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Bilinear forecast risk assessment for non-systematic inflation: Theory and evidence

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Bilinear forecast risk assessment for non-systematic inflation: Theory and evidence

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Abstract

The paper aims at assessing the forecast risk and the maximum admissible forecast horizon for the non-systematic component of inflation modeled autoregressively, where a distortion is caused by a simple first-order bilinear process. The concept of the guaranteed upper risk of forecasting and the δ-admissible distortion level is defined here. In order to make this concept operational we propose a method of evaluation of the p-maximum admissible forecast risk, on the basis of the maximum likelihood estimates of the bilinear coefficient. It has been found that for the majority of developed countries (in terms of average GDP per capita) the maximum admissible forecast horizon is between 5 and 12 months, while for the poorer countries it is either shorter than 5 or longer than 12. There is also a negative correlation of the maximum admissible forecast horizon with the average GDP growth.

1. Introduction

The literature on inflation forecasting has, so far, focused on identification and further analysis of its systematic part, often described as the core or underlying inflation. This component of inflation is loosely defined as the dynamics of prices being neutral regarding to output in the medium and long-run. The literature on this subject is huge (see e.g. the seminal works by Eckstein, 1981, Cecchetti, 1996, Quah and Vahey, 1995, Cristadoro *et al*. 2005, current critical reviews and advances by Silver, 2007, Rich and Steindel, 2007, Bodenstein, 2008, Siviero and Veronese, 2011, Wynne, 2008 and Bermingham, 2010). In fact forecasting core inflation has become a common practice at many central banks and other financial institutions. There is, however, a limited interest in investigation of the non-systematic part of inflation, described as the difference between the headline (or observed) inflation and its core component. It is usually acknowledged that the non-systematic inflation is stationary and short-term forecastable. Nevertheless, the specific forecasting techniques have not been researched so far and, in particular, the length of the admissible forecasting horizon has usually been defined here rather vaguely.

In this paper we aim to assess the forecast risk and the maximum admissible forecast horizon for the non-systematic component of inflation, where there is a certain type of non-linearity in the process, defined (or approximated) by a simple first-order bilinear process. The presence in such component creates misspecification in forecasting for periods longer than one. This prompts the question about the maximum admissible forecast horizon, for which distortions caused to the forecast due to such misspecification are not substantial. Obviously, what is 'substantial' here is arbitrary and has to be defined prior to any investigation. Here we start with the concept of the guaranteed upper risk of forecasting. The δ-admissible distortion level defined as the maximal value of the bilinear coefficient for which the forecast instability does not exceed *a priori* given admissible risk level δ (Section 2). In order to make the concept of the admissible risk level operational, in Section 3 we propose a method for evaluation of the p-maximum admissible forecast risk, which corresponds to the p^{th} fractile of the distribution of a statistic used for evaluation of the null hypothesis of no bilinearity. After a series of Monte Carlo experiments, we suggest to use, as such statistic, a Student-t ratio for the maximum likelihood estimates of the bilinear coefficient. After computing the p-maximum admissible forecast risk, it is possible to evaluate the maximum forecast horizon for which, under such level of risk, the estimated bilinear coefficient is equal to its maximal admissible value.

Section 4 contains a description of empirical results for time series of monthly data on inflation for 122 countries. The maximum span of the series is from 1957 to April 2011 (some series are shorter). This section also discusses the relationship between the GDP (in terms of levels and growths) and the maximum admissible forecast horizon.

2. Risk assessment problem

Suppose that the non-systematic inflation, π_t, that is a difference between the headline and core inflations at time *t*, *t* = 0, 1,…,*T*, is described by a simple stationary bilinear autoregressive *BL*(1,0,1,1) process:

$$\pi_t = \alpha\pi_{t-1} + \beta\pi_{t-1}u_{t-1} + u_t \quad , \tag{1}$$

where α and β are the parameters and $\{u_t\}$ is a sequence of *i.i.d.* random variables with zero expected value (both unconditional and conditional on past information) and finite higher moments. The rationale for the existence of the bilinear term in (1) can be grounded, for instance, within the theory of speculative inflation (see e.g. Schmitt-Grohé, 2004, Sims, 2004 for economies with inflationary targeting) and within the modern hyperinflation theories (see Vázquez, 1998, Jha *et al*., 2002, Adam *et al.*, 2006 and Arce, 2009).

Let us consider forecasting from (1) outside *T*, initially assuming the knowledge of α but not β. In this case the forecasting scheme is analogous to that from a linear *AR*(1) model, that is:

$$\pi^f_{T+\tau} = \alpha\pi^f_{T+\tau-1} = \alpha^\tau\pi_T \text{ , for } \tau = 1, 2, 3, \ldots. \tag{2}$$

The absence of information regarding β leads in the above forecasting scheme causes a distortion and creates a forecasting risk. Let us define such risk as the mean-square error (*MSE*) of the forecast, that is:

$$MSE(\tau) = E(\pi_{T+\tau} - \pi^f_{T+\tau})^2 \quad .$$

Theorem 1 (see Appendix A) gives the asymptotic expansion of *MSE*(τ) in terms of model parameters. In order to evaluate a possible impact of the bilinear distortion on *MSE*(τ), let us define the *guaranteed upper risk* of forecast as the maximum admissible mean square forecast error for a given set of the bilinear parameters, that is (see Kharin, 1996):

$$MSE_+(\tau) = \sup_{\beta\in[-\beta_+,\beta_+]} MSE(\tau) \, .$$

Let us also define the forecast instability coefficient $\kappa(\tau)$ as:

$$\kappa(\tau) = \frac{MSE_+(\tau) - MSE_0(\tau)}{MSE_0(\tau)} \quad ,$$

where $MSE_0(\tau) = \dfrac{\sigma^2(1-\alpha^{2\tau})}{1-\alpha^2}$

is the minimally admissible risk value for the situation without bilinear distortions, and σ^2 is variance of u_t. Following Kharin (1996), we can define the δ-admissible distortion level $\beta^+(\delta,\tau)$ as the maximal distortion level β_+ for which the instability coefficient $\kappa(\tau)$ does not exceed *a priori* given admissible risk level δ. It can be shown (see Theorem 2 in the Appendix A) that, under additional assumption of normality for u_t, the following asymptotic expansions are true:

$$MSE_+(\tau) = \frac{\sigma^2(1-\alpha^{2\tau})}{1-\alpha^2} + \beta_+^2\sigma^4\Upsilon + o(\beta_+^2) \quad , \tag{3}$$

$$\kappa(\tau) = \beta_+^2\sigma^2\Upsilon + o(\beta_+^2) \quad , \tag{4}$$

$$\beta_+(\delta,\tau) = \frac{\sqrt{\delta}}{\sigma\sqrt{\Upsilon}} + o(\beta_+^2) \quad , \tag{5}$$

where: $\Upsilon = 2\dfrac{1-\alpha^\tau}{(1-\alpha)^2} + \dfrac{1-\alpha^{2\tau}}{(1-\alpha^2)^2} - 2\dfrac{\alpha^{2\tau-1}}{(1-\alpha)}$.

With the use of the formula above one might evaluate the potential distortion to the means square error of forecast due to omitted bilinearity. Figures 1a,b and c show the results of a numerical evaluation of $MSE_+(\tau)$, $\kappa(\tau)$ and $\beta^+(\delta,\tau)$ for values of α varying from -0.99 to 0.99, τ = 1,2,3 and 5, $\sigma^2 = 1$ and δ = 1.

Figures 1a-c suggest that nonlinear and asymmetric responses of the guaranteed forecast risk and instability coefficients might cause practical problems in establishing the admissible risk level δ. The fact that for large α's (typical for inflationary processes), the $MSE_+(\tau)$ rapidly approaching infinity makes it particularly cumbersome.

This is illustrated by relating the admissible risk level δ to a range of $AR(1)$ α coefficients corresponding to a certain level of the nonstationarity which is defined as:

$$\Phi = \alpha^2 + \sigma^2\beta_+^2(\tau) \quad . \tag{6}$$

Figure 1a: Dependence of $MSE_{+}(\tau)$ on autoregressive parameter and forecast horizon

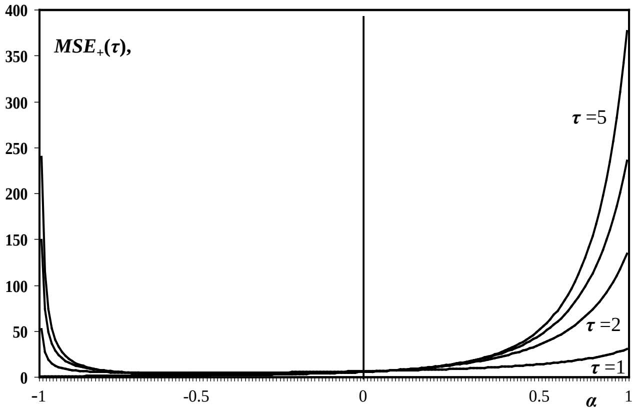


Figure 1a: Dependence of $\kappa(\tau)$ on autoregressive parameter and forecast horizon

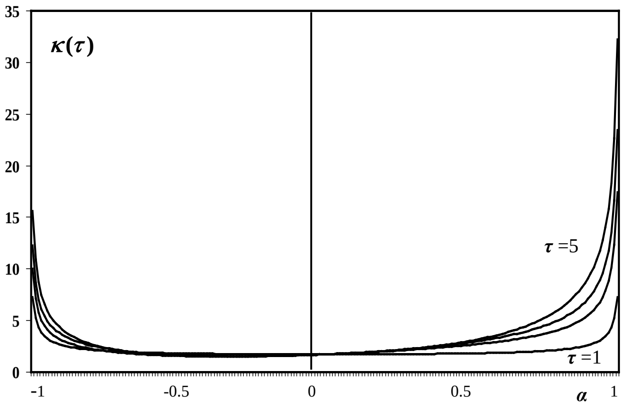
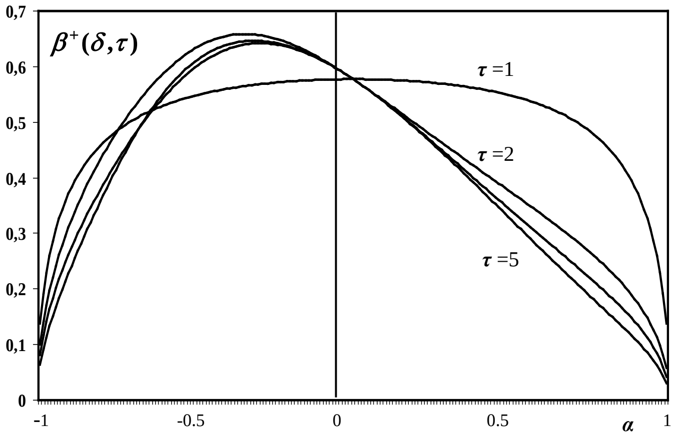


Figure 1c: Dependence of $\beta^{+}(\delta,\tau)$ on autoregressive parameter and forecast horizon



If Φ = 1, (6) constitutes the stationarity limit for (1) (see e.g. Granger and Anderson, 1978). For $0 \le \Phi < 1$ it is a general measure of time-dependence (predictability) of a stationary process (1) so that Φ = 0 refers to a purely random unpredictable process (white noise). For a given Φ, σ^2, τ and α, values of δ can be solved out from (5) and plotted against α. Figure 2 shows δ as a function of $\alpha \in [-\Phi, \Phi]$ for Φ equal respectively to 0.25, 0.75 and 0.95, $\sigma^2 = 1$ and τ = 2, where $\beta_+^2(\tau)=(\Phi-\alpha^2)/\sigma^2$.

Figure 2: Dependence of δ on the degree of predictability

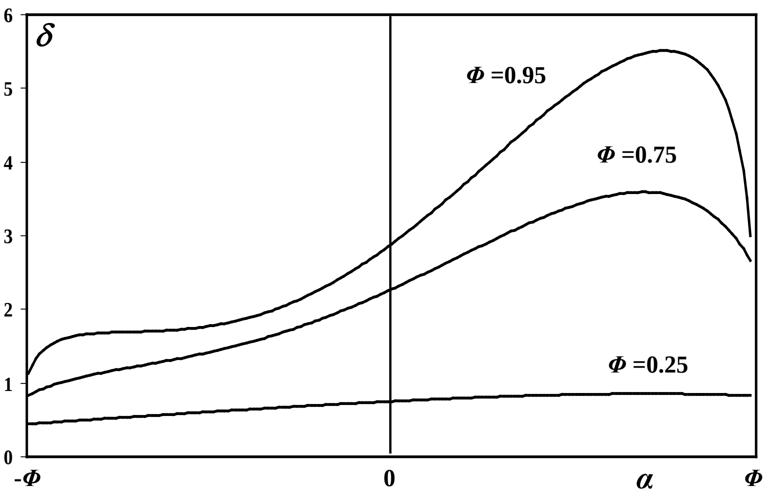


Figure 2 shows that the increase in the admissible risk for a given predictability is not linear and not even monotonous if regarded as function of the degree of predictability. For large predictability and large α (in excess of 0.8) the level of admissible risk falls. So that, establishing the appropriate admissible risk level in inflation forecasting might be difficult.

3. Econometric problem

The problem with establishing the admissible risk level, outlined in section 2, might be to some extent relaxed if it is possible to estimate the parameters of (1) econometrically. Let us assume that there exists statistical data on inflation for the period t = 0, 1,…,T, and, prior to forecasting for the periods $T + \tau$, it is possible to estimate the parameters α and β. Denoting these estimates respectively by $\hat{\alpha}$ and $\hat{\beta}$ and using some initial values π_0 and u_0, it is possible to obtain the estimates onf u_t recursively as:

$$\hat{u}_t = \pi_t - \hat{\alpha}\pi_{t-1} - \hat{\beta}\pi_{t-1}\hat{u}_{t-1} \quad .$$

This might help in constructing a one-step ahead forecast as:

$$\pi^f_{T+1} = \alpha\pi_T + \beta\pi_t u_T \quad ,$$

However, for forecast horizons longer than one, there is no possibility of recovering $u_{T+\tau}$, τ = 2, 3… . In this case forecast from the estimated equation (1) coincides with the forecast from a simple *AR*(1) model and is based upon information on a single parameter α, that is on:

$$\pi^f_{T+\tau} = \hat{\alpha}\pi^f_{T+\tau-1} = \alpha^\tau \pi_T + \alpha^{\tau-1}\hat{\beta}\pi_T \hat{u}_T \quad .$$

However, the econometric estimates can, to some extent, help with establishing the admissible risk level, which can, in turn, lead to establishing the maximum admissible forecast horizon (*MAF*), that is the maximum value of τ for which, given δ, the absence of $\hat{\beta}$ in the forecasting process does not lead to the increase of the expected $MSE(\tau)$ over the $MSE_+(\tau)$.

Let ξ_β be a well-defined statistic for β with the argument $\hat{\beta}$. In particular it can be the Student-*t* statistic for β, that is $(\hat{\beta}-\beta)/S(\hat{\beta})$, where $S(\hat{\beta})$ is the standard deviation of $\hat{\beta}$, or the normalised estimate of β, that is $\hat{\beta}\cdot S(u_t)$, where $S(u_t)$ is the estimated standard deviation of u_t. Denote by $\hat{\beta}^p_{\xi|\beta=0}$ such value of $\hat{\beta}$ which corresponds to the p^{th} fractile of the distribution of ξ_β for β=0. Knowing $\hat{\beta}^p_{\xi|\beta=0}$ and $\hat{\alpha}$, it is possible to find the *p*-maximum admissible forecast risk $\delta^p_{\hat{\beta}}(\tau)$ which can be obtained by solving (5) for δ with $\beta_+(\delta,\tau) = \hat{\beta}^p_{\xi|\beta=0}$ and $S(u_t)$. Since, in practice, a normalisation for a unitary variance of u_t is required, it can be achieved by using $\tilde{\beta}_+(\delta,\tau) = \hat{\beta}^p_{\xi|\beta=0}\cdot S(u_t)$ rather than $\beta_+(\delta,\tau)$. It is convenient to interpret the *p*-maximum admissible forecast risk $\delta^p_{\hat{\beta}}(\tau)$ as the risk which is associated with ignoring, in the forecasting scheme, the β parameter if it is equal to the unusually high (or low) estimate of β , in the case where the hypothesis that β = 0 is true. Whether the value of $\hat{\beta}$ is ‘unusually’ high (or low) is decided by using tail percentiles like 0.05 or 0.95.

The concept of *p*-maximum admissible forecast risk requires knowledge of the distribution of the statistic ξ_β, which is usually either the distribution of $\hat{\beta}$, or its Student-*t* ratio. If β is estimated by the maximum likelihood (ML) method, the asymptotic normality of the estimates allows for approximation of the normalised statistics by the standard normal distribution. However, the behaviour of the statistics in finite samples depends on the speed of convergence.

In order to investigate the finite sample properties of the statistics, the following Monte Carlo experiments have been performed. The data generating process is (1) with β = 0 and $u_t \sim i.i.d\ N(0,1)$, t = 0, 1,…,T, which reduces it to a simple AR(1) process with a random initial value. The parameter α varies as 0.25, 0.5 and 0.75, T varies as 75, 100 and 250 and, for each sets of parameters and each T, 10,000 replications are generated. In each replication the parameters α and β are estimated by the constrained maximum likelihood method used for the Kalman Filter representation of (1), where the constraint is the stationarity condition.[1]

Table 1 shows the Bera-Jarque measures of normality for the empirical distributions of the estimates of β and their Student-t statistics, $t(\hat{\beta})$ with p-values in the parentheses. It indicates that the convergence to normality is relatively slow here. This prompts the question whether the percentiles of the standard normal distribution can be used as the critical values for the t-ratios of the estimated β parameters. Table 2 shows the empirical percentiles of the simulated distributions of the t-ratios for the ML β estimates in comparison with the percentiles of the standard normal distribution, which is the asymptotic distribution for the ML estimates.

Results in Table 2 suggest that, although the finite sample distributions of the Student-t ratios are not normal and the tails of the distributions are heavy, especially for the large values of α and small samples, the differences are not very substantial. With some caution, percentiles of normal distribution can be used here for testing the significance of the estimates of β.

Figures 3a-3c show the computed values of $\delta_{\hat{\beta}}^{0.95}(\tau)$ obtained by solving (5) for δ, that is:

$$\delta_{\hat{\beta}}^{p}(\tau) = \left(\hat{\beta}_{\xi|\beta=0}^{p}\right)^2 \sigma^2 \Upsilon \quad ,$$

where $\hat{\beta}_{\xi|\beta=0}^{p}$ has been selected alternatively by three criteria: percentiles of $\hat{\beta}$ (Figure 3a), percentiles of $t(\hat{\beta})$ (Figure 3b) and percentiles of normalised $\hat{\beta}$, that is $\tilde{\beta} = \hat{\beta} \cdot S(u_t)$, where $S(u_t)$ is the estimated standard deviation of u_t. (Figure 3c). These are compared with their sample estimates $\hat{\delta}_{\hat{\beta}}^{p}(\tau)$, that is:

[1] Computations were performed in Aptech GAUSS using the constrained maximum likelihood package (CML) and Roncalli (1995) Kalman Filter routines.

$$\hat{\delta}^{p}_{\hat{\beta}}(\tau) = \left(\hat{\beta}^{p}_{\xi|\beta=0}\right)^{2} S^{2}(u)\hat{\Upsilon} \quad , \tag{7}$$

where $\hat{\Upsilon}$ is computed as Υ in (3)-(5), except that the estimates $\hat{\alpha}$ are used here rather than α.

Table 1: Bera-Jarque statistics for the ML estimates of $\boldsymbol{\beta}$ and their t ratios

T	α = 0.25	α = 0.50	α = 0.75
		for $\hat{\beta}$	
75	326.6 (0.00)	777.6 (0.00)	341.8 (0.00)
100	71.12 (0.00)	86.42 (0.00)	127.3 (0.00)
250	1.64 (0.44)	1.76 (0.41)	3.00 (0.22)
500	2.20 (0.33)	1.03 (0.60)	0.25 (0.88)
		for $t(\hat{\beta})$	
75	61.32 (0.00)	7843 (0.00)	839900 (0.00)
100	8.25 (0.02)	40.26 (0.00)	1317 (0.00)
250	0.57 (0.75)	0.86 (0.65)	0.90 (0.64)
500	3.36 (0.18)	1.91 (0.36)	0.56 (0.75)

Table 2: Simulated percentiles of $t(\hat{\beta})$

				percentiles		
		99%	97.5%	95%	90%	50%
T=75	α=0.25	2.39	1.97	1.67	1.28	0.00
	α=0.50	2.46	2.01	1.68	1.29	0.00
	α=0.75	2.54	2.11	1.75	1.33	0.00
T=100	α=0.25	2.35	1.95	1.61	1.27	0.00
	α=0.50	2.35	1.98	1.65	1.27	0.00
	α=0.75	2.44	2.05	1.68	1.30	0.00
T=250	α=0.25	2.29	1.95	1.65	1.26	-0.02
	α=0.50	2.33	1.85	1.60	1.25	0.01
	α=0.75	2.39	1.89	1.61	1.25	-0.01
∞		**2.33**	**1.96**	**1.64**	**1.28**	**0.00**

True and estimated 5% maximum admissible forecast risk

T = 100, α = 0.5, 10,000 replications

Figure 3a:
criterion: percentiles of $\hat{\beta}$

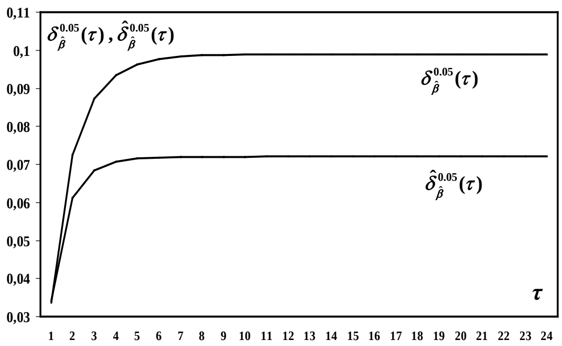


Figure 3b:
criterion: percentiles of $t(\hat{\beta})$

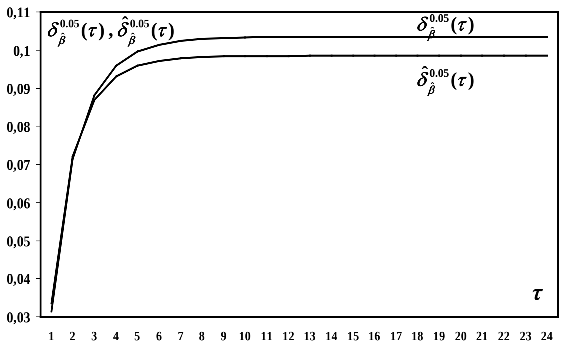
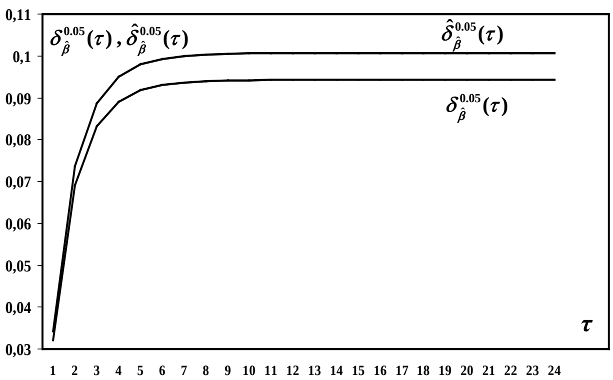


Figure 3c:
criterion: percentiles of $\tilde{\beta}$



Figures 3a-c indicate that, although the estimates of the 5% admissible forecast risk are biased (either negatively, as in Figure 3a, or positively, as in Figures 3b and 3c), its values stabilizes quickly with the increase of forecast horizon and, for the horizons greater than 9, they are virtually constant. Similar is observed for different sample sizes and different values of α. Generally, it appears that the criterion of selecting $\hat{\beta}^{p}_{\xi|\beta=0}$ according to the percentiles of $t(\hat{\beta})$ is most advisable, since the bias of the estimates is usually the smallest.

4. Risk assessment and forecast horizon for worldwide inflation

The concept of p-maximum admissible forecast risk can be applied in practice for assessing the rationale of forecasting of the non-systematic part of inflation and, in particular, evaluating the maximum forecast horizon for which the bilinear distortions do not cause the risk in excess of the admissible value. For the empirical analysis a panel of monthly time series of annual inflation rates (that is, on the basis of the corresponding month of the previous year) for a wide number of countries have been used. The data are taken from the *International Monetary Fund* database (see http://www.imfstatistics.org/imf, can be accessed e.g. through the ESDC database at http://esds80.mcc.ac.uk/wds_ifs/ReportFolders/reportFolders.aspx). Out of the data set for 170 countries, series for 122 countries have been selected with the maximum time coverage of the data set is from January 1957 to April 2011 (for most countries the series have been shorter). The series which were incomplete, with a substantial number of missing or systematically repeated observations, have been eliminated. For the remaining series, in a few obvious cases infrequent missing values have been interpolated and some evident typos in data corrected. From the original data the monthly series of annual (*y*/*y*) inflation have been computed which gives the maximum length of the series of 591 observations. Outliers greater than 5 standard deviations of the series have been truncated (there were very few of them). The systematic part of inflation has been eliminated by smoothing the data by the Hodrick-Prescott filter with the smoothing constant equal to 16,000. For each country the parameters of equation (1) have been estimated by the constrained ML Kalman Filter method (see Section 3).

Appendix B contains the results of the ML estimates of coefficients α and β for individual series. Tables B1 shows the estimation results. In columns (1)-(4), after the country codes and number of observations, the estimates of the $AR(1)$ coefficients, $\hat{\alpha}$, are given and followed by their t ratios. In column (5) the significance of the $AR(1)$ the $AR(1)$ coefficients which are significant at the 0.01 level are marked by (3) and those with p-values smaller than 0.01 by (0). Columns (6) –(9) describe the estimates of the bilinear coefficient; columns (6) and (7) give the non-normalised and normalised estimates correspondingly, column (8) shows the t-ratios for the non-normalised estimators and the last column (9) indicates the significance.

Table B2 present the forecast risk assessment characteristics. Column (3) gives the stationarity measures computed as: $\hat{\Phi} = \hat{\alpha}^2 + \hat{\beta}^2 S^2(u_t)$. Column (4) presents the $\hat{\beta}^{0.90}_{\xi|\beta=0}$ coefficients computed as in (7), with the selection criteria being the 90th percentile of the $t(\hat{\beta})$ statistic. The corresponding $\hat{\delta}^{0.90}_{\hat{\beta}}(\tau^*)$ values, where $\tau^* = 24$ and represents the most remote forecast horizon, for which the values of $\delta^{0.90}_{\hat{\beta}}(\tau)$ are virtually independent from τ, are shown in column (5). These values are halved, in order to allow for the symmetry of positive and negative bilinearity. Column (6) shows the estimates of the maximum admissible forecast horizon for which the effect of bilinearity does not exceed the maximal admissible distortion level computed at risk equal to $\hat{\delta}_{\hat{\beta}}^{0.90}(\tau^*)$. More precisely, $\tau_{\max}$ is defined as such forecast horizon τ for which $\tilde{\beta} \approx \beta_+[\hat{\delta}_{\hat{\beta}}^{0.90}(\tau^*), \tau]$.

In order to assess the poolability of the panel and to decide whether particular series in the panel can be analysed separately, a simple correlation analysis between the pairs of ML residuals of the estimated equations (1) have been performed. For 7,381 correlations the percentage of significant correlations at 5% equal to 8.89%. Although this is more than the expected 5%, nevertheless this percentage is not very high, so that the possible distortions to the estimates for the individual countries due to interdependence within the panel are likely not substantial. The estimated bilinear coefficients are, in most cases, insignificant; there are only 24 significant (at the 5% level of significance) bilinear coefficients.

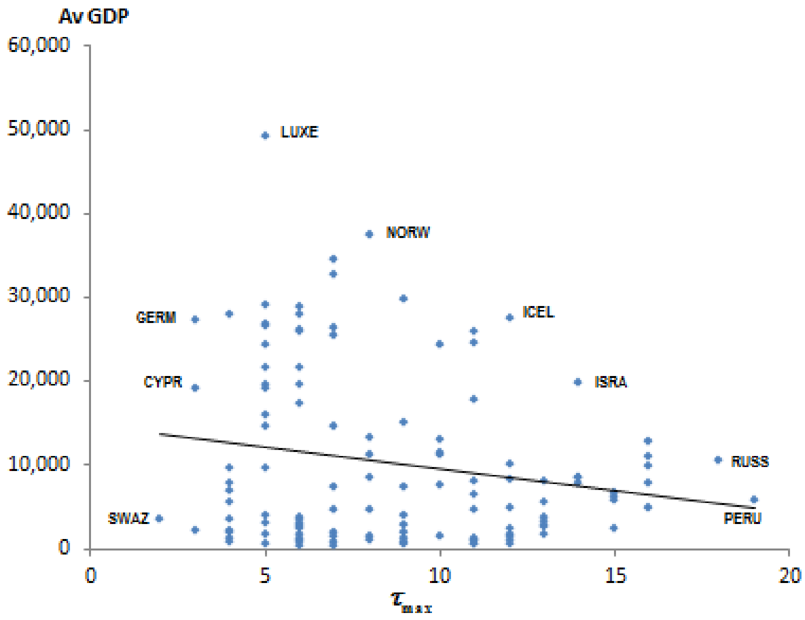
The distribution of countries according to the maximum admissible forecast horizon is given in Table 3.

Table 3: distribution of $\tau_{\max}$ for non-systematic inflation

$\tau_{\max}$	**No. of countries**
smaller than 6	29
between 6 and 9	45
Between 10 and 14	36
Greater than 14	12

There is an interesting regularity between the World Bank estimates of the annual GDP level per capita adjusted for purchasing power disparities measured at constant 2005 international dollars (see http://esds80.mcc.ac.uk/WDS_WB/) and $\tau_{\max}$. Figure 4 shows a scatter diagram of the average GDP per capita and $\tau_{\max}$. The periods for which means of the GDP have been computed correspond to the periods used for computing $\tau_{\max}$. Some visible outliers on the diagrams have been marked by country symbols. There is also a linear regression line presented at this figure.

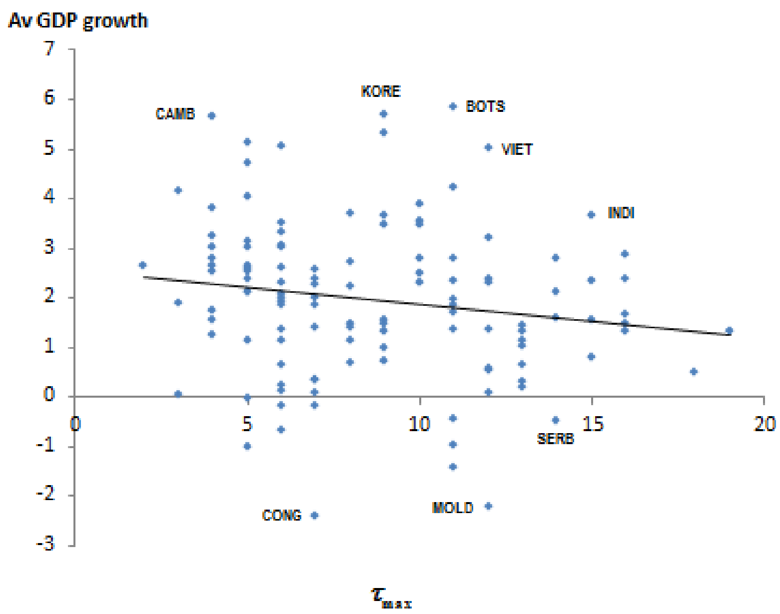
Figure 4: Average levels of GDP and the maximum admissible forecast horizons



There is a visible, albeit not very strong, negative relationship between the maximum admissible forecast horizon and the average GDP level. The correlation coefficient is equal to -0.194, with Student-t ratio equal to 2.143 and a one-sided *p*-value 0.01606. The triangular shape of the scatter points suggests a nonlinearity of the dependence pattern. Out of 12 countries with $\tau_{\max} < 5$, 9 have average per capita GDP level below the level of 10,000 International \$. Similarly, out of 19 countries with $\tau_{\max} > 12$, 8 has the average per capita GDP below the 10,000 international \$. For the countries with $\tau_{\max}$ between 5 and 12, the proportion of richer countries is greater. If there is a relation between the level of development of a country measured by its GDP per capita and the maximum admissible forecast horizon it can be stated that the developed countries have usually the linearly forecastable inflation with the moderate forecast horizons, while the poorer countries usually have inflation linearly forecastable for either very short, or very long periods.

The concept of the maximum admissible forecast horizon might also add to the empirical evidence of GDP convergence. Figure 5 depicts the relationship between $\tau_{\max}$ and the average rate of growth of the 122 countries analysed here. The data for growth have been obtained from the World Bank sources at http://esds80.mcc.ac.uk/WDS_WB/.

Figure 4: Average levels of GDP and the maximum admissible forecast horizons



There is a significant negative correlation between $\tau_{\max}$ and the average GDP growth (the correlation coefficient is equal to -0.1648, with Student t-ratio equal to 1.820 and one-sided *p*-value 0.03438). Detailed interpretation is beyond the scope of this paper, but it seems possible that it may contribute to further discussion on the empirical evidence for convergence in growth.

5. Concluding remarks

The paper presents a relatively simple method of assessing the maximal admissible forecast horizon for non-systematic inflation when an autoregressive forecasting model is used. The empirical results indicate the plausibility of the method which might be implemented in practice by monetary policy authorities and forecasting institutions. It can also be used as an auxiliary tool for evaluation the rationale of inflation smoothing and for assessing the quality of linear autoregressive forecasting models. However, the bilinear model used here is relatively simple and its extension (for instance, by allowing for more complicated lags structure) is likely to increase the practical relevance of the method proposed.

Appendix A

Proofs of Theorems 1 and 2 and corollaries

Lemma. If the time series π_t satisfies the bilinear model (1), $\alpha^2+\beta^2\sigma^2<1$, $\tau\in N$, $\beta\to 0$, then the following asymptotic expansions for the second order moments hold:

$$E\{\pi_t^2\}=\sigma^2\frac{1}{1-\alpha^2}+2\beta\mu_3\frac{\alpha}{1-\alpha^2}+\beta^2\sigma^4\left(\frac{\alpha^2}{(1-\alpha^2)^2}+\frac{4\alpha}{(1-\alpha)(1-\alpha^2)}\right)+\beta^2\mu_4\frac{1}{1-\alpha^2}+o(\beta^2),$$

$$E\{\pi_T\pi_{T+\tau}\}=\sigma^2\frac{\alpha^\tau}{1-\alpha^2}+\beta\mu_3\alpha^{\tau-1}\frac{1+\alpha^2}{1-\alpha^2}+\beta^2\sigma^4\left(\frac{3\alpha^{\tau+2}+\alpha^{\tau+1}+\alpha^{\tau-1}}{(1-\alpha^2)^2}+\frac{1}{(1-\alpha)^2}\right)+\beta^2\mu_4\frac{\alpha^\tau}{1-\alpha^2}+o(\beta^2).$$

Proof. 1) Using the decomposition of (1), analogous to the moving average decomposition of the A*R*(1) process, that is:

$$\pi_t=u_t+\sum_{i=1}^{\infty}u_{t-i}\prod_{k=1}^{i}(\alpha+\beta u_{t-k})$$

and applying the assumption of independence of u_t and u_{t-i} at $i\geq 1$, we have:

$$E\{\pi_t^2\}=\sigma^2+E\left\{\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}u_{t-i}u_{t-j}\prod_{k=1}^{i}(\alpha+\beta u_{t-k})\prod_{m=1}^{j}(\alpha+\beta u_{t-m})\right\}=\sigma^2+E\left\{\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}u_{t-i}u_{t-j}\left(\alpha^{i+j}\right.\right.$$
$$+\beta\alpha^{i+j-1}\sum_{m=1}^{j}u_{t-m}+\beta\alpha^{j+i-1}\sum_{k=1}^{i}u_{t-k}+\beta^2\alpha^{i+j-2}\sum_{m=1}^{j-1}\sum_{p=m+1}^{j}u_{t-m}u_{t-p}+\beta^2\alpha^{j+i-2}\sum_{k=1}^{i-1}\sum_{l=k+1}^{i}u_{t-k}u_{t-l}$$
$$\left.\left.+\beta^2\alpha^{i-1+j-1}\sum_{k=1}^{i}\sum_{m=1}^{j}u_{t-k}u_{t-m}+\alpha^{i+j-3}o(\beta^2)\right)\right\}.$$

Considering that $E\{u_t\}=0$, $Var\{u_t\}=\sigma^2<+\infty$, and u_{t-i} are independent at $i\neq j$, and using the fact that $E\{u_{t_1}u_{t_2}u_{t_3}u_{t_4}\}\neq 0$ only for the situations where either $t_1=t_2=t_3=t_4$ or where these four indices are pairwise equal), we get:

$$E\{\pi_t^2\}=\sigma^2+E\{\sum_{i=1}^{\infty}u_{t-i}^2\alpha^{2i}\}+2\beta E\{\sum_{i=1}^{\infty}u_{t-i}^3\alpha^{2i-1}\}+\beta^2E\{\sum_{j=2}^{\infty}\sum_{i=1}^{j-1}\alpha^{i+j-2}u_{t-i}^2u_{t-j}^2\}$$
$$+\beta^2E\{\sum_{i=2}^{\infty}\sum_{j=1}^{i-1}\alpha^{i+j-2}u_{t-i}^2u_{t-j}^2\}+\beta^2E\{\sum_{i=2}^{\infty}\sum_{k=1}^{i-1}\alpha^{2i-2}u_{t-i}^2u_{t-k}^2+\sum_{i=1}^{\infty}\sum_{j=1,j\neq i}^{\infty}\alpha^{i+j-2}u_{t-i}^2u_{t-j}^2+\sum_{i=1}^{\infty}\alpha^{2i-2}u_{t-i}^4\}$$
$$+\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\alpha^{i+j-3}o(\beta^2)=\sigma^2+\sigma^2\sum_{i=1}^{\infty}\alpha^{2i}+2\beta\mu_3\sum_{i=1}^{\infty}\alpha^{2i-1}+2\beta^2\sigma^4\sum_{j=2}^{\infty}\sum_{i=1}^{j-1}\alpha^{i+j-2}$$
$$+\beta^2\sigma^4\sum_{i=2}^{\infty}(i-1)\alpha^{2i-2}+\beta^2\sigma^4\sum_{i=1}^{\infty}\sum_{j=1,j\neq i}^{\infty}\alpha^{i+j-2}+\beta^2\mu_4\sum_{i=1}^{\infty}\alpha^{2i-2}+o(\beta^2)=\sigma^2+\sigma^2\frac{\alpha^2}{1-\alpha^2}$$
$$+2\beta\mu_3\frac{\alpha}{1-\alpha^2}+2\beta^2\sigma^4\frac{\alpha}{(1-\alpha)(1-\alpha^2)}+\beta^2\sigma^4+\beta^2\sigma^4\left(-\frac{1}{1-\alpha^2}\right)+\beta^2\mu_4\frac{1}{1-\alpha^2}+o(\beta^2)$$
$$=\sigma^2\frac{1}{1-\alpha^2}+2\beta\mu_3\frac{\alpha}{1-\alpha^2}+\beta^2\sigma^4\left(\frac{\alpha^2}{(1-\alpha^2)^2}+\frac{4\alpha}{(1-\alpha)(1-\alpha^2)}\right)+\beta^2\mu_4\frac{1}{1-\alpha^2}+o(\beta^2)$$

2) From (5), as $\tau\geq 1$, we have:

$$E\{\pi_T\pi_{T+\tau}\}=E\left\{u_T\sum_{j=1}^{\infty}u_{T+\tau-j}\prod_{m=1}^{j}(\alpha+\beta u_{T+\tau-m})\right\}+E\left\{\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}u_{T-i}u_{T+\tau-j}\prod_{k=1}^{i}(\alpha+\beta u_{T-k})\prod_{m=1}^{j}(\alpha+\beta u_{T+\tau-m})\right\}$$

Using independence of $\{u_t\}$ and selecting nonlinear elements in the first summand, we find:

$$E\{u_T\sum_{j=1}^{\infty}u_{T+\tau-j}\prod_{m=1}^{j}(\alpha+\beta u_{T+\tau-m})\}=$$

$$=E\left\{u_T\sum_{j=1}^{\infty}u_{T+\tau-j}(\alpha^j+\alpha^{j-1}\beta\sum_{m=1}^{j}u_{T+\tau-m})+\alpha^{j-3}o(\beta^2)\right\}+u_T\sum_{j=2}^{\infty}u_{T+\tau-j}\alpha^{j-2}\beta^2\sum_{m=1}^{j-1}\sum_{p=m+1}^{j}u_{T+\tau-m}u_{T+\tau-p}$$

$$=\sigma^2\alpha^\tau+\alpha^{\tau-1}\beta\mu_3+\beta^2E\left\{u_T^2\sum\nolimits_{j=\tau+1}^{\infty}u_{T+\tau-j}^2\alpha^{j-2}\right\}+o(\beta^2)=\sigma^2\alpha^\tau+\alpha^{\tau-1}\beta\mu_3$$

$$+\beta^2\sigma^4\alpha^{\tau-1}(1-\alpha)^{-1}+o(\beta^2)\,.$$

Selecting nonlinear elements in the second summand, we get:

$$E\left\{\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}u_{T-i}u_{T+\tau-j}\prod_{k=1}^{i}(\alpha+\beta u_{T-k})\prod_{m=1}^{j}(\alpha+\beta u_{T+\tau-m})\right\}=E\left\{\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}u_{T-i}u_{T+\tau-j}(\alpha^{i+j}+\beta\alpha^{i+j-1}\right.$$

$$\times\sum_{m=1}^{j}u_{T+\tau-m}+\beta\alpha^{j+i-1}\sum_{k=1}^{i}u_{T-k}+\beta^2\alpha^{i+j-2}\sum_{m=1}^{j-1}\sum_{p=m+1}^{j}u_{T+\tau-m}u_{T+\tau-p}+\beta^2\alpha^{j+i-2}\sum_{k=1}^{i-1}\sum_{l=k+1}^{i}u_{T-k}u_{T-l}$$

$$\left.+\alpha^{i+j-3}o(\beta^2)+\beta^2\alpha^{i+j-2}\sum_{k=1}^{i}\sum_{m=1}^{j}u_{T-k}u_{T+\tau-m})\right\}=\sigma^2\sum\nolimits_{i=1}^{\infty}\alpha^{2i+\tau}+2\beta\mu_3\sum\nolimits_{i=1}^{\infty}\alpha^{2i+\tau-1}+o(\beta^2)$$

$$+\beta^2\sigma^4\sum\nolimits_{i=1}^{\infty}\sum\nolimits_{j=i+\tau+1}^{\infty}\alpha^{i+j-2}+\beta^2\sigma^4\sum\nolimits_{i=2}^{\infty}\sum\nolimits_{j=\tau+1}^{i+\tau-1}\alpha^{i+j-2}+\beta^2E\left\{\sum\nolimits_{i=2}^{\infty}\sum\nolimits_{k=1}^{i-1}u_{T-i}^2u_{T-k}^2\alpha^{2i+\tau-2}\right.$$

$$\left.+\sum\nolimits_{i=1}^{\infty}\sum\nolimits_{j=1,j\neq i+\tau}^{\infty}u_{T-i}^2u_{T+\tau-j}^2\alpha^{i+j-2}+\sum\nolimits_{i=1}^{\infty}u_{T-i}^4\alpha^{2i+\tau-2}\right\}=\sigma^2\frac{\alpha^{2+\tau}}{1-\alpha^2}+\beta\mu_3+\beta^2\sigma^4\frac{\alpha^{2+\tau}}{(1-\alpha^2)^2}$$

$$+\beta^2\sigma^4\frac{2\alpha^{1+\tau}}{(1-\alpha)(1-\alpha^2)}+\beta^2\sigma^4\left(\frac{1}{(1-\alpha)^2}-\frac{\alpha^\tau}{1-\alpha^2}\right)+\beta^2\mu_4\frac{\alpha^\tau}{1-\alpha^2}+o(\beta^2)\,.$$

Then:

$$E\{\pi_T\pi_{T+\tau}\}=\sigma^2\alpha^\tau(1+\frac{\alpha^2}{1-\alpha^2})+\alpha^{\tau-1}\beta\mu_3\frac{1+\alpha^2}{1-\alpha^2}+\beta^2\sigma^4\alpha^{\tau-1}\left(\frac{1}{1-\alpha}+\frac{2\alpha^2}{(1-\alpha)(1-\alpha^2)}+\frac{\alpha^3}{(1-\alpha^2)^2}-\frac{\alpha}{1-\alpha^2}\right)$$

$$+\beta^2\sigma^4\frac{1}{(1-\alpha)^2}+\beta^2\mu_4\frac{\alpha^\tau}{1-\alpha^2}+o(\beta^2)$$

$$=\sigma^2\frac{\alpha^\tau}{1-\alpha^2}+\beta\mu_3\alpha^{\tau-1}\frac{1+\alpha^2}{1-\alpha^2}+\beta^2\sigma^4\left(\frac{3\alpha^{\tau+2}+\alpha^{\tau+1}+\alpha^{\tau-1}}{(1-\alpha^2)^2}+\frac{1}{(1-\alpha)^2}\right)+\beta^2\mu_4\frac{\alpha^\tau}{1-\alpha^2}+o(\beta^2)\,.\ \blacksquare$$

Theorem 1. If the time series π_t satisfies the bilinear model (1) with $\beta\to 0$, $\alpha^2+\beta^2\sigma^2<1$, $\tau\in N$, and the forecasting procedure (2) is used, then the mean square risk satisfies the asymptotic expansion:

$$MSE(\tau)=\sigma^2\frac{1-\alpha^{2\tau}}{1-\alpha^2}+2\beta\mu_3\frac{\alpha(1-\alpha^{2(\tau-1)})}{1-\alpha^2}$$
$$+\beta^2\sigma^4\left(\frac{4\alpha+5\alpha^2-\alpha^{2\tau+2}}{(1-\alpha^2)^2}-2\frac{\alpha^{2\tau-1}}{1-\alpha^2}-2\frac{\alpha^\tau}{(1-\alpha)^2}\right)+\beta^2\mu_4\frac{1-\alpha^{2\tau}}{1-\alpha^2}+o(\beta^2). \quad \text{(A1)}$$

Proof. Using (2), we have $MSE(\tau)=\alpha^{2\tau}E\{x_T^2\}-2\alpha^\tau E\{x_Tx_{T+\tau}\}+E\{x_{T+\tau}^2\}$.
By Lemma we get:

$$MSE(\tau)=\sigma^2\frac{\alpha^{2\tau}}{1-\alpha^2}+\beta\mu_3\frac{2\alpha^{2\tau+1}}{1-\alpha^2}+\beta^2\sigma^4\left(\frac{\alpha^{2\tau+2}}{(1-\alpha^2)^2}+\frac{4\alpha^{2\tau+1}}{(1-\alpha)(1-\alpha^2)}\right)+\beta^2\mu_4\frac{\alpha^{2\tau}}{1-\alpha^2}-2\left(\sigma^2\frac{\alpha^{2\tau}}{1-\alpha^2}\right.$$

$$\left.+\beta\mu_3\alpha^{2\tau-1}\frac{1+\alpha^2}{1-\alpha^2}+\beta^2\sigma^4\left(\frac{3\alpha^{2\tau+2}+\alpha^{2\tau+1}+\alpha^{2\tau-1}}{(1-\alpha^2)^2}+\frac{\alpha^\tau}{(1-\alpha)^2}\right)+\beta^2\mu_4\right)+\sigma^2\frac{1}{1-\alpha^2}+2\beta\mu_3\frac{\alpha}{1-\alpha^2}$$

$$+\beta^2\sigma^4\left(\frac{\alpha^2}{(1-\alpha^2)^2}+\frac{4\alpha}{(1-\alpha)(1-\alpha^2)}\right)+\beta^2\mu_4(1-\alpha^2)^{-1}+o(\beta^2)=\sigma^2\frac{1-\alpha^{2\tau}}{1-\alpha^2}+\beta\mu_3\frac{2\alpha\left(1-\alpha^{2(\tau-1)}\right)}{1-\alpha^2}$$

$$+\beta^2\sigma^4\left(\frac{4\alpha+5\alpha^2-2\alpha^{2\tau-1}+2\alpha^{2\tau+1}-\alpha^{2\tau+2}}{(1-\alpha^2)^2}-\frac{2\alpha^\tau}{(1-\alpha)^2}\right)+\beta^2\mu_4\frac{1-\alpha^{2\tau}}{1-\alpha^2}+o(\beta^2)\,.\ \blacksquare$$

Corollary 1. If the random errors $\{u_t\}$ in (1) have the Gaussian probability distribution $N_1(0,\sigma^2)$, then:

$$MSE(\tau)=\sigma^2\frac{1-\alpha^{2\tau}}{1-\alpha^2}+\beta^2\sigma^4\Big(2\frac{1-\alpha^\tau}{(1-\alpha)^2}+\frac{1-\alpha^{2\tau}}{(1-\alpha^2)^2}-\frac{2\alpha^{2\tau-1}}{1-\alpha}\Big)+o(\beta^2). \qquad \text{(A2)}$$

Proof. For the Gaussian probability distribution $N_1(0,\sigma^2)$ we have $\mu_3=0$, $\mu_4=3\sigma^4$. Then (A2) follows from (A1). ■

Note, that the risk functional in (A2), (A1) has an additive form: the first summand is the risk value for the non-distorted model ($\beta=0$), i.e. for the autoregression model; the second term proportional to β^2 is generated by the bilinear distortion.

Corollary 2. Under Theorem 1 the condition at $\tau=1$ is:

$$MSE(\tau)=\sigma^2+\mu_3\frac{2\alpha}{1-\alpha^2}\beta+\Big(\sigma^4\frac{\alpha^2}{1-\alpha^2}+\mu_4\Big)\beta^2+o(\beta^2).$$

Theorem 2. If the time series π_t satisfies the bilinear model (1), $\beta\in[-\beta_+,\beta_+]$, $\beta_+\to 0$, $\alpha^2+\beta_+^2\sigma^2<1$, $\tau\in N$, random errors $\{u_t\}$ have the Gaussian probability distribution $N_1(0,\sigma^2)$, and the forecasting procedure (2) is used, then the guaranteed upper risk, the instability coefficient and the δ-admissible distortion level satisfy the asymptotic expansions:

$$MSE_+(\tau)=\sigma^2\frac{1-\alpha^{2\tau}}{1-\alpha^2}+\beta_+^2\sigma^4\Big(2\frac{1-\alpha^\tau}{(1-\alpha)^2}+\frac{1-\alpha^{2\tau}}{(1-\alpha^2)^2}-\frac{2\alpha^{2\tau-1}}{1-\alpha}\Big)+o(\beta_+^2),$$

$$\kappa(\tau)=\beta_+^2\sigma^2\Big(2\frac{1-\alpha^\tau}{(1-\alpha)^2}+\frac{1-\alpha^{2\tau}}{(1-\alpha^2)^2}-\frac{2\alpha^{2\tau-1}}{1-\alpha}\Big)+o(\beta_+^2), \qquad \text{(A3)}$$

$$\beta^+(\delta,\tau)=\delta^{\frac{1}{2}}\sigma^{-1}\Big(2\frac{1-\alpha^\tau}{(1-\alpha)^2}+\frac{1-\alpha^{2\tau}}{(1-\alpha^2)^2}-\frac{2\alpha^{2\tau-1}}{1-\alpha}\Big)^{-\frac{1}{2}}+o(\beta_+^2).$$

Proof. 1. The coefficient at $\beta^2\sigma^4$ in (A2) equals to: $K_{\beta^2}=2\frac{1-\alpha^\tau}{(1-\alpha)^2}+\frac{1-\alpha^{2\tau}}{(1-\alpha^2)^2}-2\frac{\alpha^{2\tau-1}}{1-\alpha}$.

It can be shown that this coefficient is positive: if $\alpha = 0$, then $K_{\beta^2} = 3$; if $-1 < \alpha < 0$, then for $\tau \in N$ we have $\frac{1-\alpha^{\tau}}{(1-\alpha)^2} > 0$, $\frac{1-\alpha^{2\tau}}{(1-\alpha^2)^2} > 0$, $\frac{\alpha^{2\tau-1}}{1-\alpha} < 0$. This is why $K_{\beta^2} > 0$; if $0 < \alpha < 1$, then for $\tau \in N$ we have: $\frac{1-\alpha^{\tau}}{(1-\alpha)^2} - \frac{\alpha^{2\tau-1}}{1-\alpha} = \frac{(1-\alpha^{\tau-1}) + \alpha^{\tau-1}(1-\alpha^{\tau})(1-\alpha)}{(1-\alpha)^2} > 0$, therefore $K_{\beta^2} > 0$.

2. From (A2) and the definition of $MSE_+(\tau)$ it can be shown that:

$$MSE_+(\tau) = \sigma^2 \frac{1-\alpha^{2\tau}}{1-\alpha^2} + \max_{\beta \in [-\beta_+, \beta_+]}(K_{\beta^2}\beta^2\sigma^4 + o(\beta^2)) = \sigma^2 \frac{1-\alpha^{2\tau}}{1-\alpha^2} + K_{\beta^2}\beta_+^2\sigma^4 + o(\beta_+^2).$$

3. The second and the third expansions in (A3) follow from the expansion of the guaranteed risk.■

Corollary 3. Under Theorem 2 conditions at $\tau = 1$:

$$MSE_+(\tau) = \sigma^2 + \beta_+^2\sigma^4 \frac{3-2\alpha^2}{1-\alpha^2} + o(\beta_+^2), \kappa(\tau) = \beta_+^2\sigma^2 \frac{3-2\alpha^2}{1-\alpha^2} + o(\beta_+^2),$$

$$\beta^+(\delta, \tau) = \sigma^{-1}\sqrt{\delta \frac{1-\alpha^2}{3-2\alpha^2}} + o(\beta_+^2).$$

Appendix B

Table B1: ML Kalman Filter estimates

Country	No.obs.	$AR(1)$ coefficient			Bilinear coefficient			
		$\hat{\alpha}$	$t(\alpha)$	signif	$\hat{\beta}$	$\tilde{\beta}$	$t(\beta)$	signif
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ALBA	183	0.793	16.97	(3)	0.720	0.007	0.213	(0)
ARGE	591	0.951	72.07	(3)	-3.067	-0.012	-2.870	(3)
ARME	156	0.855	21.49	(3)	0.387	0.006	0.459	(0)
AUST	591	0.701	23.29	(3)	-33.362	-0.044	-3.131	(3)
BARB	483	0.753	24.89	(3)	-0.534	-0.002	-0.222	(0)
BELG	591	0.755	27.95	(3)	7.781	0.008	0.747	(0)
BENI	171	0.771	15.68	(3)	-0.279	-0.003	-0.095	(0)
BOLI	590	0.911	53.25	(3)	0.766	0.003	0.385	(0)
BOTS	370	0.884	29.22	(3)	-21.670	-0.039	-2.373	(3)
BRAZ	315	0.943	49.89	(3)	1.604	0.012	2.256	(3)
BULG	183	0.860	22.43	(3)	0.328	0.009	0.630	(0)
BURK	566	0.693	22.72	(3)	2.355	0.022	1.441	(0)
BURU	383	0.789	24.57	(3)	-1.512	-0.012	-0.836	(0)
CAMB	137	0.703	11.44	(3)	-1.283	-0.007	-0.199	(0)
CAME	455	0.853	33.05	(3)	5.980	0.035	2.267	(3)
CANA	590	0.789	31.02	(3)	-4.014	-0.005	-0.444	(0)
CAPE	171	0.706	12.76	(3)	5.553	0.039	1.017	(0)
CENT	299	0.781	21.65	(3)	0.000	0.000	0.000	(0)
CHAD	274	0.812	23.01	(3)	1.954	0.029	1.034	(0)
CHHK	304	0.731	18.52	(3)	-5.509	-0.013	-0.479	(0)
CHMC	218	0.780	18.38	(3)	0.000	0.000	0.000	(0)
COLO	591	0.884	43.85	(3)	0.336	0.000	0.369	(0)
CONG	496	0.802	29.30	(3)	0.947	0.011	1.796	(3)
COTE	550	0.796	30.53	(3)	0.888	0.004	0.156	(0)
CROA	243	0.948	45.53	(3)	0.934	0.011	1.875	(3)
CYPR	591	0.653	20.98	(3)	-2.530	-0.007	-0.431	(0)
CZEC	159	0.884	23.75	(3)	4.309	0.010	0.459	(0)
DENM	471	0.774	26.51	(3)	-10.670	-0.015	-0.924	(0)
DOMR	591	0.940	65.23	(3)	-0.483	-0.002	-0.109	(0)
EQUA	591	0.932	60.62	(3)	-2.568	-0.006	-1.405	(0)
EGYP	591	0.790	31.04	(3)	5.898	0.019	1.795	(3)
ELSA	591	0.835	36.50	(3)	4.853	0.011	0.903	(0)
ESTO	171	0.859	23.59	(3)	-13.709	-0.055	-2.032	(3)
ETHI	482	0.846	34.26	(3)	-1.126	-0.009	-0.531	(0)
FIJI	446	0.788	26.85	(3)	2.363	0.007	0.408	(0)
FINL	591	0.742	26.07	(3)	-4.947	-0.006	-0.108	(0)
FRAN	591	0.803	32.78	(3)	-12.762	-0.011	-0.565	(0)
GAMB	542	0.826	33.61	(3)	3.806	0.015	1.108	(0)

Country	No.obs.		*AR*(1) coefficient				Bilinear coefficient	
		$\hat{\alpha}$	$t(\alpha)$	signif	$\hat{\beta}$	$\tilde{\beta}$	$t(\beta)$	signif
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
GEOR	147	0.723	9.06	(3)	1.949	0.034	2.577	(3)
GERM	183	0.639	11.17	(3)	-24.200	-0.030	-1.014	(0)
CHAN	517	0.895	47.29	(3)	0.563	0.002	0.115	(0)
GREE	591	0.741	23.77	(3)	-7.579	-0.009	-0.262	(0)
GREN	359	0.716	19.13	(3)	7.837	0.025	1.048	(0)
GUAT	591	0.908	50.64	(3)	-7.763	-0.015	-2.050	(3)
GUIN	241	0.766	18.47	(3)	-0.277	-0.004	-0.215	(0)
GUYA	143	0.660	10.34	(3)	1.447	0.006	0.147	(0)
HAIT	586	0.879	44.34	(3)	7.445	0.022	1.865	(3)
HOND	591	0.902	49.75	(3)	0.000	0.000	0.001	(0)
HUNG	363	0.871	33.27	(3)	1.369	0.002	0.192	(0)
ICEL	279	0.895	34.67	(3)	-1.265	-0.003	-0.223	(0)
INDI	589	0.860	40.12	(3)	-3.707	-0.008	-0.499	(0)
INDI	459	0.938	57.46	(3)	0.504	0.002	0.376	(0)
IREL	111	0.865	16.92	(3)	-20.044	-0.029	-0.322	(0)
ISRA	591	0.914	54.48	(3)	0.423	0.001	0.111	(0)
ITAL	591	0.877	44.39	(3)	-1.733	-0.001	-0.568	(0)
JAMA	590	0.931	59.29	(3)	-5.669	-0.012	-1.722	(3)
JAPA	590	0.781	30.35	(3)	0.000	0.000	-0.001	(0)
JORD	363	0.745	21.27	(3)	2.755	0.014	0.639	(0)
KAZA	159	0.930	36.55	(3)	8.694	0.043	2.598	(3)
KENY	459	0.854	34.52	(3)	0.852	0.004	0.409	(0)
KORE	434	0.847	32.84	(3)	10.128	0.020	1.175	(0)
KYRG	134	0.904	24.66	(3)	4.772	0.029	1.043	(0)
LATV	171	0.697	13.20	(3)	-7.024	-0.035	-2.265	(3)
LITH	167	0.837	19.73	(3)	-2.107	-0.009	-0.370	(0)
LUXE	591	0.751	23.90	(3)	46.486	0.056	4.563	(3)
MACE	159	0.799	16.66	(3)	-0.615	-0.005	-0.304	(0)
MADA	506	0.881	392.40	(3)	0.217	0.001	0.011	(0)
MALA	314	0.883	32.90	(3)	3.506	0.012	0.717	(0)
MALY	590	0.817	33.94	(3)	5.953	0.010	0.712	(0)
MALT	590	0.743	26.73	(3)	8.537	0.027	1.573	(0)
MAUT	246	0.749	17.64	(3)	1.725	0.011	0.507	(0)
MAUR	525	0.850	36.72	(3)	5.782	0.015	1.125	(0)
MEXI	591	0.947	71.98	(3)	3.795	0.005	1.051	(0)
MOLD	148	0.894	25.44	(3)	-0.209	-0.001	-0.125	(0)
MORO	590	0.761	28.39	(3)	-2.887	-0.008	-0.485	(0)
MOZA	153	0.887	23.51	(3)	0.306	0.002	0.167	(0)
NEPA	505	0.844	35.18	(3)	1.277	0.004	0.472	(0)
NETH	591	0.731	26.03	(3)	1.922	0.002	0.111	(0)
NICA	83	0.636	7.09	(3)	-39.927	-0.086	-0.765	(0)
NIGE	459	0.736	23.23	(3)	2.639	0.028	1.613	(0)
NIGR	554	0.829	34.70	(3)	3.412	0.015	1.735	(3)
NORW	591	0.819	34.10	(3)	-4.970	-0.007	-0.722	(0)

Country	No.obs.	$AR(1)$ coefficient			Bilinear coefficient			
		$\hat{\alpha}$	$t(\alpha)$	signif	$\hat{\beta}$	$\tilde{\beta}$	$t(\beta)$	Signif
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
PAKI	591	0.768	29.11	(3)	-3.376	-0.006	-0.534	(0)
PANA	380	0.672	17.72	(3)	-5.090	-0.013	-0.844	(0)
PERU	591	0.997	43.77	(3)	27.078	0.065	10.088	(3)
PHIL	591	0.910	53.22	(3)	1.762	0.004	0.600	(0)
POLA	219	0.871	26.52	(3)	8.793	0.024	1.022	(0)
PORT	591	0.791	31.21	(3)	-8.363	-0.010	-0.649	(0)
ROMA	186	0.889	26.47	(3)	2.890	0.009	0.462	(0)
RUSS	170	0.983	41.13	(3)	16.815	0.108	10.427	(3)
SAMO	469	0.786	26.47	(3)	4.528	0.036	1.969	(3)
SAUD	313	0.770	21.44	(3)	5.499	0.017	0.990	(0)
SENE	459	0.802	28.49	(3)	4.460	0.031	2.239	(3)
SERB	146	0.917	27.27	(3)	4.713	0.034	2.371	(3)
SEYC	442	0.750	23.83	(3)	-0.877	-0.008	-0.625	(0)
SING	542	0.843	28.19	(3)	-1.377	-0.004	-0.122	(0)
SLOA	159	0.835	19.15	(3)	3.894	0.013	0.378	(0)
SLOE	172	0.765	25.86	(3)	-47.457	-0.119	-3.962	(3)
SOLO	328	0.767	21.59	(3)	-0.722	-0.003	-0.248	(0)
SOUT	591	0.913	53.44	(3)	-6.225	-0.009	-1.283	(0)
SPAI	591	0.748	27.41	(3)	17.835	0.018	1.147	(0)
SRIL	591	0.770	29.33	(3)	0.233	0.001	0.315	(0)
STKI	323	0.721	18.25	(3)	-6.068	-0.022	-0.828	(0)
STLU	501	0.708	22.01	(3)	-1.825	-0.008	-0.218	(0)
SSAF	458	0.899	42.21	(3)	15.002	0.024	2.061	(3)
SURI	444	0.913	46.65	(3)	-0.070	0.000	-0.325	(0)
SWAZ	470	0.516	13.03	(3)	0.460	0.003	0.351	(0)
SWED	591	0.792	30.88	(3)	-21.449	-0.036	-3.327	(3)
SWIT	591	0.796	31.94	(3)	-3.960	-0.005	-0.111	(0)
THAI	495	0.874	39.60	(3)	10.180	0.020	1.287	(0)
TONG	194	0.667	11.36	(3)	-14.147	-0.058	-1.627	(0)
TRIN	590	0.801	32.40	(3)	1.753	0.003	0.112	(0)
TUNI	225	0.897	29.19	(3)	-7.236	-0.011	-0.408	(0)
TURK	447	0.919	47.03	(3)	-6.139	-0.013	-1.865	(3)
UGAN	169	0.872	21.54	(3)	7.979	0.052	1.361	(0)
UNIK	591	0.884	46.00	(3)	10.587	0.012	0.971	(0)
UNIS	591	0.794	30.66	(3)	-26.021	-0.026	-1.187	(0)
URUG	591	0.924	58.39	(3)	5.608	0.010	1.694	(3)
VENE	591	0.899	47.68	(3)	-4.098	-0.007	-0.791	(0)
VIET	131	0.892	23.03	(3)	2.750	0.007	0.229	(0)
ZAMB	250	0.880	28.15	(3)	12.067	0.032	1.245	(0)

Table B2: ML Kalman Filter forecast measures

Country (1)	**No.obs.** (2)	$\hat{\Phi}$ (3)	$\hat{\beta}^{0.90}_{\xi\|\beta=0}$ (4)	$\hat{\delta}^{0.90}_{\hat{\beta}}(24)$ (5)	$\tau_{\max}$ (6)
ALBA	183	0.629	0.009	0.002	7
ARGE	591	0.904	-0.016	0.084	16
ARME	156	0.731	0.007	0.003	9
AUST	591	0.493	-0.057	0.042	4
BARB	483	0.566	-0.003	0.000	5
BELG	591	0.570	0.011	0.002	5
BENI	171	0.595	-0.003	0.000	6
BOLI	590	0.830	0.003	0.002	13
BOTS	370	0.784	-0.050	0.200	11
BRAZ	315	0.890	0.016	0.066	16
BULG	183	0.739	0.011	0.007	10
BURK	566	0.481	0.028	0.010	4
BURU	383	0.623	-0.015	0.006	6
CAMB	137	0.494	-0.009	0.001	4
CAME	455	0.730	0.045	0.106	9
CANA	590	0.623	-0.007	0.001	6
CAPE	171	0.501	0.050	0.034	4
CENT	299	0.610	0.000	0.000	6
CHAD	274	0.660	0.038	0.046	7
CHHK	304	0.535	-0.017	0.005	5
CHMC	218	0.608	0.000	0.000	6
COLO	591	0.781	0.000	0.000	11
CONG	496	0.644	0.014	0.006	7
COTE	550	0.633	0.005	0.001	7
CROA	243	0.898	0.014	0.065	16
CYPR	591	0.427	-0.009	0.001	3
CZEC	159	0.782	0.013	0.014	11
DENM	471	0.600	-0.020	0.009	6
DOMR	591	0.883	-0.002	0.001	16
EQUA	591	0.869	-0.008	0.013	15
EGYP	591	0.624	0.024	0.015	6
ELSA	591	0.697	0.015	0.009	8
ESTO	171	0.741	-0.070	0.277	10
ETHI	482	0.716	-0.011	0.006	9
FIJI	446	0.622	0.009	0.002	6
FINL	591	0.551	-0.007	0.001	5
FRAN	591	0.645	-0.014	0.006	7
GAMB	542	0.682	0.019	0.013	8
GEOR	147	0.524	0.043	0.028	5
GERM	183	0.409	-0.038	0.013	3
CHAN	517	0.801	0.002	0.000	12

Country (1)	No.obs. (2)	$\hat{\Phi}$ (3)	$\hat{\beta}^{0.90}_{\xi\|\beta=0}$ (4)	$\hat{\delta}^{0.90}_{\hat{\beta}}(24)$ (5)	$\tau_{\max}$ (6)
GREE	591	0.549	-0.011	0.002	5
GREN	359	0.514	0.032	0.015	4
GUAT	591	0.825	-0.020	0.047	13
GUIN	241	0.586	-0.005	0.000	6
GUYA	143	0.436	0.008	0.001	4
HAIT	586	0.774	0.029	0.062	11
HOND	591	0.814	0.000	0.000	13
HUNG	363	0.759	0.003	0.001	10
ICEL	279	0.801	-0.004	0.002	12
INDI	589	0.740	-0.010	0.006	10
INDI	459	0.879	0.003	0.002	15
IREL	111	0.750	-0.037	0.084	10
ISRA	591	0.835	0.001	0.000	14
ITAL	591	0.770	-0.001	0.000	11
JAMA	590	0.867	-0.015	0.047	15
JAPA	590	0.610	0.000	0.000	6
JORD	363	0.556	0.018	0.006	5
KAZA	159	0.867	0.055	0.583	15
KENY	459	0.729	0.005	0.001	9
KORE	434	0.717	0.026	0.032	9
KYRG	134	0.818	0.038	0.160	13
LATV	171	0.487	-0.045	0.026	4
LITH	167	0.701	-0.011	0.005	8
LUXE	591	0.567	0.072	0.097	5
MACE	159	0.638	-0.006	0.001	7
MADA	506	0.776	0.001	0.000	11
MALA	314	0.780	0.015	0.018	11
MALY	590	0.668	0.013	0.006	8
MALT	590	0.553	0.034	0.021	5
MAUT	246	0.561	0.015	0.004	5
MAUR	525	0.723	0.019	0.019	9
MEXI	591	0.897	0.006	0.012	16
MOLD	148	0.800	-0.001	0.000	12
MORO	590	0.579	-0.011	0.002	6
MOZA	153	0.787	0.003	0.001	12
NEPA	505	0.713	0.005	0.001	9
NETH	591	0.534	0.003	0.000	5
NICA	83	0.412	-0.111	0.110	3
NIGE	459	0.543	0.035	0.021	5
NIGR	554	0.687	0.019	0.014	8
NORW	591	0.670	-0.009	0.003	8
PAKI	591	0.590	-0.008	0.001	6
PANA	380	0.452	-0.016	0.003	4
PARA	591	0.706	0.000	0.000	9
PERU	591	0.998	0.083	57.523	19

Country (1)	**No.obs.** (2)	$\hat{\Phi}$ (3)	$\hat{\beta}^{0.90}_{\xi\mid\beta=0}$ (4)	$\hat{\delta}^{0.90}_{\hat{\beta}}(24)$ (5)	$\tau_{\max}$ (6)
PHIL	591	0.829	0.005	0.003	13
POLA	219	0.760	0.030	0.061	10
PORT	591	0.625	-0.013	0.004	6
ROMA	186	0.790	0.012	0.012	12
RUSS	170	0.978	0.138	26.580	18
SAMO	469	0.620	0.047	0.055	6
SAUD	313	0.593	0.022	0.011	6
SENE	459	0.645	0.040	0.048	7
SERB	146	0.841	0.044	0.277	14
SEYC	442	0.563	-0.010	0.002	5
SING	542	0.711	-0.005	0.001	9
SLOA	159	0.698	0.017	0.012	8
SLOE	172	0.599	-0.153	0.489	6
SOLO	328	0.589	-0.004	0.000	6
SOUT	591	0.833	-0.012	0.019	13
SPAI	591	0.560	0.023	0.010	5
SRIL	591	0.593	0.001	0.000	6
STKI	323	0.520	-0.028	0.011	5
STLU	501	0.502	-0.010	0.001	4
SSAF	458	0.809	0.030	0.095	12
SURI	444	0.833	-0.001	0.000	13
SWAZ	470	0.267	0.004	0.000	2
SWED	591	0.629	-0.047	0.058	7
SWIT	591	0.633	-0.006	0.001	7
THAI	495	0.765	0.026	0.047	11
TONG	194	0.448	-0.074	0.059	4
TRIN	590	0.642	0.004	0.001	7
TUNI	225	0.805	-0.014	0.020	12
TURK	447	0.844	-0.017	0.044	14
UGAN	169	0.763	0.067	0.302	11
UNIK	591	0.782	0.015	0.018	11
UNIS	591	0.631	-0.033	0.029	7
URUG	591	0.854	0.013	0.029	14
VENE	591	0.808	-0.009	0.008	12
VIET	131	0.795	0.009	0.008	12
ZAMB	250	0.775	0.041	0.127	11

List of country names and codes

Country name	code	Country name	code
Albania	ALBA	Kyrgyz Republic	KYRG
Argentina	ARGE	Latvia	LATV
Armenia	ARME	Lithuania	LITH
Austria	AUST	Luxembourg	LUXE
Barbados	BARB	Macedonia, FYR	MACE
Belgium	BELG	Madagascar	MADA
Benin	BENI	Malawi	MALA
Bolivia	BOLI	Malaysia	MALY
Botswana	BOTS	Malta	MALT
Brazil	BRAZ	Mauritania	MAUT
Bulgaria	BULG	Mauritius	MAUR
Burkina Faso	BURK	Mexico	MEXI
Burundi	BURU	Moldova	MOLD
Cambodia	CAMB	Morocco	MORO
Cameroon	CAME	Mozambique	MOZA
Canada	CANA	Nepal	NEPA
Cape Verde	CAPE	Netherlands	NETH
Central African Rep.	CENT	Nicaragua	NICA
Chad	CHAD	Niger	NIGE
China,P.R.:Hong Kong	CHHK	Nigeria	NIGR
China,P.R.:Macao	CHMC	Norway	NORW
Colombia	COLO	Pakistan	PAKI
Congo, Dem. Rep. of	CONG	Panama	PANA
Côte d'Ivoire	COTE	Paraguay	PARA
Croatia	CROA	Peru	PERU
Cyprus	CYPR	Philippines	PHIL
Czech Republic	CZEC	Poland	POLA
Denmark	DENM	Portugal	PORT
Dominican Republic	DOMR	Romania	ROMA
Ecuador	EQUA	Russian Federation	RUSS
Egypt	EGYP	Samoa	SAMO
El Salvador	ELSA	Saudi Arabia	SAUD
Estonia	ESTO	Senegal	SENE
Ethiopia	ETHI	Serbia	SERB
Fiji	FIJI	Seychelles	SEYC
Finland	FINL	Singapore	SING
France	FRAN	Slovak Republic	SLOA
Gambia	GAMB	Slovenia	SLOE
Georgia	GEOR	Solomon Islands	SOLO
Germany	GERM	South Africa	SOUT
Ghana	CHAN	Spain	SPAI
Greece	GREE	Sri Lanka	SRIL
Grenada	GREN	St. Kitts and Nevis	STKI

Country name	code	Country name	code
Guatemala	GUAT	St. Lucia	STLU
Guinea-Bissau	GUIN	Sub-Saharan Africa	SSAF
Guyana	GUYA	Suriname	SURI
Haiti	HAIT	Swaziland	SWAZ
Honduras	HOND	Sweden	SWED
Hungary	HUNG	Switzerland	SWIT
Iceland	ICEL	Thailand	THAI
India	INDI	Tonga	TONG
Indonesia	INDI	Trinidad and Tobago	TRIN
Ireland	IREL	Tunisia	TUNI
Israel	ISRA	Turkey	TURK
Italy	ITAL	Uganda	UGAN
Jamaica	JAMA	United Kingdom	UNIK
Japan	JAPA	United States	UNIS
Jordan	JORD	Uruguay	URUG
Kazakhstan	KAZA	Venezuela	VENE
Kenya	KENY	Vietnam	VIET
Korea, Republic of	KORE	Zambia	ZAMB

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