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ABSTRACT. We investigate the problem of reordering agents starting from an existing queue. First, we introduce four important axioms of the problem, budget balance (BB), outcome efficiency (OE), strategyproofness (SP), and individual rationality (IR). Unfortunately, it is easy to show that these four axioms are incompatible in the current setup. Given this negative result, we examine the consequences of relaxing BB, OE and SP, one at a time. Our results are as follows: (i) There is no mechanism satisfying OE, SP and IR which runs a non-negative surplus at all profiles. (ii) When there are two agents, the only non-trivial mechanisms satisfying BB, SP and IR are fixed price trading mechanisms but there are additional mechanisms when there are more than two agents. We identify an intuitive mechanism which we call the *median price exchange mechanism* and characterize its maximal level of inefficiency. (iii) By weakening SP to ‘one-sided’ strategyproofness, we identify two mechanisms, the *buyers’ mechanism* and the *sellers’ mechanism*, and characterize them on the basis of independence axioms.

JEL Classifications: C72, D63, D82.

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1. INTRODUCTION

In a queueing environment, agents want to access a service but they can be served only one at a time. One of the early papers to examine this problem was Naor [20] who addressed the issue of social efficiency for the *first come, first serve* (fcfs) queueing protocol. Dolan [9] considered queueing problems with both stochastic and deterministic arrivals of the agents and addressed the problem of asymmetric information. Assuming deterministic arrivals of the agent, queueing problems have been analyzed extensively in the literature (see Chun [3], [4], Chun, Mitra and Mutuswami [5], Curiel, Pederzoli and Tijs [8], Gershkov and Schweinzer [10], Kayi and Ramaekers [14], Maniquet [15], Mitra [16] and Mitra and Mutuswami [17]).

With the exception of the works of Curiel, Pederzoli and Tijs [8] and Gershkov and Schweinzer [10], the standard assumption has been that there

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is no initial order of the agents. However, we do face queueing situations with an initial order.¹ In most such cases, the fcfs rule is used to determine the order in which agents are served. While this rule is simple to implement, it is not desirable when waiting in a queue is costly for agents and the objective is to minimize the aggregate waiting cost. In such instances, some reordering of the initial queue may be desirable. The fact that trading of queue slots is allowed in some cases points to the desirability of reordering the initial queue.²

In this paper, we address the problem of reordering an existing queue from a mechanism design viewpoint. In our setup, agents have quasi-linear preferences and waiting costs are linear in time. We are interested in mechanisms which are *budget balanced* (the sum of the transfers to agents is zero), *outcome efficient* (the selected queue minimizes aggregate waiting costs) and *strategyproof* (no agent can benefit strictly by reporting untruthfully).

For the queueing problem without any initial order, papers by Mitra [16] and Kayi and Ramaekers [14] reveal the existence of “first-best” mechanisms which satisfies all three properties. Since one can always ignore the initial queue, it follows that our problem is non-trivial only if the presence of the initial queue imposes some additional requirements. One natural requirement is *individual rationality*. This requires that any reordering must give an agent at least the utility she would have obtained in the first-come, first-serve protocol. Not only is this requirement natural, it is present in some practical situations too.³

The problem that we are interested in has been examined by Gershkov and Schweinzer [10] from a Bayesian mechanism design viewpoint. Our work differs from theirs in two ways. Firstly, they use *Bayesian incentive compatibility* as their strategic notion while we use *strategyproofness*. In contrast to Bayesian incentive compatible mechanisms, strategyproof mechanisms offer the advantage that their specification is not dependent on the particular prior chosen by the mechanism designer.⁴

Secondly, Gershkov and Schweinzer [10] are concerned with conditions under which the efficient queue correspondence can be implemented. They show that if the default service protocol is fcfs then there is no mechanism for efficient reordering satisfying Bayesian incentive compatibility, budget

¹See Gershkov and Schweinzer [10] for some examples.

²Gershkov and Schweinzer [10] provide two instances: trading landing slots at some U. S. airports and trading queue positions in the British National Health Service system.

³For instance, Gershkov and Schweinzer [10] note that the British NHS follows the *universality principle* wherein patients have the right to accept or not accept the offered payments for switching queue positions.

⁴See Bergemann and Morris [2] for a discussion of the need to find mechanisms which are robust to the “assumed structure of the environment.”

balance and individual rationality. However, if the default service protocol is a *random queue*, then there is such a mechanism. Since individual rationality is defined with respect to the (expected) utility an agent gets from the default service protocol, these results can be interpreted as saying that efficient reordering is possible if the property rights over queue positions are “small.” In contrast, we are interested in the extent to which aggregate waiting cost can be reduced by reordering when property rights are “strong” (meaning that the default protocol is *fcfs*).

We start by showing that budget balance, outcome efficiency, strategyproofness and individual rationality are incompatible. In light of Gershkov and Schweinzer’s results, this is not surprising.⁵ Given the negative result, we proceed in three different directions by relaxing budget balance, outcome efficiency and strategyproofness, one at a time. We retain individual rationality throughout for two reasons. Firstly, we know that if we relax the individual rationality requirement to the utility from the random queue protocol, then there is a mechanism satisfying the remaining three properties. Secondly, when there is an initial queue, it makes sense to think that an agent will not agree to move to a different queue position if this leaves her worse-off than staying at her current queue position.

We first drop budget balance. We identify the class of mechanisms satisfying outcome efficiency, strategyproofness and individual rationality. It is a sub-class of the VCG mechanisms.⁶ What is noteworthy is that there is no mechanism in this sub-class which always runs a non-negative budget surplus. Hence, any mechanism which is outcome efficient, strategyproof and individually rational must run a budget deficit at some profile. Additionally, the magnitude of budget deficit can be unbounded. We thus do not gain much by dropping budget balance.

When we drop outcome efficiency, there are many mechanisms satisfying the remaining axioms, including those that are trivial.⁷ For the case of two agents, we show that the only “non-trivial” mechanisms are *fixed price trading mechanisms*. In such mechanisms, a price is chosen *a priori* and agents can, if they want, trade at that price. In any trade, the two agents exchange positions and the one moving forward pays the *a priori* chosen price to the one moving back. It is not difficult to see that such mechanisms

⁵However, our result is not implied by that of Gershkov and Schweinzer [10] nor *vice versa*. We discuss the difference in Section 3.

⁶The VCG mechanisms are due to Vickrey [21], Clarke [6] and Groves [12].

⁷The mechanism that selects the initial queue and assigns zero transfers to all agents at all profiles is an example.

will not be outcome efficient. The cost of achieving strategyproofness is therefore efficiency.⁸

When there more than two agents, obtaining a complete characterization of the set of mechanisms satisfying budget balance, strategyproofness and individual rationality becomes difficult mainly because of our domain restriction (quasi-linearity of preferences in a specific form).⁹ Rather than prove a characterization result, we identify a strategyproof mechanism, budget balanced and individually rational mechanism and put a bound on its level of inefficiency. Our measure of inefficiency is based on the distance between the agent's queue position in the actual queue (after reordering) and the agent's queue position in the outcome efficient queue.

The particular mechanism that we identify is the *median price exchange mechanism* and is based on the “median voter” type mechanism. When the outcome efficient queue is unique, this mechanism works as follows for a set of n agents. We first consider all $n(n-1)/2$ distinct pairs of agents (i, j) and fix an order on these pairs. Given a profile (or vector of waiting costs), we select the median waiting cost as the trading price. Agents are then allowed to trade their queue positions in the fixed order provided they mutually agree to trade at this median price. It can be shown that median price exchange mechanism is $(n-1)/2$ -inefficient meaning that at any generic profile, no agent is more than $(n-1)/2$ positions away from where the agent would be in the outcome efficient queue.¹⁰

Finally, we weaken strategyproofness. A natural weakening of strategyproofness is Bayesian incentive compatibility but this requires specifying priors for each agent.¹¹ Instead of going that route, we consider another form of weakening strategyproofness that requires that agents not have incentives to misreport in one direction: *upward strategyproofness* and *downward strategyproofness*. Upward (Downward) strategyproofness requires that agents do not have incentives to report higher (lower) waiting costs. We identify two “one-directional” strategyproof mechanisms that also satisfy budget balance, outcome efficiency and individual rationality that we dub the *buyers' mechanism* and the *sellers' mechanism* respectively.

⁸Similar results obtain in other contexts also, for instance, the classical exchange economy. See Barberà and Jackson [1].

⁹See Goswami, Mitra and Sen [11] for a detailed discussion of the difficulties that arise while characterizing strategyproof social choice functions even in the presence of Pareto efficiency in the context of quasi-linear exchange economies.

¹⁰This definition of median price exchange mechanism assumes that n is odd. If n is even, the corresponding mechanism can be adjusted appropriately.

¹¹Additionally, this approach has already been examined by Gershkov and Schweitzer [10].

The buyers' mechanism satisfies downward strategyproofness while the sellers' mechanism satisfies upward strategyproofness. The dynamics underlying these mechanisms is the following: starting from the initial queue and given a profile of waiting costs, we sequentially reach the outcome efficient queue by pairwise cost reducing interchange of agents in consecutive queue positions. For all pairwise switches, the trading price is the minimum (maximum) of the two waiting costs under the buyers' (sellers') mechanism. We also characterize these two mechanisms by replacing one-sided strategyproofness with 'appropriate' independence axioms. The reason for this replacement is that one-sided strategyproofness is very weak and fails to give restrictions which are adequate for the purposes of characterization.

2. THE MODEL

Let $N = \{1, \dots, n\}$, $n \geq 2$, be the set of agents. Each agent has one job to process and the processing time for all jobs is the same. Without loss of generality, the processing time is normalized to one. There is a server which can process only one job at a time.

A *queue* is an onto function $\sigma : N \rightarrow \{1, \dots, n\}$ denoting the order in which jobs are processed. Agent i 's position in the queue σ is denoted by σ_i . Let $\Sigma(N)$ be the set of all possible queues of agents in N . Given a queue $\sigma \in \Sigma(N)$, the set of all predecessors of i is $P_i(\sigma) = \{j \in N \mid \sigma_j < \sigma_i\}$ and the set of all followers of i is $F_i(\sigma) = \{j \in N \mid \sigma_j > \sigma_i\}$. When the context is clear, we shall simply refer to P_i and F_i . There is an *initial queue* σ^0 which determines the order in which jobs will be processed if no reordering is done.

If agent i 's queue position is σ_i , then she incurs a waiting cost of $(\sigma_i - 1)\theta_i$ where θ_i is the waiting cost per unit of time.¹² The agent's net utility u_i depends on the agent's waiting costs and the transfer that the agent receives. We assume that preferences are quasi-linear and hence $u_i(\sigma_i, t_i; \theta_i) = t_i - (\sigma_i - 1)\theta_i$.

A *profile* of waiting costs, $\theta \equiv (\theta_1, \dots, \theta_n) \in \mathbb{R}_+^n$, is a collection of the waiting costs of all agents. The profile of waiting costs in a coalition S , $(\theta_i)_{i \in S}$, is denoted by θ_S . We call the profiles θ and θ' *i-variants* if $\theta_k = \theta'_k$ for all $k \neq i$. The *aggregate waiting cost* associated with a queue σ and a profile of waiting costs θ is $\sum_{i \in N} (\sigma_i - 1)\theta_i$. A queue σ is *efficient* at the profile θ if $\sigma = \operatorname{argmin}_{\sigma' \in \Sigma(N)} (\sigma'_i - 1)\theta_i$. The set of all efficient queues at the profile θ is denoted $E(\theta)$.

¹²This formulation implies that no waiting cost is incurred by the agent while the agent's job is being processed. Relaxing this assumption makes no difference to the results.

The *queueing problem with an initial order* $G \equiv (N, \theta, \sigma^0)$ is a triple where N is the set of agents, θ is the profile of waiting costs of the agents and σ^0 is the initial queue. The natural objective is to reorder the original queue so as to lower the aggregate waiting cost. The problem receives its bite from the fact that an agent's per unit waiting cost is known only to the agent.

A *mechanism* $\mu = (\sigma, t)$ associates to each profile θ , a tuple $\mu(\theta) \equiv (\sigma(\theta), t(\theta))$ where $\sigma(\theta)$ is the reordered queue and $t(\theta)$ is the vector of transfers to the agents. An agent's own allocation is denoted $\mu_i(\theta) = (\sigma_i(\theta), t_i(\theta))$. Since the announcements of the agents need not be truthful, we let $u_i(\mu_i(\theta'); \theta_i) = -(\sigma_i(\theta') - 1)\theta_i + t_i(\theta')$ denote i 's utility when the announced profile is θ' and the agent's true waiting cost is θ_i .

2.1. Axioms. We now define some properties of mechanisms that are of interest. The first is *budget balance* which requires that the transfers to the agents sum to zero meaning that no money flows into or out of the system.

Definition 2.1. A mechanism μ is *budget balanced* (BB) if for all profiles θ , $\sum_{i=1}^n t_i(\theta) = 0$.

The second property is *outcome efficiency* which requires that the selected queue at any profile be efficient, i.e., the queue minimizes the aggregate waiting cost of all agents.

Definition 2.2. A mechanism μ is *outcome efficient* (OE) if for all profiles θ , $\sigma(\theta) \in E(\theta)$.

Remark 2.3. Outcome efficiency requires agents to be served in the non-increasing order of their waiting costs. The outcome efficient queue is not unique when two or more agents have the same waiting cost. In what follows, we assume that there is a tie-breaking rule to select among queues in $E(\theta)$ whenever the set is not a singleton. Any tie-breaking rule will suffice for our purpose.

The third property is *strategyproofness* which requires that truth-telling be a dominant strategy for all agents. This is a desirable property because it ensures that no agent can benefit by misrepresenting her waiting cost no matter what she believes other agents to be doing.

Definition 2.4. A mechanism μ is *strategyproof* (SP) if for all i -variants θ and θ' , $u_i(\mu_i(\theta); \theta_i) \geq u_i(\mu_i(\theta'); \theta_i)$.

Finally, *individual rationality* requires that each agent's utility in the re-ordered queue be at least as large as the utility she would get if the jobs were processed according to the order in the initial queue (and no transfers

were given). If a mechanism does not satisfy this property, then agents may not agree to trade their positions.

Definition 2.5. A mechanism μ is *individually rational* (IR) if for all profiles θ and all $i \in N$, $u_i(\mu_i(\theta); \theta_i) \geq -(\sigma_i^0 - 1)\theta_i$.

2.2. An Impossibility Result. We start with a negative result. We do not provide a proof here because in the next section, we prove a stronger result. To be precise, we show in Theorem 3.6 that BB, OE, SP and *feasibility* are incompatible. Feasibility relaxes budget balance by allowing a mechanism to run a budget surplus but not a deficit.

Theorem 2.6. *BB, OE, SP and IR are incompatible.*

Proof. Follows from Theorem 3.6. □

Remark 2.7. For the queueing problems without any initial order, Mitra [16] identified the complete class of mechanisms satisfying BB, OE and SP and, among this class of mechanisms, Kaya and Ramaekers [14] identified the “unique” mechanism that also satisfy equal treatment of equals. It is easily verified that this unique mechanism also satisfies the “identical preference lower bound” which defines the lower bound with respect to the expected utility an agent gets if every queue is chosen with equal probability. Hence, one way of interpreting Theorem 2.6 is that it is not possible to design budget balanced, strategyproof and efficient mechanisms when property rights in queue positions are “strong.” A similar point has also been made by Gershkov and Schweinzer [10] who also note the relationship to the work of Cramton, Gibbons and Klemperer [7].

3. RELAXING BUDGET BALANCE

The obvious implication of Theorem 2.6 is that at least one of the four axioms of BB, OE, SP and IR have to be dropped or relaxed to obtain positive results. In this section, we examine the consequences of relaxing BB.

We start by investigating the class of mechanisms satisfying OE, SP and IR. For any mechanism μ satisfying OE, we define the *social benefit from trade at a profile θ* as $B(\theta) = \sum_{j \in N} (\sigma_j^0 - \sigma_j(\theta))\theta_j$.

Now suppose that agent i leaves the initial queue σ^0 . We can then imagine that all agents behind i move up a place, so the initial queue now looks as follows:

$$\sigma_j^{0,-i} = \begin{cases} \sigma_j^0 & \text{if } \sigma_j < \sigma_i, \\ \sigma_j^0 - 1 & \text{if } \sigma_j > \sigma_i. \end{cases}$$

We define the *social benefit from trade in the absence of agent i at the profile θ* as¹³

$$B_{-i}(\theta) = \sum_{j \in N \setminus \{i\}} \left(\sigma_j^{0,-i} - \sigma_j(\theta_{N \setminus \{i\}}) \right) \theta_j.$$

Proposition 3.1. *Let μ be a mechanism satisfying OE. Then, for any profile θ and any $i \in N$, $B(\theta) \geq B_{-i}(\theta)$.*

Proof. By definition,

$$B(\theta) - B_{-i}(\theta) = (\sigma_i^0 - \sigma_i(\theta))\theta_i + \sum_{j \in N \setminus \{i\}} A_{ij}(\theta)\theta_j$$

where for each $j \in N \setminus \{i\}$,

$$\begin{aligned} A_{ij}(\theta) &= [\sigma_j^0 - \sigma_j(\theta)] - [\sigma_j^{0,-i} - \sigma_j(\theta_{N \setminus \{i\}})] \\ &= [\sigma_j^0 - \sigma_j^{0,-i}] - [\sigma_j(\theta) - \sigma_j(\theta_{N \setminus \{i\}})]. \end{aligned}$$

By OE, it follows that

$$\begin{aligned} \sum_{j \in N \setminus \{i\}} A_{ij}(\theta) &= \sum_{j \in F_i(\sigma^0)} (1) - \sum_{j \in F_i(\sigma(\theta))} (1) \\ &= (n - \sigma_i^0) - (n - \sigma_i(\theta)) \\ &= -(\sigma_i^0 - \sigma_i(\theta)). \end{aligned}$$

Using the above expression, we can write

$$(3.1) \quad B(\theta) - B_{-i}(\theta) = \sum_{j \in N \setminus \{i\}} A_{ij}(\theta)(\theta_j - \theta_i).$$

To complete the proof we show that $A_{ij}(\theta)(\theta_j - \theta_i)$ is always non-negative.

- (i) If $j \in F_i(\sigma^0) \cap F_i(\sigma(\theta))$ or $j \in P_i(\sigma^0) \cap P_i(\sigma(\theta))$, then $A_{ij}(\theta) = 0$ and so, $A_{ij}(\theta)(\theta_j - \theta_i) = 0$.
- (ii) If $j \in F_i(\sigma^0) \cap P_i(\sigma(\theta))$, then $A_{ij}(\theta) = 1$ and from OE, we get $\theta_j \geq \theta_i$. Hence, $A_{ij}(\theta)(\theta_j - \theta_i) \geq 0$.
- (iii) If $j \in P_i(\sigma^0) \cap F_i(\sigma(\theta))$, then $A_{ij}(\theta) = -1$ and from OE, we get $\theta_j \leq \theta_i$. Hence, $A_{ij}(\theta)(\theta_j - \theta_i) \geq 0$. $\square$

For what follows, we define $U_i(\sigma_i, t_i; \theta_i) = u_i(\sigma_i, t_i; \theta_i) + (\sigma_i^0 - 1)\theta_i = (\sigma_i^0 - \sigma_i)\theta_i + t_i$ as the net utility of agent i when she receives the allocation (σ_i, t_i) . Note that IR implies $U_i(\sigma_i(\theta), t_i(\theta); \theta_i) \geq 0$ for all θ and all $i \in N$.

Definition 3.2. A mechanism μ is a *Vickrey-Clarke-Groves (VCG) mechanism* if it satisfies OE and for all profiles θ and all $i \in N$,

$$(3.2) \quad t_i(\theta) = \sum_{j \in N \setminus \{i\}} (\sigma_j^0 - \sigma_j(\theta))\theta_j + g_i(\theta_{N \setminus \{i\}}).$$

¹³In the below equation, $\sigma(\theta_{N \setminus \{i\}})$ is the efficient queue for the $N \setminus \{i\}$ economy.

By writing $\sum_{j \in N \setminus \{i\}} (\sigma_j^0 - \sigma_j(\theta)) \theta_j = B(\theta) - (\sigma_i^0 - \sigma_i(\theta)) \theta_i$ and, without loss of generality, writing $g_i(\theta_{N \setminus \{i\}}) = -B_{-i}(\theta) + h_i(\theta_{N \setminus \{i\}})$, we can define the VCG mechanism in the following way.¹⁴

Definition 3.3. A mechanism μ is a VCG mechanism if it satisfies OE and for all profiles θ and all $i \in N$,

$$(3.3) \quad t_i(\theta) = B(\theta) - (\sigma_i^0 - \sigma_i(\theta)) \theta_i - B_{-i}(\theta) + h_i(\theta_{N \setminus \{i\}}).$$

Holmström [13] shows that the VCG mechanisms are the only ones satisfying OE and SP when the domain of preferences is convex. Since the domain of preferences in our context is $\mathbb{R}_+^n$, it follows that the VCG mechanisms are the only ones satisfying OE and SP.

If $h_i(\theta_{N \setminus \{i\}}) = 0$ for all $i \in N$ and all profiles θ , then we have the *pivotal mechanism*. For this mechanism, $U_i(\sigma_i(\theta), t_i(\theta); \theta_i) = B(\theta) - B_{-i}(\theta) \geq 0$. The inequality follows from Proposition 3.1 and this shows that there is at least one mechanism satisfying OE, SP and IR.

Our next proposition completely characterizes the class of mechanisms satisfying OE, SP and IR. For this characterization, we use the following definition. Let $B_i(\theta)$ be the maximal social surplus if agent i remains in her initial queue position and reordering takes place only among the agents in $N \setminus \{i\}$. Formally,

$$B_i(\theta) = \min_{y \in \mathbb{R}_+} B(y, \theta_{N \setminus \{i\}}).$$

It is easy to see that $B(y, \theta_{N \setminus \{i\}})$ is minimized for that type of agent i for whom there is an efficient queue such that $\sigma_i(y, \theta_{N \setminus \{i\}}) = \sigma_i^0$.

Since the notation can be confusing, let us mention that $B_{-i}(\theta)$ is the social surplus if agent i is totally removed from the economy. This removal means that the initial queue is shortened after which efficient reordering takes place. On the other hand $B_i(\theta)$ is the social surplus if agent i is present but remains in her initial queue position and (efficient) reordering takes place among the agents in $N \setminus \{i\}$.

Proposition 3.4. A mechanism μ satisfies OE, SP and IR if and only if it is a VCG mechanism with $h_i(\theta_{N \setminus \{i\}}) \geq B_{-i}(\theta) - B_i(\theta)$ for all $i \in N$ and all profiles θ .

Proof. As observed before, any mechanism μ satisfying OE and SP is a VCG mechanism and therefore, has a transfer given by (3.3). Choose $i \in N$ and a profile $\theta_{N \setminus \{i\}}$ for agents in $N \setminus \{i\}$. Let $\theta = (x_i, \theta_{N \setminus \{i\}})$. The net utility of agent i is

$$U_i(\mu_i(\theta); x_i) = B_i(\theta) - B_{-i}(\theta) + h_i(\theta_{N \setminus \{i\}}).$$

¹⁴Observe that $B_{-i}(\theta)$ does not depend on θ_i .

For IR, it is both necessary and sufficient that the right hand side of the above is non-negative for all $x_i \in \mathbb{R}_+$. Hence, it is necessary and sufficient that $B_i(\theta) - B_{-i}(\theta) + h_i(\theta_{N \setminus \{i\}}) \geq 0$, or equivalently, $h_i(\theta_{N \setminus \{i\}}) \geq B_{-i}(\theta) - B_i(\theta)$. Since the selection of agent i was arbitrary, the result follows. $\square$

We now use Proposition 3.4 to examine whether we can obtain positive results by relaxing BB to *feasibility*. Feasibility allows a mechanism to run a surplus but not a deficit. So long as there is no problem with disposing off the surplus, this should not lead to any difficulties. Since there are many strategyproof mechanisms like the pivotal mechanism which are not budget balanced, this relaxation is not trivial.

Definition 3.5. A mechanism μ satisfies *feasibility* (F) if for all profiles θ , $\sum_{i \in N} t_i(\theta) \leq 0$.

Unfortunately, the impossibility result persists even though BB is weakened to F.

Theorem 3.6. *OE, SP, IR and F are incompatible.*

Proof. Let $\mu^* = (\sigma, t^*)$ be a mechanism that satisfies OE, SP and IR and for which

$$h_i^*(\theta_{N \setminus \{i\}}) = B_{-i}(\theta) - B_i(\theta) \quad \text{for all } i \in N.$$

For all $i \in N$ and all profiles θ , the resulting VCG transfer is

$$(3.4) \quad t_i^*(\theta) = -(\sigma_i^0 - \sigma_i(\theta))\theta_i + B(\theta) - B_i(\theta).$$

From Proposition 3.4 we know that for any mechanism $\mu = (\sigma, t)$ satisfying OE, SP and IR, the associated VCG transfer must be such that

$$h_i(\theta_{N \setminus \{i\}}) \geq B_{-i}(\theta) - B_i(\theta) = h_i^*(\theta_{N \setminus \{i\}}).$$

Hence, for any $\mu = (\sigma, t)$ satisfying OE, SP and IR, $\sum_{i \in N} t_i(\theta) \geq \sum_{i \in N} t_i^*(\theta)$ for all profiles θ . To prove the incompatibility of the axioms, it is sufficient to show that μ^* fails to satisfy F, that is, there exists a profile θ such that $\sum_{i \in N} t_i^*(\theta) > 0$.

Without loss of generality, suppose that σ^0 is such that $\sigma_i^0 = n - i + 1$ for all $i \in N$. Consider the profile θ such that $\theta_1 > \theta_n > \theta_{n-1} > \dots > \theta_2$. The unique efficient queue for this profile is

$$\sigma_j(\theta) = \begin{cases} 1 & \text{if } j = 1, \\ n - j + 2 & \text{if } j \neq 1. \end{cases}$$

Using (3.4), we get $t_1^*(\theta) = -\sum_{j=2}^n \theta_j$, $t_n^*(\theta) = \theta_1$ and, for $n \geq 3$, $t_i^*(\theta) = \theta_{i+1}$ for all $i \in \{2, \dots, n-1\}$. Therefore, $\sum_{i \in N} t_i^*(\theta) = -\sum_{j=2}^n \theta_j + \sum_{j=3}^n \theta_j + \theta_1 = \theta_1 - \theta_2 > 0$ and we have the required incompatibility. $\square$

Remark 3.7. Theorem 3.6 shows that we do not gain anything by relaxing BB. Though we characterize a family of mechanisms that satisfy OE, SP and IR, none of these mechanisms satisfy F and hence, run a budget deficit. Furthermore, the extent of the budget deficit can be unbounded both in absolute terms and also in per capita terms. Note that in the Theorem 3.6, the extent of the budget deficit is the difference between θ_1 (the highest waiting cost) and θ_2 (the second highest waiting cost) and this difference can be made as large as one wants.

4. RELAXING OUTCOME EFFICIENCY

If we drop OE, then the set of mechanisms satisfying BB, SP and IR is large and includes trivial mechanisms. An example is the mechanism which at all profiles select the initial queue σ^0 and assigns zero transfers to every agent. Hence, a natural axiom to start with is non-triviality.

Definition 4.1. A mechanism μ is *non-trivial* (NT) if there exists a profile θ with $\theta_i > 0$ for all $i \in N$ such that $\sigma(\theta) \neq \sigma^0$.

For the two agent problem, this axiom (along with BB, SP and IR) suffices to give a complete characterization result. In this case, there are only two possible initial queues, one where agent 1 is in the first position (call this σ^0) and the other where agent 2 is in the first position (call this σ^1). We shall focus on the case where σ^0 is the initial queue. This is without loss of generality as the analysis for the two cases is identical.

Definition 4.2. Let $N = \{1, 2\}$ and σ^0 be the initial queue. The mechanism μ^f is a *fixed price trading mechanism* if there exists $p \geq 0$ such that the following conditions hold.

- (FP1) For all θ such that $\theta_1 < p < \theta_2$, $\sigma(\theta) = \sigma^1$.
- (FP2) For all θ such that either $\theta_1 > p$ or $\theta_2 < p$, $\sigma(\theta) = \sigma^0$.
- (FP3) For all θ such that $\theta_1 = p$ and $\theta_2 \geq p$, if there is θ such that $\sigma(\theta) = \sigma^1$, then for all $\theta'_2 > p$, $\sigma(\theta_1, \theta'_2) = \sigma^1$.
- (FP4) For all θ such that $\sigma(\theta) = \sigma^1$, $t_1(\theta) = -t_2(\theta) = p$, and for all θ such that $\sigma(\theta) = \sigma^0$, $t_1(\theta) = t_2(\theta) = 0$.

Theorem 4.3. Let $N = \{1, 2\}$ and σ^0 be the initial queue. A mechanism μ satisfies BB, SP, IR and NT if and only if $\mu = \mu^f$.

Proof. Since sufficiency is easy to verify, we only prove the necessity part. By NT, there exists a profile θ' such that $\sigma(\theta') = \sigma^1$. Denote $p = t_1(\theta')$; by BB, $t_2(\theta') = -p$. Now consider any profile $\theta \neq \theta'$. The proof strategy is to show that if trade (an exchange of positions) occurs, then it must be at the price p . Otherwise, no trade occurs and the transfers are zero. We

start by assuming that $p > 0$. The various cases that we discuss below are illustrated in Figure 1.

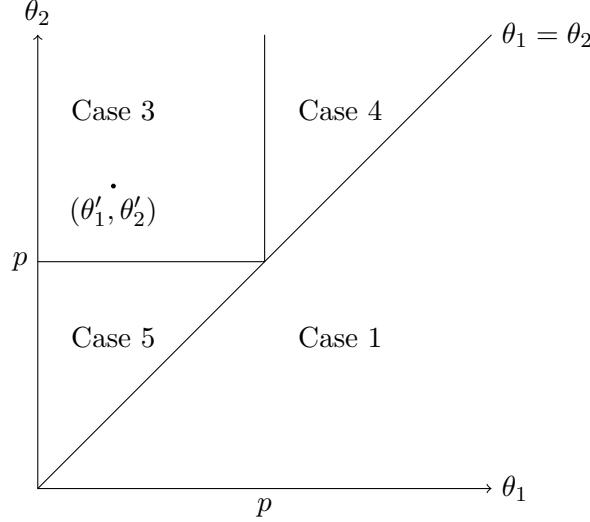


FIGURE 1. Cases for $n = 2$. Case 2 is the horizontal line at p excluding (p, p) , Case 6 is the 45° line excluding (p, p) and Case 7 is the vertical line at p above (p, p) .

Case 1: $\theta_1 > \theta_2$

Suppose trade occurs at the price p' . IR for agent 1 implies that $p' - \theta_1 \geq 0$ or $p' \geq \theta_1$. IR for agent 2 implies that $-p' \geq -\theta_2$ or $p' \leq \theta_2$. So we have $\theta_1 \leq p' \leq \theta_2$ which is a contradiction since $\theta_1 > \theta_2$. Hence, no trade occurs and by BB and IR, $t_1(\theta) = t_2(\theta) = 0$.

Case 2: $\theta_1 \in [0, p), \theta_2 = \theta'_2$.

Suppose trade does not occur. By the same argument as in Case 1, $t_1(\theta) = t_2(\theta) = 0$ and so, $u_1(\sigma_1(\theta), t_1(\theta); \theta_1) = 0$. If agent 1 deviates by announcing θ'_1 , then $u_1(\sigma_1(\theta'), t_1(\theta'); \theta_1) = p - \theta_1 > 0$. This violates SP. Hence, trade must occur and $\sigma(\theta) = \sigma^1$. If $t_1(\theta) = p' \neq p$, then 1 will manipulate at either θ or θ' and so, $t_1(\theta) = p$. By BB, $t_2(\theta) = -p$.

Case 3: $\theta_2 > p > \theta_1$.

If there is no trade, then $u_2(\sigma_2(\theta), t_2(\theta); \theta_2) = 0$. If agent 2 deviates by announcing θ'_2 , then by Case 2, a trade will take place (at price p) and this benefits agent 2 strictly because $p < \theta_2$. If $t_2(\theta) \neq -p$, then agent 2 will manipulate at either the profile θ or the profile (θ_1, θ'_2) . Hence, $t_2(\theta) = -p$ and by BB, $t_1(\theta) = p$.

Case 4: $\theta_2 > \theta_1 > p$.

Assume that trade takes place at the price p' . By BB and IR, $p' \in [\theta_1, \theta_2]$ and so $p' \geq \theta_1 > p$. This violates SP because agent 1 can unilaterally deviate from a profile of Case 3 to the profile θ .

Case 5: $p > \theta_2 > \theta_1$.

If trade takes place at the price p' , then by BB and IR, $p' \in [\theta_1, \theta_2]$ and so $p' < p$. This violates SP because agent 2 can now deviate profitably from a profile of Case 3 to the profile θ .

Case 6: $\theta_1 = \theta_2 \neq p$.

If trade takes place, then by BB and IR, it must be at the price $p' = \theta_1 = \theta_2$. If $p' > p$ then agent 1 can unilaterally deviate from a profile of Case 3 to θ . If $p' < p$, then agent 2 can profitably deviate from a profile of Case 3 to θ . Hence, no trade can take place.

Case 7: $\theta_2 \geq \theta_1 = p$.

At such a profile, agent 1 cannot profitably deviate no matter what the allocation (so long as IR is satisfied). As can be seen from Figure 1, a deviation will lead to one of cases 1, 2, 3, 4 or 6. In cases 2 and 3, there is trade but at a price of p . In the other cases, there is no trade. In all cases, the net utility after deviation is zero and this shows that the agent does not benefit strictly.

Consider agent 2. We claim that if trade takes place at any one profile, say $(p, \bar{\theta}_2)$, $\bar{\theta}_2 > p$, then it must take place at *all* profiles (p, θ_2) where $\theta_2 > p$. Indeed, if trade does not take place at some (p, θ_2) where $\theta_2 > p$, then agent 2 can deviate to $(p, \bar{\theta}_2)$ and this deviation is profitable because $\theta_2 > p$. There are therefore three possibilities for allocations at such profiles which are enumerated below.

- (1) For all profiles such that $\theta_2 \geq \theta_1 = p$, no trade takes place.
- (2) For all profiles such that $\theta_2 \geq \theta_1 = p$, trade takes place at a price p .
- (3) For all profiles such that $\theta_2 > \theta_1 = p$, trade takes place at the price p but no trade takes place when $\theta_1 = \theta_2 = p$.

We have now exhausted all profiles and it is trivial to verify that the resulting allocations correspond to a fixed price trading mechanism at the price p . To complete the proof, we need to consider the final case when $p = 0$. It is straightforward to verify (see Figure 1) that trade can take place—if at all—only when $\theta_1 = 0$, corresponding to Case 7. No trade takes place otherwise. $\square$

When there are more than two agents, getting a characterization result is difficult because the “price” for a trade between two players can depend on the waiting cost of a third agent. What we do is to identify a mechanism, which we call the *median price exchange mechanism*, that satisfies BB, SP and IR. We propose a measure of inefficiency based on the distance between the agent’s position in the actual queue and the agent’s position in the

outcome efficient queue. Using this measure of inefficiency, we analyze the median price exchange mechanism.

Definition 4.4. Let $k \in \{1, \dots, n-1\}$. The mechanism μ is k -inefficient if at every profile θ such that the outcome efficient queue, denoted by $\sigma^e(\theta)$, is unique, $|\sigma_i(\theta) - \sigma_i^e(\theta)| \leq k$ for all $i \in N$.

Remark 4.5. When the efficient queue is unique, OE requires that agent with the r th highest waiting cost to be in the r th position. Our axiom relaxes OE by allowing the agent with the r th highest waiting cost to be in any position from $\{r-k, \dots, r+k\}$. Since the maximal distance between an agent's "efficient" position and her actual position is at most $n-1$, every queue is $(n-1)$ -inefficient.

Let $\succ$ be an order on $(N \times N) \setminus \{(i, i) | i \in N\}$. Assume, to start with, that n is odd.

Definition 4.6. Given a profile θ , the median waiting cost θ_m is such that

$$|\{i : \theta_i \geq \theta_m\}| \geq \frac{n+1}{2} \quad \text{and} \quad |\{i : \theta_i \leq \theta_m\}| \geq \frac{n+1}{2}.$$

Let $M(\theta) \equiv \{i \in N | \theta_i > \theta_m\}$ be the set of agents with waiting costs strictly greater than the median waiting cost. Similarly, let $m(\theta) \equiv \{i \in N | \theta_i < \theta_m\}$ be the set of agents with waiting costs strictly smaller than the median waiting cost.

Definition 4.7. Let θ be a profile and σ a queue. Let (i, j) be a pair of agents such that $i \neq j$. An exchange is *feasible* between i and j if

- (1) $\theta_j > \theta_m > \theta_i$ and $\sigma_i < \sigma_j$, or
- (2) $\theta_j = \theta_m > \theta_i, \sigma_i < \sigma_j$ and $M(\theta) = \emptyset$, or
- (3) $\theta_j > \theta_m = \theta_i, \sigma_i < \sigma_j$ and $m(\theta) = \emptyset$.

Remark 4.8. A feasible exchange requires two agents with different waiting costs. In general, it also requires one agent whose waiting cost is *strictly greater* than the median waiting cost and another whose waiting cost is *strictly smaller* than the median waiting cost. This means that all agents whose waiting costs are equal to the median waiting cost are not part of any feasible exchange. The only exception to this general rule is if the median waiting cost is also the highest or the lowest waiting cost.

Definition 4.9. The *median price exchange mechanism* μ^m with an odd number of agents $n \geq 3$ is defined by the following algorithm. At stage $k \geq 1$, the input to the algorithm is the profile θ , the queue σ^{k-1} and the k th element of the order $\succ$, say, (i_k, j_k) .

- (1) If there is no feasible exchange between i_k and j_k , let $\sigma^k = \sigma^{k-1}$ and move to stage $k + 1$.
- (2) Otherwise, define the queue σ^k by

$$\sigma_i^k = \begin{cases} \sigma_i^{k-1} & \text{if } i \neq i_k, j_k, \\ \sigma_{i_k}^{k-1} & \text{if } i = j_k, \\ \sigma_{j_k}^{k-1} & \text{if } i = i_k. \end{cases}$$

The transfers to the agents are defined by

$$t_i^k = \begin{cases} 0 & \text{if } i \neq i_k, j_k, \\ (\sigma_{i_k}^{k-1} - \sigma_{j_k}^{k-1})\theta_m & \text{if } i = j_k, \\ (\sigma_{j_k}^{k-1} - \sigma_{i_k}^{k-1})\theta_m & \text{if } i = i_k. \end{cases}$$

Remark 4.10. The algorithm terminates since we have a finite number of agents. Observe that the mechanism is budget balanced by construction and is individually rational since a pairwise exchange takes place only if it (weakly) benefits both agents.

Remark 4.11. An important property of the algorithm is the following: If $\theta_i > \theta_m$, then $\sigma_i^{k+1}(\theta) \leq \sigma_i^k$ and if $\theta_i < \theta_m$, then $\sigma_i^{k+1}(\theta) \geq \sigma_i^k$. This straightaway implies that if $\theta_i > \theta_m$ then $\sigma_i(\theta) \leq \sigma_i^0$ and if $\theta_i < \theta_m$ then $\sigma_i(\theta) \geq \sigma_i^0$. If $\theta_i = \theta_m$, then $\sigma_i(\theta) = \sigma_i^0$ except in the two cases noted above.

Remark 4.12. Since all pairwise exchanges occur at the “price” of θ_m , it follows that, for the median price exchange mechanism, the (ex-post) utility of agent i at the profile θ is

$$u_i(\sigma_i(\theta), t_i(\theta); \theta_i) = -(\sigma_i(\theta) - 1)\theta_i + (\sigma_i(\theta) - \sigma_i^0)\theta_m.$$

We can show the following.

Theorem 4.13. *The median price exchange mechanism μ^m with an odd number of agents satisfies BB, SP and IR and is generically $(n - 1)/2$ -inefficient.*

Proof. BB and IR follow by Remark 4.10. To see that μ^p is strategyproof, consider any agent $i \in N$ and profile θ . Suppose that agent i deviates by announcing θ'_i inducing the profile θ' . Let the median waiting cost for the profiles θ and θ' be θ_m and θ'_m respectively. In the proof, we conclude that agent i does not benefit from the deviation by showing that her utility from misrepresentation is less than or equal to her status quo utility $-(\sigma_i^0 - 1)\theta_i$ (by IR, it is less than or equal to her utility from the truthful report).

We have the following cases.

- (1) $\min\{\theta_i, \theta'_i\} > \theta_m$ or $\max\{\theta_i, \theta'_i\} < \theta_m$: In both cases, $\theta_m = \theta'_m$. It follows from the algorithm that $\sigma_i(\theta) = \sigma_i(\theta')$ and $t_i(\theta) = t_i(\theta')$. Hence, the deviation does not benefit agent i .

- (2) $\theta'_i > \theta_m > \theta_i$: Note that we must have $\theta'_i \geq \theta'_m \geq \theta_m$. If $\theta'_i > \theta'_m$, then it follows from the rules for a feasible exchange (see Remark 4.11) that $\sigma_i(\theta') \leq \sigma_i^0$. Since $\theta_i < \theta_m \leq \theta'_m$, it follows that i 's utility after deviation is $u_i(\sigma_i(\theta'), t_i(\theta'); \theta_i) = -(\sigma_i(\theta') - 1)\theta_i + (\sigma_i(\theta') - \sigma_i^0)\theta'_m \leq -(\sigma_i^0 - 1)\theta_i$. This shows that agent i does not benefit from the deviation.

If $\theta'_i = \theta'_m$, then the rule (3) of a feasible exchange implies that $\sigma_i(\theta') > \sigma_i^0$ is possible only if $m(\theta') = \emptyset$. This implies that $\theta'_m = \theta_m$. Hence, $\theta'_i = \theta'_m = \theta_m < \theta'_i$, a contradiction. Therefore, $\sigma_i(\theta') \leq \sigma_i^0$. By using the same argument as in the previous paragraph, we conclude that agent i does not benefit from the deviation.

- (3) $\theta_i > \theta_m > \theta'_i$: Here, we must have $\theta_m \geq \theta'_m \geq \theta'_i$. If $\theta'_m > \theta'_i$, then the rules for a feasible exchange imply that $\sigma_i(\theta') \geq \sigma_i^0$. Since $\theta_i > \theta'_m$, we have $u_i(\sigma_i(\theta'), t_i(\theta'); \theta_i) = -(\sigma_i(\theta') - 1)\theta_i + (\sigma_i(\theta') - \sigma_i^0)\theta'_m \leq -(\sigma_i^0 - 1)\theta_i$. This shows that agent i does not benefit from the deviation.

If $\theta'_i = \theta'_m$, then $\sigma_i(\theta') < \sigma_i^0$ is possible only if $M(\theta') = \emptyset$. This implies that $\theta'_m = \theta_m$ and hence, $\theta'_i = \theta'_m = \theta_m > \theta'_i$, a contradiction. Therefore, $\sigma_i(\theta') \geq \sigma_i^0$. By using the same argument as in the previous paragraph, we conclude that agent i does not benefit from the deviation.

- (4) $\theta_i > \theta'_i = \theta_m$: Note that $\theta'_m = \theta_m$. The utility from reporting truthfully is $u_i(\sigma_i(\theta), t_i(\theta); \theta_i) = -(\sigma_i(\theta) - 1)\theta_i + (\sigma_i(\theta) - \sigma_i^0)\theta_m$ and the utility from deviating is $u_i(\sigma_i(\theta'), t_i(\theta'); \theta_i) = -(\sigma_i(\theta') - 1)\theta_i + (\sigma_i(\theta') - \sigma_i^0)\theta_m$. Hence, the benefit from deviation is equal to $(\sigma_i(\theta') - \sigma_i(\theta))(\theta_m - \theta_i)$. Since $\theta_i > \theta_m$, the deviation is profitable only if $\sigma_i(\theta') < \sigma_i(\theta)$. This is not possible. To see this, first note that $m(\theta') = m(\theta)$. If $m(\theta') = m(\theta) = \emptyset$, then $\sigma_i(\theta') \geq \sigma_i^0$ and $\sigma_i(\theta) \leq \sigma_i^0$ by the rules of feasible exchange. Hence, $\sigma_i(\theta') \geq \sigma_i(\theta)$. Now, assume that $m(\theta') = m(\theta) \neq \emptyset$. If there is a feasible exchange between i and $j \in m(\theta)$ in the profile θ' , then the same exchange is feasible in the profile θ since $\theta_i > \theta_m$. So, $\sigma_i(\theta') \geq \sigma_i(\theta)$. Altogether, it follows that agent i does not gain from the deviation.

- (5) $\theta_i < \theta'_i = \theta_m$: Since the analysis of this case closely follows the previous one, it is omitted.

- (6) $\theta_i = \theta_m > \theta'_i$: Note that $\theta'_i \leq \theta'_m \leq \theta_m = \theta_i$. If $\theta'_i < \theta'_m$ then $\sigma_i(\theta') \geq \sigma_i^0$ (by the rules of a feasible exchange) and i 's utility from the deviation is $u_i(\sigma_i(\theta'), t_i(\theta'); \theta_i) = -(\sigma_i(\theta') - 1)\theta_i + (\sigma_i(\theta') - \sigma_i^0)\theta'_m \leq -(\sigma_i^0 - 1)\theta_i$ where the last inequality follows because $\theta'_m \leq \theta_m = \theta_i$. This shows that agent i does not benefit from the deviation.

If $\theta'_i = \theta'_m < \theta_i$, then note that $M(\theta') \neq \emptyset$. Hence, by the rules of feasible exchange, $\sigma_i(\theta') \geq \sigma_i^0$. Since $\theta'_i < \theta_i$, we have $u_i(\sigma_i(\theta'), t_i(\theta'); \theta_i) = -(\sigma'_i - 1)\theta_i + (\sigma_i(\theta') - \sigma_i^0)\theta'_i \leq -(\sigma_i^0 - 1)\theta_i$. This shows that agent i does not benefit from the deviation.

- (7) $\theta_i = \theta_m < \theta'_i$: Note that $\theta_i = \theta_m \leq \theta'_m \leq \theta'_i$. If $\theta'_i > \theta'_m$ then $\sigma_i(\theta') \leq \sigma_i^0$ (by the rules of a feasible exchange). Since $\theta'_m \geq \theta_m = \theta_i$, we have $u_i(\sigma_i(\theta'), t_i(\theta'); \theta_i) = -(\sigma_i(\theta') - 1)\theta_i + (\sigma_i(\theta') - \sigma_i^0)\theta'_m \leq -(\sigma_i^0 - 1)\theta_i$. This shows that agent i does not benefit from the deviation.

If $\theta'_i = \theta'_m > \theta_i$, then $m(\theta') \neq \emptyset$. By the rules of feasible exchange, $\sigma_i(\theta') \leq \sigma_i^0$. Since $\theta'_i > \theta_i$, we have $u_i(\sigma_i(\theta'), t_i(\theta'); \theta_i) = -(\sigma'_i - 1)\theta_i + (\sigma_i(\theta') - \sigma_i^0)\theta'_i \leq -(\sigma_i^0 - 1)\theta_i$. This shows that agent i does not benefit from the deviation.

To show generic $(n-1)/2$ -inefficiency, consider a profile θ such that the efficient queue is unique.¹⁵ Suppose $\theta_i > \theta_m$. We claim that $\sigma_i(\theta) \leq (n+1)/2$. Suppose not: then $\sigma_i(\theta) > (n+1)/2$. Then there exists j such that $\theta_j < \theta_m$ and $\sigma_j(\theta) \leq (n+1)/2$.¹⁶ Consider the pair (i, j) . Suppose this pair is k th in the order $\succ$. If at stage k of the algorithm there was a feasible exchange between i and j , then $\sigma_i^k = \sigma_j^{k-1} < \sigma_i^{k-1} = \sigma_j^k$. Since $\theta_i > \theta_m$, it follows from Remark 4.11 that $\sigma_i^k \geq \sigma_i^{k+1} \geq \dots \geq \sigma_i(\theta)$. Similarly, since $\theta_j < \theta_m$, $\sigma_j^k \leq \sigma_j^{k+1} \leq \dots \leq \sigma_j(\theta)$. Hence, $\sigma_i(\theta) < \sigma_j(\theta)$ but this contradicts the fact that $\sigma_j(\theta) \leq (n+1)/2 < \sigma_i(\theta)$.

So there can be no feasible exchange at stage k which implies that $\sigma_i^{k-1} < \sigma_j^{k-1}$. (Otherwise, since $\theta_i > \theta_m > \theta_j$, there would be a feasible exchange.) Since there is no feasible exchange $\sigma^k = \sigma^{k-1}$. By Remark 4.11, it follows that $\sigma_i^k < \sigma_j^k \leq \sigma_j^{k+1} \leq \sigma_j^{k+2} \leq \dots \leq \sigma_j(\theta)$. Hence, $\sigma_i^k < \sigma_j(\theta)$. Since $\theta_i > \theta_m$, we also have $\sigma_i^k \geq \sigma_i^{k+1} \geq \dots \geq \sigma_i(\theta)$. Therefore, $\sigma_i(\theta) < \sigma_j(\theta)$. But this leads to another contradiction as we have started with the assumption that $\sigma_j(\theta) \leq (n+1)/2 < \sigma_i(\theta)$.

We can use the same logic to show that if $\theta_i < \theta_m$, then $\sigma_i(\theta) \geq (n+1)/2$. Now note that efficiency dictates that any agent i such that $\theta_i > \theta_m$ should occupy queue positions $\{1, \dots, (n-1)/2\}$. Similarly, any agent i such that $\theta_i < \theta_m$ should occupy queue positions $\{(n+3)/2, \dots, n\}$. The median agent should occupy queue position $(n+1)/2$.

¹⁵The efficient queue is unique whenever $\theta_i \neq \theta_j, i, j \in N, i \neq j$. The set of profiles where the efficient queue is unique is an open and dense set (in the set of all profiles) and hence any property which holds for this set is a generic property.

¹⁶If not, then $\theta_k \geq \theta_m$ for all k such that $\sigma_k(\theta) \leq (n+1)/2$. Hence, $|\{k : \theta_k \geq \theta_m\}| \geq (n+1)/2 + 1 = (n+3)/2$. Since the efficient queue is unique for θ , there are no ties, and hence $|\{k : \theta_k \leq \theta_m\}| \leq n - (n+3)/2 + 1 = (n-1)/2 < (n+1)/2$. But this means that θ_m is not the median waiting cost.

Therefore, if $\theta_i > \theta_m$, then the maximal inefficiency for agent i is $(n + 1)/2 - 1 = (n - 1)/2$. Similarly, if $\theta_i < \theta_m$, then the maximal inefficiency is $n - (n + 1)/2 = (n - 1)/2$. Finally for the median agent, the maximal efficiency is when she is either in the first or the last queue position; in both cases, the inefficiency is $(n + 1)/2 - 1 = (n - 1)/2$. This shows that the reordered queue is at most $(n - 1)/2$ -inefficient. $\square$

Example 4.14. The following example shows that $(n - 1)/2$ -inefficiency may not be satisfied when the efficient queue is not unique. Suppose that the initial queue σ^0 is given by $\sigma_i^0 = i$. Consider the profile $\theta_7 > \theta_6 > \theta_2 = \theta_3 = \theta_4 = \theta_5 > \theta_1$. Suppose that the order on pairs of agents is (partially) given by $(1, 7) \succ \dots$ (We do not need the whole order.)

The median waiting cost for this profile is $\theta_2 = \theta_3 = \theta_4 = \theta_5$. At the first stage of the algorithm, there is a feasible exchange between agents 1 and 7: $\theta_7 > \theta_2 > \theta_1, \sigma_1^0 = 1 < \sigma_7^0 = 7$. So the queue gets transformed to σ^1 where $\sigma_1^1 = 7, \sigma_2^1 = 2, \sigma_3^1 = 3, \sigma_4^1 = 4, \sigma_5^1 = 5, \sigma_6^1 = 6$ and $\sigma_7^1 = 1$. After this, there is no feasible exchange. To see this, note that the median agents cannot be part of any feasible exchange. Agent 7 is already in the first queue position and there can be no feasible exchange involving her. The only remaining possibility is an exchange between agents 1 and 6 but this is not feasible because $\sigma_6^1 = 6 < \sigma_1^1 = 7$. However, σ^1 is not 3-inefficient: Agent 6's queue position is 2 in all efficient queues but her position in σ^1 is 6.

So far we have assumed that the number of agents is odd. If the number of agents is even, we face a problem because the median waiting cost is typically not well-defined. We can deal with this problem in one of two ways. We can introduce a *phantom agent* (as in Moulin [18]) or drop one agent *a priori*.¹⁷ Either method amounts to converting the problem into one where the median is well-defined.

In the latter case, the best we can do (in terms of minimizing inefficiency) is to drop the agent i such that $\sigma_i^0 = n/2$ (irrespective of the agent's waiting cost) and apply the median price exchange mechanism to the remaining $(n - 1)$ agents.¹⁸ Call this mechanism μ_e^m . We show now that the mechanism μ_e^m is generically at most $n/2$ -inefficient.

Corollary 4.15. *The mechanism μ_e^m with even number of agents $n \geq 4$ satisfies BB, SP, IR and is at most $n/2$ -inefficient generically.*

¹⁷Note that the choice of which agent to drop cannot depend on the profile θ as this will violate strategyproofness.

¹⁸One could, of course, drop the agent in the queue position $(n/2) + 1$.

Proof. The fact that the mechanism satisfies BB, SP and IR follows from the fact that μ^m (the median price exchange mechanism with an odd number of agents) has these properties.

To show generic $n/2$ -inefficiency, let θ be a profile such that the outcome efficient queue is unique. Let i^* be such that $\sigma_i^0 = n/2$. Let i_m^* be the agent with the median waiting cost in the profile θ_{-i^*} (the profile of waiting costs excluding θ_{i^*}).

- (1) If $i = i^*$, then $\sigma_i(\theta) = \sigma_i^0$ and the maximal inefficiency associated with this agent is $n - n/2 = n/2$.
- (2) If $i = i_m^*$, then also $\sigma_i(\theta) = \sigma_i^0$, and the maximal inefficiency associated with this agent is either $\{(n/2)+1\}-1 = n/2$ or $n-(n/2) = n/2$.
- (3) Suppose that $\sigma_{i_m^*}^0 < n/2$. In this case, we can use the argument in Theorem 4.13 to show that if $\theta_i > \theta_{i_m^*}$ then $\sigma_i(\theta) \leq (n/2) + 1$ and if $\theta_i < \theta_{i_m^*}$, then $\sigma_i(\theta) \geq (n/2) + 1$. Let $I = \{i \in N \setminus \{i^*\} : \theta_i > \theta_{i_m^*}\}$ and $J = \{i \in N \setminus \{i^*\} : \theta_i < \theta_{i_m^*}\}$. Since θ is a profile where the efficient queue is unique, it follows that there are exactly $(n-2)/2$ elements in each set.

Denote by σ_i^e the queue position of i in the efficient queue for the profile θ . Now note that for any $i \in I$, $\sigma_i^e \leq n/2$. Similarly, for any $i \in J$, $\sigma_i^e \geq (n/2) + 1$. For any $i \in I$, the maximal inefficiency is therefore $\{(n/2) + 1\} - 1 = n/2$. Similarly, for any $j \in J$, the maximal inefficiency is $n - \{(n/2) + 1\} = (n/2) - 1 < n/2$.

A similar analysis applies if $\sigma_{i_m^*}^0 > n/2$.

□

As can be seen, the median price exchange mechanisms μ^m and μ_e^m are inefficient. Are there mechanisms satisfying BB, SP and IR with better efficiency properties? This is a difficult open question.

5. WEAKENING STRATEGYPROOFNESS

A natural weakening of strategyproofness requires that agents do not have incentives to misreport in one direction.

Definition 5.1. A mechanism $\mu = (\sigma, t)$ is *upward strategyproof* (USP) if for all $i \in N$, all i -variants θ and θ' such that $\theta'_i > \theta_i$, $u_i(\sigma_i(\theta'), t_i(\theta'); \theta_i) \leq u_i(\sigma_i(\theta), t_i(\theta); \theta_i)$.

Definition 5.2. A mechanism $\mu = (\sigma, t)$ is *downward strategyproof* (DSP) if for all $i \in N$, all i -variants θ and θ' such that $\theta'_i < \theta_i$, $u_i(\sigma_i(\theta'), t_i(\theta'); \theta_i) \leq u_i(\sigma_i(\theta), t_i(\theta); \theta_i)$.

Obviously, strategyproofness implies and is implied by the combination of upward and downward strategyproofness. It turns out that there are many

mechanisms satisfying BB, OE, IR and either upward or downward strategyproofness. Among them are the *buyers' mechanism* which sets the price to be the minimum of two waiting costs and the *sellers' mechanism* which sets the price to be the maximum of two waiting costs. The following example shows that in some contexts, these mechanisms have desirable properties.

Example 5.3. Let $N = \{1, 2\}$ and $\sigma_i^0 = i, i = 1, 2$. In the buyers' mechanism, both agents report their types and the two agents exchange their positions if $\theta_2 > \theta_1$. If an exchange takes place, then agent 2 pays θ_1 to agent 1. In the sellers' mechanism, trade again takes place if $\theta_2 > \theta_1$ but in this case, agent 2 pays θ_2 to agent 1. Note that in the sellers' mechanism agent 2 has an incentive to understate but not overstate the agent's waiting cost while, in the buyers' mechanism, agent 1 has an incentive to overstate but not understate.

In the buyers' mechanism, agent 1 can benefit from overstating the waiting cost but only in the case when trade is desirable, that is, when $\theta_2 > \theta_1$.¹⁹ If agent 1 reports $\theta_2 > \theta'_1 > \theta_1$, then agent 1 will get a higher compensation for moving back. Observe however, that if $\theta'_1 \geq \theta_2$, then no trade takes place and agent 1 is strictly worse-off from the manipulation. Thus, in the instances when agent 1 can successfully manipulate, we really don't care about it because the outcome efficient queue is still implemented. If agent 1 manipulates in a way which results in an inefficient queue, then agent 1 is strictly worse-off. Note also that there is no incentive for agent 2 to manipulate. This shows that in the two-agent context, the buyers' mechanism is still a "desirable" mechanism even though it is not strategyproof. A similar point can be made about the sellers' mechanism.

The buyers' and sellers' mechanisms are the end result of the following dynamic procedure. Starting from the initial order σ^0 and given a profile of waiting costs, under both the buyers' and the sellers' mechanisms, we sequentially reach the outcome efficient queue by pairwise *cost reducing* interchange of agents in consecutive queue positions. For all pairwise switches, if the trading price is the minimum of the two waiting costs, then we have the buyers' mechanism and if the trading price is the maximum of the two waiting costs, then we have the sellers' mechanism. Observe that we are allowing for cost reducing switches only and hence, for each profile θ , we will always have a single selection $\sigma^*(\theta)$ as the outcome efficient queue from the efficiency correspondence $E(\theta)$. Moreover, for each θ , $\sigma^*(\theta)$ satisfies

¹⁹It is straightforward to verify that agent 1 cannot strictly benefit from overstating or understating the waiting cost if $\theta_1 \geq \theta_2$.

the following property: if $\theta_i = \theta_j$ then $\sigma_i^*(\theta) < \sigma_j^*(\theta)$ whenever $\sigma_i^0 < \sigma_j^0$.²⁰ Throughout this section we assume that for OE we have the profile contingent selection σ^* . Incorporating all these features we get the following definitions of the two mechanisms.

Definition 5.4. A mechanism $\mu^B = (\sigma^*, t^B)$ is a *buyers' mechanism* if for each profile θ , $\sigma^*(\theta) \in E(\theta)$ and all $i \in N$,

$$u_i(\mu_i^B(\theta); \theta_i) = -(\sigma_i^*(\theta) - 1)\theta_i + t_i^B(\theta),$$

where $t_i^B(\theta) = -\sum_{k \in P_i(\sigma^0) \cap F_i(\sigma^*(\theta))} \theta_k + |F_i(\sigma^0) \cap P_i(\sigma^*(\theta))| \theta_i$.

In words, the buyers' mechanism μ^B selects an efficient queue at each profile θ . Agent i has to pay the waiting cost of each agent k who is placed behind agent i in the outcome efficient queue but who was in front of agent i in the initial queue. Moreover, agent i also receives θ_i from each agent who is in front of agent i in the outcome efficient queue and who was behind agent i in the initial queue.

Definition 5.5. A mechanism $\mu^S = (\sigma^*, t^S)$ is a *sellers' mechanism* if for each profile θ , $\sigma^*(\theta) \in E(\theta)$ and all $i \in N$,

$$u_i(\mu_i^S(\theta); \theta_i) = -(\sigma_i^*(\theta) - 1)\theta_i + t_i^S(\theta),$$

where $t_i^S(\theta) = -|P_i(\sigma^0) \cap F_i(\sigma^*(\theta))| \theta_i + \sum_{k \in F_i(\sigma^0) \cap P_i(\sigma^*(\theta))} \theta_k$.

The sellers' mechanism is similar to the buyers' mechanism except that for any profile θ , agent i has to pay θ_i to each agent who is placed behind agent i in the outcome efficient queue but who was in front of agent i in the initial queue. Moreover, agent i receives θ_k from each agent k who is in front of agent i in the outcome efficient queue and who was behind agent i in the initial queue.

Proposition 5.6. The buyers' mechanism $\mu^B = (\sigma^*, t^B)$ is DSP while the sellers' mechanism $\mu^S = (\sigma^*, t^S)$ is USP.

Proof. Consider the buyers' mechanism μ^B . Let θ and θ' be i -variants such that $\theta'_i < \theta_i$. Let agent i 's true waiting cost be θ_i and θ'_i be a misreport. By OE, $\sigma_i^*(\theta) \leq \sigma_i^*(\theta')$, $F_i(\sigma^*(\theta')) \subseteq F_i(\sigma^*(\theta))$ and $P_i(\sigma^*(\theta)) \subseteq P_i(\sigma^*(\theta'))$. Define $B^1 \equiv F_i(\sigma^*(\theta)) \setminus F_i(\sigma^*(\theta'))$, $B^2 \equiv P_i(\sigma^*(\theta')) \setminus P_i(\sigma^*(\theta))$ and also define the benefit of agent i from such a deviation by $\mathcal{D}^B \equiv u_i(\mu_i^B(\theta'); \theta_i) - u_i(\mu_i^B(\theta); \theta_i)$. Using the transfer of the buyers' mechanism, we get

²⁰It is important to note that in what order we do the pairwise cost reducing switches does not matter. As long as we start from the initial queue σ^0 and, given θ , we continue doing this pairwise cost reducing switches till no further cost reducing switch is possible, we will end up in the same outcome efficient queue $\sigma^*(\theta)$.

$$(5.1) \quad \mathcal{D}^B = -(\sigma_i^*(\theta') - \sigma_i^*(\theta))\theta_i + \sum_{k \in P_i(\sigma^0) \cap B^1} \theta_k + |F_i(\sigma^0) \cap P_i(\sigma^*(\theta))|(\theta'_i - \theta_i) + |F_i(\sigma^0) \cap B^2|\theta'_i.$$

Observe first that $B^1 = B^2 = F_i(\sigma^*(\theta)) \cap P_i(\sigma^*(\theta')) \equiv \bar{B}$ (say) and that $|\bar{B}| = \sigma_i^*(\theta') - \sigma_i^*(\theta)$. Therefore, condition (5.1) can be rewritten as

$$(5.2) \quad \mathcal{D}^B = -|\bar{B}|\theta_i + \sum_{k \in P_i(\sigma^0) \cap \bar{B}} \theta_k + |F_i(\sigma^0) \cap P_i(\sigma^*(\theta))|(\theta'_i - \theta_i) + |F_i(\sigma^0) \cap \bar{B}|\theta'_i.$$

If the deviation from θ_i to θ'_i is such that $\theta'_i < \theta_i$ and $\sigma_i^*(\theta) = \sigma_i^*(\theta')$, then $\bar{B} = \emptyset$, and from (5.2), we get $\mathcal{D}^B = |F_i(\sigma^0) \cap P_i(\sigma^*(\theta))|(\theta'_i - \theta_i) < 0$. Hence, agent i does not benefit from such a deviation. Therefore, assume that the deviation from θ_i to θ'_i is such that $\theta'_i < \theta_i$ and $\sigma_i^*(\theta) < \sigma_i^*(\theta')$, so that $\bar{B} \neq \emptyset$. Since $[P_i(\sigma^0) \cap \bar{B}] \cup [F_i(\sigma^0) \cap \bar{B}] \subseteq \bar{B}$ and OE implies that for all $k \in \bar{B}$, $\theta_k \leq \theta_i$, from (5.2), it follows that

$$\mathcal{D}^B = \sum_{k \in P_i(\sigma^0) \cap \bar{B}} (\theta_k - \theta_i) + |F_i(\sigma^0) \cap P_i(\sigma^*(\theta))|(\theta'_i - \theta_i) < 0.$$

Hence, such a deviation is also not beneficial for agent i . Since the selection of i and the i -variant profiles were arbitrary, the downward strategyproofness of the buyers' mechanism is established.

The proof for the sellers' mechanism satisfying upward strategyproofness follows similar lines and is left to the readers. $\square$

We now give characterizations of the buyers' and sellers' mechanisms. The characterizations do not use upward or downward strategyproofness as axioms. The technical reason for this is that upward and downward strategyproofness give us inequalities and these are not sufficiently strong to obtain characterizations.

The first set of axioms require that an agent should not be affected by an increase (decrease) in the waiting cost of another agent whose waiting cost is higher (lower) than his own waiting cost.

Definition 5.7. A mechanism μ satisfies *independence of higher waiting costs* (IH) if for all i -variants θ and θ' such that $\theta'_i \geq \theta_i$, and all $j \in N$ such that $\theta_j \leq \theta_i$, $u_j(\mu_j(\theta'); \theta_j) = u_j(\mu_j(\theta); \theta_j)$.

Definition 5.8. A mechanism μ satisfies *independence of lower waiting costs* (IL) if for all i -variants θ and θ' such that $\theta'_i \leq \theta_i$, and all $j \in N$ such that $\theta_j \geq \theta_i$, $u_j(\mu_j(\theta'); \theta_j) = u_j(\mu_j(\theta); \theta_j)$.

For a queue σ , we say that agents i and j are *adjacent* if $|\sigma_i - \sigma_j| = 1$. The next axiom is motivated by the idea that if two adjacent agents interchange their positions, then such an interchange should not affect the utilities of any other player. We define this formally in the following way. Fix an initial order σ^0 . For all profiles θ and all $i, j \in N$ such that i and j are adjacent in σ^0 , let $\theta^{ij} = (\theta_1^{ij}, \dots, \theta_n^{ij})$ be the waiting cost vector defined by

$$\theta_k^{ij} = \begin{cases} \theta_k & \text{if } k \neq i \neq j \neq k, \\ \theta_j & \text{if } k = i, \\ \theta_i & \text{if } k = j. \end{cases}$$

Definition 5.9. A mechanism μ satisfies *independence of irrelevant position interchanges* (IPI) if for all profiles θ , all $i, j \in N$, all θ^{ij} , and all $k \in N \setminus \{i, j\}$, $u_k(\mu_k(\theta^{ij}); \theta_k) = u_k(\mu_k(\theta); \theta_k)$.

Given σ^0 , for each profile θ , let $\ell(\theta, \sigma^0)$ be the number of pairwise interchanges of queue positions required to reach the outcome efficient queue $\sigma^*(\theta)$ from σ^0 . We will assume that the interchanges occur only if it is cost reducing for the two adjacent agents. If for some profile θ , $\sigma^*(\theta) = \sigma^0$ then $\ell(\theta, \sigma^0) = 0$ and if for some profile $\bar{\theta}$, $\sigma^*(\bar{\theta})$ is such that $\sigma_i^*(\bar{\theta}) = n + 1 - \sigma_i^0$ for all $i \in N$, then $\ell(\bar{\theta}, \sigma^0) = n(n-1)/2$. Hence the maximum number that $\ell(\theta, \sigma^0)$ can take is $n(n-1)/2$. The proof of our next characterization use an induction argument on the number of such pairwise position interchanges $\ell(\theta, \sigma^0)$.

Theorem 5.10.

- (i) A mechanism μ satisfies BB, OE, IR, IH, and IPI if and only if $\mu = \mu^B$.
- (ii) A mechanism μ satisfies BB, OE, IR, IL, and IPI if and only if $\mu = \mu^S$.

Proof. (i) It is easy to check that the buyers' mechanism μ^B satisfies BB, OE, IR, IH, and IPI. Conversely, let μ be a mechanism satisfying the five axioms. Given σ^0 , consider the profile θ . As described earlier, we will use an induction argument on the number of pairwise position interchanges to obtain an efficient queue starting from σ^0 . Suppose that $\ell(\theta, \sigma^0) = \ell \in \{0, 1, \dots, n(n-1)/2\}$.

Step 1: If $\ell = 0$, then the outcome efficient queue is $\sigma(\theta) = \sigma^0 = \sigma^*(\theta)$. By IR, $t_i(\theta) \geq 0$ for all $i \in N$ and hence by BB, $t_i(\theta) = 0 = t_i^B(\theta)$ for all $i \in N$. Thus, $u_i(\mu_i(\theta); \theta_i) = u_i(\mu_i^B(\theta); \theta_i)$ for all $i \in N$.

Step 2: If $\ell = 1$, then, for the profile θ , let i and j be the two adjacent agents who *need* to interchange their queue positions at σ^0 to reach the outcome efficient queue $\sigma^*(\theta)$. Without loss of generality, suppose that $\sigma_i^0 < \sigma_j^0$, which

implies that, for the profile $\theta = (\theta_i, \theta_j, \theta_{-i-j})$, $\theta_i < \theta_j$. Let $\tilde{\theta} = (\theta_i, \theta_i, \theta_{-i-j})$ be the waiting cost vector obtained from $\theta = (\theta_i, \theta_j, \theta_{-i-j})$ by replacing θ_j with θ_i . Since $\ell(\theta, \sigma^0) = 1$, for θ^{ij} the initial queue σ^0 is outcome efficient and hence for all $k \in P_j(\sigma^0)$, $\theta_k \geq \theta_j$ and all $k \in F_i(\sigma^0)$, $\theta_k \leq \theta_i$. Moreover, since $\theta_j > \theta_i$ we also have that for $\tilde{\theta}$, σ^0 is outcome efficient. Therefore $\ell(\tilde{\theta}, \sigma^0) = 0$ and from Step 1, for all $k \in N$, $u_k(\mu_k(\tilde{\theta}); \tilde{\theta}_i) = u_k(\mu_k^B(\tilde{\theta}); \tilde{\theta}_i)$. By IH, $u_i(\mu_i(\theta); \theta_i) = u_i(\mu_i(\tilde{\theta}); \tilde{\theta}_i) = u_i(\mu_i^B(\tilde{\theta}); \tilde{\theta}_i) = u_i(\mu_i^B(\theta); \theta_i)$. Moreover, for the profile θ^{ij} , $\ell(\theta^{ij}, \sigma^0) = 0$ and hence by Step 1, for all $k \in N$, $u_k(\mu_k(\theta^{ij}); \theta_k^{ij}) = u_k(\mu_k^B(\theta^{ij}); \theta_k^{ij})$ and by IIPI, for all $k \in N \setminus \{i, j\}$, $u_k(\mu_k(\theta); \theta_k) = u_k(\mu_k(\theta^{ij}); \theta_k) = u_k(\mu_k^B(\theta^{ij}); \theta_k) = u_k(\mu_k^B(\theta); \theta_k)$. Finally, using BB, we get $u_j(\mu_j(\theta); \theta_j) = u_j(\mu_j^B(\theta); \theta_j)$. Hence, if $\ell = 1$, then we have $\mu = \mu^B$.

Step 3: As an induction hypothesis, suppose that the conclusion holds for all $\ell(\theta, \sigma^0) \in \{0, \dots, \ell - 1\}$. Now we show that the conclusion holds for the problem with $\ell(\theta, \sigma^0) = \ell$. Let i and j be the two adjacent agents who need to change their positions to reach the outcome efficient queue $\sigma^*(\theta)$. By using the same argument as in Step 2, we obtain the desired conclusion.

Since the proof of (ii) of this theorem is similar to (i), it is left to the readers. $\square$

Remark 5.11. In this paper, we have analyzed the queueing problem with an initial order when there is one facility (server) and the processing time of all the jobs are identical. As a topic for future research, this model can be extended in many directions. One can consider multiple identical or non-identical facilities, or extend the analysis by dropping the identical processing time assumption. All these extensions certainly have real life applications and hence are worth pursuing.

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