Optimal Institutional Design when there is a Zero Lower Bound on Interest Rates*

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Abstract

Given the recent experience, there is a growing interest in the liquidity trap; which occurs when the nominal interest rate reaches its *zero lower bound*. We outline the surprising policy recommendations when there is the possibility of a zero lower bound. Then, using the Dixit-Lambertini (2003) framework of strategic policy interaction between the Treasury and the Central Bank, we find that the optimal institutional response to the possibility of a liquidity trap has two main components. First, an optimal inflation target is given to the Central Bank. Second, the Treasury, who retains control over fiscal policy and acts as Stackelberg leader, is given optimal output and inflation targets. This institutional solution achieves the optimal rational expectations pre-commitment solution.

Keywords: liquidity trap, strategic monetary-fiscal interaction, optimal Taylor rules.

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1. Introduction

In its classical form, the *liquidity trap*, a term coined by Keynes (1936), is a situation where an economy is caught up in a deflation and the nominal interest rate has been driven down to zero (the so called 'zero lower bound'). The source of a liquidity trap, in most circumstances, is a sharp fall in aggregate demand; see Keynes (1936), Bernanke (2002).

Interest in the liquidity trap has revived in recent years due, in no small measure, to the experience of Japan since 1990. Woodford (2005, p.29) discusses the near miss of the US economy from a liquidity trap in the summer of 2003. The era of successful delegation of monetary policy to independent central banks with low inflation targets opens up the possibility that sufficiently large negative demand shocks might push an economy into a liquidity trap with huge associated welfare consequences. Recent policy discussions on either side of the Atlantic have expressed concerns about the nominal interest rate hitting the zero lower bound.[1]

In a liquidity trap, traditional monetary policy loses its effectiveness because nominal interest rates can be reduced no further in order to boost the interest sensitive components of aggregate demand. Hence, reliance must be placed on other, possibly more expensive, policies. Keynes (1936), in the first policy prescription for a liquidity trap, suggested the use of fiscal policy, which works through the multiplier effect to boost output and employment.

1.1. The modern literature on the liquidity trap

The recent literature has largely focussed on the role of inflation expectations in a liquidity trap. To see this, note that the real interest rate, r, is given by $r = i - \pi^e$, where i is the nominal interest rate and π^e is expected inflation. In a liquidity trap, by definition, $i = 0$, and due to deflationary expectations, typically $\pi^e < 0$, hence, $r > 0$. To expand economic activity, the government needs to lower r.

One possible solution, suggested by Krugman (1998, 1999), is to generate positive inflationary expectations, $\pi^e > 0$, so that the real interest rate $r < 0$. This, in turn, creates a need for a credible commitment to the future level of actual inflation because

[1] In an article on Bloomberg.com on 6 July 2009, the Federal Reserve Bank of San Francisco President, Janet Yellen, is reported to have told reporters after a speech in San Francisco: "We have a very serious recession, we have a 9.4 percent unemployment rate... Given the recession's severity, we should want to do more. If we were not at zero, we would be lowering the funds rate." The Economist wrote on November 13, 2008 that: "Remember Japan's zero interest rates? America is almost there too. Since October 29th, ...the "effective rate", has averaged around 0.25%..." The Telegraph, a national daily in Britain reported in January 2009 that the interest rate has been cut to it's lowest level in the last 300 years to 1.5% and went on to say that: "Further cuts are expected in the next few months and the base rate may be reduced to zero this year."

after the economy has escaped from the liquidity trap it is in the interest of all parties to reduce inflation. A forward looking private sector will anticipate this and expect low future inflation. But then the real interest rate remains high, keeping the economy in a liquidity trap. The era of low inflation targets exacerbates the problem, because such targets lead to low values of π^e and, so, the real interest cannot be reduced low enough to boost recovery in a liquidity trap.

The literature on the liquidity trap has considered a range of solutions, e.g., exchange rate policies such as currency depreciation, integral stabilization, a carry tax on currency, open market operations in long term bonds, price level targets, and money growth rate pegs. The surveys in Svensson (2003) and Blinder (2000) consider these policies in detail. However, these policies have important limitations[2,3]. Eggertsson (2006a, 2006b) recommends abandonment of an independent central bank and a return to discretionary policy by a unitary monetary-fiscal authority. A debt financed fiscal expansion during a liquidity trap results, via the government budget constraint, in higher expectations of future inflation. Eggertsson shows that this solution is superior to either monetary policy alone or uncoordinated monetary and fiscal policy. However, as Eggertsson shows, even optimal discretion is inferior to the fully optimal rational expectations solution with commitment. Moreover, abandoning delegation of monetary policy to an independent central bank with a narrow mandate, in favor of a return to discretion, appears to be a retrograde step.[4]

1.2. Some relevant questions to ask in modeling a liquidity trap

What kind of questions should economists ask when they wish to model a liquidity trap? Perhaps it would be useful to be guided by the recent Japanese experience. Based on

[2] Variants of the devaluation approach can be found in McCallum (2000) and Svensson (2003). There are several potential problems with the devaluation option. First, calibrated models show that the magnitude of the devaluation required to get out of the liquidity trap might be too high. Second, using the uncovered interest rate parity condition, when the domestic interest rate is zero, the expected appreciation of the home currency is fully locked-in by the foreign interest rate. Third, current devaluation will generate expectations of future appreciation of currency when the economy moves out of the liquidity trap, generating counter flows that frustrate attempts to devalue. Fourth, devaluations may bring about competitive devaluations or retaliations in the form of other barriers to trade.

[3] In a liquidity trap, zero nominal interest rates make bonds and money perfect substitutes. Hence, it might be difficult to engineer a price level increase. Furthermore, increases in money supply, suggested, for instance, in Clouse et al. (2003) and in Orphanides and Wieland (2000), for a long enough period that exceeds the duration of the liquidity trap, creates problems of credibility. While short term interest rates might be zero, long term interest rates might be strictly positive (this has been true of Japan during its deflationary experience). Hence, several authors such as Bernanke (2002) and Auerbach and Obstfeld (2005) have suggested open market operations in long term bonds. However, moving the long run yield curve on securities is confounded by the presence of the risk premium term whose behavior in a liquidity trap is not well known. A carry tax on money, suggested by Buiter and Panigirtzoglu (2003), works in theory but substantial practical problems of implementation are likely.

[4] Central bank independence has other benefits. For example, it shields monetary policy from political interference and allows the delegation of policy to the most competent experts etc.

several recent works on the Japanese experience[5] we cull out the following three stylized facts, S1-S3, about the Japanese experience which, we believe, must be respected, at the minimum, by any model of a liquidity trap that makes economic policy prescriptions.[6]

S1. *Fiscal policy is potent in a liquidity trap*: Whenever the net fiscal stance was expansionary, it worked well and the deficit spending multipliers were significant, and large. However, the net fiscal stance was typically contractionary or neutral.[7]

S2. *The response to a liquidity trap is inadequate if the Treasury or the Central Banks are not given appropriate targets*: There were no explicit inflation or output targets given to either the Treasury or to the Central Bank. So, the fiscal stance was typically contractionary, or neutral, and the monetary policy reaction was too little, too late.

S3. *Lack of coordination between the Treasury and the Central Bank hampers policy response in a liquidity trap*: The Treasury and the Central Bank may disagree about an appropriate response and also differ about their respective spheres of responsibility, if not well defined. They might end up taking policy actions that are counter to each other or cancel each other out and so, on net, are not expansionary.[8]

Furthermore, models of a liquidity trap must make the following modelling choices, M1-M2.

M1. Models belong to either one of two categories. In the first category are models with an *ex-ante perspective* that recognize the possibility of a liquidity trap in the future. The focus of such models is on the optimal design of institutions in order to reduce/eliminate the consequences arising from a liquidity trap. In the second category are models with an *ex-post perspective* that propose policy solutions, conditional on an economy having already slipped into a liquidity trap.

M2. Economics recommends aiming for an optimal level of 'economic bads' (e.g., externalities), rather than their complete elimination. Clearly, the optimal policy must balance the marginal cost of removal of the economic bads against the marginal benefits. In the context of a liquidity trap, the optimal policy recommendation might, therefore, turn out to be one that lets the economy fall into a liquidity trap with some probability rather than completely eliminating the possibility of a liquidity trap.[9]

[5] See, for instance, Posen (1998), Kuttner and Posen (2001), Iwamura et al. (2005) and Ball (2005).

[6] See the working paper version of the paper, Dhami and al-Nowaihi (2007), for the details.

[7] See, for instance, Posen (1998), Kuttner and Posen (2001) and Eggertsson (2006b).

[8] See, for instance, the empirical evidence in Iwamura et al (2005) and Eggertsson (2006b).

[9] A dental analogy might be appropriate here. Tooth decay can be prevented by extracting all the child's teeth. But, normally, the optimal policy is not to extract; tooth decay then occurs with some probability.

1.3. Our approach to modeling a liquidity trap

Our model of a liquidity trap takes explicit account of all the three stylized facts, S1, S2, S3. We consider strategic interaction between monetary and fiscal authorities in a simple aggregate supply - aggregate demand model similar to the one in Dixit and Lambertini (2003) and Lambertini and Rovelli (2003) but extended to allow for a liquidity trap and the effect of inflationary expectations in the aggregate supply curve. Our perspective is *ex-ante*, i.e., we focus on the design of institutions, namely, the appropriate incentives for the Treasury and the Central Bank, prior to the occurrence of a liquidity trap (M1). In our model, we find it optimal to allow the economy to fall into a liquidity trap with some probability (M2).

Issues of *strategic policy interaction* between monetary and fiscal authorities are completely ignored by the theoretical work on the liquidity trap. Typically, the only policy considered is monetary policy and so issues of strategic interaction do not arise[10]. On the other hand, when several policies are simultaneously considered, their strategic interaction is not allowed for.[11] One strand of the literature considers policies that could mitigate the effects of liquidity traps. The other strand prescribes policies that would prevent the economy from ever falling into a liquidity trap.[12] Some papers in the literature take an ex-ante perspective, while others take an ex-post perspective.[13].

1.4. Some results and intuition

As pointed out above, Krugman identified the solution to a liquidity trap as creating high enough inflationary expectations. However, under discretion, promises of high inflation will not be believed. This is because, outside a liquidity trap, the correct value for the real interest rate can be achieved more cheaply with zero inflation. Therefore, if the economy turns out not to be liquidity trapped, the Treasury has an incentive to renege on its promise

[10] Examples are Krugman (1998), Eggerston and Woodford (2003), Nishiyama (2003), Clouse et al. (2003), Buiter- Panigirtzoglou (2003), and Auerbach and Obstfeld (2005). Ball (2005) considers fiscal policy alone.

[11] Examples include (1) monetary and fiscal policy in Benhabib, Schmitt-Grohe and Uribe (2002), Iwamura et al. (2005), and (2) monetary and exchange rate policy in Orphanides and Wieland (2000), McCallum (2000) and Svensson (2003). Bernanke (2002) considers both monetary and fiscal policy but there is no theoretical analysis.

[12] In the first group are Krugman (1998), Eggertsson and Woodford (2003), Orphanides and Wieland (2000), McCallum (2000), and Svensson (2003). In the second group are Benhabib, Schmitt-Grohe and Uribe (2002), Nishiyama (2003), Clouse et al. (2003), Buiter- Panigirtzoglou (2003), and Auerbach and Obstfeld (2005).

[13] In the first group are Krugman (1998), Eggertsson and Woodford (2003), Benhabib, Schmitt-Grohe and Uribe (2002), Shin-Ichi (2003), Clouse et al. (2003), Buiter- Panigirtzoglou (2003). In the second group are papers by Ball (2005), Auerbach and Obstfeld (2005). Finally there are papers that touch on both ex-ante and ex-post issues, for instance, Orphanides and Wieland (2000), McCallum (2000), Bernanke (2002), Svensson (2003).

of high inflation. A rational forward looking private sector will anticipate this. The result is low inflation expectations, keeping the real interest rate too high in a liquidity trap. Notice that unlike the standard analysis conducted in the absence of a liquidity trap, the discretionary outcome can be suboptimal relative to the precommitment outcome because it creates *too little* inflation (Eggertsson (2006a,b) calls this the deflation bias).

We suggest an institutional solution, the *optimal delegation regime*, that achieves the optimal rational expectations precommitment solution for all parameter values in our model. This regime has three components. First, the Treasury acts as the Stackelberg leader and the Central Bank as the follower. Second, an inflation target is given to a Central Bank who has exclusive control over monetary policy. Outside a liquidity trap, where monetary policy is effective, the Treasury would rather not use the relatively more costly fiscal stabilization policy, leaving the Central Bank to perform the stabilization function. Because the Central Bank is operationally independent and its sole objective is achieving monetary stability, this type of delegation provides a commitment to the necessary inflation level when the economy is not in a liquidity trap. Our third component is to give the Treasury, who retains control of fiscal policy, something like a *Taylor rule*, which penalizes deviations of output from an output target and inflation from an inflation target. This gives the Treasury the correct incentive to undertake the appropriate (but costly) fiscal stimulus in a liquidity trap where monetary policy is ineffective. Consequently, inflation expectations are at the right level to produce the correct value for the real interest rate in a liquidity trap.

Furthermore, the *optimal delegation regime* achieves the optimal mix between monetary and fiscal policy as we now explain. Theoretically, society could give a sufficiently high inflation target to the Central Bank which in turn generates sufficiently high inflation expectations so that the nominal interest rate never hits its zero floor (see modelling choice M2, above). While this policy would always avoid the liquidity trap, it is not optimal because inflation is costly. Analogously, it is not optimal to give the Treasury too high an output target because if a liquidity trap occurs, it would use the costly fiscal policy excessively. The optimal solution then is to have a mix of both, i.e., some inflation outside a liquidity trap and some dependence on fiscal policy in a liquidity trap.

The first best is achieved if one could remove the distortions that cause the liquidity trap. The second best obtains with the optimal rational expectations commitment solution. The third best is achieved with various institutional design features introduced into policy making. The fourth best obtains under discretion.[14] It is well known that, in the absence of a liquidity trap, 'optimal institution design', such as Walsh contracts, can achieve the second best.[15] Our suggested institutional design achieves the second best in the presence

[14] See, for example, Rogoff (1985).
[15] See Walsh (1995) and al-Nowaihi and Levine (1996).

of a liquidity trap; see section 4, below.

1.5. Schematic outline

The model is formulated in Section 2. Section 3 derives the two benchmark solutions: the *optimal rational expectations precommitment solution* and the *discretionary solution*. Section 4 derives the *optimal delegation solution*. Section 5 concludes. Proofs can be found in the appendix.

2. Model

We describe the most parsimonious version of the model in this paper. At substantial costs in terms of complexity, and much reduced pedagogical clarity, the model can be shown to be robust with respect to the following extensions.[16] The full set of parameters in the aggregate demand and supply curves, persistent demand shocks, a general probability distribution over the two states of nature, and further considerations about the Treasury's objectives.

2.1. Aggregate Demand and Aggregate Supply

We use an aggregate demand and supply framework that is similar to recent work on strategic monetary-fiscal policy interaction, e.g., Ball (2005), Dixit and Lambertini (2003) and Lambertini and Rovelli (2003). The aggregate demand and supply equations are given by, respectively[17]

$$AD : y = f - (i - \pi^e) + \epsilon. \tag{2.1}$$

$$AS : y = \pi - \pi^e, \tag{2.2}$$

where y is the deviation of output from its natural rate and f captures fiscal policy[18]. For example, $f > 0$ could denote a fiscal deficit (either debt financed or money financed[19]) while $f < 0$, denotes a fiscal surplus. But f could also denote a temporary balanced

[16] See the working paper version, Dhami and al-Nowaihi (2007), for a full proof of these claims. Some of the expressions run into more than a page and so are not convenient to print.

[17] Extension to a full set of parameters for the AS and AD curves is possible at substantial increase in complexity; see Dhami and al-Nowaihi (2007).

[18] To be more precise, f is the *stabilization* component of fiscal policy (which varies over the business cycle). *Total* fiscal policy can be represented by $F = f_0 + f$, where f_0 denotes the *fixed* fiscal commitments of the government.

[19] In principal, these alternative modes of finance need not be equivalent. However, in the context of a liquidity trap, Ball (2005) shows that there are no long run differences arising from these alternative modes of finance.

budget reallocation of taxes and subsidies that has a net expansionary effect; for instance Dixit and Lambertini (2000). $i \geq 0$ is the nominal interest rate, π is the rate of inflation, π^e is expected inflation and, in keeping with the modern literature on the liquidity trap, ϵ is a demand shock[20]. We assume rational expectations on behalf of the public. So, if x is a random variable and $E(x)$ is its mathematical expectation, then the expectation, x^e, of x formed by a decision maker is

$$x^e = E\left(x\right). \tag{2.3}$$

The instruments of policy are the nominal interest rate, i, and the stabilization component of fiscal policy, f.

For simplicity, let the demand shock, ϵ, be independently distributed and take only two values, $a, -a$, with equal probability, where $a > 0$, hence[21]

$$E\left(\epsilon\right) = 0,\ Var\left(\epsilon\right) = a^2. \tag{2.4}$$

The aggregate demand equation reflects the fact that demand is increasing in the fiscal impulse, f, and decreasing in the real interest rate, $r = i - \pi^e$. Demand is also affected by demand shocks. The aggregate supply equation shows that deviations of output from the natural rate are caused by unexpected movements in the rate of inflation.

Equating aggregate demand and supply we get from (2.1) and (2.2), our reduced form equations for output and inflation.

$$y = f - i + \pi^e + \epsilon. \tag{2.5}$$

$$\pi = f - i + 2\pi^e + \epsilon. \tag{2.6}$$

Hence, fiscal policy, monetary policy and inflation expectations (in the spirit of New Keynesian models) have an affect on output (and so also on unemployment) and inflation.[22]

2.2. Notation and the rational expectations condition

We shall write a variable with a subscript (sometimes a superscript) 'H', for example, y_H and U_T^H, to denote the realization of these variables in the good (or high) state of the

[20] Our framework can be easily extended to incorporate supply, as well as demand, shocks.

[21] The results do not depend on the demand shock taking only two possible values, although it allows for a particularly clear exposition. The results hold for a general probability distribution over demand shocks that, furthermore, exhibit persistence, see Dhami and al-Nowaihi (2007)

[22] The analogy with New Keynesian models is quite strong. There are no structural dynamics in our model. Hence, if policy rules are also stationary (as they will be in our model), then $\pi_t^e = \pi_{t+1}^e$. The AS-curve (2.2) can then be written as $y_t = \pi_t - \pi_{t+1}^e$, i.e., the expected change in the inflation rate is related to the output gap, as in the New Keynesian Phillips curve. Similarly, the AD-curve (2.1) can be written in a form reminiscent of the New Keynesian IS-curve. We are grateful to Referee 2 for these observations.

world, $\epsilon = a$. Analogously, to denote the realization of the same variables in the bad (or low) state of the world, $\epsilon = -a$, we use a subscript (sometimes a superscript) 'L', for example, y_L and U_T^L. Since the states $\epsilon = a$ and $\epsilon = -a$ occur with equal probability, the average or expected value of any stochastic variable, x, is $E(x) = \frac{1}{2}x_L + \frac{1}{2}x_H$. The rational expectations assumption (2.3) becomes $x^e = \frac{1}{2}x_L + \frac{1}{2}x_H$. In particular,

$$\pi^e = \frac{1}{2}\pi_L + \frac{1}{2}\pi_H. \tag{2.7}$$

From (2.6) we get $\pi_L = f_L - i_L + 2\pi^e - a$ and $\pi_H = f_H - i_H + 2\pi^e + a$. Substituting into (2.7) gives $\pi^e = \frac{1}{2}(f_H + f_L) - \frac{1}{2}(i_H + i_L) + 2\pi^e$ and, hence,

$$\pi^e = \frac{1}{2}(i_H + i_L) - \frac{1}{2}(f_H + f_L). \tag{2.8}$$

2.3. Microfoundations

Our model is inspired by the microfounded dynamic model of monopolistic competition and staggered price setting in Dixit and Lambertini (2000, 2003). Our structural model in (2.1), (2.2) (or its variant with the full set of parameters in Dhami and al-Nowaihi, 2007) is similar to the Dixit and Lambertini model.[23] In the Dixit and Lambertini framework, unexpected movements in inflation have real effects because prices are staggered. Alternatively, a range of 'rational inattention' theories currently compete as potential explanations for the presence of the unexpected inflation term in (2.2). For instance, see Sims (2003).[24]

To simplify the dynamic *game-theoretic* analysis Dixit-Lambertini follow the tradition, established in the time-inconsistency literature[25], of abstracting from *structural* dynamic issues, notably, capital formation, the term structure of interest rates, exchange rate policy and the financing of the stabilization component of fiscal policy. Concentrating on the aggregate demand consequences of investment expenditure, but abstracting from its

[23] However, our model has the following differences from the Dixit-Lambertini model. (1) If the natural rate of output is normalized to zero, then, the additive shock ϵ (in (2.1) or in (2.5)) can also be interpreted as a shock to the natural rate of output. (2) Our model has the New Keynesian feature that expected inflation, π^e, also affects actual inflation, π. (3) Our stochastic structure allows persistence (see Dhami and al-Nowaihi, 2007). While there is no persistence in Dixit-Lambertini, they allow all parameters to be stochastic, hence, considering the possibility of non-additive shocks. (4) In our model, a fiscal impulse acts on the demand side, creating greater output and inflation. However, in the Dixit-Lambertini model, fiscal policy works on the supply side and takes the form of a subsidy to imperfectly competitive firms that increases output but reduces prices.

[24] Most dynamic structural models used in the analysis of a liquidity trap are forward looking New Keynesian models. Gertler (2003), Mankiw (2002) note dissatisfaction with this model in terms of its inability to explain persistence in the data. Recent work, for instance, Rudd and Whelan (2006), casts doubt even on the hybrid variant proposed by Gali and Gertler (1999). Of course, similar criticisms apply to the version of our model microfounded along the lines of Dixit and Lambertini (2003). Thus, all current macroeconomic models lack satisfactory microfoundations.

[25] See, for example, Romer (2006, chapter 10) and Walsh (2003, chapter 8).

contribution to growth, is standard in models of the business cycle, and is, as far as we know, a feature of all existing models of the liquidity trap.

Eggertsson and Woodford (2003), in a structurally dynamic model of monetary policy with a financial sector and a zero lower bound on interest rates, show that the short-run interest rate (which is the instrument of policy) determines all other interest rates and exchange rates. As they clearly explain, open market operations only work to the extent that they enhance the credibility of policy. Thus, and in common with many models, we take the short-run interest rate as directly affecting aggregate demand and we abstract from open economy aspects.

We offer two arguments that mitigate not explicitly modelling the government budget constraint. First, we shall assign a higher welfare loss to the use of fiscal policy relative to monetary policy. The cost of using fiscal policy could include deadweight losses, costs of servicing debt and a risk premium for default. Second, in all equilibria of our model, fiscal policy is not used for stabilization purposes outside a liquidity trap. In a calibrated model of Japan, Ball (2005) shows that the combination of higher output, higher tax revenues and higher inflation outside the liquidity trap is more than adequate to finance the extra fiscal spending during the liquidity trap. See subsection 4.3, below, for further discussion of this issue.

2.4. Social Preferences

Society's preferences over output and inflation are given by the social welfare function,

$$U_S = -\frac{1}{2}\left(y - \overline{y}\right)^2 - \frac{1}{2}\pi^2 - f^2. \tag{2.9}$$

The first term shows that departures of output from its desired level, $\overline{y}$ (note that $\overline{y}$ is the difference between desired output and the natural rate), are costly. We assume that

$$\overline{y} \geq 0. \tag{2.10}$$

This captures the fact that the natural level of output is socially suboptimal (unless $\overline{y} = 0$)[26].

The second term in (2.9) makes the standard assumption that inflation reduces social welfare. The third term captures the fact that the exercise of fiscal policy is more costly

[26] The microfoundations for this in Dixit and Lambertini (2000, 2003) rest on the presence of monopolistic competition. Monopoly power in the product market reduces output below the efficient level, hence, giving policy makers an incentive to raise output. There are also a large number of other well known reasons for (2.10) but the ultimate cause, argue Alesina and Tabellini (1987), is the absence of non-distortionary taxes. For if they were available then other market failures could be corrected.

than that of monetary policy[27]. We model this as imposing a strictly positive cost of fiscal policy, f^2, but no cost of using the monetary policy[28]. The cost of using fiscal policy could include deadweight losses, as in Dixit and Lambertini (2003), costs of servicing debt and a risk premium for default.[29]

From (2.9), we see that the first best obtains when $\pi = 0$, $f = 0$, and $y = \overline{y}$. However, from (2.1) and (2.2), it follows that this cannot be an outcome of a rational expectations equilibrium (unless $\overline{y} = 0$). On the microfoundations of the social welfare function, in (2.9), see Dixit and Lambertini (2000, 2003), Rotemberg and Woodford (1999).

Substituting (2.5) and (2.6) into (2.9), we get

$$U_S = -\frac{1}{2}\left(f - i + \pi^e + \epsilon - \overline{y}\right)^2 - \frac{1}{2}\left(f - i + 2\pi^e + \epsilon\right)^2 - f^2. \tag{2.11}$$

In the light of subsection 2.2, and using (2.11), the expected social welfare is

$$\begin{aligned} E\left(U_S\right) = &\frac{1}{2}\left[-\frac{1}{2}\left(f_H - i_H + \pi^e + a - \overline{y}\right)^2 - \frac{1}{2}\left(f_H - i_H + 2\pi^e + a\right)^2 - f_H^2\right] \\ &+\frac{1}{2}\left[-\frac{1}{2}\left(f_L - i_L + \pi^e - a - \overline{y}\right)^2 - \frac{1}{2}\left(f_L - i_L + 2\pi^e - a\right)^2 - f_L^2\right]. \end{aligned} \tag{2.12}$$

Substituting (2.8) in (2.12), the expected social welfare becomes

$$\begin{aligned} E\left(U_S\right) = &-\frac{1}{4}\left[\frac{1}{2}\left(f_H - f_L\right) - \frac{1}{2}\left(i_H - i_L\right) + a - \overline{y}\right]^2 - \frac{1}{4}\left(i_L - f_L + a\right)^2 - \frac{1}{2}f_H^2 \\ &-\frac{1}{4}\left[\frac{1}{2}\left(i_H - i_L\right) - \frac{1}{2}\left(f_H - f_L\right) - a - \overline{y}\right]^2 - \frac{1}{4}\left(i_H - f_H - a\right)^2 - \frac{1}{2}f_L^2. \end{aligned} \tag{2.13}$$

2.5. Sequence of Moves

In the first stage, the economy designs its institutions, which assign to one or two independent policy makers (i.e., the Treasury and the Central Bank) their respective domains of decision making. This is followed by the formation of inflationary expectations, π^e, and the signing of nominal wage contracts in anticipation of future inflation. Next, the demand shock, ϵ, is realized. Conditional on the actual realization of the shock, the relevant policy makers then decide on the optimal values of the policy variables, f and i.

[27] Fiscal policy is typically more cumbersome to alter, on account of the cost of changing it (balanced budget requirements, lobby groups etc.). Indeed the 'monetary policy committee' in the UK or the 'Fed' in the USA meet on a regular basis to make decisions on the interest rate while changes to the tax rates are much less frequent.

[28] Strictly speaking, for our qualitative results to hold, we only require that fiscal policy be relatively more expensive than the (possibly strictly positive) cost of using monetary policy. Normalizing the cost of using monetary policy to zero, however, ensures greater tractability and transparency of the results.

[29] See Eser et al. (2009) for a microfounded discussion of why fiscal policy is a relatively more costly stabilization instrument.

We shall also derive the *optimal rational expectations solution* (precommitment benchmark) in which the last stage is conducted up-front, i.e., the (state contingent) policy variables f and i are announced to the economy prior to the resolution of demand uncertainty.

3. The Precommitment and Discretionary Solutions

We first calculate the globally optimal solution in the class of all rational expectations solutions in subsection 3.1[30]. Strictly speaking, this is a second best solution. The first best obtains if the imperfections responsible for the liquidity trap are removed. But because it is only second best, there is 'room for improvement'. Specifically, once the public has formed its expectations of inflation, the government has the incentive to launch a 'surprise inflation' or, in the case of a liquidity trap, a 'surprise deflation', that is potentially welfare improving. This destroys the credibility of the optimal solution in the eyes of a rational public. Thus, if the government retains control over monetary policy (the discretionary regime), the outcome is even worse (third best). This is considered in subsection 3.2, below. However, the global optimality of the precommitment solution serves as a useful benchmark.

3.1. The Precommitment Regime (The optimal rational expectations solution)

The sequence of moves is described in figure 3.1, below.

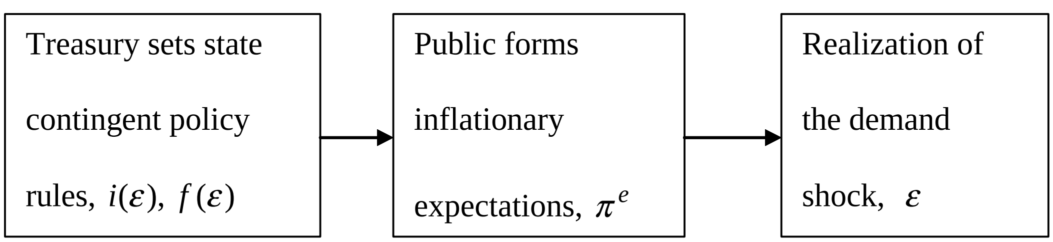

Figure 3.1: **Sequence of moves for the precommitment regime**

The solution method is to find state contingent rules for the policy variables, (i_L, f_L), (i_H, f_H), that maximize the expected value of the social welfare (2.13). The results are summarized in Proposition 1.

[30] It is variously referred to as the 'precommitment solution', the 'optimal rational expectations solution', the 'second best solution' or simply the 'optimal solution'.

Proposition 1 *: Under the optimal state-contingent rational expectations precommitment solution, the economy is always liquidity trapped when $\epsilon = -a$ but not when $\epsilon = a$. The expected utility in the precommitment regime is given by $E\left(U_{S\leq}^{Opt}\right) = -\frac{1}{5}a^2 - \frac{1}{2}\overline{y}^2$. The full solution is given in the Table below.*

	$\epsilon = -a < 0$	$\epsilon = a > 0$	$\epsilon^e = -0.5a + 0.5a = 0$
Interest rate	$i_L = 0$	$i_H = 6a/5$	$i^e = 3a/5$
Fiscal policy	$f_L = 2a/5$	$f_H = 0$	$f^e = a/5$
Output	$y_L = -a/5$	$y_H = a/5$	$y^e = 0$
Inflation	$\pi_L = a/5$	$\pi_H = 3a/5$	$\pi^e = 2a/5$
Real interest rate	$i_L - \pi^e = -2a/5$	$i_H - \pi^e = 4a/5$	$i^e - \pi^e = a/5$

Since the economy is liquidity trapped when $\epsilon = -a$, monetary policy is ineffective in this case, $i_L = 0$. Hence, the government resorts to using the expensive fiscal policy, $f_L = \frac{2}{5}a$, in order to 'lean against the wind'. By contrast, when $\epsilon = a$, monetary policy is effective, $i_H = \frac{6}{5}a$, and the government has no need for the expensive fiscal instrument, so $f_H = 0$.[31]

Recalling that y denotes deviation of output from the natural level, output is below the natural rate in the liquidity trap ($\epsilon = -a$) but above it otherwise ($\epsilon = a$). On average, it equals the natural rate. Inflation is positive in both states of the world. The real interest rate is negative[32] in the liquidity trap but positive otherwise and on average.

Recall that the shocks take the two possible values, $-a$ and a and, so, $Var(\epsilon) = a^2$. Thus, on average, ceteris paribus, inflation, interest rates and the fiscal instrument of the government will display greater variability in economies where demand shocks exhibit greater variability and precommitment is possible. Furthermore, the magnitude of policy instruments employed in the two states of the world, $f_L = \frac{2}{5}a$ and $i_H = \frac{6}{5}a$, are increasing in the size of the shock. This is not surprising as each of these policies fulfills a stabilization role and a larger shock elicits a greater effort in "leaning against the wind".

The solution is independent of $\overline{y}$, society's desired output relative to the natural rate. As in time consistency models in the absence of the liquidity trap, this occurs because even if society has a high $\overline{y}$, the precommitment technology allows it to counter expectations of ex-post surprise inflation (designed to push output towards the high target).

The magnitude of social welfare in this regime depends negatively on the variance of shocks hitting the economy, a^2, and negatively on the output target of society, $\overline{y}$.

Finally, note that the values of the instruments, i_H, i_L, f_H, f_L, are optimal *ex-ante*. However, after the realization of the shock, $\epsilon = -a$ or $\epsilon = a$, the *ex-post* optimal values

[31] Recall that f refers only to the stabilization component of fiscal policy, hence, $f_H = 0$ is consistent with a strictly positive level of government expenditure on other items such as redistribution etc.

[32] We conjecture that the combination of rigid wages-prices and a flexible nominal interest rate has the effect that the real interest rate, $i - \pi^e$, overshoots so as to equilibrate the economy.

of i, f will, in general, be different from these. Thus, for successful implementation, this optimal rational expectations solution needs a precommitment technology. We discuss this in Section 4 below. Next, we turn to the second regime in the paper, discretion.

3.2. Discretionary Regime

In the discretionary regime, the monetary instrument, i, and the fiscal instrument, f, are both assigned to the Treasury. We calculate the time consistent discretionary policy. The sequence of moves is described in figure 3.2, below.

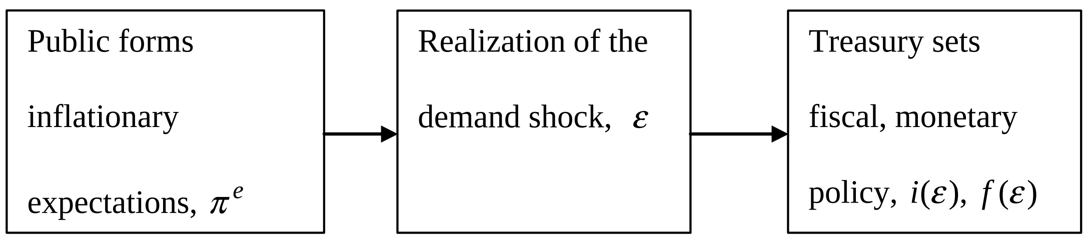

Figure 3.2: **Sequence of moves when Treasury controls i, f.**

To find the discretionary solution, we first need to find the values of the policy variables $i_L(\pi^e)$, $f_L(\pi^e)$ and $i_H(\pi^e)$, $f_H(\pi^e)$ that maximize social welfare (2.11) conditional on given π^e, ϵ. This allows the computation of the state-contingent inflation rates $\pi_L(\pi^e)$ and $\pi_H(\pi^e)$. Then we find the fixed-point π^e by solving

$$\pi^e = \frac{1}{2}\pi_L(\pi^e) + \frac{1}{2}\pi_H(\pi^e). \tag{3.1}$$

Finally, substitute the value for π^e back into the state-contingent policy variables $i_L(\pi^e)$, $f_L(\pi^e)$ and $i_H(\pi^e)$, $f_H(\pi^e)$ to find the solution under discretion. Depending on the parameter values, a liquidity trap may, or may not, arise. Proposition 2 below summarizes the results when a liquidity trap arises, which is the focus of this paper[33].

Proposition 2 : *For $\overline{y} < a \leq 2\overline{y}$, the economy is liquidity trapped for $\epsilon = -a < 0$ but not liquidity trapped for $\epsilon = a > 0$. The expected social welfare is given by $E\left(U_S^{Disc}\right) = 12a\overline{y} - 8\overline{y}^2 - 5a^2$. The full solution under discretion is given in the Table below.*

[33] The full set of results under discretion is given in Appendix-B of Dhami and al-Nowaihi (2007).

	$\epsilon = -a < 0$	$\epsilon = a > 0$	$\epsilon^e = -0.5a + 0.5a = 0$
Interest rate	$i_L = 0$	$i_H = 4\overline{y} - 2a$	$i^e = 2\overline{y} - a$
Fiscal policy	$f_L = 2(a - \overline{y})$	$f_H = 0$	$f^e = a - \overline{y} > 0$
Output	$y_L = \overline{y} - a < 0$	$y_H = a - \overline{y}$	$y^e = 0$
Inflation	$\pi_L = 4\overline{y} - 3a$	$\pi_H = 2\overline{y} - a$	$\pi^e = 3\overline{y} - 2a$
Real interest rate	$i_L - \pi^e = 2a - 3\overline{y}$	$i_H - \pi^e = \overline{y}$	$i^e - \pi^e = a - \overline{y}$

For stabilization purposes, the costly fiscal policy is used only in a liquidity trap when the monetary policy looses effectiveness. As in the precommitment solution, deviations of output from the natural rate are zero on average, i.e., $y^e = 0$. The following proposition compares expected social welfare under *Precommitment* with that under *Discretion.*

Proposition 3 *: For $\overline{y} < a \leq 2\overline{y}$, $E\left(U_S^{Opt}\right) - E\left(U_S^{Disc}\right) = \frac{3}{10}\left(5\overline{y} - 4a\right)^2 \geq 0$.*

As one would expect, the presence of a liquidity trap does not alter the ranking between the Precommitment and the Discretion regimes, from a social welfare point of view. In fact, the ranking $E\left(U_S^{Disc}\right) < E\left(U_S^{Opt}\right)$ holds for all parameter values. However, we have only reported it for the most interesting case, namely, when a liquidity trap occurs.

3.3. Alice through the looking glass

Krugman (1998) observed that 'applying conventional modelling to liquidity trap conditions produces unconventional conclusions and policy recommendations'. To which he added (1999) 'The whole subject of the liquidity trap has a sort of Alice-through-the-looking-glass quality'. And indeed, our model exhibits these features, as we will now see.

3.3.1. Precommitment can have higher inflation than Discretionary

It is well known in the traditional time inconsistency literature, in the absence of a liquidity trap, that the optimal level of average inflation is zero (given (2.9)) while under discretion it is positive (unless $\overline{y} = 0$, in which case it is also zero). The reason is that under discretion, agents perceive (correctly) that the government has an ex-post incentive to create surprise inflation, while under precommitment, ex-post surprise inflation is institutionally ruled out. When a liquidity trap occurs with a positive probability, this result changes dramatically.

From Proposition 1, we see that the optimal level of average inflation under precommitment now is positive ($\pi^e = 2a/5$), rather than zero. Under discretion, π^e depends on $\overline{y}$. For $\overline{y} = a/2$, Proposition 2 gives a negative average expected inflation rate ($\pi^e = -a/2$), rather than a positive one. Eggertsson (2006a, 2006b) calls this the deflation bias**.**

The intuitive explanation is as follows. Under precommitment, it is optimal to have positive inflation on average ($\pi^e = 2a/5$), despite its cost, to be able to deliver negative real interest rates ($i_L - \pi^e = -2a/5$) in the bad state of the world ($\epsilon = -a$). However, this optimal policy is time inconsistent. If ex-post, the economy is in the good state ($\epsilon = a$) then the optimal real interest rate is positive ($i_H - \pi^e = 4a/5$) which can be achieved more cheaply with zero inflation. Hence, the policy maker has the incentive to renege on its commitment to positive inflation. The rational private sector will perceive this and expect low future inflation. This destroys the credibility of the announcement of high inflation, unless a commitment technology is available.

3.3.2. Higher output targets are a good thing

In the standard textbook model in the absence of a liquidity trap, a higher value of desired output relative to the natural rate, $\overline{y} > 0$, is bad because it leads to high inflation and no gain in output ($y^e = 0$). The reverse occurs with a liquidity trap. $\overline{y} > 0$ is now good! The intuition is that a higher $\overline{y}$ increases inflationary expectations (see Proposition 2), which by reducing the real interest rate in a liquidity trap, reduces the need for using the expensive fiscal instrument.

If society has a high enough output target (and the Treasury follows it) then, in the discretionary regime, ex-post, a liquidity trap will not arise. However, this outcome might require using the costly fiscal instrument excessively, which could be suboptimal. In section 4, below, we show this to be precisely the case.

4. Institutions and Delegation

In the delegation regime considered in this section, society gives the Central Bank the mandate of achieving an inflation target π_B. The monetary instrument, which is the nominal interest rate, i, is assigned to the Central Bank whose objective is to attain the inflation target π_B. We formalize this by assigning the following objective function to the Central Bank,

$$U_B = -\frac{1}{2}\left(\pi - \pi_B\right)^2. \tag{4.1}$$

The fiscal instrument, f, is controlled by the Treasury whose objective function is similar to that of society in (2.9) but with, possibly, different inflation and output targets,

$$U_T = -\frac{1}{2}\left(y - y_T\right)^2 - \frac{1}{2}\left(\pi - \pi_T\right)^2 - f^2 \ . \tag{4.2}$$

The output target of the Treasury is given by y_T. It is important to bear in mind the difference between the socially desirable output level, $\overline{y}$, and the Treasury's output target, y_T. In order to maximize the social objective function in (2.9), society might assign some target value, y_T^*, of y_T to the Treasury. The Treasury is assumed to comply with the target

y_T^* even if $y_T^* \neq \overline{y}$. It seems reasonable, though not necessary, to assign the same inflation target to the Treasury, $\pi_T = \pi_B$, as assigned to the Central Bank.[34]

4.1. The Optimal Delegation Regime

Under optimal delegation, the game has the following five *sequential* stages.

1. Society optimally assigns an inflation target, π_B, to the Central Bank and the output, inflation targets, y_T, π_T to the Treasury.

2. The public observes π_B, y_T, π_T and then forms inflationary expectations, π^e.

3. The demand shock, ϵ, which takes two possible values, $a, -a$, is realised.

4. The Treasury (Stackelberg leader) sets state contingent fiscal policy $f(\pi^e, \epsilon)$.

5. The Cental Bank (Stackelberg follower) sets state contingent monetary policy $i(\pi^e, \epsilon)$.

Given the sequence of moves, the Central Bank sets monetary policy taking as given the fiscal policy set by the Treasury. The Treasury sets fiscal policy, taking into account the anticipated response of the Central Bank. We solve the game backwards. First we obtain the Central Bank's optimal reaction function $i = i(\pi_B, \pi^e, f, \epsilon)$ by maximizing U_B. Second, we find the Treasury's optimal reaction function $f = f(y_T, \pi_T, \pi^e, \epsilon)$ by maximizing U_T. This allows us to derive output and inflation as functions of y_T, π_T, π_B, π^e, ϵ. Third, we determine π^e, assuming rational expectations on the part of the private sector. Fourth, we find the expected social welfare, EU_S, as a function of y_T, π_T, π_B, which we maximize to find the optimal values of y_T, π_T, π_B, denoted by y_T^*, π_T^*, π_B^*. We assume that the Treasury and Central Bank fully comply with the objectives assigned to them. We now state our main result in Proposition 4, i.e., the optimal delegation regime achieves the optimal rational expectations solution.

Proposition 4 *: Assume that monetary policy is delegated to an independent Central Bank with inflation target $\pi_B^* = \frac{3}{5}a$. Fiscal policy is retained by the Treasury with output target $y_T^* = \frac{1}{5}a$. The Treasury acts as the Stackelberg leader. Then the optimal rational expectations (precommitment) solution (see Proposition 1) is achieved. Society's expected utility in the optimal delegation regime is given by $E\left(U_S^{OD}\right) = -\frac{1}{5}a^2 - \frac{1}{2}\overline{y}$. The economy is liquidity trapped only under adverse demand shocks. Inflation and output targets are achieved in the good state but not in the bad state.*[35]

[34] Our analysis allows one to consider both the cases: $\pi_T = \pi_B$ and $\pi_T \neq \pi_B$.

[35] As stressed by Eggertsson and Woodford (2003), failure to meet the inflation target in the liquidity trap does not signify failure of policy. A similar remark can be made with respect to the output target.

The next proposition shows that our suggested institutional solution is very flexible in terms of the mixture between optimal output and inflation contracts for the Treasury. We discuss this proposition in section 5 below.

Proposition 5 *: Any mixture of output and inflation targets for the Treasury, y_T, π_T, that satisfy $y_T + \pi_T = \frac{4}{5}a$, will achieve the optimal rational expectations (precommitment) solution (see Proposition 1 and 4), provided $\pi_B = \pi_B^* = \frac{3}{5}a$.*

4.2. Credibility[36]

There is an infinity of pairs of output-inflation targets for the Treasury that implement the optimal rational expectations (precommitment) solution. However, one particular pair in which the Central Bank's and Treasury's optimal inflation targets coincide, i.e., $(y_T^*, \pi_T^*) = (y_T^*, \pi_B^*) = \left(\frac{1}{5}a, \frac{3}{5}a\right)$ has special salience.

To see this, compare (y_T^*, π_B^*) with the other candidate in which output equals its natural level, i.e., $(y_T^*, \pi_T^*) = \left(0, \frac{4}{5}a\right)$.[37] Note the following features of the candidate $\left(0, \frac{4}{5}a\right)$: (1) $\left(0, \frac{4}{5}a\right)$ sets the Treasury an inflation target, $\frac{4}{5}a$, *greater* than that assigned to the Bank, $\pi_B^* = \frac{3}{5}a$. (2) The inflation target, $\frac{4}{5}a$, is *never* achieved (in any state of the world). (3) The output target, $y = 0$, is *only achieved on average* but neither when the economy is liquidity trapped ($\epsilon = -a$) nor when non-liquidity trapped ($\epsilon = a$). (4) It would appear strange to the public that the Treasury and Bank are assigned *different* inflation targets.

By contrast, the target $(y_T^*, \pi_B^*) = \left(\frac{1}{5}a, \frac{3}{5}a\right)$ has the following attractive features. (1) It is *achieved* during normal times, i.e., when the economy is not liquidity trapped (recall Proposition 1, which is implemented by these targets). (2) The Bank and the Treasury are assigned the *same* inflation target which is important for reasons of *transparency* and *credibility* of policy. For these reasons, we believe that $(y_T^*, \pi_B^*) = \left(\frac{1}{5}a, \frac{3}{5}a\right)$ is the superior choice from the set of all targets consistent with Proposition 5.[38]

Why does the optimal delegation regime perform so well? The inflation target given to the Central Bank provides a commitment to the necessary inflation level when the economy is not in a liquidity trap. In the absence of such a target, there will be an incentive to reduce inflation outside a liquidity trap. Such a commitment affects the (ex-ante) inflation expectations ensuring the correct value for the real interest rate in a liquidity trap. Furthermore, inflationary expectations are also influenced correctly by the output and inflation targets given to the Treasury. The latter provide the Treasure with the incentive to use the appropriate level of fiscal policy in a liquidity trap. Such an institutional regime achieves the optimal balance between fiscal and monetary policy by

[36] We are grateful to both referees for raising the issues discussed here.
[37] Note that both $(y_T^*, \pi_B^*) = \left(\frac{1}{5}a, \frac{3}{5}a\right)$ and $\left(0, \frac{4}{5}a\right)$ satisfy condition of Proposition 5.
[38] These issues are treated formally in al-Nowaihi and Levine (1996).

neither having to rely too much on costly inflation outside the liquidity trap nor relying too much on costly fiscal policy in a liquidity trap.

Dhami and al-Nowaihi (2007) show that these results generalize to more general probability distributions of the shocks, persistence in shocks, full set of parameters of the aggregate demand and aggregate supply curves and more general objectives of the Treasury. These generalizations serve to assure us that the results of this model do not arise because of the simplicity of the model. Rather, the results are more fundamental. However, the extensions come at a very substantial cost in terms of complexity. The simpler model is pedagogically much superior. While we allow for the economy to fall in a liquidity trap with some probability, this probability, in the more general model can be quite small.

4.3. Debt[39]

Recall, from Proposition 1, that for the optimal solution, fiscal policy is expansionary in a liquidity trap but neutral outside it. Hence, it is expansionary on average. An important question, therefore, relates to the implications for government debt. In particular, is this solution compatible with a bounded debt to GDP ratio? Since the optimal delegation regime implements the optimal solution (Proposition 4), the same issue arises for the optimal solution as well.[40] Our answer is three-fold.

First, as remarked in subsection 2.1, above, f could be a temporary balanced budget reallocation of taxes and subsidies that has a net expansionary effect; as in Dixit and Lambertini (2000).

Second, the total fiscal budget, F, could have two (or more) components: $F = f_0 + f + ...$, where f_0 is a *fixed* budget surplus (hence, in our notation, $f_0 < 0$) and f is the variable component that is positive in a liquidity trap but zero outside it. f_0 can then be chosen so that $E(f_0 + f) = 0$ which ensures that the government budget holds in expected terms; see Varian (1980).

Third, suppose that f is financed entirely from debt (and assume no surpluses or deficits are generated out of a liquidity trap). Then f is the magnitude of new nominal debt. Assume that nominal GDP is normalized to one and, so, f is also the ratio of new debt to GDP. Now let us look at inflation, π, relative to f. From Proposition 1, $f^e = a/5$ but $\pi^e = 2a/5$. Hence, on average, the new debt, arising from debt financing of the fiscal expansion in a liquidity trap, could (if desired) be financed entirely from inflation.

Thus, the fact that fiscal policy is, on average, expansionary under optimal delegation is consistent with a bounded debt to GDP ratio.

[39] We are grateful to Referee 2 for raising this issue.
[40] We are grateful to Referee 2 for raising this issue.

4.4. Related literature

The theoretical literature has considered some aspects of our *optimal delegation regime*, that achieves the precommitment solution. For instance, inflation targets have been suggested in Krugman (1998), Nishiyama (2003), and Iwamura et al. (2005). Other variants of monetary policy commitment have also been considered. Benhabib Schmitt-Grohe and Uribe (2002) consider a commitment to switch from an interest rate rule to a money growth rate peg in a liquidity trap. Eggertsson and Woodford (2003) propose a commitment to adjust nominal interest rates to achieve a time varying price level target. Bernanke (2002) suggests a commitment to a buffer zone for the inflation rate. Svensson (2003) advocates a price level target (as part of a larger set of policies). However, none of these models allow for the possibility of strategic interaction between monetary and fiscal authorities nor jointly derive the optimal set of targets and instruments of the two policy making authorities.

Eggertsson (2006a, 2006b) studies the liquidity trap within a new Keynesian stochastic general equilibrium model with a government budget constraint and explicit microfoundations. Eggertsson recommends abandonment of an independent central bank and a return to discretionary policy by a unitary monetary-fiscal authority. A debt financed fiscal expansion during a liquidity trap results, via the government budget constraint, in higher expectations of future inflation. Eggertsson shows that this solution is superior to either monetary policy alone or uncoordinated monetary and fiscal policy. However, as Eggertsson shows, even optimal discretion is inferior to the fully optimal rational expectations solution with commitment. Moreover, abandoning delegation of monetary policy to an independent central bank with a narrow mandate, in favor of a return to discretion, appears to be a retrograde step.

Dixit and Lambertini (2003) and Lambertini and Rovelli (2003) consider strategic interaction between fiscal and monetary authorities, but in the absence of a liquidity trap. Lambertini and Rovelli (2003) show that the equilibrium with the fiscal authority acting as leader is superior to the Nash equilibrium. Dixit and Lambertini (2003) show that this regime can achieve the optimal precommitment rational expectations solution.

5. Conclusions

In a liquidity trap, with nominal interest rates bound below by zero, an expectation of positive inflation is needed to reduce the real interest rate and, so, to boost economic activity. This, in turn, needs a credible commitment to a future level of positive actual inflation. The credibility problem comes about because after the economy has escaped from the liquidity trap, it is in the interest of all parties to renegotiate and reduce inflation. A forward looking private sector will anticipate this and expect low future inflation. With low expected future inflation, the real interest rate remains positive, keeping the economy in the liquidity trap; see for instance Krugman (1998).

The first best solution obtains when the rigidities that give rise to the liquidity trap are removed. But removal of these distortions is usually slow and difficult (witness the experience of Japan). Therefore, macroeconomic policy can play an important role. Furthermore, the Japanese experience suggests that issues of strategic monetary fiscal policy interaction and of appropriate institutional design, assume even greater importance in a liquidity trap.

In the solution considered in this paper, society delegates monetary policy to an operationally independent Central Bank with an inflation target. Fiscal policy is delegated to the Treasury with inflation and output targets. Furthermore, the Treasury acts as a leader and the Central Bank is the follower. The required institutional arrangements are quite natural and are able to achieve the second best solution, namely, the optimal rational expectations precommitment solution. This institutional setting provides (1) the appropriate level of inflation and, hence, inflation expectations, and (2) the optimal balance between monetary and fiscal policy.

6. Appendix

Proof of Proposition 1 (Precommitment)

We seek those values of f_L, f_H, i_L and i_H that maximize $E(U_S)$ (as given by (2.13)) subject to $i_H \geq 0$ and $i_L \geq 0$. The first order conditions for this are:

$$\frac{\partial E(U_S)}{\partial f_H} = 0, \tag{6.1}$$

$$\frac{\partial E(U_S)}{\partial f_L} = 0, \tag{6.2}$$

$$\frac{\partial E(U_S)}{\partial i_H} \leq 0,\ i_H \left(\frac{\partial E(U_S)}{\partial i_H}\right) = 0, \tag{6.3}$$

$$\frac{\partial E(U_S)}{\partial i_L} \leq 0,\ i_L \left(\frac{\partial E(U_S)}{\partial i_L}\right) = 0. \tag{6.4}$$

From (2.13), (6.1) and (6.2) we get,

$$f_H = \frac{5}{12} i_H - \frac{1}{12} i_L - \frac{1}{2} a, \tag{6.5}$$

$$f_L = \frac{5}{12} i_L - \frac{1}{12} i_H + \frac{1}{2} a. \tag{6.6}$$

From (2.13), (6.3) and (6.4) we get,

$$i_H = \max\left\{0,\ f_H - \frac{1}{3}f_L + \frac{1}{3}i_L + \frac{4}{3}a\right\}, \tag{6.7}$$

$$i_L = \max\left\{0,\ f_L - \frac{1}{3}f_H + \frac{1}{3}i_H - \frac{4}{3}a\right\}. \tag{6.8}$$

Substituting from (6.5) and (6.6) into (6.7) and (6.8), we get,

$$i_H = \max\left\{0,\ \frac{4}{9}i_H + \frac{1}{9}i_L + \frac{2}{3}a\right\}, \tag{6.9}$$

$$i_L = \max\left\{0,\ \frac{4}{9}i_L + \frac{1}{9}i_H - \frac{2}{3}a\right\}. \tag{6.10}$$

Suppose the economy was not liquidity trapped in either state. Then (6.9) and (6.10) would imply that $i_H = a$ and $i_L = -a$; which cannot be since $i_L \geq 0$ and $a > 0$. Suppose the economy was liquidity trapped in the high state ($\epsilon = a$). Then, $i_H = 0$ and, from (6.9), $\frac{1}{3}i_L + 2a \leq 0$; which cannot be, since $i_L \geq 0$ and $a > 0$. Hence, the only solution to (6.9) and (6.10) is that the economy is liquidity trapped in the low state ($\epsilon = -a$) but not in the high state ($\epsilon = a$), with

$$i_L = 0,\ i_H = \frac{6}{5}a. \tag{6.11}$$

Substituting from (6.11) into (6.5) and (6.6), gives

$$f_H = 0,\ f_L = \frac{2}{5}a. \tag{6.12}$$

From (2.8), (6.11) and (6.12), we get

$$\pi^e = \frac{2}{5}a. \tag{6.13}$$

From (2.5), (2.6), (6.11), (6.12) and (6.13), we get

$$y_H = \frac{1}{5}a,\ y_L = -\frac{1}{5}a,\ \pi_H = \frac{3}{5}a,\ \pi_L = \frac{1}{5}a. \tag{6.14}$$

From (2.13), (6.11) and (6.12), we get

$$E\left(U_S\right) = -\frac{1}{2}\overline{y}^2 - \frac{1}{5}a^2.\ \blacksquare. \tag{6.15}$$

Proof of Proposition 2 (Discretion)

Since f is unrestricted but $i \geq 0$, the first order conditions for maximizing U_S (given by (2.11)) are as follows.

$$\frac{\partial U_S}{\partial f} = \overline{y} - 2\epsilon - 4f - 3\pi^e + 2i = 0, \tag{6.16}$$

$$\frac{\partial U_S}{\partial i} = 2f - 2i + 3\pi^e + 2\epsilon - \overline{y} \leq 0,\ i \geq 0 \text{ and } i\frac{\partial U}{\partial i} = 0 \text{ (liquidity trapped).} \tag{6.17}$$

From (6.16) we get

$$f = \frac{1}{4}\overline{y} + \frac{1}{2}i - \frac{3}{4}\pi^e - \frac{1}{2}\epsilon, \tag{6.18}$$

and from (6.17) we get

$$i = \max\left\{0,\ f - \frac{1}{2}\overline{y} + \frac{3}{2}\pi^e + \epsilon\right\}. \tag{6.19}$$

Recall that values in the liquidity trap (when $\epsilon = -a$) are distinguished by a 'L' subscript and those in the complementary case (when $\epsilon = a$) by the 'H' subscript. Hence, from (6.18), we get

$$f_H = \frac{1}{4}\overline{y} + \frac{1}{2}i_H - \frac{3}{4}\pi^e - \frac{1}{2}a, \tag{6.20}$$

$$f_L = \frac{1}{4}\overline{y} + \frac{1}{2}i_L - \frac{3}{4}\pi^e + \frac{1}{2}a, \tag{6.21}$$

and, from (6.19), we get

$$i_H = \max\left\{0,\ f_H - \frac{1}{2}\overline{y} + \frac{3}{2}\pi^e + a\right\}, \tag{6.22}$$

$$i_L = \max\left\{0,\ f_L - \frac{1}{2}\overline{y} + \frac{3}{2}\pi^e - a\right\}. \tag{6.23}$$

Substitute from (6.20), (6.21) into (6.22) and (6.23), to get

$$i_H = \max\left\{0,\ \frac{3}{2}\pi^e - \frac{1}{2}\overline{y} + a\right\}, \tag{6.24}$$

$$i_L = \max\left\{0,\ \frac{3}{2}\pi^e - \frac{1}{2}\overline{y} - a\right\}. \tag{6.25}$$

Substitute from (6.20), (6.21) into (2.8), to get

$$\pi^e = i_H + i_L - \overline{y}. \tag{6.26}$$

Suppose the economy is liquidity trapped only in the state $\epsilon = -a$. Then, from (6.24), (6.25):

$$\frac{3}{2}\pi^e - \frac{1}{2}\overline{y} + a \geq 0, \tag{6.27}$$

$$\frac{3}{2}\pi^e - \frac{1}{2}\overline{y} - a < 0, \tag{6.28}$$

$$i_L = 0, \tag{6.29}$$

$$i_H = \frac{3}{2}\pi^e - \frac{1}{2}\overline{y} + a. \tag{6.30}$$

Substituting from (6.29) and (6.30) into (6.26), and solving for π^e, gives

$$\pi^e = 3\overline{y} - 2a. \tag{6.31}$$

Substituting from (6.31) into (6.27) and (6.28) gives the necessary and sufficient conditions for this case:

$$\overline{y} < a \leq 2\overline{y}. \tag{6.32}$$

Substituting from (6.31) into (6.30) gives

$$i_H = 4\overline{y} - 2a. \tag{6.33}$$

Substituting from (6.29), (6.31) and (6.33) into (6.20) and (6.21) gives

$$f_H = 0, \tag{6.34}$$

$$f_L = 2a - 2\overline{y}. \tag{6.35}$$

Substituting from (6.29), (6.33), (6.34) and (6.35) into (2.13) gives

$$E\left(U_S^{Disc}\right) = 12a\overline{y} - 8\overline{y}^2 - 5a^2. \tag{6.36}$$

The two other cases, when the economy is liquidity trapped in both states or in none, are solved in exactly the same way. In each case it is straightforward to check that expected utility under discretion is less than under commitment. ■.

Proof of Proposition 3. Substitute for the values of $E\left(U_S^{Opt}\right)$ and $E\left(U_S^{Disc}\right)$ from Propositions 1 and 2 into $E\left(U_S^{Opt}\right) - E\left(U_S^{Disc}\right)$, then complete the square to get the required result. ■.

Proof of Proposition 4 (Solution under the optimal delegation regime)

Monetary authority's reaction function

The monetary authority's reaction function can be found by maximizing U_B in (4.1). Since $i \geq 0$, the first order conditions for maximizing U_B are $\frac{\partial U_B}{\partial i} \leq 0$, $i \geq 0$, $i\frac{\partial U_B}{\partial i} = 0$. Using (2.6), (4.1) this gives

$$i\left(f - i + 2\pi^e - \pi_B + \epsilon\right) = 0. \tag{6.37}$$

We start with the case where the economy is liquidity trapped in the bad state $(\epsilon = -a)$ only. The other cases will be considered at the end.

Case-I: The economy is in a liquidity trap $(\epsilon = -a)$

In this case, $\epsilon = -a$ and $i \geq 0$. Using (6.37), $f_L + 2\pi^e - \pi_B - a < 0$ implies that

$$i_L = 0, \text{ with } f_L + 2\pi^e - \pi_B - a < 0. \tag{6.38}$$

Case-II: The economy is not in a liquidity trap ($\epsilon = a$)

In this case $\frac{\partial U_B}{\partial i} = 0$ is satisfied with $i \geq 0$. Solving out for i at $\epsilon = a$, gives

$$i_H = f_H + 2\pi^e - \pi_B + a, \text{ with } f_H + 2\pi^e - \pi_B + a \geq 0. \tag{6.39}$$

The state contingent reaction function of the monetary authority is given by (6.38) and (6.39).

Fiscal authority's reaction function

The Treasury now chooses its state contingent fiscal policy f to maximize the objective function (4.2) after observing π^e and ϵ and knowing that the state contingent reaction function of the monetary authority is given by (6.38) and (6.39).

Case-I: Liquidity trapped ($\epsilon = -a$)

In this case, the subsequent monetary policy is $i_L = 0$, hence, using (2.5), (2.6), (4.2), the Treasury maximizes

$$U_T^L = -\frac{1}{2}\left(f_L + \pi^e - a - y_T\right)^2 - \frac{1}{2}\left(f_L + 2\pi^e - a - \pi_T\right)^2 - f_L^2. \tag{6.40}$$

Maximizing U_T^L (unrestrictedly) with respect to f_L gives

$$f_L = \frac{1}{2}a + \frac{1}{4}y_T + \frac{1}{4}\pi_T - \frac{3}{4}\pi^e. \tag{6.41}$$

Case-II: Not liquidity trapped ($\epsilon = a$)

In this case, the subsequent monetary policy is given by (6.39), hence, using (2.5), (2.6), (4.2), the government maximizes

$$U_T^H = -\frac{1}{2}\left(\pi_B - \pi^e - y_T\right)^2 - \frac{1}{2}\left(\pi_B - \pi_T\right)^2 - f_H^2. \tag{6.42}$$

Maximizing U_T^H with respect to f_H gives

$$f_H = 0. \tag{6.43}$$

The optimal state contingent reaction function of the fiscal authority is given by (6.41) and (6.43) respectively.

Substituting the state contingent monetary and fiscal policy reaction functions in (2.5) and (2.6) one obtains

$$y_L = -\frac{1}{2}a + \frac{1}{4}y_T + \frac{1}{4}\pi_T + \frac{1}{4}\pi^e, \tag{6.44}$$

$$\pi_L = -\frac{1}{2}a + \frac{1}{4}y_T + \frac{1}{4}\pi_T + \frac{5}{4}\pi^e, \tag{6.45}$$

$$y_H = \pi_B - \pi^e, \tag{6.46}$$

$$\pi_H = \pi_B. \tag{6.47}$$

Calculation of expected inflation

Since the two states of the world are equally probable, π^e is simply a weighted average of inflation in (6.45) and (6.47) respectively

$$\pi^e = \frac{1}{3}y_T + \frac{1}{3}\pi_T + \frac{4}{3}\pi_B - \frac{2}{3}a. \tag{6.48}$$

Substituting π^e in (6.39), (6.41), (6.44)-(6.46), one obtains

$$f_L = a - \pi_B, \tag{6.49}$$

$$y_L = \frac{1}{3}y_T + \frac{1}{3}\pi_T + \frac{1}{3}\pi_B - \frac{2}{3}a, \tag{6.50}$$

$$\pi_L = \frac{2}{3}y_T + \frac{2}{3}\pi_T + \frac{5}{3}\pi_B - \frac{4}{3}a, \tag{6.51}$$

$$i_H = \frac{2}{3}y_T + \frac{2}{3}\pi_T + \frac{5}{3}\pi_B - \frac{1}{3}a, \tag{6.52}$$

$$y_H = \frac{2}{3}a - \frac{1}{3}y_T - \frac{1}{3}\pi_T - \frac{1}{3}\pi_B. \tag{6.53}$$

Calculation of the optimal inflation target

From (2.9) and (6.43), we get that the expected social welfare is given by

$$E\left(U_S^{OD}\right) = -\frac{1}{4}\left(y_L - \overline{y}\right)^2 - \frac{1}{4}\pi_L^2 - \frac{1}{2}f_L^2 - \frac{1}{4}\left(y_H - \overline{y}\right)^2 - \frac{1}{4}\pi_H^2. \tag{6.54}$$

Differentiating (6.54) with respect to the targets y_T, π_T and π_B we get, respectively,

$$\frac{\partial EU_S^{OD}}{\partial y_T} = -\frac{1}{6}\left(y_L - \overline{y}\right) - \frac{1}{6}\pi_L + \frac{1}{6}\left(y_H - \overline{y}\right), \tag{6.55}$$

$$\frac{\partial EU_S^{OD}}{\partial \pi_T} = -\frac{1}{6}\left(y_L - \overline{y}\right) - \frac{1}{3}\pi_L + \frac{1}{6}\left(y_H - \overline{y}\right), \tag{6.56}$$

$$\frac{\partial EU_S^{OD}}{\partial \pi_B} = -\frac{1}{6}\left(y_L - \overline{y}\right) - \frac{5}{6}\pi_L + f_L + \frac{1}{6}\left(y_H - \overline{y}\right) - \frac{1}{2}\pi_H. \tag{6.57}$$

Setting $\frac{\partial EU_S^{OD}}{\partial y_T} = \frac{\partial EU_S^{OD}}{\partial \pi_T} = \frac{\partial EU_S^{OD}}{\partial \pi_B} = 0$ and solving simultaneously, using (6.47), (6.49), (6.50), (6.51) and (6.53), we get

$$\pi_B = \frac{3}{5}a, \tag{6.58}$$

$$y_T + \pi_T = \frac{4}{5}a. \tag{6.59}$$

Substituting (6.58) and (6.59) in (6.54), using (6.47), (6.49), (6.50), (6.51) and (6.53), gives the final expression for expected social welfare in the optimal delegation case

$$E\left(U_S^{OD}\right) = -\frac{1}{5}a^2 - \frac{1}{2}\overline{y}. \tag{6.60}$$

Comparing (6.15) and (6.60), we see that the output targets (6.58) and (6.59) achieve the same level of social welfare as the optimal solution. Moreover, substituting from (6.58) and (6.59) into (6.38), (6.43) and (6.47)-(6.53), then comparing with Proposition 1, we see that these inflation and output targets achieve the optimal solution, with the economy liquidity trapped in the bad state only. Hence, the two other cases, when the economy is never liquidity trapped and when the economy is liquidity trapped in both states, need not be considered; thus the proof is complete. ■

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