Social Interactions, the Evolution of Trust, and Economic Growth

Dimitrios Varvarigos, University of Leicester
Guangyi Xin, University of Leicester

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Dimitrios Varvarigos‡ Guangyi Xin
University of Leicester University of Leicester

Abstract
We present a model where the dynamics of trust and the process of capital accumulation are jointly determined. Trust evolves intergenerationally, as the process of social interactions with people from different backgrounds creates experiences and forms opinions that are bequeathed to the next generation, thus shaping their level of trust. The provision of public goods and services is also a supporting factor towards the formation of trust. A key result is the possibility of social segregation if the level of trust is below a critical threshold. As a result, long-run equilibria are path-dependent. Both the current level of trust and the current stock of capital are important in determining the economy’s long-term prospects.

Keywords: Trust; Cultural Externalities; Economic Growth

JEL Classification: O41; Z13

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‡ Corresponding Author: Dimitrios Varvarigos. Address: Department of Economics, University of Leicester, Astley Clarke Building, University Road, Leicester LE1 7RH, UK. Email: dvar@le.ac.uk. Tel: ++44 (0) 116 252 2184
1 Introduction

Throughout the course of human history, societies have been composed of people who are heterogeneous in their ethnic, linguistic, religious, cultural or political/ideological characteristics. Irrespective of the source of such heterogeneity however, its very presence evokes the significance of interpersonal trust, i.e., the belief/confidence that a person attaches to the integrity and the reliability of others. This is particularly important in circumstances when these ‘others’ are individuals who, having a different background in terms of the aforementioned characteristics, do not seem, on the outset, to share the same values, attitudes, or moral codes. This notion of generalised trust (Uslaner 2002) portrays people’s abilities to overcome the boundaries imposed by such differences, thus encouraging mutual approach and interactions that facilitate integration and social cohesion.

In recent years, an idea that is gaining momentum is that trust has far-reaching implications that are not confounded solely to social aspects. Instead, they can permeate many facets of economic performance. Putnam (1993) was one of the first to initiate the idea by including trust among the components that constitute social (as opposed to physical or human) capital – a form of capital that a burgeoning literature of empirical investigations have attempted to associate with a broad range of economic outcomes.1

![Figure 1. Cross-country correlation between trust and (real) GDP per capita](chart.png)

Figure 1. Cross-country correlation between trust and (real) GDP per capita

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1 See Algan and Cahuc (2013) and the references therein.
In Figure 1, we provide a scatterplot on the cross-country correlation between GDP per capita and trust. As an index of trust, we employ the percentage of respondents who chose “Most people can be trusted” as an answer to the following question in the World Value Survey: “Generally speaking, would you say that most people can be trusted or that you need to be very careful in dealing with people?”\(^2\) Even though Figure 1 depicts a simple correlation, it is still suggestive of a positive link between economic development and the level of trust. In any case, there is a plethora of empirical analyses that employ more sophisticated methods in order to shed more light on the relation between trust and per capita GDP. A causal, positive effect of trust on growth and economic development is reported by Algan and Cahuc (2010) and Tabellini (2010). Bjørnskov and Méon (2010) and de Biek (2014) employ cross-country data and find that increased trust has a positive effect on productivity. In Knack and Keefer (1997) the negative impact of reduced trust on productivity is partially attributed to the idea that the lack of trust may be associated with the reallocation of resources from production to activities designed to protect against the effects of polarisation and lack of social cohesion (e.g., rent seeking, violence, theft etc.). Zak and Knack (2001) and Dearmon and Grier (2009) report that trust has a positive effect on investment and the accumulation of physical capital, while Bützer et al. (2013) attribute almost one-fifth of macroeconomic imbalances within the Eurozone to differences in interpersonal trust among the Eurozone countries.

There are also equally intuitive arguments (supported by evidence) to suggest that differences in interpersonal trust may also be, to some extent, symptomatic of differences in broader economic conditions. Bjørnskov (2006) finds that income inequality is a significant component of lower trust, whereas Alesina and La Ferrara (2002) report evidence that low income and low levels of education cause a reduction in interpersonal trust. Delhey and Newton (2005) interpret their finding of a positive effect of public expenditures on trust by alluding to the idea that the provision of public goods increases the sense of citizenship and community among the population.

If anything, the evidence that we have just summarised advocates the view that the relation between interpersonal trust and economic performance is two-way causal. Naturally,

\(^2\) We use data from the 5th wave of the World Value Survey (2005-2008) which can be accessed electronically via [www.worldvaluessurvey.org/WVSDocumentationWV5.jsp](http://www.worldvaluessurvey.org/WVSDocumentationWV5.jsp). Data on real GDP per capita (PPP adjusted) for the corresponding years were retrieved from Penn World Table version 7.1 (Alan Heston, Robert Summers and Bettina Aten, Center for International Comparisons of Production, Income and Prices, University of Pennsylvania) and can be downloaded from [pwt.sas.upenn.edu/php_site/pwt71/pwt71_form.php](http://pwt.sas.upenn.edu/php_site/pwt71/pwt71_form.php).
two-way causal effects are conducive to the existence of persistent differences in socio-
-economic outcomes. Insofar as social trust fuels, and at the same time is fuelled, by
economic conditions, then we can envisage circumstances where the current conditions may
determine, to a great extent, the long-term prospects of the economy. On the one hand, we
may have a vicious circle where reduced trust impedes economic performance which,
subsequently, nurtures the attitudes that ingrain widespread mistrust into the fabric of
society. On the other hand, there is the possibility of a virtuous circle whereby improved
economic conditions are supportive to increased trust which, by itself, fosters productivity,
economic growth and the overall standard of living.

The purpose of this paper is to address and analyse these issues by focusing on the joint
determination of trust and economic development. We build a model where interpersonal
trust and capital accumulation interact with each other, thus generating the joint evolution of
their dynamics. Increased trust fosters productivity, thus increasing saving and
accommodating the formation of capital through its positive effect on labour income.
Interpersonal trust evolves by means of an intergenerational externality. Specifically, the
current generation of adults engage in social interactions based on their inherited level of
trust. These interactions generate experiences and form opinions that are bequeathed to the
next generation, hence forming their level of trust. This process is also supported by the
provision of public goods and services. Two results prove critical for the co-evolution of
economic development and trust. Firstly, individuals optimally devote effort to establish
social ties with people that possess different characteristics, only if the current state of trust
is above an (endogenously derived) threshold. Secondly, the transition equation for trust
generates two equilibria above the aforementioned threshold, the lower of which is unstable
and decreasing in the stock of capital. In other words, economic development makes it more
likely that, for given current conditions, trust will increase over time. We show that the
dynamic path that determines the economy’s convergence to the long-run equilibrium
depends on current conditions where both the existing level of trust and the existing stock
of capital play a key role. On the one hand, for given levels of economic development, the
current state of interpersonal trust can be important in shaping the dynamic path of socio-
economic outcomes. On the other hand, for a given level of trust, the dynamic path of such

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3 There is strong evidence in favour of the intergenerational component in the evolution of trust (e.g., Algan
and Cahue 2010; Dohmen et al. 2012; Ljunge 2014; Moschion and Tabasso 2014).
outcomes depends critically on the current stock of capital, thus highlighting the importance of economic conditions for shaping long-term prospects in terms of both economic and social characteristics. Put differently, the feedback that imbues the joint evolution of trust and economic development can transform current imbalances among economies into permanent fixtures of their long-term characteristics.

Our model is broadly related to other analyses that have endeavoured to elucidate the theoretical underpinning behind the social trust-economic development nexus. Francois and Zabojnik (2005) model an economy where modern production requires a matching process between entrepreneurs and contractors who may be trustworthy or opportunistic. Opportunistic contractors are the ones who find optimal to cheat the entrepreneur, rather than facilitating production and sharing the joint surplus of a business venture. Successive generations of contractors may become trustworthy or opportunistic through a stochastic intergenerational socialisation process à la Bisin and Verdier (2001). The authors find multiple, path-dependent equilibria whereby convergence to the long-run equilibrium depends on the initial stock of social capital, captured by the share of trustworthy individuals among the population of contractors. Conceptually closer to our analysis is the paper by Growiec and Growiec (2014). They employ a representative agent framework in which multiple equilibria emerge as a result of the complementarity between social capital and trust. Particularly, they assume that trust is increasing in the stock of social capital, through the use of a step function, while the formation of social capital is supported by increased trust because the latter increases the pool of trusted people with whom an individual can socialise. The equilibrium multiplicity in trust and social capital is transmitted to the economy’s output because, in their model, individuals respond optimally to increased trust by actually reducing their socialisation effort. Given the trade-off between socialisation and labour, this effect leads to an increase in labour supply and, therefore, production.

Apart from the obvious differences in terms of both the set-up and the mechanisms that lead to the main results, other major differences of our model in comparison to the aforementioned analyses stem from our modelling of economic dynamics through an explicit process of saving and capital accumulation, as well as the explicit consideration of how the capital stock impinges on the formation of interpersonal trust. These differences are not mere theoretical curios. On the contrary, they have important implications as they emphasise a salient point: When considering the potential path of socio-economic outcomes, it is not
only the current state of social trust, but also the current state of the economy (in terms of the stage of economic development) that is crucial in dictating both the social and the economic prospects of a country.

The remaining analysis is organised as follows. In Section 2 we present the set-up of our economy and show the mechanisms that govern capital accumulation and the evolution of interpersonal trust. Section 3 analyses the dynamic equilibrium and discusses the main results. Section 4 shows that the main message from our analysis survives under an alternative specification for the economy’s production technology. We summarise and conclude in Section 5.

2 The Economy

Consider an economy that is populated by an infinite sequence of overlapping generations of three period-lived individuals. The first period of each individual’s lifetime is her childhood while the two subsequent periods are her youth (the first period of adulthood) and maturity (the second period of adulthood). Agents are active only during their adulthood. Although they are largely inactive during their childhood, it is the period where they form the set of personality traits (values, beliefs, attitudes etc.) that determine their level of trust, in a manner that will be described later.

The population mass of each age cohort is denoted $N > 0$ and is assumed to be constant over time. Once she reaches the first period of adulthood, each person is endowed with a unit of labour which she inelastically supplies to final good producing firms in exchange for the wage $w_i$. She pays (lump-sum) taxes $T_i$ and then allocates her disposable income between consumption expenditures (denoted $c_i$) and saving (denoted $s_i$). The latter is deposited to financial intermediaries that return it next period, augmented by the (gross) interest rate $R_{i+1}$. The individual uses her saving in order to finance her consumption expenditures during maturity (consumption during the second period of adulthood is denoted $d_{i+1}$) – a period during which she does not have any labour endowment, therefore no other source of income other than the one that accrues from saving.
2.1 Socialisation and Trust

In addition to the standard consumption-saving choice, individuals can also enjoy utility through social interactions with their peers. Particularly, young individuals build social ties with other members of their age group – ties that are retained over the lifetime and allow individuals to socialise. We are going to assume that the population is divided in two groups \( i \) and \( j \). The former group has a population mass of \( M < N \) while the latter group has a population mass of \( N - M \). This distinction may capture characteristics such as cultural, ethnolinguistic, religious, or ideological ones. These differences have no bearing on any of the economic characteristics of agents, i.e., their ability to perform labour, the rate at which they discount future outcomes etc.\(^4\) It only affects their attitudes towards socialisation.

Consider a person belonging to group \( i \). This person can interact costlessly with a fraction \( \pi \in (0,1) \) of individuals belonging to her own group, while each of these interactions yields \( b > 1 \) units of utility.\(^5\) She can also potentially establish a social tie with a fraction \( p \in (0,1) \) of people belonging to group \( j \). To minimise notation, we shall be making use of \( p(N-M) \equiv \tilde{n} \) hereafter. Interactions with ‘outsiders’ are costly to establish. Particularly, establishing a social tie with \( \varphi' \) (\( 0 \leq \varphi \leq \tilde{n} \)) individuals entails an effort cost that is captured by the function \( \Phi(\varphi', m, \tilde{n}) \). The variable \( m \in [0,1] \) measures the level of trust among agents that do not belong to groups that share common characteristics, thus they need to devote effort in order to approach and interact with each other. The effort function satisfies \( \Phi_{\varphi'} > 0, \Phi_{\varphi''} > 0, \Phi(0, m, \tilde{n}) = 0, \lim_{\varphi' \to \tilde{n}} \Phi(\varphi', m, \tilde{n}) = +\infty \) and \( \Phi_{\varphi} < 0 \) (the latter assumption capturing the idea that is relatively easier to interact and/or socialise when the number of people with whom one can potentially establish a social tie is higher).

Trust is an attribute that individuals build during their childhood, thus it is taken as given once they reach their adulthood.\(^6\) It is also an attribute that affects the attitudes of individuals

\(^4\) Klansing and Milionis (2014) develop a model that shows how the intergenerational transmission of attitudes regarding patience affects economic growth. This is an issue that goes beyond the scope of our analysis.

\(^5\) As it will become clear, \( b > 1 \) is required for a meaningful solution to an individual’s maximisation problem. Later we shall introduce a specific restriction on this parameter so as to guarantee a non-trivial dynamic equilibrium (see Assumption 3).

\(^6\) This assumption is in accordance to the existing evidence. According to Delhey and Newton (2003), trust is a personality trait that “is learned in early childhood and tends to persist in later life” (Delhey and Newton 2003, p. 95). A similar assumption on inherited trust being persistent throughout an individual’s lifespan is employed by Francois and Zabojnik (2005).
across different groups.\(^7\) Trust affects socialisation efforts as follows: Once established, an interaction with any of the \(\phi^j\) individuals from group \(j\) yields the same units of utility that accrue from interactions with people with whom the individual shares common characteristics, i.e., \(b\).\(^8\) Nevertheless, increased trust reduces the effort cost associated with establishing such ties. In other words, it is relatively easier to establish some type of relation with ‘outsiders’ when the level of trust is higher. These arguments may capture the idea that trust promotes the attempts of individuals to approach people from different backgrounds, understand that their underlying differences should not be detrimental to their effort to communicate and share common goals and interests, thus inhibiting prejudice and intolerance. Assuming that higher \(m_i\) is indicative of more trust, we capture these ideas through \(\Phi_{m_i} < 0, \Phi_{m_i m_j} > 0\) and \(\lim_{m_i \to 0} \Phi(\phi^j, m_i, \bar{m}) = +\infty\).

A specific functional form that satisfies all the aforementioned properties for the effort function, and therefore it shall be employed in our analysis, is given by

\[
\Phi(\phi^j, m_i, \bar{m}) = \frac{\bar{m}\phi^j}{(\bar{m} - \phi^j)\zeta(m_i)},
\]

where the function \(\zeta(m_i) \in [0,1]\) satisfies \(\zeta'(m_i) > 0, \zeta''(m_i) > 0, \zeta(0) = 0\) and \(\zeta(1) = 1\).

### 2.2 The Individual’s Problem

The objective of a young individual that belongs to group \(i\) is to choose her saving, which will also dictate her intertemporal consumption profile, as well as her socialisation effort in order to maximise her lifetime utility

\[
u'_i = (1 - \delta) \ln(\epsilon_i) + \delta \ln(d_{i+1}) + b(\pi M + \phi^j) - \Phi(\phi^j, m_i, \bar{m}),
\]

subject to \(0 \leq \phi^j \leq \bar{m}\) and the budget constraints for youth and maturity that are given by

\[
\epsilon_i = w_i - T_i - s_i,
\]

and

\[
d_{i+1} = R_{i+1} \phi^j,
\]

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\(^7\) Hence we abscond from the issue of differentiated trust levels across different groups.

\(^8\) Nothing will change qualitatively from our subsequent results, if we assume that the utility accruing from such interactions differs from the one corresponding to socialisation with people who are more akin to the individual who is establishing social ties.
respectively.\textsuperscript{9} Note that the parameter \( \delta \in (0,1) \) quantifies the relative weight attached to the utility that accrues from consumption during the second period of the individual’s adulthood. Substituting (1), (3) and (4) in (2), we can express the individual’s problem as follows:

\[
\max_{\phi_i} \mu' = (1 - \delta) \ln(w_i - T_i - s_i) + \delta \ln(R_i + s_i) + b(\pi M + \phi_i) - \frac{\tilde{\mu} \phi_i}{(\tilde{a} - \phi_i) \zeta(m_i)}.
\] (5)

In terms of optimal saving behaviour, this problem leads to the familiar solution

\[
s_i' = \delta(w_i - T_i),
\] (6)
i.e., young individuals will save a fraction of their disposable income in order to finance their future consumption needs. This fraction (corresponding to the marginal propensity to save) is equal to the preference weight that people attach to consumption during maturity. With regard to socialisation, it is straightforward to establish that the solution to the individual’s problem results in

\[
\phi_i' = \max \{0, [1 - \beta(\zeta(m_i))^{-1}] \theta \},
\] (7)

where \( \beta \equiv (\sqrt{b})^{-1} \). According to the result in Eq. (7), individuals will try to establish social ties with a fraction \( 1 - \beta(\zeta(m_i))^{-1} \) of the total number \( \tilde{n} \) of ‘outsiders’ with whom they can potentially interact. Given that \( \zeta'(\cdot) > 0 \), it is obvious that this fraction is increasing in the level of trust \( m_i \). In other words, increased trust will induce individuals to seek more social interactions with people that do not belong to the group of individuals with similar characteristics.

What is also important is the possibility of a corner solution that is embedded to the result in (7). With the purpose of illustrating this point and improving the clarity and analytical convenience of the subsequent analysis, without any significant loss of generality, henceforth we shall be making use of a functional form for \( \zeta(m_i) \) that satisfies all the properties that were outlined previously.\textsuperscript{10} Particularly, for the remaining analysis we shall specify

\textsuperscript{9} The reason we do not use a subscript on consumption and saving is because, as it will transpire later, these choices will not be affected by social traits.

\textsuperscript{10} The results remain qualitatively similar even without the specific function form in (8), as long as \( \zeta(m_i) \) satisfies the properties outlined in the main part of the analysis. We employ this function for analytical convenience and expositional purposes.
\[ \zeta(m_i) = m_i^2. \] (8)

Combining (7) and (8), we can express the optimal decision regarding \( \varphi_i \) according to

\[
\varphi_i^{\ast} = \begin{cases} 
0 & \text{if } m_i \leq \beta \\
\left(1 - \frac{\beta}{m_i}\right) \bar{n} & \text{if } m_i > \beta.
\end{cases}
\] (9)

As it is obvious from this expression, individuals find it worthwhile to devote effort in engaging socially with ‘outsiders’ only if the level of trust is above the threshold characterised by the parameter term \( \beta \). Trust levels that are below this threshold imply that the utility benefit of such social interactions falls short of the effort cost that is necessary in order to establish them. As a result, an individual will opt to interact only with people who are more akin to her specific attributes.

It should be noted that, by analogy, the similar analysis and results apply for a person that belongs to group \( j \). Assume that each individual can interact costlessly with a fraction \( \pi \) of people within her group, whereas potential interactions with a fraction \( p \) of people from group \( i \) require effort. Denoting \( pM = \bar{\pi} \) then the effort function is the same as in (1), after replacing \( \bar{n} \) with \( \bar{\pi} \) and \( \varphi_i \) with \( \varphi_i' \). The problem of this person can be described as

\[
\max_{\varphi_i', \varphi_i} u' = (1 - \delta)\ln(w_i - T_i - s_i) + \delta \ln(R_i, s_i) + b[\pi(N - M) + \varphi_i'] - \frac{\bar{\pi} \varphi_i'}{\bar{\pi} - \varphi_i' \zeta(m_i)}.
\]

It is straightforward to establish that the solution in (6) is the same while, after applying the function in (8), the optimal socialisation effort is summarised in

\[
\varphi_i^{\ast, \prime} = \begin{cases} 
0 & \text{if } m_i \leq \beta \\
\left(1 - \frac{\beta}{m_i}\right) \bar{\pi} & \text{if } m_i > \beta.
\end{cases}
\] (10)

The results in (9) and (10) indicate that unless trust is above a critical threshold, there will be some form of segregation in the sense that people will not form social ties with individuals from different backgrounds. This is a result that will prove important for the economy’s long-term prospects, as we shall see later.\(^{11}\)

\(^{11}\) We do not explicitly consider the issue of reciprocity. Besides contributing significant technical complication, without changing the main message of our analysis, there is no widespread consensus on the connection between reciprocity and interpersonal trust. In fact, Uslaner (2000) argues that "generalized trusters’ moral
2.3 Firms and Production

Young individuals are employed by perfectly competitive firms (whose mass we normalise to one) who combine units of labour (denoted \( l_t \)) and capital (denoted \( K_t \)) in order to produce \( Y_t \) units of the economy’s single commodity by utilising a technology \( F(K_t, l_t) \) such that \( F_{K_t} > 0 \), \( F_{K_t K_t} < 0 \), \( F_{l_t} > 0 \), \( F_{l_t l_t} < 0 \) and \( F_{l_t K_t} > 0 \). Furthermore, in line with the existing literature, we are assuming that, for given productivity variables, the technology displays unit constant returns, i.e., \( F(xK_t, xl_t) = xF(K_t, l_t) \). For the purposes of our analysis, we shall be employing the following production technology:

\[
Y_t = F(K_t, l_t) = \Theta l_t + K_t^\eta (A_t l_t)^{1-\eta}, \quad \eta \in (0,1).
\]  (11)

The term \( A_t \) introduces two external effects on production according to

\[
A_t = H_t^\lambda G_t^{1-\lambda}, \quad 0 < \lambda < 1.
\]  (12)

The variable \( H_t \) is a learning-by-doing externality (see Arrow 1962, Frankel 1962, and Romer 1986), capturing the idea that workers gain knowledge and become more productive by handling more capital goods. Hence, following the existing literature, we shall assume that \( H_t \) is related to average stock of capital per person according to

\[
H_t = H_t k_t, \quad H > 0.
\]  (13)

where \( k_t = K_t / N \). The variable \( G_t \) follows the existing literature (Barro 1990; Alesina and Rodrik 1994) by introducing the beneficial effect of productive public spending per capita on aspects such as education and research, infrastructure, health etc., aspects that can improve productivity. As it is customary in many models of economic growth, we are going to assume that government spending per person is measured relative to the economy’s capital stock (e.g., Alesina and Rodrik 1994). Particularly, it is assumed that \( G_t \) is proportional to capital per worker \( (k_t = K_t / N) \) according to

\[
G_t = g k_t, \quad 0 < g < 1.
\]  (14)

Additionally, we shall consider the scenario where low levels of trust entail costs to the society in terms of a loss in productivity and output. This scenario may capture the idea that codes are not simple reflections of their expectations of how others are likely to behave...they are committed to others in the society beyond anticipation of reciprocity” (Uslaner 2000, p. 579).
low levels of trust among different groups of people with distinct identities, could lead to social tension, conflict and disorderly behaviour that can impede productivity both directly (e.g., rioting, crime etc.) and indirectly. Examples that can be associated with the indirect effects on productivity are either the psychological impact on the affected segments of the population (e.g., racial abuse, fear etc.) or the increased resources required to maintain some degree of law and order under such tense conditions. Note that support for this idea is found in the empirical analysis of Rodrik (1999): He uses the lack of trust as one of the components of social conflict and finds that the latter can explain, to a large extent, incidences of economic collapse. To introduce the supporting impact of high levels of trust on productivity, we let $\Theta(t) = \Theta(m_t)$ such that $\Theta(m_t) > 0$. Specifically, we shall employ
\[
\Theta(m_t) = \theta(1 - q + q m_t), \quad \theta > 0, \quad 0 < q < 1.
\]

At this point, we should note that our choice of production technology is made in order to guarantee analytical solutions throughout. In Section 4, we present an example with a more standard Cobb-Douglas technology where we show that the main results of our analysis remain qualitatively intact. This is because the absence of an impact from trust to the marginal product of capital is innocuous in a model where logarithmic preferences imply that saving behaviour is not affected by the interest rate. However, in that case the transition equation for trust becomes so complicated that it is not possible to obtain closed form solutions for one of the possible pairs of steady state equilibria.

Using Eq. (11)-(15) together with the labour market clearing condition $I_t = N$ and the condition $k_t = \bar{k}$, we can solve the firms’ profit maximisation problem according to which each input is paid its marginal product. Formally,
\[
w_i = \theta(1 - q + q m_t) + (1 - \eta)(H^i g^{1-i})^{1-q} k_t, \quad \eta \in (0, 1) \quad \text{and} \quad R_{t+1} = \eta(H^i g^{1-i})^{1-q} \equiv \bar{R} \quad \forall \tau.
\]

\subsection*{2.4 The Dynamics of Trust}

We consider a scenario whereby the social interactions of a generation of adults create experiences that affect their perspective on the qualities of individuals from other groups. The next generation of individuals are inculcated with this perspective while developing the
personality traits that shall ultimately determine their level of trust. We view this as a mechanism of intergenerational transmission of trust, a mechanism that can describe either the vertical (i.e., ideas and beliefs passed on from parents to their offspring) or the oblique transmission (i.e., imitation of a role model; instruction by a religious or political leader) of opinions, beliefs, or other such traits. Particularly, the mechanism we propose works as follows. Individuals form their social ties with people from different backgrounds, based on the level of trust with which they were endowed. Once formed, these interactions will expose them to different characteristics, hence generating experiences that will be transmitted to the next generation of individuals when they form those attributes that shape their trust level. Based on this, once the next generation reaches their adulthood, they will form their own social ties with people from different backgrounds and so on.

Furthermore, we shall assume that the government’s resources devoted to productive public spending also have a positive external effect in the sense that they can improve the degree of tolerance and trust, in addition to their positive effect on productivity to which we alluded earlier. Indeed, Delhey and Newton (2005) report evidence of a significant positive effect of public expenditures (e.g., health and education) on trust, attributing it to the idea that such public services “generate a sense of citizenship and social trust” (Delhey and Newton 2005, p. 318). We may also appeal to other arguments that relate to the specific issue of public spending on education. For example, education improves social skills (Glaeser et al. 2000) and those cognitive skills that increase the levels of acceptance and trust among segments of the population that possess different characteristics (Dehley and Newton 2005).

Taking account of the previous arguments, the level of trust of the next generation is formed by the current generation’s experiences from social interactions and the economy’s spending on public goods and services. Since trust is a society-wide characteristic, we shall

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12 Liebkind and McAlister (1999) present experimental evidence, suggesting that the contact between people from different ethnic groups can promote tolerance.

13 A good example is the ‘Promoting a Culture of Trust’ (PACT) grant scheme available to Northern Irish schools by the Integrated Education Fund (IEF) – a charitable trust that was partly established with public funds from the European Union and the Department of Education in Northern Ireland, in addition to private donations. Through this scheme, the IEF supports “projects that promote a culture of trust and the development of paths of reconciliation through…the development of skills, structures and relationships that enable schools, pupils and their parents…to increase their understanding, acceptance and respect for political, cultural and religious differences.” (http://www.ief.org.uk/grants/pact/)

13
assume that it is driven by the socialisation efforts of both groups i and j, with each group’s impact being weighted by its relative size over the whole population. Hence

\[ m_{x, t} = \frac{M}{N} y'(p_{x}^{*, i}, G_i) + \frac{N - M}{N} y'(p_{x}^{*, j}, G_j), \]

such that \( y_{x}^{*, i}, y_{x}^{*, j} > 0 \) for \( x \in \{i, j\} \).

For the remainder of our analysis, we are going to employ the following functional forms:

\[ y'(p_{i}^{*, i}, G_i) = \frac{1}{1 + G_i (p_{i}^{*, i} / \overline{\bar{p}})} \quad \text{and} \quad y'(p_{j}^{*, j}, G_j) = \frac{1}{1 + G_j (p_{j}^{*, j} / \overline{\bar{p}})}, \]

where \( \overline{\bar{p}} \) (\( \overline{\bar{p}}' \)) gives the total number of social ties with ‘outsiders’, for an agent of the current generation of young adults in group i (group j), had her degree of trust been the highest possible, i.e., \( m_i = 1 \). Therefore, given (9) and (10), it is \( \overline{\bar{p}} = (1 - \beta)\bar{n} \) and \( \overline{\bar{p}}' = (1 - \beta)\bar{u} \) respectively. Combining (18) and (19), it is evident that \( 0 \leq m_{x, t+1} \leq 1 \), which is the permissible range of values for the trust variable.

Substituting (9), (10), (14), \( y = (1 - \beta)\bar{n} \) and \( y' = (1 - \beta)\bar{u} \) in (18) and (19), we get a transition equation for the trust variable. That is

\[ m_{x, t+1} = M(k_x, m_x) = \begin{cases} 0 & \text{if} \quad m_x \leq \beta \\ \frac{(1 + g_k)\beta}{1 - \beta + g_k[1 - (\beta / m_x)]} & \text{if} \quad m_x > \beta \end{cases} \]

Given the expression in (20) we can derive the result summarised in

**Proposition 1.** \( M_{k} \geq 0 \) and \( M_{m} > 0 \), as long as \( m_x > \beta \).

**Proof.** We can use (20) to establish that, as long as \( m_x > \beta \), we have

\[ \frac{\partial M(k_x, m_x)}{\partial k_x} = \frac{1}{m_x - 1} \left( 1 - \frac{\beta}{m_x} \right) \frac{\beta g}{\{1 - \beta + g_k[1 - (\beta / m_x)]\}^2} \geq 0, \]

\[ \frac{\partial M(k_x, m_x)}{\partial m_x} = \frac{\beta}{m_x^2 \{1 - \beta + g_k[1 - (\beta / m_x)]\}^2} > 0, \]

\[ 14 \text{ Effectively, the presence of } \overline{\bar{p}} \text{ and } \overline{\bar{p}}' \text{ introduces the maximum number of social ties that a person can establish with ‘outsiders’. Naturally, there is scope for creating new experiences that can improve trust intergenerationally as long as the existing interactions fall short of this number.} \]
given that \( m_t \leq 1 \) holds by assumption. □

As we established in the analysis related to the results in (9) and (10), scenarios for which \( m_t \leq \beta \) entail some form of social segregation in the sense that individuals will avoid establishing relations and ties with people from different backgrounds. In this sense, any trust among different groups will eventually disappear. Individuals will seek to socialise and interact with people of different characteristics only if the level of trust is above a certain threshold (i.e., whenever \( m_t > \beta \)). When this is the case, the extent of social interactions is increasing in the level of trust. Nevertheless, as we indicated previously, such interactions create experiences that are transmitted to the next generation of young agents, thus forming the personality traits that ultimately determine their trust levels. In other words, if the current generation is more trustful and hence more willing to engage socially, then the processes of vertical and oblique transmission will endow the next generation with increasing levels of trust, thus motivating them to establish more social ties with different people, and so on. This is the intuition behind \( M_m > 0 \). The intuition behind \( M_k \geq 0 \) is also straightforward. When the capital stock is higher, there is an increase in resources that are devoted towards public goods and services. As we have argued before, their provision can cultivate higher trust levels for the next generation, either because it increases their sense of citizenship and community, or because it has a direct benefit through increased tolerance (e.g., cultivated through the public education system).

2.5 The Dynamics of Capital Accumulation

The savings of young workers are deposited to perfectly competitive financial intermediaries who access a technology that transforms units of time \( t \) output into units of time \( t+1 \) capital on a one-to-one basis. They rent the capital to firms at a unit price of \( R_{t+1} \). The total amount of deposited savings is \( Ns_t^* \), implying that \( K_{t+1} = Ns_t^* \). Given \( k_{t+1} = K_{t+1} / N \) denotes capital per worker, we can write the equation that links capital formation to saving according to \( k_{t+1} = s_t^* \). Substituting (6) and (16), we get

\[^{15}\text{Note that capital depreciates completely within one period; therefore the (gross) interest rate on saving is equal to the rental rate of capital.}\]
\[ k_{t+1} = \delta \{ \theta(1 - q + \eta m_t)(H^t g^{t-1})^{1-\eta} k_t - T_t \} . \quad (21) \]

Recall that the government devotes resources towards productive public spending. We shall assume that it finances public spending by utilising tax revenues according to a continuously balanced budget. This implies that total revenues, \( NT_t \), are equal to total spending, \( NG_t \). Therefore, we can use (14) to write
\[ T_t = g k_t . \quad (22) \]

Substituting (22) in (21) yields the transition equation for the stock of capital per worker. That is
\[ k_{t+1} = K(k_t, m_t) = \delta(1 - q + \eta m_t) + bk_t \],

where \[ b \equiv g \left( \frac{(1 - \eta)H^t}{G^{t+1}(1-\eta)} - 1 \right) \] is a composite term. In order to focus on the more familiar (and more widely analysed) case where capital formation is positively monotonic, we shall employ a parameter restriction in the form of 16

Assumption 1. \( H > \left[ \frac{G^{t+1}(1-\eta)}{1-\eta} \right] \Leftrightarrow b > 0 . \)

Furthermore, to guarantee that the long-run equilibrium for the capital stock is bounded, we shall also impose the following condition:

Assumption 2. \( \delta h < 1 . \)

We can use (23) to derive the results that identify the effects of the current capital stock and trust on the process of capital formation. These are summarised in

Proposition 2. \( K_{k_t} > 0 \text{ and } K_{m_t} > 0 . \)

Proof. From (23) it is straightforward to establish that

16 Removing Assumption 1 by considering \( b < 0 \) would imply the presence of complex dynamics, through endogenous (limit) cycles, and quite possibly chaotic (aperiodic) dynamic behaviour. As these are issues that go way beyond the scope of our paper, we have chosen to abscond from them.
\[ \frac{\partial K(k_t, m_t)}{\partial k_t} = \delta h > 0, \]
\[ \frac{\partial K(k_t, m_t)}{\partial m_t} = \delta h q > 0, \]

thus completing the proof. □

Once more the intuition behind these results is straightforward. The explanation behind \( K_0 > 0 \) is that (disposable) labour income is higher in an economy with more capital stock. However, labour income determines total saving, hence the extent of future capital formation. In addition, increased trust improves labour productivity for the reasons that where outlined in the formal description of the economy’s production characteristics (see Section 2.3). Consequently, the intuition behind \( K_0 > 0 \) is that the higher productivity increases the wage per unit of labour. Therefore, trust is a factor that promotes saving and capital accumulation.

3 The Dynamic Equilibrium

As we have seen from the preceding analysis, the economy’s equilibrium is characterised by the planar system of first-order difference equations with two stock variables \( k_t \) and \( m_t \), displayed in (20) and (23). This system of transition equations will facilitate us in tracing the economy’s transitional dynamics for given initial conditions \( m_0 \in (0,1) \) and \( k_0 > 0 \), as well as deriving its long-run equilibrium. However, in order to avoid a situation where the long-run equilibrium is uniquely characterised by a degenerate solution for which \( \lim_{i \to \infty} m_i = 0 \)
\[ \forall \ m_0 \in (0,1) \text{ and } k_0 > 0, \] we need to impose a condition on the value of the parameter that quantifies the utility accruing from social interactions. This condition comes in the form of

**Assumption 3.** \( b > 4 \Leftrightarrow \beta < \frac{1}{2}. \)

We shall begin the analysis with the derivation of the steady state solutions. These are summarised in
Lemma 1. There are three pairs of steady state equilibria \((k^*, m^*)\), \((k^{**}, m^{**})\) and \((k^{***}, m^{***})\), such that \(k^{***} > k^{**} > k^*\) and \(m^{***} > m^{**} > m^*\).

Proof. See Appendix A1. □

The formal proof that is provided in Appendix A1 offers explicit solutions for these steady state equilibria. Defining the composite parameter terms \(\psi = \delta h\) and \(\epsilon = g \theta\), these solutions are the following:

\[ m^* = 0, \quad (24) \]
\[ m^{**} = -\frac{(1-\psi)(1-\beta) + \epsilon[1-q(1+\beta)]}{2qe} \]
\[ + \frac{\sqrt{\{(1-\psi)(1-\beta) + \epsilon[1-q(1+\beta)]\}^2 + 4qe[1-q + (1-q)\epsilon]}}{2qe}, \quad (25) \]
\[ m^{***} = 1, \quad (26) \]
\[ k^* = \frac{\delta \theta(1-q)}{1-\psi}, \quad (27) \]
\[ k^{**} = \frac{\delta \theta(1-q + q m^{**})}{1-\psi}, \quad (28) \]
\[ k^{***} = \frac{\delta \theta}{1-\psi}. \quad (29) \]

It should also be noted that Appendix A1 shows \(m^{**} \in (\beta, 1)\).

The next step of our analysis is to examine the stability of the three equilibrium pairs. This is something we do in

Lemma 2. The pairs \((k^*, m^*)\) and \((k^{***}, m^{***})\) are locally asymptotically stable whereas the pair \((k^{**}, m^{**})\) is a saddle point.

Proof. See Appendix A2. □
The implication from Lemma 2 is that we can establish the economy’s long-run equilibrium for given initial conditions $0 < m_0 < 1$ and $k_0 > 0$. This analysis is formally presented in

**Proposition 3.** The long-run equilibrium of the economy is path-dependent. Particularly, depending on the initial values $(k_0, m_0)$, the economy may converge to either the equilibrium characterised by the pair $(k^*, m^*)$ or the equilibrium characterised by the pair $(k^{***}, m^{***})$.

*Proof.* It follows from Lemma 2. □

In order to get a better understanding of the intuition and the mechanisms leading to the result in Proposition 3, we need to recall two issues. Firstly, at the beginning of any time period $t$ there are two predetermined variables – the stock of capital $k_t$ and the level of trust $m_t$ – implying that for a given stock of $k_t$ ($m_t$) there is only one value for $m_t$ ($k_t$), out of an infinite range of possible ones, that will converge to the saddle point $(k^*, m^*)$; all the other paths diverge away from it. In essence, the pair $(k^*, m^*)$ is not a stable equilibrium. Secondly, the fact that individuals engage in social interactions with ‘outsiders’ only if the level of trust is sufficiently high (see Eq. 9 and 10) implies that the interior solution \( m_{t+1} = m_t \) that one derives (for given $k_t$) from Eq. (20) acts like a threshold (see Figure 2). Given this, higher values of $k_t$ make it more likely that (for given $m_t$) the dynamics of trust will eventually converge to $\lim_{t \to \infty} m_t = 1$. This is the reason why the TL locus, illustrating combinations of $k_t$ and $m_t$ for which $\Delta m_t = 0$ in (20), is downward sloping in the phase diagram of Figure 3.

The idea that economic development (captured by the stock of capital $k_t$) makes it less likely that the economy will degenerate to a situation of complete segregation – the latter owing to the lack of trust among different groups of people – is important for the long-term prospects of the economy. Particularly, we can anticipate the result that the current stocks of both $m_t$ and $k_t$ will be critical for the equilibrium to which the economy will converge in the long-run.
Given the above, we can use the phase diagram to identify the forces governing the economy’s convergence to the long-run equilibrium. Before doing so, note that the CS locus depicts combinations of $k_t$ and $m_t$ for which $\Delta k_t = 0$ in (23). Let us begin by considering two scenarios entailing the same initial value for $m_0$ but different initial values for $k_0$ – a relatively low one (point D) and a relatively high one (point B). At point D the stock of capital, and its effect through public spending, is not sufficient to support increasing levels of trust. Despite the fact that the capital stock may increase temporarily, the corresponding level of trust is still below the threshold required to support increasing trust over time. As trust decreases, at some point the capital stock will start decreasing as well due to the negative effect of low trust on productivity, saving, and capital accumulation. Eventually the economy will converge to the equilibrium $(k_*, m_*)$. At point B however, the dynamics are different despite the fact that the initial trust level is the same in both scenarios. In this case, the current stock of capital (affecting the dynamics of trust through the effect of public spending) supports an increasing $m_t$ as the current level of trust is above the threshold defined by the TL locus. Although the capital stock may decrease temporarily, the increasing trust will support productivity, saving, and capital accumulation to such an extent that the
capital stock will eventually increase and the economy will converge to a long-run equilibrium characterised by \((k^{**}, m^{**})\).

Now, let us consider two different scenarios entailing the same initial value for \(k_0\) but different initial values for \(m_0\) — a relatively low one (point C) and a relatively high one (point A). At point C, the level of trust is below the threshold depicted by the TL locus. Trust will be decreasing over time, having a detrimental effect on capital accumulation due to the loss in productivity, and the economy will eventually converge to the equilibrium characterised by \((k^*, m^*)\). At point A however, despite the fact that the capital stock is still the same initially, the level of trust is above the threshold defined implicitly by the TL locus. Trust will be increasing over time, thus supporting capital formation due to the beneficial effect on productivity. Eventually, the economy will converge to \((k^{***}, m^{***})\).

Figure 3. Phase diagram
4 An Alternative Specification

In this section we consider two modifications to our set-up, thus bringing it closer to more conventional approaches. Firstly, we shall consider income proportional taxation, i.e., labour income is taxed at a flat rate \( \tau \in (0,1) \). In this case, the saving function in (6) is replaced by

\[
      s_t^* = \delta(1-\tau)w_t. \tag{30}
\]

The second modification applies to the production technology for which we replace (11) with

\[
      Y_t = A_tK_t^\gamma L_t^\nu, \tag{31}
\]

where now it is assumed that

\[
      A_t = \Theta(m_t)[H_t^\gamma G_t^{1-\gamma}]^\zeta, \quad \zeta \in (0,1). \tag{32}
\]

The ideas behind the effect of trust on productivity and the learning-by-doing mechanism remain the same. The variable \( G_t \) is once more the value of public goods and services per person, for which it is assumed that they are financed by tax revenues according to a continuously-balanced budget, i.e.,

\[
      NG_t = N\tau w_t. \tag{33}
\]

Furthermore, we assume that \( \zeta + \eta < 1 \) to guarantee the existence of a bounded steady state solution for the capital stock.\(^{17}\)

With these assumptions, it is straightforward to establish that the transition equation for capital accumulation, originally in (21), will be replaced by

\[
      k_{t+1}^* = K(k_t, m_t) = \bar{\Lambda}[\Theta(m_t)]^{\frac{1}{1-\eta}} k_t^{\frac{\lambda\tau+\eta}{1-\eta}} , \tag{34}
\]

where \( \bar{\Lambda} \equiv \delta(1-\tau)(1-\eta)^{\frac{1}{1-\eta}}(H_t^\gamma \tau^{1-\gamma})^\zeta \). Using (34) we can establish that the results in Proposition 2 still hold. As for the dynamics of trust, originally in (20), these are now described by

\[
      m_{t+1} = M(k_t, m_t) = \begin{cases} 
      0 & \text{if } m_t \leq \beta \\
      \frac{1}{1-\beta} \left[ 1 - \bar{\Lambda}[\Theta(m_t)]^{\frac{1}{1-\eta}} k_t^{\frac{\lambda\tau+\eta}{1-\eta}} \frac{1}{1-\eta} \right] [1 - (\beta / m_t)] & \text{if } m_t > \beta 
   \end{cases}, \tag{35}
\]

\(^{17}\) If \( \zeta = 1-\eta \) the economy will exhibit ever increasing levels of output per worker over time.
where $\tilde{A} = [r(1-\eta)]^{(1-\lambda)\xi} H^{\lambda}$. Again, it is straightforward to verify that the results of Proposition 1 still hold.

Using the planar system of Eq. (34) and (35), we can also see that the results in Lemma 1 remain the same qualitatively. That is, after substituting Eq. (15), there are three equilibrium pairs $(k^*, m^*)$, $(k^{**}, m^{**})$ and $(k^{***}, m^{***})$, such that $m^* = 0$, $m^{**} = 1$, $k^* = (\tilde{\lambda})^{1-(1-\lambda)\xi/(1-\xi-\eta)(1-q)}$, $1/(1-\xi-\eta)$ and $k^{**} = (\tilde{\lambda})^{1-(1-\lambda)\xi/(1-\xi-\eta)}$. The issue is that it is impossible to get analytical solutions for $m^{**}$ and $k^{**}$. Nevertheless, it is clear that the behaviour of the equilibrium pair $(k^{**}, m^{**})$ is the same as with the one illustrated in Figure 2, meaning that once more the TI locus in the phase diagram will be similar to the one in Figure 3, acting as a threshold that determines the equilibrium path for given initial conditions. The CS locus will also be monotonically increasing, as in the original phase diagram (Figure 3), the only difference now is that it is going to be non-linear. The dynamic implications, as these are summarised in Proposition 3, will remain unaffected though.

5 Conclusion

The view that the social and economic dimensions of a nation, rather than being independent, are closely interlinked is by no means a new one. Nevertheless, it is receiving increased attention in recent years. With this paper our purpose was to contribute to this emerging literature by adding to the current understanding on the conditions that underpin the relation between trust and economic development, and the implications from it. This was done by means of a dynamic model where the evolution of trust and the formation of capital are endogenous and mutually dependent. The characteristics of the model’s equilibrium suggest that current imbalances among nations can cast their shadow over their long-term socio-economic prospects. In other words, the positive feedback in the co-evolution of trust and economic activity may perpetuate these imbalances and establish them as permanent fixtures. In this respect, both the current state of interpersonal trust and the current stage of economic development may be vital in perpetuating these differences. Economies at a similar stage of economic development, but different levels of trust, may experience strikingly different long-term prospects. Equally important, however, is the
likelihood that countries with similar levels of trust may yet experience drastically opposite socio-economic paths if they differ in terms of their economic conditions.

Methodologically, our approach was to analyse these issues in the most tractable manner so as to enhance the clarity of the mechanisms involved and to avoid blurring their intuition. One of the means to achieve this was the careful selection of functional forms to allow the derivation of closed-form solutions. Furthermore, in order to maintain a sharp focus on the joint dynamics of trust and capital accumulation, without undermining our story by making its intuition impenetrable, we absconded from other issues that could provide a broader perspective in terms of both social capital and economic performance. For example, in addition to trust, other components of social capital that can be transmitted through successive generations of individuals are social norms. Their importance in relation to economic growth has already been identified by researchers (e.g., Cole et al. 1992; Palivos 2001) but without considering a mechanism for intergenerational transmission of such norms. Furthermore, besides the dynamics of income per capita, social trust could impinge on other characteristics of the economy such as income inequality or demographics (e.g., fertility behaviour). Finally, social trust could interact with other engines of long-run growth such as education/human capital and R&D/technological progress. All these issues certainly merit attention and offer a large scope for future research on the co-evolution of economic and social characteristics.

Appendix

A1 Proof of Lemma 1

We are looking for solutions satisfying \( k_{t+1} = k_t = k \) and \( m_{t+1} = m_t = m \). Applying the steady state condition in (23) yields,

\[
k = \frac{\delta \theta (1 - q + qm)}{1 - \delta b}.
\]

(A1.1)

Recall that from the expression in (19), we have \( m_{t+1} = 0 \) \( \forall t \) whenever \( m_t \leq \beta \). Together with (A1.1) this implies that the pair

\[
m = 0 \quad \text{and} \quad k = \frac{\delta \theta (1 - q)}{1 - \delta b},
\]

24
is a steady state. Next, substitute (A1.1) in the part of (20) that applies for \( m, > \beta \) and use the steady state condition to write

\[
m = \left[ 1 + \frac{g\delta \theta(1-q+qm)}{1-\delta b} \right] \frac{[1-(\beta/m)]}{1-\beta + \frac{g\delta \theta(1-q+qm)}{1-\delta b} [1-(\beta/m)]}.
\]  

(A1.2)

Defining the composite parameter terms \( \psi \equiv \delta h \) and \( \varepsilon \equiv g\delta \theta \), the equation in (A1.2) can be manipulated algebraically to derive

\[
(1-\psi)(1-\beta)m^2 = (1-\psi)(m-\beta) + \varepsilon(1-q+qm)(m-\beta)(1-m).
\]  

(A1.3)

The cubic expression in (A1.3) has three roots, only two of them being acceptable in the sense that they lie on the interval \( (\beta, 1] \). These are \( m^{***} = 1 \) and

\[
m^{**} = -\frac{(1-\psi)(1-\beta) + \varepsilon[1-q(1+\beta)]}{2q\varepsilon} \\
+ \frac{\sqrt{\left[ (1-\psi)(1-\beta) + \varepsilon[1-q(1+\beta)] \right]^2 + 4q\varepsilon(1-\eta)(1-q)\varepsilon}}{2q\varepsilon},
\]

which are the solutions in Eq. (26) and (25) respectively. Using the solution in (25), it is straightforward to establish that \( m^{**} > \beta \Leftrightarrow \beta(1-\psi) > 0 \) (which holds given \( 0 < \psi < 1 \) by virtue of Assumption 2) and \( m^{**} < 1 \Leftrightarrow (1-\psi)(1-2\beta) + \varepsilon(1-\beta) > 0 \) (which holds given \( \beta < 1/2 \) by virtue of Assumption 3). Thus, the pairs

\[
m = m^{**} \quad \text{and} \quad k = \frac{\delta \theta (1-q+qm^{**})}{1-\delta b},
\]

and

\[
m = 1 \quad \text{and} \quad k = \frac{\delta \theta}{1-\delta b},
\]

are steady state equilibria. Finally, after verifying that \( \frac{\partial k}{\partial m} > 0 \) from (A1.1), it follows that \( k^{***} > k^{**} > k^* \).

A2 Proof of Lemma 2

Consider the solutions that satisfy \( k_{r+1} = k_r = k \) and \( m_{r+1} = m_r = m \). The Jacobian matrix associated with the planar system of difference equations in (23) and (20) is the following:
\[
\begin{bmatrix}
K_k & K_m \\
M_k & M_m
\end{bmatrix}
\]

Note that \(K_k = \delta b \equiv \psi \in (0,1)\) and \(K_m = \delta q\), whereas for \(m_i \leq \beta\) it is \(M_k = M_m = 0\). Therefore, with the equilibrium pair \(m^* = 0\) and \(k^* = \frac{\delta \theta (1-q)}{1-\psi}\), the trace and the determinant are respectively given by

\[
TR = \psi \in (0,1) \text{ and } DET = 0.
\] (A2.1)

Since \(1 + TR + DET = 1 + \psi > 0, \text{ } 1 - TR + DET = 1 - \psi > 0, \text{ } |TR| < 2 \) and \(|DET| < 1\), we conclude that the pair \((k^*, m^*)\) is a stable equilibrium. For \(m_i > \beta\) it is

\[
M_k = \left(\frac{1}{m} - 1\right) \left(1 - \frac{\beta}{m}\right) \frac{\beta g}{\{1 - \beta + gk[1 - (\beta / m)]\}^2},
\] (A2.2)

and

\[
M_m = \frac{\beta}{m} \frac{(1 - \beta)(1 + gk)}{\{1 - \beta + gk[1 - (\beta / m)]\}^2}.
\] (A2.3)

Focusing on the equilibrium pair \(m^{***} = 1\) and \(k^{***} = \frac{\delta \theta}{1-\psi}\), we can see that \(M_k = 0\) and

\[
M_m = \frac{\beta (1 - \beta)(1 + gk)}{[1 - \beta + gk(1 - \beta)]^2} = \frac{\beta}{(1 - \beta)(1 + gk)} \equiv \zeta,
\] (A2.4)

where the composite term \(\zeta\) satisfies \(\zeta \in (0,1)\) by virtue of \(\beta < 1/2\). Therefore,

\[
TR = \psi + \zeta \text{ and } DET = \psi \zeta,
\] (A2.5)

meaning that \(1 + TR + DET = (1 + \psi)(1 + \zeta) > 0, \text{ } 1 - TR + DET = (1 - \psi)(1 - \zeta) > 0, \text{ } |TR| < 2 \) and \(|DET| < 1\). Thus, the pair \((k^{***}, m^{***})\) is also a stable equilibrium.

The complexity of the steady state solutions for \(m^*\) and \(k^*\), together with the complex expressions in (A2.2) and (A2.3) impose an insurmountable difficulty in evaluating analytically the expressions for \(1 + TR + DET\) and \(1 - TR + DET\). For this reason, we shall evaluate these expressions by means of numerical examples. Doing so, we shall allow \(\beta\) and \(q\) to range freely within their permissible values and set a baseline parameter configuration for the remaining parameters, making sure that the conditions in Assumptions 1-3 hold. Subsequently, we shall deviate from the baseline case by choosing (in turns) different values for some of these parameters.
The baseline scenario sets $\delta = 0.5$, $g = 0.4$, $\eta = 0.6$, $H = 10$, $\lambda = 0.5$ and $\theta = 1$. Given these, below we plot $1 + TR + DET$ and $1 - TR + DET$ against $\beta \in [0, 1/2]$ and $q \in [0, 1]$ using three-dimensional diagrams. As we can see from the plots, $1 + TR + DET > 0$ and $1 - TR + DET < 0$ meaning the pair $(k^{**}, m^{**})$ is a saddle point.

As we can see from the Figures below (A2 to A9), the result that the pair $(k^{**}, m^{**})$ is not a stable equilibrium survives under a wide range of parameter values deviating from the baseline case. We do this in turns, considering different parameter values for $H$, $g$, $\delta$, and $\theta$. The resulting plots verifying that $1 + TR + DET > 0$ and $1 - TR + DET < 0$ in all cases.
Figure A3. $H = 80$

Figure A4. $g = 0.1$

Figure A5. $g = 0.55$
Figure A6. $\delta = 0.2$

Figure A7. $\delta = 0.8$

Figure A8. $\theta = 0.5$
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