop our concept of median dominance and

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2The latter term captures some notion of social justice. Others have included such a term to incorporate social justice, e.g., Charness and Rabin (2002). However, they posit preferences, different from Galasso (2003), that are a convex combination of the total payoff of the group (this subsumes selfishness, in so far as one’s own payoff is part of the total, and altruism) and a Rawlsian social welfare function. These sorts of models are able to explain positive levels of giving in dictator games, and reciprocity in trust and gift exchange games. However, they are not able to explain situations where an individual tries to punish others in the group at some personal cost, for instance, punishment in public good games.
explore the relation between changes in income distribution and the extent of redistribution. Some brief conclusions are given in Section 5. Proofs are in section 6.

2. Model

We now embed voters with fair preferences in the RRMR model to formulate the fair voter model. In this section, the basic model that we describe is the one used in Dhami and al-Nowaihi (2010a,b).

Let there be \( n = 2m - 1 \) voter-worker-consumers (henceforth, voters). Let the skill level of voter \( j \) be \( s_j, j = 1, 2, ..., n \), where

\[
0 < s_i < s_j < 1, \text{ for } i < j, \quad (2.1)
\]

Hence, \( s_m \) is the median skill level. Denote the skill vector by \( s = (s_1, s_2, ..., s_n) \). Each voter has a fixed time endowment of one unit and supplies \( l_j \) units of labor and so enjoys \( 1 - l_j \) units of leisure, where

\[
0 \leq l_j \leq 1. \quad (2.2)
\]

Labour markets are competitive and each firm has access to a linear production technology such that production equals \( s_j l_j \). Hence, the wage rate offered to each voter coincides with the marginal product, i.e., the skill level, \( s_j \). Thus, the before-tax income of voter \( j \) is given by

\[
y_j = s_j l_j. \quad (2.3)
\]

Note that ‘skill’ here need not represent any intrinsic talent, just ability to translate labour effort into income. Let the average before-tax income be

\[
\bar{y} = \frac{1}{n} \sum_{j=1}^{n} y_j. \quad (2.4)
\]

We make the empirically plausible assumption that the income of the median-skill voter, \( y_m \), is less than the average income,\(^3\)

\[
y_m < \bar{y}. \quad (2.5)
\]

The government operates a linear progressive income tax that is characterized by a constant marginal tax rate, \( t, t \in [0, 1] \), and a uniform transfer, \( b \), to each voter that equals the average tax proceeds,

\[
b = t\bar{y}. \quad (2.6)
\]

\(^3\)The assumption that \( y_m < \bar{y} \) is needed for Propositions 4, 5, 6, 7 and 8 but not for Propositions 1, 2 or 3. The necessary and sufficient condition on exogenous parameters for \( y_m < \bar{y} \) to be true is given by (2.28) below.
Hence, the tax rate is also the ratio of social spending to aggregate income,

\[ t = \frac{nb}{\sum_{i=1}^{n} y_i}. \]  

(2.7)

**Remark 1**: The net tax collected from an individual with income \( y \) is \( ty - b \), hence, the average tax paid \( (t - (b/y)) \) is increasing in the level of income. This is the sense in which the tax system is progressive in the RRMR model. From (2.7), changes in the tax rate can equivalently be viewed as changes in the ratio of social spending to aggregate income.

The budget constraint of voter \( j \) is given by \( 0 = c_{j} + b \) which, in view of (2.3), can be written as

\[ 0 \leq c_{j} \leq (1 - t) y_{j} + b. \]  

(2.8)

### 2.1. Preferences of Voters

We define a voter’s preferences in two stages. First, let voter \( j \) have a continuous own-utility function, \( \bar{u}_{j}(c_{j}, 1 - l_{j}) \), defined over own-consumption, \( c_{j} \), and own-leisure, \( 1 - l_{j} \).

Second, and for the reasons stated in the introduction, voters have other-regarding preferences as in Fehr-Schmidt (1999). Let \( c_{-j} \) and \( l_{-j} \) be the vectors of consumption and labour supplies, respectively, of voters other than voter \( j \). Under Fehr-Schmidt preferences the FS-utility of voter \( j \) is

\[
\tilde{U}_j (c_j, l_j; c_{-j}, l_{-j}, t, b, \alpha, \beta, s) = \bar{u}_j (c_j, 1 - l_j; t, b, s_j)
- \frac{\alpha}{n - 1} \sum_{k \neq j} \max \{0, \bar{u}_k (c_k, 1 - l_k; t, b, s_k) - \bar{u}_j (c_j, 1 - l_j; t, b, s_j)\}
- \frac{\beta}{n - 1} \sum_{i \neq j} \max \{0, \bar{u}_j (c_j, 1 - l_j; t, b, s_j) - \bar{u}_i (c_i, 1 - l_i; t, b, s_i)\},
\]

(2.9)

where

for selfish voters \( \alpha = \beta = 0 \), so \( \tilde{U}_j = \bar{u}_j \)

(2.10)

for fair voters \( \alpha > 0 \) and \( 0 < \beta < 1 \), so \( \tilde{U}_j \neq \bar{u}_j \).

(2.11)

Thus, \( \bar{u}_j \) is also the utility function of a selfish voter, as in the standard textbook model. The RRMR is a special case of our model when \( \alpha = \beta = 0 \). However, throughout, we make the assumption that \( \alpha > 0 \) and \( 0 < \beta < 1 \).

From (2.9), the fair voter cares about own payoff (first term), payoff relative to those where inequality is disadvantageous (second term) and payoff relative to those where inequality is advantageous (third term). The second and third terms which capture respectively, envy and altruism, are normalized by the term \( n - 1 \). Notice that in FS preferences, inequality is self-centered, i.e., the individual uses her own payoff as a reference point with
which everyone else is compared to. From (2.11), \( \beta \) is bounded below by 0 and above by 1: \( \beta > 1 \) would imply that an individual could increase utility by simply destroying all his/her wealth; this is counterfactual.

We take utility differences rather than differences in monetary payoffs as the source of envy and altruism. It is well known that while the Fehr and Schmidt (1999) model is presented in terms of differences in monetary payoffs (the linear version), the model also bears interpretation in terms of utility differences (the non-linear version). Hence, one of the contributions of our model is also to show how the non-linear inequity aversion model can deliver tractable results.

We make the standard assumption that \( \frac{\partial u_j}{\partial c_j} > 0 \). Since \( 0 < \alpha < 1 \), it follows, from (2.9), that \( \tilde{U}_j(c_j, l_j; c_{-j}, l_{-j}, t, b, \alpha, \beta, s) \) is also a strictly increasing function of \( c_j \). Hence, the budget constraint (2.8) holds with equality:

\[
c_j = (1 - t) s_j l_j + b, \tag{2.12}
\]

or, in the light of (2.3),

\[
c_j = (1 - t) y_j + b. \tag{2.13}
\]

From (2.4), (2.6) and (2.13) we get

\[
\sum_{i=1}^{n} c_i = \sum_{i=1}^{n} y_i. \tag{2.14}
\]

Thus, total pretax income is equal to total post tax income. This is simply a consequence of the fact that all the tax revenue is returned to consumers. Substituting for \( c_j \) from (2.12) into the utility function, \( \tilde{u}_j(c_j, 1 - l_j) \), gives the following form for utility

\[
u_j(l_j; t, b, s_j) = \tilde{u}_j(\alpha (1 - t) s_j l_j + b, 1 - l_j) \tag{2.15}
\]

Correspondingly, the FS-utilities take the form

\[
U_j(l_j; l_{-j}, t, b, \alpha, \beta, s) = u_j(l_j; t, b, s_j) - \frac{\alpha}{n - 1} \sum_{k \neq j} \max \{ 0, u_k(l_k; t, b, s_k) - u_j(l_j; t, b, s_j) \} - \frac{\beta}{n - 1} \sum_{i \neq j} \max \{ 0, u_j(l_j; t, b, s_j) - u_i(l_i; t, b, s_i) \}, \tag{2.16}
\]

\footnote{As Fehr et al. (2009, p.11) clearly write "For example, if some subjects have preferences characterized by nonlinear inequality aversion they do not perfectly equalize monetary payoffs but they still move in the direction of more equality. We explicitly abstained from using this more general model because it is less tractable ..." However, this less tractable model can explain some findings that the linear model (with monetary payoff differences only) cannot explain; this is noted explicitly in Fehr et al (2006). Neilson (2006) gives an axiomatization of a non-linear version of the Fehr-Schmidt (1999) model.}
Remark 2: (Weighted utilitarian preferences) First define the sets $A_j$ and $D_j$ as the set of voters with respect to whom voter $j$ has respectively, advantageous and disadvantageous inequity. So

$$A_j = \{ i : i \neq j \text{ and } u_i (l_i; t, b, s_i) \leq u_j (l_j; t, b, s_j) \} ,$$

$$D_j = \{ k : k \neq j \text{ and } u_k (l_k; t, b, s_k) > u_j (l_j; t, b, s_j) \} .$$

Denote the respective cardinalities of these sets by $|A_j|$ and $|D_j|$. Then FS-utility (2.16) can be written in a way that is reminiscent of the weighted utilitarian form:

$$U_j (l_j; l_{-j}, t, b, \alpha, \beta, s) = \omega_{jj} u_j (l_j; t, b, s_j) + \sum_{i \neq j} \omega_{ji} u_i (l_i; t, b, s_i) ,$$

where

$$i \in A_j \Rightarrow \omega_{ji} = \frac{\beta}{n-1} > 0,$$

$$i = j \Rightarrow \omega_{jj} = 1 - \frac{|A_j| \beta}{n-1} + \frac{|D_j| \alpha}{n-1} > 0,$$

$$k \in D_j \Rightarrow \omega_{jk} = -\frac{\alpha}{n-1} < 0.$$ 

Furthermore, the weights sum up to one i.e.

$$\sum_{i=1}^{n} \omega_{ji} = 1.$$

In particular, for selfish voters (2.10) and (2.20) give

If voter $j$ is selfish, then $\omega_{jj} = 1$ and $\omega_{ji} = 0 \ (i \neq j).$ 

2.2. Sequence of moves

We consider a two-stage game. In the first stage, all voters vote directly and sincerely on the redistributive tax rate. Should a median voter equilibrium exist, then the tax rate preferred by the median voter is implemented. In the second stage, all voters make their labour supply decision, conditional on the tax rate chosen by the median voter in the first stage. On choosing their labour supplies in the second stage, the announced first period tax rate is implemented and transfers made according to (2.6).

In the second stage, the voters play a one-shot non-cooperative Nash game. Each voter, $j$, chooses his/her labour supply, $l_j$, given the vector, $l_{-j}$, of labour supplies of the other voters, so as to maximize his/her FS-utility (2.16). In the first stage, each voter votes for his/her preferred tax rate, correctly anticipating second stage play.

The solution is by backward induction. We first solve for the (second stage problem) of the Nash equilibrium in labour supply decisions of voters, conditional on the announced tax rates and transfers (which are determined in the first stage). The second stage decision is then fed into the first stage FS-utilities to arrive at the indirect utilities of voters, which are purely in terms of the tax rate. Voters then choose their most desired tax rates which maximize their indirect FS-utilities, with the proposal of the median voters being the one that is implemented.
2.3. Labour supply decision of taxpayers (second stage problem)

Given the tax rate, $t$, and the transfer, $b$, both determined in the first stage (see Section 2.5, below), the voters play a one-shot Nash game (in the subgame determined by $t$ and $b$). Each voter, $j$, chooses own labour supply, $l_j$, so as to maximize his/her FS-utility (2.16), given the labour supplies, $l_{-j}$, of all other voters.

Since in (2.19), $u_i(l_i; t, b, s_i)$, $i \neq j$, enter additively, and $\omega_{jj} > 0$, it follows that maximizing the FS-utility, $U_j(l_j; l_{-j}, t, b, \alpha, \beta, s)$, with respect to $l_j$, given $l_{-j}$, $t$, $b$ and $s$, is equivalent to maximizing own-utility, $u_j(l_j; t, b, s_j)$, with respect to $l_j$, given $t$, $b$ and $s_j$. We summarize this in the following proposition, which is an immediate consequence of Proposition 1 in Dhami and al-Nowaihi (2010a).

**Proposition 1**: In the second stage of the game, voter $j$, whether fair or selfish, chooses own labour supply, $l_j$, so as to maximize own-utility, $u_j(l_j; t, b, s_j)$, given $t$, $b$ and $s_j$.

In general, one’s intuition might suggest that fairness concerns should affect labour supply. One could believe that fair high-skill workers might reduce their labour supply (compared to selfish workers) to become relatively less rich. And, on the other hand, fair low-skill workers might put in an extra effort (again, compared to selfish workers) to reduce the utility gap with higher-skill workers. That this intuition is incorrect, is established by Proposition 1.

In particular, a fair voter, despite having social preferences, chooses labour supply exactly like a selfish voter who does not have social preferences. However, when making a decision on the redistributive tax rate, the same fair voter uses social preferences to choose the tax rate in a manner that the selfish voter does not. In other words, in two separate domains, labour supply and redistributive voting choice, the fair voter behaves as if he had selfish preferences in the first domain and social preferences in the second. We emphasize ‘as if’ because, of course, the voter has identical underlying social preferences in both domains.

This opens up yet another dimension to the literature on inconsistency of preferences (or context dependent preferences). For example, individuals, when making a private consumption decision might act so as to maximize their selfish interest. But in a separate role as part of the government, as a school governor or as a voter, could act so as to maximize some notion of public well being. Individuals might, for instance, send their own children to private schools (self interest) but could at the same time vote for more funding to government run schools in local or national elections (public interest). Thus, individuals can put on different hats in different situations. Further research might show that some, or all, of these might be explained using FS-preferences.

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5 Proposition 1 would not be true if the $u_i$, $i \neq j$, entered non-additively into the FS-utility function. However, the empirical evidence strongly supports the adopted form for the FS-utility function.
2.4. Simplifying assumptions

To derive the comparative static results of sections 3 and 4 we need a specific functional form for the utility function, $\tilde{u}_i$, of voter $i$. We adopt a functional form that is standard in the literature. In common with the literature, we assume that all voters have the ‘same own-utility function’, $\tilde{u}$, although, of course, their realized utility will depend on their realized consumption, $c_i$, and their realized leisure, $1 - l_i$. Thus

$$\tilde{u}_i(c_i, 1 - l_i) = \tilde{u}(c_i, 1 - l_i).$$ (2.23)

Furthermore, we assume that the own-utility function is quasi-linear, with constant elasticity of labour supply, which is the most commonly used functional form in various applications of the median voter theorems.

$$\tilde{u}(c, 1 - l) = c - \frac{\epsilon}{1 + \epsilon} l_{\epsilon}^{1+\epsilon},$$ (2.24)

where $\epsilon$ is the constant elasticity of labour supply, assumed positive. The case $\epsilon = 1$ has special significance in the literature. Meltzer and Richard (1981) use (2.24) with $\epsilon = 1$ to derive the celebrated result that the extent of redistribution varies directly with the ratio of the mean to median income. Piketty (1995) restricts preferences to the quasi-linear case with disutility of labour given by the quadratic form, (2.24) with $\epsilon = 1$. Benabou and Ok (2001) do not actually consider a production side and their model has exogenously given endowments which evolve stochastically. Benabou (2000) considers the additively separable case with log consumption and disutility of labor given by the constant elasticity case, (2.24).

Substituting $c_j = (1 - t) s_j l_j + b$ in (2.24), the own-utility function of voter $j$, we get

$$u_j (l_j; t, b, s_j) = u (l_j; t, b, s_j) = (1 - t) s_j l_j + b - \frac{\epsilon}{1 + \epsilon} l_j^{1+\epsilon}. \hspace{1cm} (2.25)$$

We list, in lemmas 1, 2, below, some useful results.

**Lemma 1** (Labour supply): Given $t, b$ and $s_j$, the unique labour supply for voter $j$, $l_j = l (t, b, s_j)$, that maximizes utility (2.25), is given by

$$l_j = l(t, b, s_j) = (1 - t)^\epsilon s_j^\epsilon,$$

and is independent of $b$ due to quasi-linear preferences.

---

6 A large number of studies suggest positive labour supply elasticities (see, for example, Pencavel (1986) and Killingworth and Heckman (1986)). Negative labour supply elasticities may be due to estimating misspecified models (see Camerer and Loewenstein (2004), Chapter 1, ‘Labor Economics’, pp33-34).
Substituting labour supply, \( l(t, b, s_j) \), given by Lemma 1, into (2.3), gives the before-tax income:

\[
y_j(t, b, s_j) = (1 - t)^\epsilon s_j^{1+\epsilon}. \tag{2.26}
\]

Define \( \overline{S} \) to be the ‘weighted average of skills’ in the following sense.

\[
\overline{S} = \frac{1}{n} \sum_{i=1}^{n} s_i^{1+\epsilon} \tag{2.27}
\]

From (2.4), (2.5), (2.26) and (2.27) we get that for the median skill level, \( s_m \),

\[
s_m^{1+\epsilon} < \overline{S}. \tag{2.28}
\]

Conversely, (2.28) implies (2.5). Substituting labour supply in (2.6) we get,

\[
b(t, s) = t (1 - t)^\epsilon \overline{S}. \tag{2.29}
\]

Substituting labour supply in (2.25) we get the indirect utility function corresponding to the own-utility of voter \( j \):

\[
v(t, b, s_j) = u(l(t, b, s_j); t, b, s_j) = b + \frac{(1 - t)^{1+\epsilon}}{1+\epsilon} s_j^{1+\epsilon}. \tag{2.30}
\]

Lemma 2 (Properties of the indirect utility function corresponding to own-utility):

\( a \) \( \frac{\partial v(t,b,s)}{\partial b} = 1 \),

\( b \) \( \frac{\partial v(t,b,s)}{\partial s} = (1 - t)^{1+\epsilon} s^\epsilon \). Hence, \( \left[ \frac{\partial v(t,b,s)}{\partial s} \right]_{t=1} = 0 \) and \( t \in [0, 1) \Rightarrow \frac{\partial v(t,b,s)}{\partial s} > 0 \).

Lemma 2 shows that an increase in transfer payment, \( b \), increase utility one for one. Furthermore, for any interior tax rate, indirect utility is strictly increasing in the level of skill. However, when \( t = 1 \), one’s entire income is taxed away, hence, an increase in skill does not increase indirect utility.

2.5. Preferences of voters over redistribution (the first stage problem)

Given the second stage choice of labor supplies by the voters (Proposition 1 and Lemma 1), the first stage problem is to choose the redistributive tax rate, \( t \) (and, consequently, the transfer, \( b \), given by (2.3), (2.4) and (2.6)). For this purpose, we calculate the voters’ indirect utility functions corresponding to their FS-preferences.

To find the indirect utility function, for voter \( j \), \( V_j = V_j(t, b, \alpha, \beta, s) \), that corresponds to his/her FS-preferences, substitute labour supply (Lemma 1) into (2.16), and take ac-
count of (2.30) and Lemma 2b to get\(^7\)

\[
V_j = u(l(t, b, s_j); t, b, s_j) - \frac{\alpha}{n-1} \sum_{k \neq j} \max \{0, u(l(t, b, s_k); t, b, s_k) - u(l(t, b, s_j); t, b, s_j)\} \\
- \frac{\beta}{n-1} \sum_{i \neq j} \max \{0, u(l(t, b, s_j); t, b, s_j) - u(l(t, b, s_i); t, b, s_i)\} \\
= v(t, b, s_j) - \frac{\alpha}{n-1} \sum_{k \neq j} \max \{0, v(t, b, s_k) - v(t, b, s_j)\} \\
- \frac{\beta}{n-1} \sum_{i \neq j} \max \{0, v(t, b, s_j) - v(t, b, s_i)\}
\]

\[
v(t, b, s_j) - \frac{\alpha}{n-1} \sum_{k > j} [v(t, b, s_k) - v(t, b, s_j)] - \frac{\beta}{n-1} \sum_{i < j} [v(t, b, s_j) - v(t, b, s_i)] \\
= b + \frac{(1-t)^{1+\epsilon}}{1+\epsilon} \left[ s_j^{1+\epsilon} - \frac{\alpha}{n-1} \sum_{k > j} (s_k^{1+\epsilon} - s_j^{1+\epsilon}) - \frac{\beta}{n-1} \sum_{i < j} (s_j^{1+\epsilon} - s_i^{1+\epsilon}) \right]
\]

(2.31)

To keep formulas down to manageable length, we define for each \(j, j = 1, 2, ..., n,\)

\[
\psi_j = (S - s_j^{1+\epsilon}) + \frac{\alpha}{n-1} \sum_{k > j} (s_k^{1+\epsilon} - s_j^{1+\epsilon}) + \frac{\beta}{n-1} \sum_{i < j} (s_j^{1+\epsilon} - s_i^{1+\epsilon})
\]

(2.32)

\[
\hat{j} = \max \{j : \psi_j > 0\}
\]

(2.33)

The interpretation of (2.32) is as follows. Recall that from (2.26) that \(y_j(t, b, s_j) = (1-t)^\epsilon s_j^{1+\epsilon}.\) Thus, for a fixed level of the tax rate, \(t,\) the first term in (2.32) is proportional to the difference of the \(j^{th}\) voter’s income from the average, the second term is proportional to the extent of disadvantageous inequity felt by voter \(j,\) while the third term is proportional to advantageous inequity felt by voter \(j.\) The intuition for defining (2.33) will become clear from Proposition 2; the set of voters indexed by \(j \geq \hat{j}\) will turn out to prefer zero redistribution.

From (2.10), (2.11), (2.28) and (2.32), for the median skill voter, \(s_m,\) it is immediate (because skills are ordered and \(s_m^{1+\epsilon} < S\) from (2.28)) that

\[
\psi_m = S - s_m^{1+\epsilon} + \frac{\alpha}{n-1} \sum_{k > m} (s_k^{1+\epsilon} - s_m^{1+\epsilon}) + \frac{\beta}{n-1} \sum_{i < m} (s_m^{1+\epsilon} - s_i^{1+\epsilon}) > 0.
\]

(2.34)

From (2.33) and (2.34) it immediately follows that

\[
m \leq \hat{j}.
\]

\(^7\)We use the standard mathematical conventions that \(\sum_{i \in \emptyset} x_i = 0,\) where \(\emptyset\) is the empty set. In particular,

\[
\sum_{k > n} (v_k - v_j) = \sum_{i < 1} (v_j - v_i) = 0.
\]
\( \psi_j \) and \( \hat{j} \) are functions of the exogenous parameters, \( n \) (number of voters), \( \epsilon \) (elasticity of labour supply), \( s \) (the skills vector), \( \alpha \) (disadvantageous inequity parameter) and \( \beta \) (advantageous inequity parameter).

Substituting from (2.32) into (2.31) we can express the FS preferences in the following tractable form

\[
V_j(t, b, \alpha, \beta, s) = b + \frac{(1 - t)^{1+\epsilon}}{1 + \epsilon} (\overline{S} - \psi_j).
\] (2.36)

When voter \( j \) votes on the tax rate, \( t \), and the transfer, \( b \), he/she takes into account the government budget constraint (2.29). Hence, substitute \( b(t, s) \) into (2.36), to get

\[
W_j(t, \alpha, \beta, s) = t (1 - t)^\epsilon \overline{S} + \frac{(1 - t)^{1+\epsilon}}{1 + \epsilon} (\overline{S} - \psi_j).
\] (2.37)

Voter \( j \) votes for that tax rate, \( t \), which maximizes social welfare from his/her own point of view, as given by his/her FS-indirect utility function (2.37). For \( 0 \leq t < 1 \), (2.37) gives

\[
\frac{\partial W_j}{\partial t} (t, \alpha, \beta, s) = (1 - t)^\epsilon \psi_j - \epsilon t (1 - t)^{\epsilon - 1} \overline{S}.
\] (2.38)

In Proposition 2, below, we give some results on the existence of optimal (or most preferred) taxes for any individual voter who at the first stage is asked to state his/her choice of the most preferred tax rate. The next section, Section 2.6, will look at the equilibrium tax rate that is actually implemented by society.

**Proposition 2 (Existence of optimal tax rates):**

(a) Given \( \alpha, \beta \) and \( s \), \( W_j(t, \alpha, \beta, s) \) attains a maximum at some \( t_j \in [0, 1) \).

(b) If \( j > \hat{j} \), then the tax rate preferred by voter \( j \), is \( t_j = 0 \).

(c) If \( j \leq \hat{j} \), then the tax rate, \( t_j \), preferred by voter \( j \), is unique, satisfies \( 0 < t_j < 1 \) and is given by

\[
t_j = \frac{\psi_j}{\epsilon \overline{S} + \psi_j},
\] (2.39)

where \( \overline{S}, \psi_j \) and \( \hat{j} \) were defined respectively in (2.27), (2.32) and (2.33).

(d) For \( j \leq \hat{j} \), \( t_j \) is strictly increasing in \( \alpha \) and \( \beta \).

(e) \( 1 > t_1 > t_2 > ... > t_{\hat{j}} > t_{\hat{j}+1} = t_{\hat{j}+2} = ... = 0 \). In particular, if \( \hat{j} = n \), then \( 1 > t_1 > t_2 > ... > t_n > 0 \).

The result of Propositions 2(d) and (e) may be deserving of further comment. The interpretation of Propositions 2(d) is that an increase in \( \alpha \) increases disutility arising from disadvantageous inequity. By increasing the redistributive tax rate, the voter reduces, relatively, the utility of anyone who is richer, hence, reducing disadvantageous inequity. On the other hand, an increase in \( \beta \) increases disutility arising from advantageous inequity.
An increase in the redistributive tax benefits everyone poorer than the voter relatively more, thus, reducing advantageous inequity. The interpretation of (e) is that the system of taxes and transfers benefit low skill voters relatively more than high skill voters. Hence, lower skill voters prefer higher tax rates than high skill voters.

2.6. Existence of a Condorcet winner

Before undertaking an equilibrium analysis, it is essential to establish the existence of an equilibrium. This is guaranteed by Proposition 3, below, which is an immediate consequence of Proposition 4 in Dhami and al-Nowaihi (2010a), and so we omit the proof.

**Proposition 3**: A majority prefers the tax rate that is optimal for the median-voter.

In light of the emerging evidence, it increasingly appears that issues of fairness and concern for others are important human motivations that play a significant part in the actual design of redistributive tax policies. Hence, the result in Proposition 3 should be useful for political economy models that seek to incorporate social preferences.

3. Comparative static results

Proposition 4, below, gives the change in the tax rate chosen by the median voter, \( t_m \), as various parameters in the model are changed.

**Proposition 4**: (a) The tax rate, \( t_m \), chosen by the median voter, is given by

\[
    t_m = \frac{\psi_m}{\epsilon S + \psi_m}. \tag{3.1}
\]

where \( S, \psi_m \) were defined respectively in (2.27), (2.32).

(b) For fair voters, \( \frac{\partial t_m}{\partial \alpha} > 0, \frac{\partial t_m}{\partial \beta} > 0 \).

(c) A fair median voter chooses a higher tax rate as compared to the case if he/she had selfish preferences instead.

From part (b), the tax rate (equivalently, the ratio of social spending to GDP, see (2.7)) is increasing in \( \alpha, \beta \). The intuition is as described in the discussion following Proposition 2. An increase in \( \alpha \) (respectively \( \beta \)) increases disutility arising from disadvantageous (respectively advantageous) inequity. By increasing the redistributive tax rate, the median voter mitigates advantageous and disadvantageous inequity. If the median voter had purely selfish preferences then he/she would have liked to redistribute only on account of the fact he/she is poorer than the average voter. Part (c) follows by simply noting that fair median voters have an additional tendency to redistribute on account of their fairness.
We now ask what happens to the optimal tax rate chosen by the median voter, $t_m$, when the individual skill levels of voters that are richer and poorer as compared to the median voter, change. Proposition 5 provides the necessary answers. All voters in our model have fair preferences, but we also compare the optimal tax rates chosen respectively by a fair-median voter and a selfish-median voter. Issues of changes in the entire skill distribution are taken up later in section 4.

**Proposition 5:**

(a) $j > m \Rightarrow \frac{\partial t_m}{\partial s_j} > 0$.

(b) $\frac{\partial t_m}{\partial s_m} < 0$.

(c) Suppose $j < m$, then

$$\frac{\partial t_m}{\partial s_j} \leq 0 \iff \frac{\alpha}{n-1} \sum_{k>m} (s_k^{1+\varepsilon} - s_m^{1+\varepsilon}) + \frac{\beta}{n-1} \left[ \sum_{i=m}^{n} s_i^{1+\varepsilon} + \sum_{i<m} (s_i^{1+\varepsilon} - s_m^{1+\varepsilon}) \right] - s_m^{1+\varepsilon} \geq 0,$$

(c(i)) for a selfish median voter, $j < m \Rightarrow \frac{\partial t_m}{\partial s_j} > 0$,

(c(ii)) for a sufficiently fair median voters, $j < m \Rightarrow \frac{\partial t_m}{\partial s_j} < 0$.

From Remark 1 and Proposition 5(a), selfish and fair voters alike, respond to increased affluence of the rich by redistributing more and so also raising the ratio of social spending to GDP. Selfish voters would like to redistribute more when the rich get richer because average incomes increase and so the lumpsum available for redistribution is higher. Fair voters have an additional motive to redistribute more, namely, that it reduces disadvantageous inequity.

Parts c(i) and c(ii) point out to an important difference in the predictions of the fair and selfish voter models. From part c(i), for a selfish voter, an increase in poverty reduces the tax rate and the ratio of social spending to GDP. The intuition is that poverty reduces average income available for redistribution, hence, reducing the marginal benefit of increasing the tax rate. For fair voters, however, the results can go either way. The reason is that the fair voter, like the selfish voter, cares about own payoff. However, in addition, the fair voter also cares about the income of poorer voters. The interplay between these two opposing factors determines if the fair voter will respond, unlike the selfish voter, by redistributing more in response to poverty. From part c(ii), for fair voters, if $\alpha$ or $\beta$ (or both) is sufficiently high, then empathy for the poorer voters (as well as envy towards richer voters) becomes stronger, which increases the tax rate and the ratio of social spending to GDP in response to increased poverty.

We can exploit the difference in the predictions of the selfish and the fair voter models in part (c) to draw some important inferences about social spending in a recession. Recall that, in our model, ‘skill’ is just a measure of the ability of a voter to translate labour
time into income. We may, therefore, identify periods of high unemployment with episodes where the ‘skills’ of below median voters receive strong negative shocks. The selfish voter model would then predict a decline in the ratio of social spending to GDP, while the fair voter model would predict an increase in this ratio. Thus, the selfish voter model predicts procyclical movement of the social spending to GDP ratio, while the fair voter model predicts a countercyclical movement. For the US data, the prediction of the selfish voter model is inconsistent with the evidence, while the prediction of the fair voter model is consistent with the evidence; see, for instance, Auerbach (2003).

4. Changes in the income distribution and redistribution

In section 3, above, we investigated the effect of a change in the level of skill of one voter on the tax rate chosen by the median-skill voter (and, hence, also chosen by society). Such a change will, necessarily, change mean income. We saw the striking difference between a fair median-skill voter and the benchmark case of a selfish median-skill voter in response to an increase in poverty as measured by a decline in the skill level of workers below the median.

We are now interested in the effect on redistribution in the fair voter model when the entire pre-tax income distribution changes, leaving the mean income unchanged. We have a discrete set of skill levels and, by implication, a discrete set of income levels. Recall from (2.26) that in any subgame that corresponds to a choice of the fiscal tuple $t, b$, incomes of the $n$ voters are ordered as $y_1, y_2, ..., y_n$. Thus, we are interested in asking: when is one set of incomes $y_1, y_2, ..., y_n$ more unequal than another, say, $x_1, x_2, ..., x_n$? This question has been posed, for the case of selfish preferences, in several notable papers, for instance, Atkinson (1970), Marshall and Olkin (1979), Preston (1990, 2006) and Zheng (2007). Let us call this as the first approach to measuring inequality. A second approach, one that we do not follow, is outlined in subsection 4.1 below.

We now propose a new concept, based on the first approach, that is more appropriate in the fair voter model. This we call as strong median dominance (SMD). Since fair voters care about advantageous and disadvantageous inequity, hence, the relevant inequality measure must specify these to pin down a preference for one income distribution over another.

**Definition 1** (Median Dominance): Consider the set of vectors:

$$I = \left\{ x : 0 < x_1 < x_2 < \ldots < x_n, \frac{1}{n} \sum_{i=1}^{n} x_i = \mu \text{ and } x_m < \mu \right\}.$$

Let $x, y \in I$.

(a) If $x_m \geq y_m$, we say that $x$ median-dominates $y$. If the inequality is strict, we say $x$
strictly median-dominates \( y \).

(b) Suppose \( x_m \geq y_m \), \( \sum_{k>m} (x_k - x_m) \leq \sum_{k>m} (y_k - y_m) \), \( \sum_{i<m} (x_m - x_i) \leq \sum_{i<m} (y_m - y_i) \) and, at least, one of these inequalities is strict. Then we say that \( x \) strongly median-dominates \( y \).

**Definition 2** (Inequality in the sense of strong median-dominance): Let \( x, y \in I \). We say that \( x \) is less unequal to \( y \), and that \( y \) is more unequal than \( x \), if \( x \) strongly median-dominates \( y \).

We now explain in words. Consider two distributions, \( x \) and \( y \), with the same mean and respective medians \( x_m, y_m \) that are both lower than the mean. Then \( x \) median-dominates \( y \) (resp. \( x \) strictly median dominates \( y \)) if, and only if, \( x_m \geq y_m \) (resp. \( x_m > y_m \)), regardless of the other components of \( x \) and \( y \). These concepts are clearly relevant for the case of a selfish median voter. For the case of a fair median voter, advantageous and disadvantageous inequality also become important. Thus \( x \) strongly median-dominates \( y \) if, and only if, the following criteria hold: (i) \( x_m \geq y_m \), (ii) advantageous inequality (measured in terms of outcomes), relative to the median, under \( x \) is lower than that under \( y \), (iii) disadvantageous inequality (measured in terms of outcomes), relative to the median, under \( x \) is lower than that under \( y \); and at least one of these three criteria holds with strict inequality. Notice that none of these observations places restrictions on the preferences of the voters.

**Definition 3** (Difference dominance): Let \( x, y \in I \) but \( x_m < \mu \) does not necessarily hold. According to Marshall and Olkin (1979), \( x \) difference-dominates \( y \) if \( i < j \Rightarrow x_j - x_i \leq y_j - y_i, \ i, j = 1, 2, ..., n \). If one of these inequalities is strict, then \( x \) strictly difference-dominates \( y \).

**Remark 3** (Comparison of difference dominance with strong median dominance): Comparing strict difference-dominance with strong median-dominance, we see that strict difference-dominance is weaker in the sense that it does not require that \( x_m < \mu \) or that \( x_m \geq y_m \). However, it is much stronger in the sense that it requires \( x_j - x_i \leq y_j - y_i \), for each \( i, j = 1, 2, ..., n \), \( i < j \). Thus if \( x \) strictly difference-dominates \( y \) and if, in addition, \( x, y \in I \) (defined in (4.1)) and \( x_m \geq y_m \), then \( x \) strongly median-dominates \( y \).

**Example 1** : Consider the three sets:

\[ x = \{0.2, 0.3, 0.7\}, \ y = \{0.1, 0.25, 0.85\}, \ z = \{0.05, 0.35, 0.8\} \]

Note that these three sets have the same mean:

\[ \frac{1}{3} \Sigma_{i=1}^{3} x_i = \frac{1}{3} \Sigma_{i=1}^{3} y_i = \frac{1}{3} \Sigma_{i=1}^{3} z_i = 0.4, \]
then
(a) \( \mathbf{x} \) strictly difference-dominates \( \mathbf{y} \) and \( \mathbf{z} \),
(b) neither \( \mathbf{y} \) difference-dominates \( \mathbf{z} \) nor does \( \mathbf{z} \) difference-dominate \( \mathbf{y} \),
(c) \( \mathbf{x} \) strictly and strongly median-dominates \( \mathbf{y} \) (hence, from Definition 2, \( \mathbf{y} \) is more unequal than \( \mathbf{x} \)),
(d) \( \mathbf{z} \) strictly, but not strongly, median-dominates \( \mathbf{x} \) and \( \mathbf{y} \).

Proposition 6, below, establishes the sense in which redistribution lowers inequality.

**Proposition 6**: At any tax rate \( t < 1 \), the disposable (post tax) income vector\(^8\), \( \mathbf{c} \), strongly median dominates the factor (pretax) income vector, \( \mathbf{y} \).

The tax rate, \( t_m \), is, of course, determined by the exogenous variables of the model \((n, s_i, \alpha, \beta)\) through equations (2.27), (2.34) and (2.39). However, these variables are not normally observable. This raises an issue of how an econometrician interested in testing our prediction, might proceed. We give some indication of the answer in Proposition 7 below, which has two purposes. First, it establishes relationships between \( t_m \) and the endogenous variables \( y_i \) (pretax incomes) and \( c_i \) (post-tax incomes); which are observable. Hence, in principle, one could find instruments for these variables and perform econometric analysis. Second, the results in Proposition 7 will be useful in the proof of Proposition 8 that follows.

**Proposition 7**: (a) Let \( \mathbf{y} \) be the vector of pretax incomes \((2.26)\). Let

\[
y = \frac{1}{n} \sum_{j=1}^{n} y_j - y_m + \frac{\alpha}{n-1} \sum_{k>m} (y_k - y_m) + \frac{\beta}{n-1} \sum_{i<m} (y_m - y_i),
\]

then

\[
t_m = \frac{1}{1 + \frac{\alpha}{ny} \sum_{j=1}^{n} y_j}.
\]

(b) Let \( \mathbf{c} \) be the vector of post tax incomes (which is also the consumption vector). Let

\[
c = \frac{1}{n} \sum_{j=1}^{n} c_j - c_m + \frac{\alpha}{n-1} \sum_{k>m} (c_k - c_m) + \frac{\beta}{n-1} \sum_{i<m} (c_m - c_i),
\]

then

\[
t_m = \frac{nc}{\epsilon \sum_{j=1}^{n} c_j}.
\]

\(^8\)Post-tax income is simply consumed, hence, our choice of \( \mathbf{c} \) for the post-tax income vector.
Proposition 8: Let $x, y \in I$, where $I$ is a set of pretax or post tax incomes, as in Definition 1. Let $t^S_m$ and $T^S_m$ be the tax rates associated with $x, y$, respectively, when the median voter is selfish ($\alpha = \beta = 0$). Let $t^F_m$ and $T^F_m$ be the tax rates associated with $x, y$, respectively, when the median voter is fair ($\alpha > 0$ and $0 < \beta < 1$). Then

(a) $t^S_m < t^F_m$ and $T^S_m < T^F_m$.

(b) If $x$ strictly median-dominates $y$, then $t^S_m < T^S_m$.

(c) If $x$ strongly median-dominates $y$, i.e., $y$ is more unequal than $x$, then $t^F_m < T^F_m$.

A consequence of Proposition 8(c) is that, controlling for fairness, higher inequality results in greater redistribution. The existing literature ignores issues of fairness. To illustrate the pitfalls that this could lead to, we plot in Figure 4.1 below, the optimal tax rate chosen by the median voter (vertical axis) against fairness (as measured by $\alpha, \beta$) and inequality (Definition 2).

In actual practice, empirical researchers could be picking up any sequence of points along the surface in Figure 4.1. This practice is likely to lead to mixed and possible contradictory results. As the figure clearly shows, low-inequality and high-fairness countries have a similar level of redistribution as high-inequality and low-fairness countries. Not controlling for fairness would then lead to absurd results. This issue, we believe, could have seriously contaminated the existing literature’s attempt at finding an empirical relation between inequality and the extent of redistribution and seems worthy of empirical investigation.

4.1. The traditional approach: Stochastic dominance

In contrast to our approach to measuring inequality, which we have called as the first approach, there is a second approach based on classical statistics that we find less appealing in our model. To outline the second approach, suppose that there is a set of $m$ random variables: $Y_1, Y_2, \ldots, Y_m$ such that each random variable is distributed identically and independently with some density $f$ and distribution $F$. Let us draw a random sample of incomes $y_1, y_2, \ldots, y_m$ which has a joint probability $\prod_{i=1}^{m} f(y_i)$. Suppose that an individual voter’s utility from the income level $y_i, u(y_i)$, is increasing and concave, $i = 1, 2, \ldots, m$. Furthermore, consider a utilitarian social planner who wishes to maximize

$$W = \sum_{i=1}^{m} u(y_i)f(y_i).$$

\footnote{Consider a three person economy: poor, middle class and rich. The plot assumes a mean income of 20. Along the fairness axis, the envy parameter, $\alpha$, moves between 0.5 and 5, while along the inequality axis, the income of the rich voter moves between 25 to 30. The tax rate, which is plotted against the vertical axis is based on a adjustments in the incomes of the poor and the middle class voters such that average income remains 20.}
Alternatively, we could have specified an individual who is behind a veil of ignorance and could be assigned to take the role of any one of the $m$ individuals. If such an individual has selfish preferences, he/she too would maximize (4.2). Let $F$ SOSD $G$ where $G$ is another distribution function with the same mean as $F$. Then, clearly, on account of SOSD,

$$\sum_{i=1}^{n} u(y_i) f(y_i) \geq \sum_{i=1}^{n} u(y_i) g(y_i), \quad (4.3)$$

where $g$ is the density function of $G$.

If the second approach to inequality were to be preferred, for whatever reasons, then it turns out that SOSD would no longer be the appropriate inequality construct if individuals have other regarding preferences. To see this, suppose that a preference for fairness took the Fehr-Schmidt (1999) form, as in this paper. Then we would define the preferences of the $i^{th}$ voter as

$$U_i(y_1, y_2, \ldots, y_m) = u(y_i) - \frac{\alpha}{n-1} \sum_{k>i} [u(y_k) - u(y_i)] - \frac{\beta}{n-1} \sum_{j<i} [u(y_i) - u(y_j)]. \quad (4.4)$$

The social welfare function, the analogue of (4.2) in this case is:

$$\sum_{i=1}^{m} \left[ u(y_i) - \frac{\alpha}{n-1} (1 - F(y_i)) \sum_{k>i} [u(y_k) - u(y_i)] - \frac{\beta}{n-1} F(y_i) \sum_{j<i} [u(y_i) - u(y_j)] \right] f(y_i) \quad (4.5)$$

Under the assumptions on $u$, SOSD implies (4.3) when preferences are selfish and so distribution $F$ is preferred to distribution $G$. However, there is no presumption that, under
the conditions specified, an individual with utility function specified in (4.4) will prefer $F$ over $G$. Of course, these considerations might deserve further development. But we have followed a more direct and plausible path in our paper that is associated with the first approach.

4.2. Redistribution in Sweden Versus US, once again

Consider again the comparison between Sweden and the US, mentioned in the introduction, that defies explanation using the RRMR framework. Namely, disposable income inequality in Sweden is only about 60% that of the US, but the ratio of social spending to GDP is twice that of the US. We compare here, how the selfish voter model and a fair voter model might explain these facts.

4.2.1. The selfish voter model’s take on the Sweden versus US comparison

When voters have selfish preferences, two different theoretical models are often invoked to explain this sort of violation of the RRMR model. The first, by Alesina and Angeletos (2005), hinges on different beliefs of Europeans and Americans about the source of income variability. Europeans are assumed to believe that luck plays the key role while the Americans are assumed to believe that effort does. These beliefs are self-fulfilling in equilibrium and outcomes governed by luck are assumed to generate greater redistribution. Benabou (2000) develops a stochastic growth model with incomplete asset markets and heterogeneous agents who vote over redistributive policies. He shows that multiple equilibria can exist, some featuring low inequality and high redistribution, while others exhibit high inequality and low redistribution.

Whether these models successfully explain the violation of the RRMR prediction is an empirical question, which is still unanswered. Three potential difficulties with explanations based on these models are the following. First, the Alesina and Angeletos (2005) model does not explain cases where the predictions of the RRMR model actually hold; see, for instance, Persson and Tabellini (2000). Second, models of multiple equilibria do not explain which of the multiple equilibria is more likely, hence, they can justify either of the two opposing predictions. Third, by not taking ORP into account, both models fail to explain the growing evidence arising from experiments that is mentioned in the introduction.

4.2.2. The fair voter model’s take on the Sweden versus US comparison

How might the fair voter model provide an explanation of the Sweden versus US comparison? We have shown in this paper that greater fairness elicits greater redistribution in society (Propositions 5, 8). If it could be shown that Swedish voters have greater fairness relative to US voters, then we might be able to account for greater redistribution in
Sweden despite its lower level of income inequality relative to the US. Measuring fairness (i.e., altruism and envy) poses problems with field data (because of the number of controls needed), and experimental data (which can lead to problems of ‘ecological validity,’ i.e., appropriate subject pools, context etc.). These problems are no different from many other empirical problems. However, little empirical work is available in this regard.

One could argue that the magnitude of charitable giving provides a measure of fairness. However, there are serious problems with this argument. The reason is that charitable giving is endogenous, so US citizens might contribute relatively more because they take account of a relatively lower size of the welfare state, say, relative to Europeans. Furthermore, a great fraction of charitable giving in the US is for religious causes, which lie outside the scope of our framework.

A more serious candidate to measure fairness is the level of multilateral aid given to developing countries, after correcting for elements of strategic giving. Clearly, such aid might also be motivated by its low aggregate volume (as in charitable giving), however, arguably, the ‘corrected’ relative giving of different countries reflects their relative concerns for fairness. Thus, ‘multilateral aid as a percentage of GDP’ is, at the very least, a crude measure of fairness. Data on this is available through OECD statistics. Alternatively one could use ‘quality adjusted aid to GDP ratio’ drawn from an index compiled by Roodman (2005). It turns out that on this measure of fairness, the US is placed close to the bottom in a list of 21 OECD economies, while Sweden is near the top. In per capita terms, for instance, the multilateral aid of Sweden relative to the US is 9 times larger! Hence, taking account of fairness considerations offers a potential and plausible resolution. Ideally, if one could find appropriate measures of fairness, a cross country econometric analysis that includes measures of inequality, and other controls, seems to hold great promise for future research.

5. Conclusions

The emerging evidence is strongly suggestive of the important role of fairness (altruism and envy) in voting contexts. Furthermore, the evidence suggests that many voters have the Fehr and Schmidt (1999) preferences (FS). Yet, with very few exceptions, the existing political economy literature deals almost exclusively with selfish voters (who derive utility solely from their own material payoffs). This raises the difficult issue of reconciling existing models with the evidence. Motivated by these considerations, we replace the self interested voters in the simple general equilibrium Romer-Roberts-Meltzer-Richard (RRMR) frame-
work with *fair voters* who have a preference for fairness in the sense of FS. We primarily explore the affect on redistribution when the income distribution changes?

Our findings are as follows. Increased fairness leads to a more redistributive outcome. Fair voters, if they are sufficiently fair, will respond to poverty by redistributing more (and not less as the selfish voter model predicts). The ratio of social spending to GDP moves countercyclically in the fair voter model but pro-cyclically in the selfish voter model. The latter is not consistent with the evidence. Controlling for fairness, higher inequality leads to greater distribution but not controlling for fairness could lead to mixed and possibly contradictory empirical results.

It turns out that the standard concepts of inequality such as a mean preserving spread are not suitable in the fair voter model. We, therefore, introduce the concept of *strong median dominance*, which is the appropriate notion of inequality when one is interested in other regarding preferences, especially in the sense of Fehr and Schmidt (1999). We anticipate that the concept of strong median dominance will open the way for greater application of models in which economic agents have social preferences. In particular, the critical tools developed in the paper may pave the way for greater exploration in the emerging, but promising, field of behavioral political economy.

6. Proofs

**Proof of Proposition 1**: This is an immediate consequence of Proposition 1 in Dhami and al-Nowaihi (2010a).

**Proof of Lemma 1**: From (2.25) we see that, given $t, b$ and $s_i$, $u(l_i; t, b, s_i)$ is a continuous function of $l_i$ on the non-empty compact set $[0, 1]$. Hence, a maximum exists. From (2.25), we get

$$\frac{\partial u}{\partial l_i}(l_i; t, b, s_i) = (1 - t) s_i - l_i^{\frac{1}{t}}, \quad (6.1)$$

$$\frac{\partial u}{\partial l_i}(0; t, b, s_i) = (1 - t) s_i, \quad (6.2)$$

$$\frac{\partial u}{\partial l_i}(1; t, b, s_i) = (1 - t) s_i - 1, \quad (6.3)$$

$$\frac{\partial^2 u}{\partial l_i^2}(l_i; t, b, s_i) = -\frac{1}{\epsilon l_i^{\frac{3}{t}}}. \quad (6.4)$$

First, consider the case $t = 1$. From (2.25), or (6.1), we see that $u(l_i; 1, b, s_i)$ is a strictly decreasing function of $l_i$ for $l_i > 0$. Hence, the optimum must be

$$l_i = 0 \text{ at } t = 1. \quad (6.5)$$
Now, suppose \( t \in [0, 1] \). From (2.1), (2.2), (6.2) and (6.3) we get that \( \frac{\partial u_i}{\partial t_i} (0; t, b, s_i) > 0 \) and \( \frac{\partial u_i}{\partial t_i} (1; t, b, s_i) < 0 \). Hence an optimal value for \( l_i \) must lie in \((0, 1)\) and, hence, must satisfy \( \frac{\partial u_i}{\partial l_i} (l_i; t, b, s_i) = 0 \). From (6.1) we then get

\[
l_i = (1 - t) s_i^*,
\]

which, therefore, must be the unique optimal labour supply (this also follows from (6.4)). For \( t = 1 \), (6.6) is consistent with (6.5). Hence, for each consumer, \( i \), (6.6) gives the optimal labour supply for each \( t \in [0, 1] \).

**Proof of Lemma 2**: The proof follows from (2.30) by direct calculation.

**Proof of Proposition 2** (existence of optimal tax rates): (a) From (2.37), we see that \( W_j(t, \alpha, \beta, s) \) is a continuous function of \( t \in [0, 1] \), for \( j = 1, 2, ..., n \). Hence, \( W_j(t, \alpha, \beta, s) \) attains a maximum at some \( t_j \in [0, 1] \). We will now show that \( t_j < 1 \). From (2.37), we get

\[
W_j(1; \alpha, \beta, s) = 0, \tag{6.7}
\]

\[
W_j(0; \alpha, \beta, s) = \frac{\overline{S} - \psi_j}{1 + \epsilon}. \tag{6.8}
\]

If \( \psi_j < \overline{S} \) then, from (6.7) and (6.8), we get \( W_j(0; \alpha, \beta, s) > W_j(1; \alpha, \beta, s) \). Hence, \( t_j < 1 \). Now, suppose \( \psi_j \geq \overline{S} \). Let

\[
t_0 = \frac{1}{2} \left[ 1 + \frac{\psi_j - \overline{S}}{\psi_j + \epsilon \overline{S}} \right]. \tag{6.9}
\]

Clearly, \( 0 < t_0 < 1 \). Substituting for \( t = t_0 \) from (6.9) into (2.37), gives

\[
W_j(t_0, \alpha, \beta, s) = \frac{1}{4} \left( \frac{1 + \epsilon}{\psi_j + \epsilon \overline{S}} \right)^\epsilon \overline{S}^{1+\epsilon} > 0. \tag{6.10}
\]

From (6.7) and (6.10) we get, again, that \( t_j < 1 \).

(b) Suppose \( j > \hat{j} \). From (2.33) we get \( \psi_j \leq 0 \). Hence, from (2.38), we see that \( \frac{\partial W_j}{\partial \alpha} < 0 \) for all \( t \in [0, 1] \). Hence, the optimal tax rate, \( t_j \), for voter \( j \), must be \( t_j = 0 \).

(c) Suppose \( j \leq \hat{j} \). From (2.33) we get \( \psi_j > 0 \). Hence, from (2.38), we see that \( \frac{\partial W_j}{\partial \alpha} > 0 \) at \( t = 0 \). Hence, the optimal tax rate, \( t_j \), for voter \( j \) satisfies \( t_j > 0 \). Combining this with \( t_j < 1 \) (from (a)), we get that, necessarily, \( \frac{\partial W_j}{\partial \alpha} = 0 \) at \( t = t_j \). From (2.38) we then get (2.39). Since an optimum, \( t_j \), exists (from (a)), since it must satisfy \( \frac{\partial W_j}{\partial \alpha} = 0 \) and since the latter has the unique solution (2.39), it follows that (2.39) gives the unique global optimum (this can also be derived by showing that \( \frac{\partial^2 W_j}{\partial \alpha^2} < 0 \) for \( t \in (0, 1) \)).

(d) Suppose \( j \leq \hat{j} \). From (2.32) we get

\[
\frac{\partial \psi_j}{\partial \alpha} = \frac{1}{n - 1} \sum_{k > j} (s_k^{1+\epsilon} - s_j^{1+\epsilon}) > 0, \tag{6.11}
\]
\[ \frac{\partial \psi_j}{\partial \beta} = \frac{1}{n-1} \sum_{i<j} (s_j^{1+\epsilon} - s_i^{1+\epsilon}) > 0, \quad (6.12) \]

and, hence, from part (c) and (2.39), we get
\[ \frac{\partial t_j}{\partial \alpha} = \frac{\epsilon \tilde{S} \sum_{i<j} (s_j^{1+\epsilon} - s_i^{1+\epsilon})}{(\epsilon \tilde{S} + \psi_j)^2} > 0, \quad (6.13) \]
\[ \frac{\partial t_j}{\partial \beta} = \frac{\epsilon \tilde{S} \sum_{i<j} (s_j^{1+\epsilon} - s_i^{1+\epsilon})}{(\epsilon \tilde{S} + \psi_j)^2} > 0. \quad (6.14) \]

Hence, \( t_j \) is strictly increasing in \( \alpha \) and \( \beta \).

(e) (2.32) gives:
\[ \psi_j - \psi_{j+1} = \left[ 1 - \frac{j \beta}{n-1} + \frac{(n-j) \alpha}{n-1} \right] (s_{j+1}^{1+\epsilon} - s_j^{1+\epsilon}) \quad (6.15) \]
\[ \psi_{n-1} - \psi_n = \left( 1 - \beta + \frac{\alpha}{n-1} \right) (s_n^{1+\epsilon} - s_{n-1}^{1+\epsilon}) > 0, \quad (6.16) \]
\[ \psi_1 - \psi_2 = \left( 1 - \frac{\beta}{n-1} + \alpha \right) (s_2^{1+\epsilon} - s_1^{1+\epsilon}) > 0, \quad (6.17) \]

(6.15), (6.16) and (6.17) can be combined to produce
\[ \psi_j - \psi_{j+1} = \left[ 1 - \frac{j \beta}{n-1} + \frac{(n-j) \alpha}{n-1} \right] (s_{j+1}^{1+\epsilon} - s_j^{1+\epsilon}) > 0, \quad 1 \leq j \leq n-1, \quad (6.18) \]

from which it follows that
\[ \psi_1 > \psi_2 > \ldots > \psi_n. \quad (6.19) \]

From (6.19) and (2.39)
\[ t_j - t_{j+1} = \frac{\epsilon \tilde{S} (\psi_j - \psi_{j+1})}{(\epsilon \tilde{S} + \psi_j) (\epsilon \tilde{S} + \psi_{j+1})} > 0, \quad j + 1 \leq \tilde{j}. \quad (6.20) \]

Combining (6.20) with parts (a), (b) and (c) gives \( 1 > t_1 > t_2 > \ldots > t_j > t_{j+1} = t_{j+2} = \ldots = 0. \) In particular, if \( \tilde{j} = n \), then \( 1 > t_1 > t_2 > \ldots > t_n > 0. \)

**Proof of Proposition 3:** This is an immediate consequence of Proposition 4 in Dhami and al-Nowaihi (2010a).

**Proof of Proposition 4:** Parts (a) and (b) follow from (2.35) and Proposition 2(c) and (d). Part (c) then follows from part (b).

**Proof of Proposition 5:** From (2.27), (2.32) and (3.1), after some algebraic manipulation, we get the following:
\[ \left( 1 + \frac{n \alpha}{n-1} \right) \tilde{S} - \psi_m > 0 \quad (6.21) \]
\[ j > m \Rightarrow \frac{\partial t_m}{\partial s_j} = \frac{\left[(1 + \frac{n\alpha}{n-1}) \bar{S} - \psi_m\right] \epsilon (1 + \epsilon) s_j^\epsilon}{n(\epsilon \bar{S} + \psi_m)^2} > 0, \] (6.22)

\[ 2(n-1) + n(\alpha - \beta)] \bar{S} + 2\psi_m > 0, \] (6.23)

\[ \frac{\partial t_m}{\partial s_m} = -\frac{\left[2(n-1) + n(\alpha - \beta)] \bar{S} + 2\psi_m\right] \epsilon (1 + \epsilon) s_m^\epsilon}{2n(\epsilon \bar{S} + \psi_m)^2} < 0, \] (6.24)

\[ j < m \Rightarrow \frac{\partial t_m}{\partial s_j} = -\frac{[\psi_m - \left(1 - \frac{n\beta}{n-1}\right) \bar{S} \epsilon (1 + \epsilon) s_j^\epsilon}{n(\epsilon \bar{S} + \psi_m)^2}, \] (6.25)

\[ \psi_m - \left(1 - \frac{n\beta}{n-1}\right) \bar{S} = \frac{\alpha}{n-1} \sum_{k > m} (s_k^{1+\epsilon} - s_m^{1+\epsilon}) + \frac{\beta}{n-1} \left[\sum_{i = 1}^{n} s_i^{1+\epsilon} + \sum_{i < m} (s_m^{1+\epsilon} - s_i^{1+\epsilon})\right] - s_m^{1+\epsilon}. \] (6.26)

From (6.22), we see that an increase in the skill of voter-workers above the median will increase the tax rate, whether the median voter is selfish or fair. This establishes part (a).

From (6.23) and (6.24), we see that an increase in the skill of the median skill voter-workers will reduce the tax rate, whether the median voter is selfish or fair. This establishes part (b). Part (c) follows from (6.25) and (6.26), from which subcases c(i) and c(ii) follow. \(\blacksquare\)

**Proof of Proposition 6:** Let \(t < 1\). Let \(y\) be the factor (pretax) income vector and let \(c\) be the disposable (post tax) income vector. From (2.5), (2.14) and (4.1), we see that \(c_i = \frac{1}{n} \sum_{i = 1}^{n} c_i = \frac{1}{n} \sum_{i = 1}^{n} y_i\). From (2.1), (2.5), (2.13), (2.26) and (2.28), we get:

\[ c_m - y_m = t (1 - t)^\epsilon (\bar{S} - s_m^{1+\epsilon}) > 0, \]

\[ \sum_{k > m} (y_k - y_m) - \sum_{k > m} (c_k - c_m) = t (1 - t)^\epsilon \sum_{k > m} (s_k^{1+\epsilon} - s_m^{1+\epsilon}) > 0, \]

\[ \sum_{i < m} (y_m - y_i) - \sum_{i < m} (c_m - c_i) = t (1 - t)^\epsilon \sum_{i < m} (s_m^{1+\epsilon} - s_i^{1+\epsilon}) > 0. \]

Hence, \(c\), strongly median dominates \(y\). \(\blacksquare\)

**Proof of Proposition 7:** Let

\[ y = \frac{1}{n} \sum_{j = 1}^{n} y_j - y_m + \frac{\alpha}{n-1} \sum_{k > m} (y_k - y_m) + \frac{\beta}{n-1} \sum_{i < m} (y_m - y_i) \] (6.27)

and

\[ c = \frac{1}{n} \sum_{j = 1}^{n} c_j - c_m + \frac{\alpha}{n-1} \sum_{k > m} (c_k - c_m) + \frac{\beta}{n-1} \sum_{i < m} (c_m - c_i) \] (6.28)

From (2.27) and (2.34), we get

\[ \psi_m = \frac{1}{n} \sum_{i = 1}^{n} s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1} \sum_{k > m} (s_k^{1+\epsilon} - s_m^{1+\epsilon}) + \frac{\beta}{n-1} \sum_{i < m} (s_m^{1+\epsilon} - s_i^{1+\epsilon}) \] (6.29)
and, hence,
\[
(1 - t_m)^\epsilon \psi_m = \frac{1}{n} \sum_{j=1}^n (1 - t_m)^\epsilon s_j^{1+\epsilon} - (1 - t_m)^\epsilon s_m^{1+\epsilon} + \frac{\alpha}{n - 1} \sum_{k > m} [(1 - t_m)^\epsilon s_k^{1+\epsilon} - (1 - t_m)^\epsilon s_m^{1+\epsilon}] + \frac{\beta}{n - 1} \sum_{i < m} [(1 - t_m)^\epsilon s_m^{1+\epsilon} - (1 - t_m)^\epsilon s_i^{1+\epsilon}].
\] (6.30)

From (2.26), (6.27) and (6.30), we get
\[
(1 - t_m)^\epsilon \psi_m = y
\] (6.31)

From (2.26), (2.27), (3.1) and (6.31), we get
\[
t_m = \frac{\psi_m}{\frac{1}{n} \sum_{j=1}^n (1 - t_m)^\epsilon s_j^{1+\epsilon} + \psi_m} = \frac{\frac{1}{n} \sum_{j=1}^n (1 - t_m)^\epsilon s_j^{1+\epsilon} + (1 - t_m)^\epsilon \psi_m}{y} = \frac{1}{\frac{1}{n} \sum_{j=1}^n y_j + y} = 1 + \frac{\epsilon}{ny} \sum_{j=1}^n y_j.
\] (6.32)

This establishes part (a).

From (6.32) we get
\[
t_m = \frac{n (1 - t_m) y}{\epsilon \sum_{j=1}^n y_j}.
\] (6.33)

From (2.13), (6.27) and (6.28), and substituting \(t_m\) for \(t\), we get
\[
(1 - t_m) y = c.
\] (6.34)

From (2.14), (6.33) and (6.34), we get
\[
t_m = \frac{nc}{\epsilon \sum_{j=1}^n c_j},
\]
which establishes part (b). ■

**Proof of Proposition 8:**

Let
\[
x^S = \mu - x_m,
\] (6.35)
\[
y^S = \mu - y_m,
\] (6.36)
\[
x^F = \mu - x_m + \frac{\alpha}{n - 1} \sum_{k > m} (x_k - x_m) + \frac{\beta}{n - 1} \sum_{i < m} (x_m - x_i),
\] (6.37)
\[
y^F = \mu - y_m + \frac{\alpha}{n - 1} \sum_{k > m} (y_k - y_m) + \frac{\beta}{n - 1} \sum_{i < m} (y_m - y_i),
\] (6.38)
First, we consider the case where \( x, y \in I \) are pretax incomes. Since \( \Sigma_{i=1}^{n} x_i = \Sigma_{i=1}^{n} y_i = \mu \) we get the following, from Proposition 7(a):

\[
\begin{align*}
    t_s^m & = \frac{1}{1 + \frac{\epsilon \mu}{nx^s}}, \\
    T_s^m & = \frac{1}{1 + \frac{\epsilon \mu}{ny^s}}, \\
    t_f^m & = \frac{1}{1 + \frac{\epsilon \mu}{nx^f}}, \\
    T_f^m & = \frac{1}{1 + \frac{\epsilon \mu}{ny^f}}.
\end{align*}
\]

From (6.35) and (6.37) it is clear that \( 0 < x^S < x^F \). Hence, from (6.39) and (6.41), it follows that \( t_s^m < t_f^m \). Similarly, from (6.36), (6.38), (6.40) and (6.42), it follows that \( T_s^m < T_f^m \). This establishes part (a).

Suppose \( x \) strictly median-dominates \( y \) (Definition 1a), it follows that \( x_m > y_m \). Hence, from (6.35) and (6.36), it follows that \( 0 < x^S < y^S \). Hence, from (6.39) and (6.40), it follows that \( t_s^m < T_s^m \). This establishes part (b).

Suppose \( x \) strongly median-dominates \( y \) (Definition 1b). It follows that \( x_m \geq y_m \), \( \sum_{k>m} (x_k - x_m) \leq \sum_{k>m} (y_k - y_m) \), \( \sum_{i<m} (x_m - x_i) \leq \sum_{i<m} (y_m - y_i) \) and, at least, one of these inequalities is strict. Hence, from (6.37) and (6.38), it follows that \( 0 < x^F < y^F \). Hence, from (6.41) and (6.42), it follows that \( t_f^m < T_f^m \). This establishes part (c).

Second, we consider the case where \( x, y \in I \) are post tax incomes. The proof in this case is similar to to that in case (I), except that (6.39)-(6.42) are replaced by \( t_s^m = \frac{nx^S}{\epsilon \mu} \), \( T_s^m = \frac{ny^S}{\epsilon \mu} \), \( t_f^m = \frac{nx^F}{\epsilon \mu} \), and \( T_f^m = \frac{ny^F}{\epsilon \mu} \), respectively, and part (b) of Lemma 7 is used instead of part (a). ■

**Acknowledgements:** We are grateful for helpful comments from Ian Preston and Hamish Low on earlier drafts of this paper and to seminar participants at the Public Economics Weekend in Leicester.

**References**


