

Heterogeneous Beliefs in Over-The-Counter Markets



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Abstract

The behavior and stability of over-the-counter markets is of central concern to regulators. Little is known, however, about how the structure of these markets determine their properties. In this paper we consider an over-the-counter market populated by boundedly rational heterogeneous traders in which the structure is represented by a network. Stability is found to decrease as the market becomes less well connected, however, the configuration of connections has a significant effect. The presence of hubs, such as those found in scale free networks increases stability and decreases volatility whilst small-world links have the opposite effect. Volatility in the fundamental value increases market volatility, however, volatility in the riskless asset returns has an ambiguous effect.

Keywords: Over-the-Counter, Boundedly Rationality, Stability, Network, Heterogeneous Agent Model

JEL codes: G10, C63

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1 Introduction

Over-the-counter (OTC) markets are key to the operation of the modern financial system. Much of the world's trade in derivatives, foreign currency, and many other assets, is conducted in these exchanges. They provide flexibility to financial institutions, however, this comes at a cost. During adverse conditions, their decentralized nature can cause traders to be unable to identify counter-parties and as a result the markets may lose liquidity and fail. This happened during the 2008 financial crisis when the sparse structure of OTC markets was blamed for a lack of transparency and an inability for institutions to identify prices of assets and to trade (Brunnermeier, 2009). As a result this has led to calls for trade to be moved away from OTC markets towards centralized exchanges to increase stability. There is, however, very little work comparing these two types of institutions. This paper aims to address a key aspect: the stability and dynamics of prices. In particular it will look at how the pattern of interactions between institutions, the structure of an OTC market, affects the market behavior. The markets considered in this paper will contain heterogeneous traders which base their trading decisions on their valuation of the asset. Each trader's valuation is itself dependent on their strategy and the information the trader gains through trading with its counterparts. As a result the structure of the market will affect the behavior of traders.

OTC markets allow investors to trade assets directly between each other rather than through centralized exchanges. They are particularly prevalent when assets are illiquid, are traded in very large quantities or when there is scope for bespoke contracts. The largest OTC markets are those for currency exchange and swaps. In these markets investors buy or sell directly from dealers, however, each customer may only know a subset of the dealers within the market, limiting their ability to observe the best price. The dealers themselves trade with each other in order to balance inventory, meet liquidity needs and speculate, again however, each dealer may only interact with a subset of the other dealers. Lyons (1997) captures this interaction in a formal model and shows that this market setup can reduce the amount of information in prices. Duffie et al. (2005) show how constrained trading opportunities and search costs in OTC markets affect prices and the resulting

bid-ask spread, whilst Koepl et al. (2012) use a mechanism design approach to examine the effect of the clearing arrangements (centralized versus bilateral) on stability in both types of markets.

Inter-bank lending markets also generally operate on an OTC basis. In this case the ‘price’ is the interest rate at which a bank or financial institution may lend or borrow. The nature of the contracts (length of borrowing, size of borrowing, time) and the participants (credit ratings of borrower, history, etc.), all affect the interest rate a particular institution will be offered. An OTC structure provides the flexibility necessary for this type of trade. Theoretical studies have shown that the linkages between banks (lending and borrowing relationships) have an important effect on stability (Allen and Gale, 2000). Several papers have considered the effect of particular network structures e.g. Battiston et al. (2009); Georg (2013); Iori et al. (2006); Ladley (2013); Lorenz and Battiston (2008) and have shown that the connectivity (the number of links between traders in the network) and the configuration of linkages both play a role in market stability.

The structure of OTC markets, as defined by the interactions of the institutions within them, may be highly complex. Network theory, offers an effective analogy to capture and analyze their detail. An OTC market may be represented as a graph in which nodes correspond to traders and edges represent potential trading relationships. Within the network each financial institution is restricted to only interact and gain information from those to whom it is directly connected. Seminal work by Watts and Strogatz (1998), Barabasi and Albert (1999), Newman (2003) and others have provided tools applicable to a wide range of systems, from friendship groups to gene regulation which may be employed in this setting.

Within this paper we represent the OTC market as a network. The traders within the market follow one of two strategies which differ in their estimation of future market prices. Chartists look at previous trends in the market price to extrapolate future price changes, whilst fundamentalists know the true value of the asset and assume that the market price will move back towards this value. We use numerical simulation in order to analyze the behavior of the model. The results show that the market structure has a significant effect

on price dynamics and market stability. The more heavily a market is connected, i.e. the more easily information may flow between traders, the more frequently stable dynamics are observed. As the number of connections is reduced, the market dynamics deviate more often from the fundamental, as sections of the market diverge in their valuation of the asset. The presence of hubs increases stability whilst the inclusion of ‘small world’ type connections has the opposite effect. Markets are also shown to be less stable if they contain an above average number of chartist traders. Volatility in the underlying fundamental or riskless asset returns are amplified by the network structure, particularly the locally connected market. In some markets, however, low levels of riskless asset return volatility were found to synchronize the traders and reduce price volatility. Overall, the model is found to have much in common with the underlying Chiarella (1992) model in terms of the parameter combinations which lead to non-equilibrium prices and the effect of those parameters on the amplitude of cycles, although the network structure has marked influence on this.

The paper proceeds as follows. Section 2 discusses literature relevant to our model. Section 3 details the model of the interaction of heterogeneous traders in OTC markets. Section 4 presents the results, first focusing on the behavior of individual traders and then looking at the role of the number of connections, network structure, parameters, compositions of traders and volatile fundamentals. Section 5 concludes.

2 Related literature

The analogy of a network has been used in a body of work looking at the effects of market structures on trade.¹ Evstigneev and Taksar (2002) show that equilibria within these markets exist and that the networks formed can maximize overall efficiency (Kranton and Minehart, 2001) although Gofman (2011) shows that with insufficient numbers of connections an inefficient allocation becomes almost certain. The dynamics of these markets are also highly dependent on network structure, e.g. Bell (1998) and Tassier and Menczer (2008). Both the number of connections and the pattern of connectivity play important

¹See Wilhite (2006) for a review.

roles. For instance, Wilhite (2001) shows that ‘small world’ connections, those connecting otherwise distantly separated sections of the market, have a large effect on reducing search costs. The behavior of traders within these markets has also been shown to vary with their location (Ladley and Bullock, 2008). Importantly, this is not just dependent on their trading opportunities but also on their information linkages (Ladley and Bullock, 2007). Babus and Kondor (2012) considers how information diffuses across OTC markets showing that with private valuations the information efficiency of prices is maximized when all traders trade with all others. Information linkages may also be external to the market. Panchenko et al. (2010) examine this issue explicitly. They extend the model of Brock and Hommes (1998) to allow traders to adopt one of two strategies based on the choices and performance of a trader’s social network. By basing this adoption on a network of connections the stability of the system is changed.²

As described above, and in a similar manner to Panchenko et al. (2010), we ground our work in the literature of market dynamics under heterogeneous beliefs. The dynamics of centralized exchanges with chartist and fundamentalist traders have been considered in detail.³ Chiarella (1992) presents one of the first versions of this type of model in which the interaction of these two types of speculative traders leads to a range of market dynamics. This class of models has been employed to answer a range of questions relating to market structure. For example: Westerhoff (2004) uses a chartist/fundamentalist model to examine trade in multiple markets. Anufriev and Panchenko (2009) contrast centralized and order book market mechanisms. Lux (1995) examines the effect of herding behavior whilst Chiarella et al. (2009b) examine the dynamics of boundedly rational traders within an order book market. Diks and Dindo (2008) investigate information costs, whilst Westerhoff (2003) looks at the role of transaction taxes. Here we extend this line of reasoning to examine the dynamic stability of OTC markets.

²A separate area of the literature examines OTC markets through search based models - see for example Duffie et al. (2005, 2009); Lagos and Rocheteau (2009).

³See Hommes (2006) and Chiarella et al. (2009a) for summaries.

3 Model

We analyze a model in which boundedly rational chartist and fundamentalist investors trade an asset in an OTC market. The majority of previous work using these types of traders has considered a single centralized exchange in which all traders are able to interact. Consequently trading opportunities and information are unconstrained and flow freely across the market. Here we consider OTC markets where this is no longer the case. We model a market architecture in which traders may only interact with a subset of individuals, their trading contacts. Each trader may only buy or sell the asset with these contacts whilst their estimation of the future asset price is based on the prices of their recent transactions. As such, different traders within the market may have access to different trading opportunities and information sets and so may have differing valuations.

The underlying behavior of the traders in this model is based on those of Chiarella (1992). There are, however, several key difference which we discuss below.⁴ The Chiarella model captured the behavior of boundedly rational speculative traders, in a relatively simple setting. This makes it an appropriate base on which to consider the potentially complex effect of the OTC network structure on market dynamics and speculative trade. Other models have included factors such as wealth constraints or strategy switching which could have interesting and important effects. We leave these additions to future work and focus here on understanding the effect of the the key element of the OTC market: the structure.

3.1 Markets and Prices

We represent an OTC market architecture as an undirected graph where traders are nodes and trading connections are edges. There are N traders in the market where L is the connection matrix. We denote by $L(i, j)$ the potential connection between trader i and trader j with $L(i, j) = 1$ if they are connected and $L(i, j) = 0$ if they are not. For all traders $L(i, i) = 0$. Traders may only interact with those individuals to whom they are directly connected on the network. Here, we restrict our analysis to networks which

⁴For reference, a description of the Chiarella (1992) model is included in Appendix A.

consist of a single component, i.e. there exists a path between all pairs of traders in the market. Whilst an individual trader's direct interactions are limited, through its indirect connections (neighbors of neighbors etc.) a trader may affect the entire market. Dis-jointed networks are excluded as without connections separate components would behave independently.

To capture the nature of interactions in OTC markets we model traders as acting asynchronously. In a centralized exchange the presence of a single auctioneer, or order book, provides some degree of coordination by spreading information to all traders simultaneously. In an OTC market, however, there is no equivalent. Traders are only connected to a subset of other institutions and so will not receive all information. As a result, without a central coordinating device, it is natural to think of participants acting asynchronously. The model, therefore, proceeds as a series of discrete time steps, in each of which a single trader is selected at random with uniform probability to act.

A second key difference between centralized exchange and OTC markets is the mechanism of trade. In a centralized exchange all trade occurs through a single mechanism giving all traders access to the same trading opportunities and prices. In contrast, in OTC markets trade occurs directly between pairs of traders. As a result there is not a unique price at which all exchange occurs. Rather, prices vary across the market depending on the valuations and demands of the traders in a particular neighborhood. In the Chiarella model a price was determined for all traders by the market maker who updated the price based on excess demand. In this model we depart from the notion of a centralized price, and the associated market maker adjusting prices, and instead define trader i 's local price at time t to be P_i^t .

Given the local nature of trades and information, a trader's estimate of their local price is based on information gained from the trades in which they have participated. We model this estimate as the volume weighted average price (VWAP): the total value of all trades the trader has participated in since the last time they were chosen, divided by the total quantity traded. This value is frequently used in practice in real financial markets as an estimate of the current price based on recent trades. For a trader who has participated

in K trades since they were last chosen, VWAP is:

$$P_i^t = \frac{\sum_{k=1}^K p_k |q_k|}{\sum_{k=1}^K |q_k|} \quad (1)$$

where p_k and q_k are the price and quantity of trade k . If there are no trades during the period the trader maintains the previous valuation. It is important to note this is the traders estimate of the local price. It is not a price at which the trader can necessarily trade, unlike that defined by the market maker in the Chiarella model. Details of how trade prices are determined are given below.

3.2 Traders

As in Chiarella (1992) the market is composed of two varieties of trader *chartists* and *fundamentalists*. The type of each of the N traders is determined at the start of the simulation and remains fixed throughout.

Fundamentalists believe that the asset price will return to the fundamental value. Fundamentalist trader i values the asset at the fundamental value $V_i^t = W$ and their demand for the asset is proportional to the difference between the asset's fundamental value and the trade price, p :

$$D_i(p) = a(W - p) \quad (2)$$

where the constant a determines how strongly the traders demand is driven by the difference between the price and the fundamental asset value, W (note we write this demand function in terms of W , to match Chiarella's notation, however, W could be replaced by V_i^t). Like Chiarella, we assume this is common to all traders.⁵ Fundamentalist demand therefore increase in magnitude as the price moves away from the fundamental.

Chartist traders do not have access to, or do not choose to use, the asset's fundamental value. Rather trader i 's prediction of the future price is based on a simple linear assessment

⁵In reality this may not be the case, fundamentalist in different parts of the market could disagree on the assets true value. We leave this extension to future work.

of the trend in the local price, ψ_i^t

$$\psi_i^t = \psi_i^{t_l} + c(P_i^t - P_i^{t_l} - \psi_i^{t_l}) \quad (3)$$

Traders only update their estimates of the price and trend when they are chosen to act. Time t is the current time, whilst t_l was the previous time the trader was chosen. The trend is calculated by an exponentially weighted moving average where the constant c expresses how quickly the chartist's assessment of the current trend is driven by recent price changes. Based on this at time t , chartist i 's valuation of the asset, V_i^t is the current local price plus the trend:

$$V_i^t = P_i^t + \psi_i^t \quad (4)$$

The chartist demand function is then given by the difference between this price and the trade price:

$$D_i(p) = h(V_i^t - p - g), \quad (5)$$

where g is the return on a riskless asset (like Chiarella (1992) the trade of this asset is not modeled) and h is defined as:

$$h(x) = \left(\frac{1}{1 + e^{-4bx}} - 0.5 \right) \quad (6)$$

h increases monotonically, has a single point of inflection and well defined lower and upper bounds $\lim_{x \rightarrow \pm\infty} h(x) = \pm 0.5$. Moreover $h(0) = 0$. The steepness of the sigmoid is parametrized with b where $b > 0$.⁶

Chiarella explains this function in terms of chartists wishing to maximize their intertemporal utility of consumption. Traders have the choice of allocating their wealth between the risky and riskless asset. In line with Merton (1971) demand is proportional to the difference in returns. In this case between the return the trader will gain on trading at a particular price based on their prediction and the return on the riskless asset. If $g > 0$ and the trade price is only a little below the traders valuation i.e. $V_i^t - p < g$ the predicted

⁶Other functions with similar properties were tested and produced qualitatively similar results.

return on the risky asset is sufficiently low that the trader would prefer to short the risky asset and go long in the riskless asset. As the trade price increases the size of the short position increases, whilst if the trade price decreases the trader may take a long position. This demand, however, is bounded above and below by non-modeled constraints such as available wealth or maximum permissible risk level.

3.3 Trades

Once the chosen trader has established their valuation they trade with each of their trading contacts in turn.⁷ Transactions occur directly between pairs of traders and each transaction is independent of all others. Each trader (i and j) has an estimated valuation of the asset (V_i and V_j) and a demand function ($D_i(p)$ and $D_j(p)$), which gives the trader's demand at price p relative to their valuation. The form of these demand functions is given above, both are monotonic in the price p and give positive demands when the price is below the trader's valuation and negative demands when the price is above. Since both functions are monotonic, for any pair of valuations there exists a price such that $D_i(p) + D_j(p) = 0$. At this price the quantity traded is $q = D_i(p) = D_j(p)$. This is an equilibrium price at which demand and supply (negative values of the demand function for the trader with higher valuation) are equal. The trader with higher valuation sells to the one with lower valuation. It is also the price at which the total expected profit, summed over the two traders, and calculated from each trader's price expectation, is maximized. As such, at any other price-quantity pair both traders could increase their expected profits by making a second simultaneous trade for additional units at the equilibrium price. We, therefore, set the quantity traded as the equilibrium quantity. We abstract from the details of the bilateral negotiation and assume this volume is traded at the equilibrium price. In a real OTC market traders could trade multiple units at different prices based on their preferences and bargaining power. Modeling this process, however, would require further assumptions regarding individual preferences which would complicate this model

⁷Like Chiarella (1992) this model does not consider wealth or net positions. In the context of traders who can offset positive and negative positions with different partners this is a complex strategic decision.

and move it away from the underlying Chiarella model.⁸

In each time step the model proceeds as follows: 1) A single trader is selected at random. 2) The chosen trader estimates the local price using Equation 1. 3) The chosen trader forms their valuation of the asset V_i^t . For fundamentalist this is the fundamental value, for chartists this is given by Equation 3. 4) The chosen trader trades with each of their contacts in turn. The price and quantity of each trade are determined by the intersection of the demand functions of the traders involved (Equations 2 and 5, as appropriate). It is important to note no other traders update their valuations in a given step except for the chosen trader.

4 Results

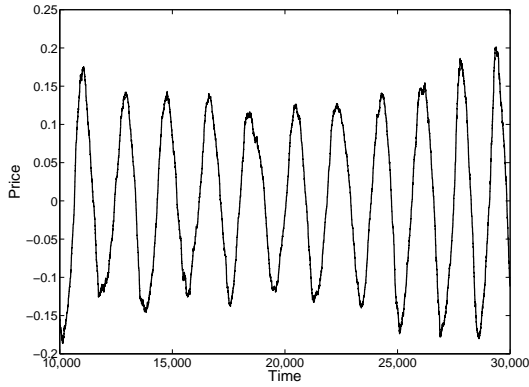
The model set out above is considered under a range of experimental settings. The effect of connectivity (the number of links between traders), the configuration (the pattern of the connections), the composition (the distribution of types of traders in the market) and volatility in the underlying fundamental price and riskless asset return are all examined. Statistics are collected between steps 10000 and 30000, giving time for the model dynamics to settle down and avoiding initialization effects. In all cases, unless otherwise stated, the fundamental value $W = 0$ and the return on the riskless asset $g = 0$. In all simulations there are 100 traders in the market and each trader starts with the same estimate of the asset price, $P_i^0 = 0.05$.⁹

4.1 Market Behavior

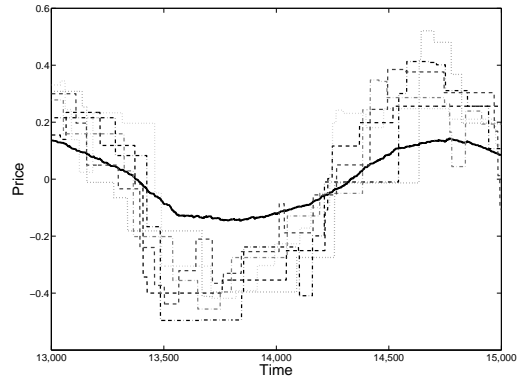
In order to understand the behavior of the model we first focus on two examples which demonstrate the possible dynamics. In each case each trader in the market is a chartist or fundamentalist with equal probability. In Figure 1 we plot the price paths of two simulations. In all cases the parameters are $a = 1.0$, $b = 4.0$ and $c = 1.0$ and the market is an Erdős-Rényi random graph (see Appendix B for details of the construction of these

⁸Other ways of determining the trade price were tested but had little qualitative effect on the results.

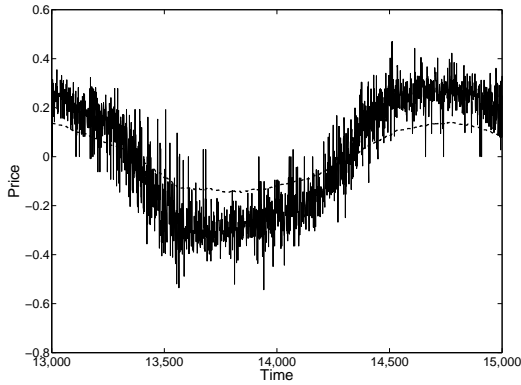
⁹This has little qualitative effect on the long term dynamics presented below.



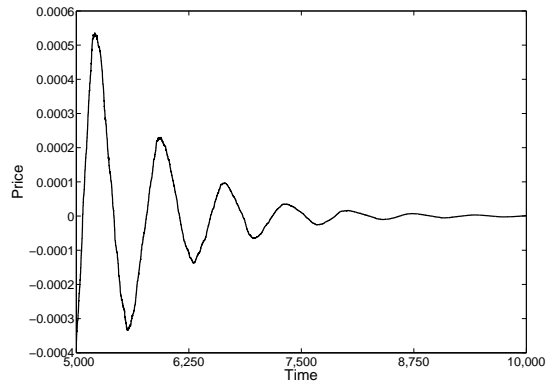
(a) Average Valuation



(b) Individual Valuations



(c) Trade Prices



(d) Average Valuation

Figure 1: Figures (a)-(c) show data from the same simulation. Figure (a) shows the average of the asset values of all traders. Figure (b) the average asset value in black whilst all other lines are individual valuations. Figure (c) shows the average asset value (dashed line) and the trade price at each point in time (solid line). Figure (d) shows the average of the local valuations for all traders for a separate simulation. In all cases $a = 1.0$, $b = 4.0$ and $c = 1.0$. Both networks are Erdős-Rényi random graphs with 100 nodes. For Figures (a)-(c) the probability of connection is 0.1 whilst for Figure (d) it is 0.6.

graphs). In Figures 1a-1c the probability of any pair of traders being connected is 0.1 whilst in Figure 1d it is 0.6.

Figure 1a shows the average of all traders asset values between periods 10,000 and 30,000. A cyclical dynamic is clearly present. There is some variation at the peaks and troughs of the cycles but generally the cycles appear smooth. This cyclical pattern is driven by the interaction of the two trader types. Fundamentalist traders wish to buy the asset for prices lower than the fundamental and sell if for prices above. As such their demand encourages the price to move back towards the fundamental. If chartist demand is strong enough, the trend, resulting from a reversion initiated by fundamentalist may

overshoot the fundamental and result in a further mispricing. As this new mispricing increases, fundamentalist demand also increases resulting in a higher proportion of trade at prices closer to the equilibrium price and so local price estimates which are closer to the fundamental. This in turn reduces the size of price changes and reduces the trend until it reverses and the process starts again. If chartist demand is strong enough this cycle may perpetuate, however, if it is not the fundamentalists may damp the swings caused by the chartists, reducing their size, and eventually resulting in a steady state (see Figure 1d). The cyclical dynamics may be related to the bubbles and crashes frequently seen in financial market. Individuals join trends, creating bubbles and overpricing until a point at which the asset is sufficiently overvalued that the trend reverses and a crash occurs. This may then result in a period of under pricing before investors again realize this and reverse the trend.

These cycles occur in a setting where not every trader is connected to every other. One of the results of this is the uneven peaks to the cycles. Connections in the market maintain a degree of synchronization, however, different parts of the market may change price at different speeds. The gradual reaction of traders may be seen in Figure 1b. The individual asset values generally follow the trend, however, they frequently depart, increasing when others are decreasing and vice versa. This is driven by the network. When there is a reversal in trend most chartists will identify this through the changes in the demands of their neighbors. The incomplete structure, however, means that chartist traders may occasionally be out of step with others in the market, predicting increases when all others are predicting the reverse. Similarly some chartists will be connected to relatively greater numbers of fundamentalists and so experience a greater demand for the asset towards the equilibrium price. These deviations in valuations create volatility in prices as observed in Figure 1c.¹⁰ This figure shows that whilst the average valuation appears smooth the trade price is much more volatile due to traders differing in their valuations of the asset. Chartists differ from fundamentalists and chartists with different neighbors and trade histories differ between themselves.

¹⁰Note, in this example, the trade prices tend to be further from the equilibrium than the average valuation because chartists have steeper demand than fundamentalists ($b > a$).

Figure 1d shows a separate case where the model converges to a steady state at the fundamental price. In this case the market is better connected and the time series starts earlier as the model converges prior to the results collection window. The figure shows that the average price exhibits a decreasing series of cycles. Individual valuations (not shown) follow a similar pattern. The higher degree of connectivity means that fundamentalist traders are able to damp the trends created by the chartists traders early in their development, ensuring that the market converges to the steady state at the fundamental value. When connectivity is lower fundamentalists are unable to do this - instead chartists can build up local trends which spread through the market leading to a divergence in trade prices and non-steady state dynamics.

In this paper we focus on the dynamics of trade prices as these are representative of the opportunities available to traders within the market. The average valuation, whilst illustrative, is not a price at which any trader is necessarily able to trade. In the remainder of this paper we associate the type of volatility seen in Figure 1c with market instability and the steady state at the fundamental price, as seen at the end of the period in Figure 1d, with stability. Volatile trade prices indicate that the market has not converged to the equilibrium valuation whereas a constant trade price indicates the opposite. It should be noted that in nearly all cases the steady state is the equilibrium price, the only exception is the extreme market composition consisting of no fundamental traders - in which case no trade occurs and trader valuations are never updated.

The dynamics of the average price were investigated. For the parameter regions considered in this paper we did not observe patterns in the average price which appeared chaotic. These patterns were only observed when $c > 2$, however, c is a moving average parameter and so is not economically meaningful in this range. This lack of chaotic behavior may at first appear strange to those used to models of chartists and fundamentalist. The trade price, however, should be the main focus. Empirical financial data and models of this type are principally concerned with the market price at which trade occurs rather than average valuations. Examination of trade prices in this model is therefore more natural and the volatile nature of this is in line with real world data. The smooth

pattern in average valuation is not readily comparable to market data.

4.2 Connectivity

Each trader within the market is connected to a number of trading contacts. As this number increases the trader may enter more trades, gaining a better picture of local trends. If every trader is connected to every other, the market essentially becomes a centralized exchange in which all participants have access to the same opportunities. Several papers, particularly in the inter-bank market literature, have considered the effect of connectivity e.g. Gai and Kapadia (2010) and Ladley (2013). These papers have shown how the collapse of one participant may spread through the network of credit linkages and affect the remainder of the market. A potentially significant question, which has received little attention, however, is how the connectivity of markets affects the dynamics of prices. It is this issue we address here.

We consider markets based on Erdős-Rényi random graphs with connectivities in the range 0.05 to 1.0. For each connectivity 1000 random networks are generated (see Appendix B for details) and 100 repetitions of the simulation for each network are run. Multiple repetitions are important because there is path dependence. Due to the network structure, the order in which traders are selected may influence the market dynamics. Markets consist of 50 chartists and 50 fundamentalists. Any network consisting of more than a single component is rejected and regenerated. We initially present a single parametrization of the model ($a = 1.0$, $b = 4.0$, $c = 0.7$). These values exhibit diverse dynamics, however, for a large range of other parameter combinations the behavior is qualitatively similar. In Section 4.3 we examine the sensitivity of the model to these parameters.

In this section we are principally interested in the number of runs which arrive at stable, steady state dynamics vs. the number that do not. In analyzing the simulations we use a simple test to identify those runs which achieve a steady state. Steady state dynamics are defined as those in which the sum of squared price changes is less than

10^{-6} .¹¹ In later sections we go on to investigate how the market structure affects other quantities of interest such as volatility and price deviations from the fundamental value.

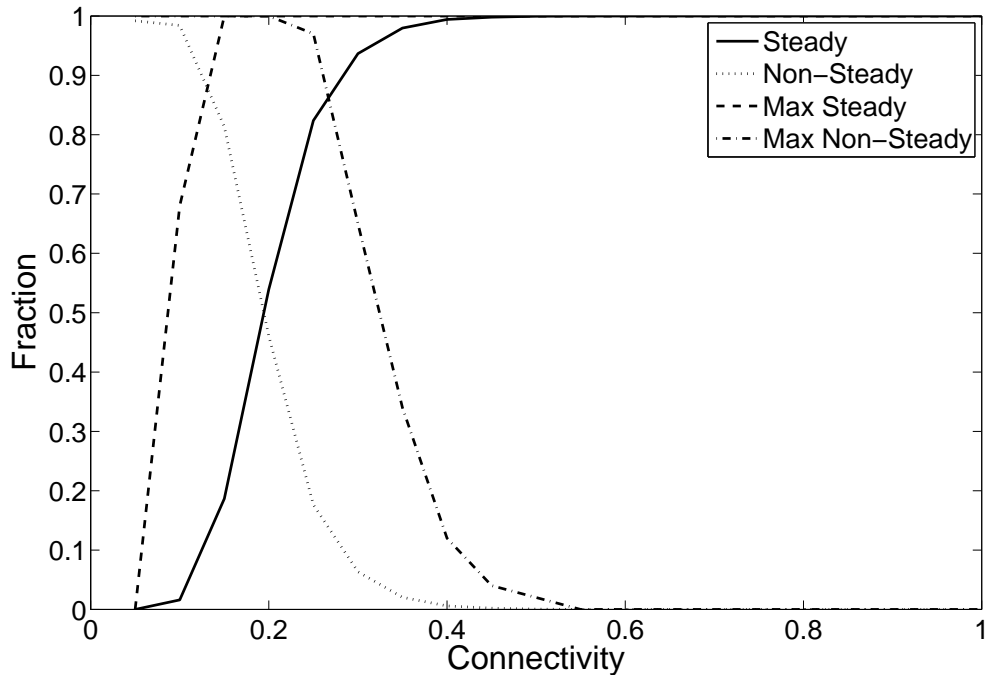


Figure 2: Graph showing the percentage of runs exhibiting steady state dynamics for parameters $a = 1.0$, $b = 4.0$ and $c = 0.7$ and for varying levels of connectivity within Erdős-Rényi markets. Each data point is calculated over 100 repetitions for each of 1000 networks (100,000 simulations for each point). Max (Non-)Steady is the largest number of runs for any one of the 1000 markets which exhibit (non-)steady state dynamics.

Figure 2 shows the percentage of runs which exhibit stable dynamics for different levels of market connectivity. There is a clear pattern: as markets become more connected their dynamics change. They exhibit more stable behavior. Increased connectivity means that traders have more trading partners. As such chartists will tend to be more synchronized in their trends and fundamentalists will have more opportunity to bring deviations back towards the fundamental. As a result, divergences are rare and the prices of traders across the market will tend to converge. In less connected markets traders will have less partners in common, producing more varied valuations and more divergence in trade prices. The

¹¹This value was chosen based on empirical observation. There is, however, a wide difference in this measure between those runs achieving a steady state, which often exhibit values close to 10^{-12} and those which do not, which often have values of 10^{-1} or higher. This measure was chosen for its similarity, however, there are many alternative measures based on aspects, such as price dispersion, volatility etc., which give qualitatively similar result.

sparser connectivity will mean that in some areas of the market fundamentalist traders will be a relative minority, and so prices away from the fundamental will be corrected more slowly allowing cycles to develop.

The results, however, are not as simple as the average implies. Figure 2 also presents the maximum percentage of steady states observed for a single market at each connectivity. These results show that, whilst poorly connected markets exhibit steady states rarely on average, this can vary a lot between market structures. For instance with connectivity equal to 0.15 only 19% of the total observations are stable, however, in one case a market exhibited steady state behavior 100% of the time. A similar story may be told for better connected markets. With connectivity equal to 0.3 on average 6% of the runs are non-steady state, however, one case exhibits non-steady state dynamics on 65% of occasions. Whilst connectivity has a large effect on stability, the configuration of connections also plays a role. The exact set of links between traders may allow information and trade to flow freely and for fundamentalist traders to damp fluctuations. Alternatively they may allow groups of chartist traders to deviate from the market and create disagreements in prices. The next section will consider the importance of the configuration of connections.¹²

4.3 Configuration

The previous section showed that whilst connectivity is important in determining market dynamics the configuration of the market also plays an important role. This finding supports previous work in this area. Both Ladley and Bullock (2008) and Wilhite (2001) find that small world type connections can reduce search costs and make the price formation process quicker, whilst Georg (2013) finds that the configuration of an inter-bank market, not just the connectivity, is important in determining the susceptibility to systemic shocks.

In this section, we investigate the role of the market configuration on price stability.

¹²An individual traders connectivity also effects how they may trade - better connected traders have access to a larger fraction of the market. Due to the cyclicity of the price dynamics and the zero sum nature of trade in this model all traders wealth's have periods of being both positive and negative. This means that whilst better connected traders trade more and so have more variable wealths, individual connectivity has no effect on the average.

We consider seven fixed market architectures with similar numbers of connections (with one exception) but qualitatively different patterns of connections in order to identify aspects of market structure, other than connectivity, that affect the dynamics of prices. The type of each trader, chartist or fundamentalist, is determined at random with the probability of them being a chartist $P_c = 0.5$. The parameters are $a = 1.0$, $b = 4.0$ and c in the range $0.1 \dots 1.0$. For each parameter combination we perform 100,000 repetitions of the model.

The first network structure we consider is completely connected - all traders are connected to all others. The second market architecture, core-periphery, has a completely connected core, comprising a fraction of the traders. The remainder of the traders form the periphery and are weakly connected to the traders in the core. The next two markets have random patterns of connections. The first is a Erdős-Rényi random graph whilst the second is based on a scale free network. This means the structure is random but the degree distribution, the distribution of node connectivity's, follows a power law. The fifth network structure is a torus in which each trader's configuration of connections is the same: each has the same number of partners connected in the same manner resulting in a homogeneous market structure. The final two structures relax the assumption of homogeneity. The sixth market exhibits local connectivity, in this case groups of ten traders form completely connected cliques. Each of these sub-markets, however, is only weakly connected to the other groups. As a result sub-markets are homogeneous but the market structure as a whole is not. The final structure introduces additional 'small world' connections between sub-markets. These connections allow trade between groups which would otherwise have been separated by a large number of links. Details of how these networks are constructed are provided in Appendix B.

We first consider the effect of configuration on the probability of steady state dynamics and then present the effect on volatility and the difference between the market price and fundamental. Table 1 shows the percentage of runs which achieve steady states for different market configurations and values of c . In all cases, when c is low the market is stable. Chartists are relatively unresponsive to recent price changes and so regardless

	Complete	Core Periphery	Erdős Rényi	Scale Free	Torus	Local	Small World
Connections	4950	505	495	485	500	480	485
0.1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.2	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.3	1.000	1.000	1.000	1.000	0.965	0.999	1.000
0.4	1.000	1.000	0.996	1.000	0.824	0.991	0.979
0.5	1.000	0.981	0.878	1.000	0.506	0.968	0.924
0.6	0.995	0.889	0.437	0.977	0.216	0.925	0.850
0.7	0.921	0.651	0.115	0.793	0.064	0.859	0.766
0.8	0.638	0.361	0.015	0.433	0.010	0.774	0.673
0.9	0.275	0.154	0.002	0.152	0.001	0.673	0.572
1.0	0.070	0.046	0.001	0.033	0.000	0.555	0.459

Table 1: Fraction of runs showing steady state dynamics for different market structures and values of c . Results calculated over 100 repetitions for each of 1000 instantiations of each market type. In all cases $P_c = 0.5$, $a = 1.0$ and $b = 4.0$. The number of connections in each market are also shown.

C	Complete	Core Periphery	Erdős Rényi	Scale Free	Torus	Local	Small World
0.1	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
0.2	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
0.3	-	-	-	-	0.001	0.000	-
	-	-	-	-	(0.006)	(0.000)	-
0.4	-	-	0.000	-	0.008	0.000	0.000
	-	-	(0.001)	-	(0.025)	(0.000)	(0.004)
0.5	-	0.000	0.004	-	0.029	0.000	0.003
	-	(0.003)	(0.014)	-	(0.049)	(0.001)	(0.017)
0.6	0.000	0.004	0.032	0.000	0.069	0.001	0.007
	(0.001)	(0.018)	(0.053)	(0.003)	(0.070)	(0.003)	(0.029)
0.7	0.002	0.018	0.099	0.006	0.117	0.001	0.012
	(0.010)	(0.046)	(0.081)	(0.020)	(0.094)	(0.006)	(0.039)
0.8	0.014	0.049	0.168	0.032	0.175	0.002	0.017
	(0.034)	(0.073)	(0.078)	(0.050)	(0.413)	(0.009)	(0.047)
0.9	0.051	0.093	0.215	0.082	0.314	0.004	0.023
	(0.062)	(0.089)	(0.063)	(0.073)	(2.504)	(0.013)	(0.054)
1.0	0.105	0.138	0.245	0.137	2.553	0.007	0.030
	(0.075)	(0.090)	(0.053)	(0.075)	(68.449)	(0.018)	(0.061)

Table 2: Absolute difference between trade price and fundamental value for different market structures and values of c . Standard deviations in parentheses. Results calculated for those runs not exhibiting steady state dynamics from 100 repetitions for each of 1000 instantiations of each market type. In all cases $P_c = 0.5$, $a = 1.0$ and $b = 4.0$.

C	Complete	Core Periphery	Erdős Rényi	Scale Free	Torus	Local	Small World
0.10	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
0.20	-	-	-	-	-	-	-
	-	-	-	-	-	-	-
0.30	-	-	-	-	0.002	0.000	-
	-	-	-	-	(0.011)	(0.000)	-
0.40	-	-	0.000	-	0.013	0.000	0.001
	-	-	(0.001)	-	(0.042)	(0.002)	(0.007)
0.50	-	0.000	0.004	-	0.045	0.001	0.005
	-	(0.003)	(0.016)	-	(0.074)	(0.005)	(0.025)
0.60	0.000	0.005	0.036	0.000	0.099	0.002	0.012
	(0.001)	(0.023)	(0.059)	(0.004)	(0.104)	(0.010)	(0.042)
0.70	0.002	0.023	0.112	0.007	0.162	0.004	0.020
	(0.011)	(0.057)	(0.090)	(0.022)	(0.178)	(0.016)	(0.057)
0.80	0.016	0.063	0.188	0.036	0.281	0.007	0.030
	(0.037)	(0.091)	(0.085)	(0.055)	(1.910)	(0.024)	(0.070)
0.90	0.056	0.121	0.240	0.092	0.874	0.012	0.041
	(0.068)	(0.112)	(0.069)	(0.081)	(13.608)	(0.032)	(0.083)
1.00	0.115	0.179	0.271	0.149	3.161	0.022	0.058
	(0.141)	(0.213)	(0.278)	(0.170)	(154.772)	(0.051)	(0.114)

Table 3: Volatility of trade price for different market structures and values of c . Standard deviations in parentheses. Results calculated for those runs not exhibiting steady state dynamics from 100 repetitions for each of 1000 instantiations of each market type . In all cases $P_c = 0.5$, $a = 1.0$ and $b = 4.0$.

of the market structure there are no trends established. As c increases, chartists become more sensitive leading to potential deviations. The size of the deviations also increase. Tables 2 and 3 show the average distance of trade prices from the fundamental and the volatility in trade prices conditional on non-stable dynamics. As traders react more strongly to trends, this causes larger deviations from the fundamental and greater price volatility. The configuration, however, has a strong effect on the pattern.

The core-periphery market may be viewed as a completely connected network with additional periphery nodes attached. The effect of these traders are to make the market less stable in comparison to the completely connected market. The periphery nodes only have a single connection to the core, limiting the signals they receive. As a result their prices may become disconnected from the rest of the market and result in them providing a destabilizing influence. These nodes also increase the size of deviations even after

controlling for the lower number of stable markets (Table 2 and 3). In comparison to the Erdős-Rényi network with the same number of connection the core-periphery market is more stable. The completely connected center reduces price divergences and allows a greater proportion of steady state dynamics.

Both the Erdős-Rényi and scale free networks exhibit steady state dynamics less frequently than the completely connected market. These markets also show higher volatility and price deviations from the fundamental. Unlike the the completely connected market some traders are not directly connected and are therefore able to trade at different prices leading to more volatility in trade prices and more frequent deviations from the fundamental. There is, however, a significant difference between the two network types, the scale free market shows stable dynamics more frequently and lower volatility than the Erdős-Rényi network. This difference is driven by differences in the degree distribution of the two networks. The scale free network is characterized by a greater number of hubs - high connectivity nodes - relative to the Erdős-Rényi network. The hub traders connect to large numbers of individuals across the market. As a result they provide a coordinating influence, bringing the valuations of different traders together towards the fundamental making them more stable and only slightly more volatile than a centralized market.

Whilst possessing approximately the same number of connections as both random networks, the torus network exhibits much higher volatility and steady state behavior less frequently than either of them. Unlike the random networks the structure of the torus is homogeneous and connections are local in the sense that the distance between pairs of nodes on opposite sides of the torus is very long. This structure means that there can be substantial deviations in valuation in different areas of the market. With traders trading in random order this produces much higher volatility than any other structure. As a result for relatively low values of c , the market does not achieve a steady state. The lack of ‘long range’ connections means that traders are unable to synchronize and bring the market price back to the fundamental or even coordinate on a single price.

The locally connected market is characterized by small groups of heavily connected traders with few connections between them. Whilst this is similar to the torus network,

in the sense that there are relatively long distances between traders, the results are very different. The locally connected market is more stable and much less volatile. Relative to the completely connected market, the locally connected market exhibits a reduction in stability for low values of c but an increase for high values. The statistics for volatility and price deviations give an indication as to why this occurs. These values are much lower than any other market, suggesting that relatively few traders trade away from the fundamental value even during non-steady state dynamics. At any given time most of the groups are in, or very near, the steady state. Deviations may occur in some areas of the market and not spread beyond a single group. Some segments may be less stable than others - perhaps containing a higher fraction of chartist traders. As a result of this heterogeneity an individual section may deviate from the fundamental more easily at lower values of c making the whole market more prone to non-steady state behavior. For higher c , however, this relative effect is reversed. Whilst local groups are increasingly prone to non-steady state behavior the limited connections mean that fluctuations are often not spread throughout the market, whereas in a completely connected network they would be. Relative to completely connected markets, cycles in locally connected markets may be damped or eliminated more easily.

The addition of small world connections to the locally connected market has an ambiguous effect. They provide a small increase in stability for most values of c but a large reduction for high c . For low c the small world connections reduce the scope of mispricings by providing additional connections to other groups, who, assuming they are also not away from the steady state, provide a small damping effect. When chartists are more sensitive to price changes, rather than enhancing the flow of information and therefore price stability, the cross linkages instead let destabilizing price changes spread more easily. The aspect of the structure which meant that the locally connected market was more stable than the completely connected market is weakened. This result is in contrast to that of Georg (2013) who finds that small world markets are more stable than random networks. Similarly Ladley and Bullock (2007, 2008) observe that small world connections stabilize the market allowing information to flow more quickly and so leading to faster

price discovery. In this previous work traders had pricing rules which did not permit trend following, instead traders learned the asset price based on local information. In this paper the ability of traders to respond to local patterns means that rather than stabilizing the market, the cross connections can propagate mispricings. In all cases volatility and price deviations increase, indicating that whilst the links may spread or damp price deviations the added connections involve more traders in this process resulting in more trades away from the steady state.

4.4 Composition

In earlier sections all markets had, on average, equal numbers of each type of trader, in reality, however, this will frequently not be the case. There may be some markets which are dominated by fundamental traders, whilst others may have higher proportions of chartist traders. There may also be variation over time as traders switch strategies, e.g. Panchenko et al. (2010) or move between asset markets, e.g. Westerhoff (2004).

The probability of a trader being a chartist, P_c , is varied in the range 0 to 1. Connectivity is varied between 0.05 and 1.0. For each connectivity level and composition, 1000 random Erdős-Rényi networks are generated. In each case 100 repetitions are performed and the aggregate statistics reported. As before, the parameters are $a = 1.0$, $b = 4.0$ and $c = 1.0$.

The effect of market composition is shown in Figure 3. More chartists traders generally lead to less frequent steady state behavior. In markets with large numbers of fundamentalists, there is greater pressure to move the asset price back towards the fundamental value producing, for large parameter ranges, a fixed price. This effect combines with market connectivity such that even relatively highly connected markets with high numbers of chartists exhibit non stable dynamics. For the market composed solely of chartists there are no individuals able to identify the initial mispricings of the asset and so no price trend develops resulting in a steady state being maintained.

Figure 4 demonstrates the effect of trader behavior by varying the three parameters a , b and c for a range of markets architectures. For brevity we restrict our presentation

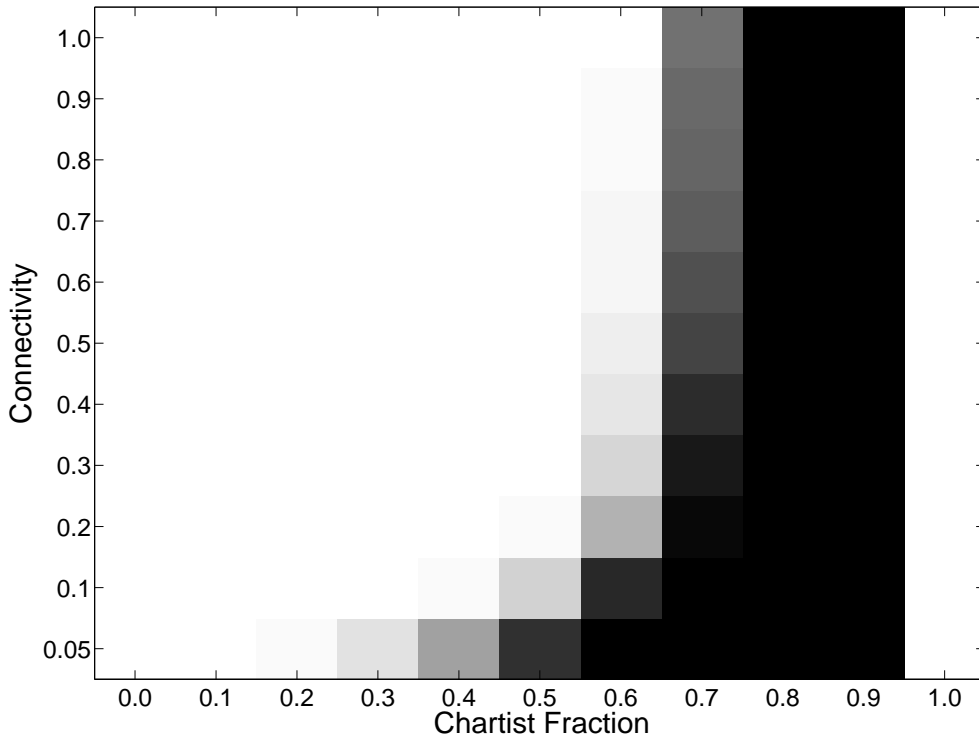


Figure 3: Fraction of simulations which result in a steady state dynamic (white indicates 100%, black 0%) for different values of connectivity and population composition P_c . In all cases $a = 1.0$, $b = 4.0$ and $c = 1.0$.

here to three architectures: completely connected, Erdős-Rényi and locally connected.¹³ Each graph shows the percentage of runs for each parameter combination which result in the steady state and the region in which the Chiarella model would exhibit steady state behavior (those parameters for which $\frac{1}{c} > \frac{b-1}{a}$). In all cases for low c non-steady state dynamics are restricted to parameter combinations with low fundamental demand and high chartist demand, i.e. when chartists only weakly follow trends, prices only deviate from the fundamental if chartists have very strong demand (to push the weak trends) and fundamentalists have weak demand (so they are unable to damp the trends). As chartists react more strongly to trends (c increases) the non-stable region also increases in size such that this dynamic is observed for lower levels of chartist demand and higher levels of fundamentalist demand.

The completely connected market demonstrates the sharpest dynamics - a greater

¹³The patterns for the other markets are qualitatively similar and are available upon request.

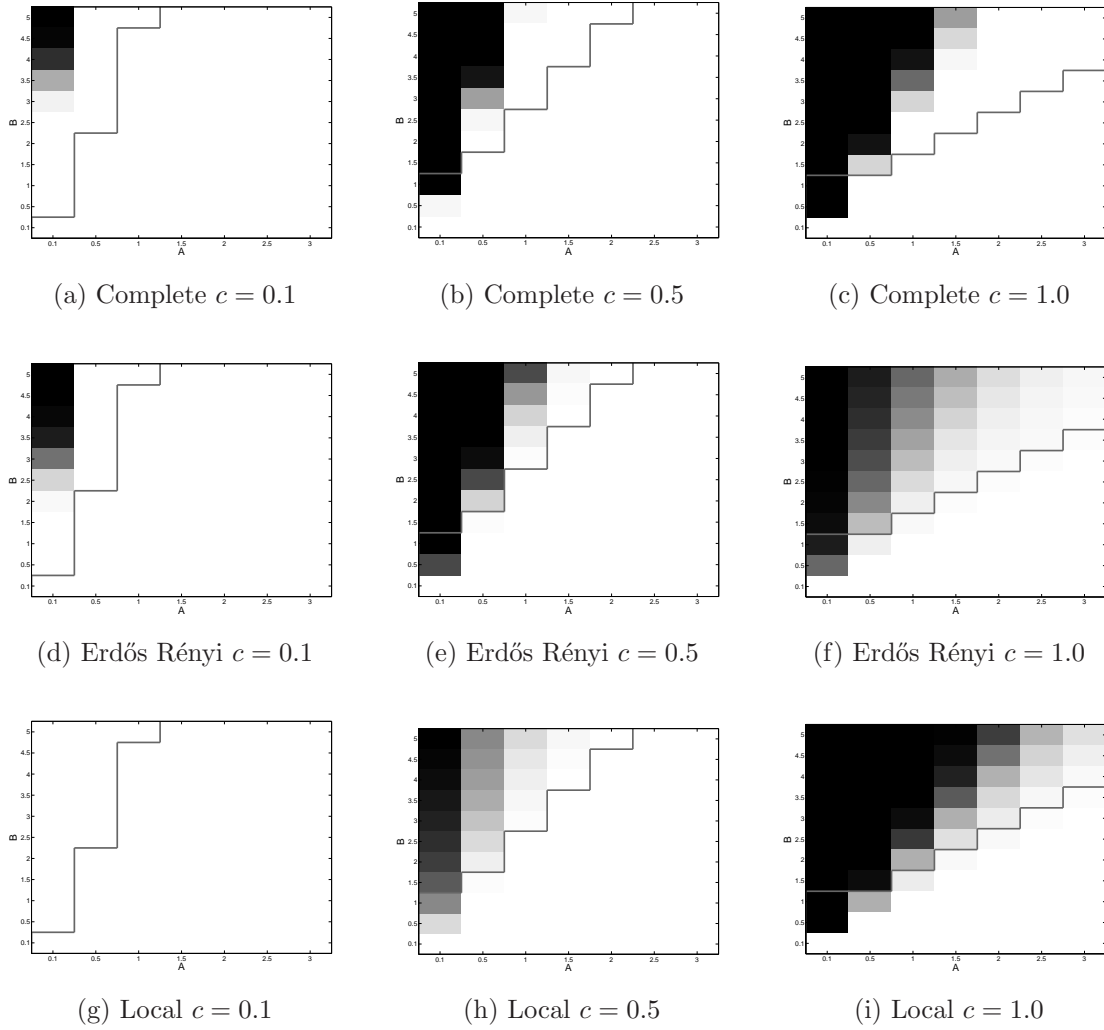
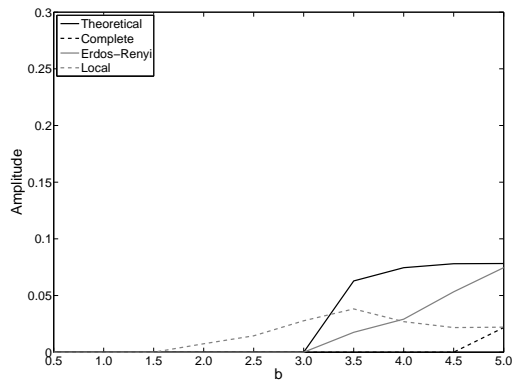


Figure 4: Each graph shows the fraction of simulations which result in a steady state dynamic (white indicates 100%, black 0%) for different values of a and b . Figures show the statistics for different values of c and different network structures. Grey line indicates the boundary of the region (below and to the right) of stability under the Chiarella model.

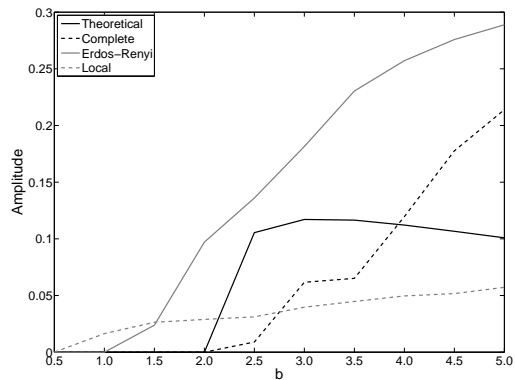
fraction of the parameter combinations exhibit either all or no steady state behavior and very few exhibit both. This is because the greatest source of variation, the configuration of links, is identical across runs. The Erdős-Rényi network exhibits a much larger region of gray cells. The exact combination of random links effect the stability resulting in a less precise, but larger, area in which both stable and non-stable dynamics are seen for particular parameter combinations. The locally connected model is most sensitive to changes in c , showing the largest area of steady state dynamics for low c and the smallest area for high c . This indicates the significance of the local groups - if the strength of trend following is weak even if there is a divergence from the equilibrium in one area, it

is unlikely to spread and may eventually be damped. If, however, c is high and trends are able to develop, the local structure has the opposite effect reducing the ability of the market to synchronize and damp prices.

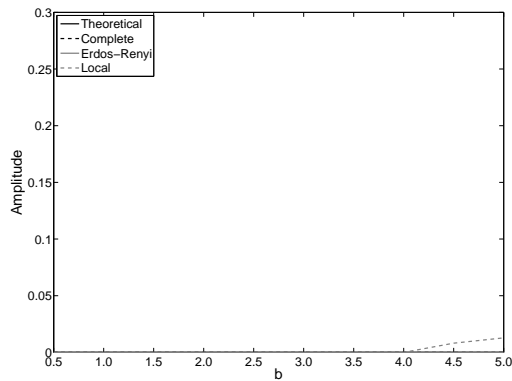
The stable regions have similar shapes to those demonstrated in the Chiarella (1992) model. The degree of correspondence is relatively high. Only for low a and b and high c are non-stable dynamics observed for parameter combinations which would result in a steady state in the Chiarella model. For high a and b , steady state dynamics are observed in some cases when a steady state would not be found in the Chiarella model.



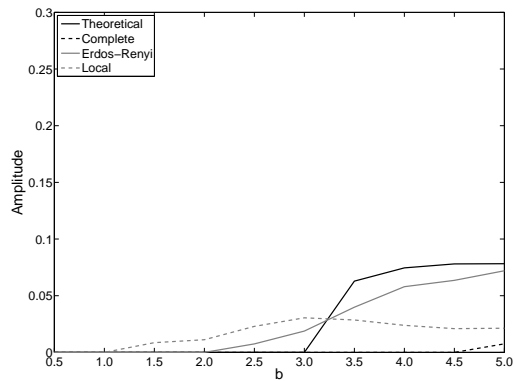
(a) $a = 1.0, c = 0.5$



(b) $a = 1.0, c = 1.0$



(c) $a = 2.0, c = 0.5$



(d) $a = 2.0, c = 1.0$

Figure 5: Each graph shows the amplitude of cycles for different market structures, along with the value for the Chiarella model. Figures show the statistics for different values of a and b . All results averaged over those runs of 100,000 simulations not resulting in steady state dynamics. Amplitude of zero indicates all runs were steady state. Standard deviations in parentheses.

Chiarella (1992) also gives an analytic expression for the amplitude of cycles¹⁴, however, we have shown that the price process in the OTC model is volatile and does not follow

¹⁴This expression is presented in Appendix A.

a simple cyclic pattern. In contrast the average of local valuations is cyclic and is linked to the trade prices in that these values are used to construct trader valuations. Given the different measures, market prices vs. average valuation, a direct quantitative comparison is not possible, however, an examination of the relative effects of the parameters on the amplitudes is still insightful.¹⁵

We estimate the amplitude as the difference between the maximum and minimum of the average valuations within each run. The dynamics of the average valuation are sufficiently smooth and regular that this is a reasonable estimate of the amplitude. Figure 5 shows these values for three networks and a range of parameter values along with the calculated Chiarella model value. In line with the previous results for price volatility the local network generally has a lower amplitude than the completely connected market whilst the random network has a higher amplitude. The behavior of the model in response to parameters a and c for all networks is in line with the theoretical predictions. The parameter a leads to a reduction in amplitude as higher fundamentalist demand leads to damped oscillations. Whilst an increase in the sensitivity to recent trends (increase in c) leads to an increase in the amplitude. The effect of b is more complex. The numerical calculations suggest this will have an ambiguous effect, sometimes increasing and sometimes decreasing the amplitude. For the OTC model, however, an increase in b nearly always increase the size of the amplitude. Greater chartist demand creates stronger trends and therefore greater oscillations. This difference in the relationships may be due to constraints on the derivation of the analytical formula underlying the theoretical predictions. Chiarella suggests that the amplitude increasing effect may dominate, however, this must be interpreted with caution.

In addition to the network structure, the OTC model presented here and the original model of centralized trade differ significantly in the representation of time: asynchronous discrete updates vs. continuous time. The qualitative similarity in their behavior, both in terms of the stable region and the response to parameter changes is, therefore, surprisingly close. This suggest that the underlying equations governing trader behavior play

¹⁵Furthermore, as Chiarella notes the formula for calculating the amplitude is derived through the method of averaging and so is only valid for a limited range of parameters.

a very significant role. Whilst the market structure itself modulates the market dynamics, the traders may be the dominant factor. It is particularly enlightening to compare the completely connected market with the theoretical predictions in this setting. In the completely connected OTC market everyone trades with everyone, effectively the market becomes centralized. Differences in behavior are then principally due to differences in the representation of time. In general this change makes the markets more stable - exhibiting steady state dynamics more frequently. This may be understood by realizing that updating traders one at a time effectively makes the valuation process more gradual which acts to smooth the overall market dynamics.

4.5 Volatile Fundamentals

Finally, we consider volatility in the underlying fundamental asset value and riskless asset return. Previously both of these components were assumed to be constant, however, this is not representative of real financial markets. In this section the fundamental price and riskless asset return follow random walks. Both variables are updated after each trader is chosen as follows:

$$W(t + 1) = W(t) + \sigma_W \eta_W^t \quad (7)$$

$$g(t + 1) = g(t) + \sigma_g \eta_g^t \quad (8)$$

where σ_W and σ_g control the scale of the volatility of the fundamental value and return of the riskless asset and η_W^t and η_g^t are standard normally distributed random variables. Initially $W(0) = 0$ and $g(0) = 0$. As before 100,000 simulations are conducted and the parameters are $a = 1.0$, $b = 4.0$, $c = 1.0$ and $P_c = 0.5$.

Table 4 presents results showing the effect of volatility in the riskless asset return and fundamental value on the volatility of the trade price.¹⁶ In all cases, increases in the volatility of the fundamental results in an increase in the price volatility by at least an order of magnitude. This may be explained as follows - Traders are on average chosen once

¹⁶The effects of the two volatility's are considered separately for clarity, however, the effects are additive. Tables of data showing this are available from the authors.

Asset Volatility	Riskless Volatility	Complete	Core Periphery	Erdős Rényi	Scale Free	Torus	Local	Small World
0.000	0.000	0.115 (0.141)	0.179 (0.213)	0.271 (0.278)	0.149 (0.170)	3.161 (154.8)	0.022 (0.051)	0.058 (0.114)
0.001	0.000	0.145 (0.157)	0.200 (0.223)	0.275 (0.281)	0.169 (0.181)	3.206 (161.1)	0.126 (0.146)	0.143 (0.166)
0.002	0.000	0.187 (0.194)	0.234 (0.250)	0.291 (0.297)	0.204 (0.212)	3.163 (161.5)	0.188 (0.202)	0.199 (0.215)
0.000	0.000	0.115 (0.141)	0.179 (0.213)	0.271 (0.278)	0.149 (0.170)	3.161 (154.8)	0.022 (0.051)	0.058 (0.114)
0.000	0.001	0.224 (0.230)	0.236 (0.243)	0.243 (0.248)	0.224 (0.230)	1.859 (50.4)	0.206 (0.215)	0.214 (0.223)
0.000	0.002	0.295 (0.300)	0.304 (0.310)	0.298 (0.302)	0.294 (0.298)	1.746 (45.8)	0.261 (0.269)	0.266 (0.274)

Table 4: Volatility of trade price for different market structures and volatilities of riskless asset return and fundamental value. Results calculated over 100 repetitions for each of 1000 instantiations of each market type. In all cases $P_c = 0.5$, $a = 1.0$, $b = 4.0$ and $c = 1.0$.

every 100 time steps. The noise is normally distributed and therefore, to an individual trader, the change in the underlying asset value between times they are active will scale with the square root of this time span i.e. $\sqrt{100}\sigma_W$. Some markets, however, exhibit much greater increases. The locally connected markets show the greatest increase in volatility. This structure was previously shown to damp variations between the weakly connected regions, however, changes in the fundamental price act to create shocks which effect all areas, bypassing the weak connections. For instance, an increase in the fundamental would create positive demand amongst all fundamentalists at the current market price and therefore an upwards trend. The Erdős-Rényi random graph shows an almost flat pattern. It should, however, be noted that this is from a very high starting point and is subject to a very high standard deviation making reliable interpretation of a pattern difficult.

The magnitude of the effect on price volatility should be an order of magnitude greater than the volatility in the riskless asset for the same reason as given for the fundamental volatility. In general this is the case, with some markets such as the locally connected market exhibiting considerably greater increases. For the Erdős-Rényi and torus networks, however, the results are more ambiguous. For the torus network riskless asset volatility always decreases price volatility whilst for the Erdős-Rényi network a large in-

crease in the riskless asset volatility increases price volatility and a small increase has the opposite effect. This patterns is due to the presence of the riskless asset return in the chartists demand functions. Changes in the riskless asset return have a common effect on the chartist traders. When they increase (decrease) all chartist traders will tend to decrease (increase) their estimate of the trend. This acts to increase the synchronization of chartist's demands and therefore prices across the network - reducing price volatility. A similar effect is not observed for volatility of the fundamental because fundamentalists all have the same valuation and are already coordinated. This only has an effect in the torus and random networks which are relatively decentralized and in which prices diverge the most (as seen by the high volatility in the base case). Of these the random network has more coordinated prices and so as the volatility of the riskless asset return increases the synchronization effect becomes secondary to the increase in underlying volatility resulting in a general increase in price volatility. The larger dispersion permitted by the regular structure of the torus network means that higher levels of riskless asset volatility still coordinate traders.

5 Conclusion

This paper has examined the effect of the structure of OTC markets on the stability and dynamics of prices. We have considered a model in which heterogeneous investors interact with a subset of individual in the market and base their strategies on local prices. It was found that the structure of an OTC market has a strong effect on the price dynamics. As markets become less heavily connected, and trade is more constrained, the market becomes less stable. The occurrence of steady states decreases and is instead replaced by volatile prices. The sparseness of connections means that different traders within the market may diverge in their estimates of the price and the fundamental traders are unable to bring the price back to the fundamental.

Connectivity, however, is only part of the story, the configuration of the connections and the composition of the market also play roles. Small world links were shown to have an ambiguous effect, in some cases increasing stability whilst in others decreasing

it. In random networks the presence of hubs leads to lower volatility and more accurate pricing in scale free networks than the equivalent Erdős-Rényi network. This was further confirmed in the torus market in which the large distances between some traders results in the greatest volatility. The composition of the market was also shown to play a role with greater numbers of chartist traders found to be a destabilizing factor. The OTC market structures were found to amplify the effect of fundamental and riskless asset volatility, however, in some cases low levels of riskless asset return volatility were found to reduce price volatility. Despite the influence of the network structure the behavior of the model has much in common with the underlying Chiarella (1992) model in terms of the relationship between parameters and stable behavior. This was particularly true for completely connected markets - suggesting that highly connected OTC markets may behave similarly to centralized exchanges. As connections are removed, the relationship became less clear cut and the exact configuration of connections was found to play a role.

Several aspects of this model invite future extensions. Here we focused on price dynamics and ignored aggregate trader positions and partner selection. An extension to consider these aspects in detail would allow the inclusion of portfolio and leverage effects. In this setting traders would be constrained in the size of the positions that they could take based on their wealth. As a result, trader demand, and therefore market dynamics, would be dependent on recent performance. A further extension would be to consider the endogenous formation of the network structure. In this paper the network was specified exogeneously, however, in reality traders may choose their connections. The model could be extended to incorporate this feature and examine equilibrium network structures, in a similar manner to Condorelli and Galeotti (2012), under speculative trade.

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A Chiarella's Model

Chiarella's (1992) model considers a market populated by two groups of traders - *fundamentalists* and *chartists*. The asset price $P(t)$ is set by an auctioneer based on excess demand $D(t)$ as follows:

$$\dot{P} = D(t) \quad (9)$$

where $D(t)$ is the sum of *fundamentalist* $D^0(t)$ and *chartist* demand $d(t)$:

$$D(t) = D^0(t) + d(t) \quad (10)$$

Fundamentalist demand is a linear function of the price difference between the market and Walrasian equilibrium price ($W(t)$):

$$D^0(t) = a(W(t) - P(t)) \quad (11)$$

Where a controls the slope of the demand function. The *chartist* demand function is given by:

$$d(t) = h(\psi(t) - g(t)), \quad (12)$$

where $\psi(t)$ is the *chartist* assessment of the current trend in $P(t)$, $g(t)$ is the return on the riskless asset, and $h()$ is a sigmoid function with the following properties: h increases monotonically, has a single point of inflection and well defined lower and upper bounds. Moreover $h(0) = 0$. The steepness of the sigmoid is parametrized with $b(b > 0)$, indeed $b = h'(-g(t))$. Assuming that the trading dynamics occur at a much faster time scale than changes in the equilibrium price W , or the return on the safe investment g , the latter two variables can be assumed to be constant. Chiarella's model is, therefore, defined by the following set of differential equations:

$$\dot{P} = a(W - P + h(\psi - g)) \quad (13)$$

$$\dot{\psi} = -acP - c\psi + ch(\psi - g) + acW \quad (14)$$

The steady state of system 14 is given by:

$$\bar{\psi} = 0 \text{ and } \bar{P} = W + h(-g)/a \quad (15)$$

The stability of the system can be analyzed in terms of the Jacobian J of system 14, showing that there is no saddle point behavior, i.e. either a stable equilibrium or a limit cycle occurs. The condition for equilibrium is:

$$\tau > (b - 1)/a. \quad (16)$$

Where $\tau = 1/c$. There is, therefore, a clear boundary between equilibrium and limit cycle behavior that is easily computable. Moreover, the amplitude of the limit cycle can be calculated analytically from:

$$\dot{A} = A(K(A) - a\epsilon/2)/\tau \quad (17)$$

where A is the amplitude, $\epsilon = \tau - \frac{b-1}{a}$ and

$$K(A) = \frac{1}{2} \int_0^{2\pi} [k'(A \sin y) - k'(0)] \cos^2 y dy \quad (18)$$

where $k(x) = h(x - g) - h(-g)$.

B Network Structures

In all descriptions below networks consist of X vertexes (in this paper $X = 100$) connected by varying configurations of edges. The network parameters were chosen to give approximately the same number of edges in different configurations.

B.1 Erdős Rényi Random Graph

Are named after the early work of Erdős and Rényi (1959). The presence of edges in Erdős-Rényi random graphs is determined by a connectivity parameter. The probability of an

edge existing between any pair of nodes is equally likely. This relatively simple structure means that many of the properties of these graphs, such as the degree distribution (the distribution of node connectivity's), may be determined analytically for large graphs. Whilst these networks are rarely found in reality their simplicity has resulted in them being used in studies in many fields. See Newman (2010) for a detailed discussion of their properties and variants.

These graphs are constructed as follows. For each pairs of nodes i and j the connecting edge is added with probability p . We consider a range of values of p .

B.2 Completely Connected

In this case every vertex is connected to every other vertex in the network. This may be considered a special case of the random graph with $p = 1$. Every trader in the market is able to interact with every other.

B.3 Core Periphery

Several systems, including inter bank markets (Becher et al., 2008; van Lelyveld and in t Veld, 2012), are characterized by a core-periphery or hierarchical structure. In this type of network a proportion of the nodes form a highly connected core. The remaining periphery nodes are weakly connected to core but there are few if any connections directly between the periphery nodes. Markets structured in this way contain a group of traders who all know of and interact with each other. The remaining traders only have one or two connections to these central parties.

This network structure consist of a core of m nodes which are completely connected. Each of the remaining $X - m$ periphery nodes is connected to a single randomly chosen core node. There are no direct connections between periphery nodes. In this paper we consider $m = 30$.

B.4 Scale Free

In scale free networks the degree distribution, the distribution of the number of nodes each node is connected to, follows a power law. This means that there are many nodes with very few connections and a small number of nodes with very large numbers of connections. There are many real world networks which have this type of structure for instance citation networks and the world wide web. These networks are also seen in economic systems, for instance Soramäki et al. (2007) finds that the network of payments on the FedWire Funds system follows a scale free distribution. They are characterized by generative processes often based on preferential attachment. This means that individuals are more likely to connect to other individuals who are already well connected. There has been a great deal of research in this area both in identifying examples of scale free networks and understanding the manner in which they come about. For a recent review see Newman (2010)

To generate scale free networks we use the preferential attachment model of Albert and Barabasi (2002). The algorithm for generating these networks is given below (as presented by Barabasi and Albert, 1999) The graph starts with a core of m_0 nodes which are completely connected. Nodes are then added to the network one at a time. When a node is added m (such that $m \leq m_0$) connections are made to existing nodes. The probability of the new node being connected to an existing node i is $\frac{x_i}{\sum_{i=1}^X x_i}$ where x_i is the number of edges connected to vertex i . As such nodes with more connections are likely to gain further connections. Here we consider $m = m_0 = 5$.

B.5 Ring

The ring is a regular network structure. Whilst not a realistic structure it provides a useful baseline as a market with low connectivity and high separation between traders. The network is constructed as follows. Consider the nodes arranged in a circle. Each trader is connected to the N nodes immediately to the left and N nodes immediately to the right.

B.6 Local Structure

This network consists of several groups of nodes. Each group is strongly connected internally but weakly connected to other groups. We can liken this to a market with several sub-markets. The majority of traders only trade in one of these submarkets. Those that trade in more than one tend to trade in submarkets which are close in some dimension. Formally we consider N groups each containing X/N nodes. The vertexes in each group form a completely connected graph. These groups are arranged in a circle. Each group is connected to the adjacent group to the left and right. For each connection M members of the group are chosen at random and connected to each of M randomly chosen members of the other group. No node may be chosen to be connected to both adjacent groups. In this paper we use $N = 10$ and $M = 2$.

B.7 Small World

Small world networks have received much attention in the scientific and popular literature. They typically feature multiple cliques of densely connected nodes which are connected by a small number of edges. In network theoretic terms this means that in comparison to random graphs they have relatively high clustering coefficients and a relatively low average path lengths. These type of properties are found in many systems including science collaboration networks and power networks (Albert and Barabasi, 2002). It is relatively easy to extend the example given for local structure to include small world connections by the addition of a small number of traders willing to trade over long distances in the separated markets.

There are many algorithms for generating small world networks (e.g. Watts and Strogatz, 1998). In this paper the small world networks are based on the local network described above. After the local network is constructed S additional edges are added. For each edge, two disconnected vertexes are randomly selected with uniform probability and an edge is added between them. In this paper we consider $S = 5$.