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ABSTRACT

This paper examines the relationship between the ‘exclusion’ type core inflation measures and the stability price index. Empirical results for Malaysia and Pakistan suggests that, if targeting core inflation index is to stabilize output, weights of the export-oriented sectors (energy for Malaysia and foodstuffs for Pakistan) should be reduces, in relation to the consumers’ price index weights, and for import-oriented sectors, increased. It also indicates that, in order to maintain real sector stability, central bankers should include the fundamental component of the stock market prices in the price index they target.

1. INTRODUCTION

Most commonly used ways of constructing a core inflation index in central banks is through exclusion the most volatile components, usually foodstuffs and energy prices (for description, critique and alternatives see Cecchetti *et. al*, 2000 and Cechcetti and Kim, 2003, Morana, 2007, Huh and Lee, 2011, Humala and Rodriguez, 2012 and others, and for a review of contemporary techniques see Aleem and Lahiani, 2011). This paper argues that, for open economy developing countries, the question of inclusion or exclusion (or, more general, weighting) of the foodstuff and energy prices from the central bankers' inflation measure should be decided by the structure of exports and imports. In particular such weighting should be different for the oil-exporting and importing countries and also for countries importing and exporting agricultural products. Appropriate weighting of these prices could result in significant reduction of output volatility. Moreover, it argues that the financial price component, in the form of the fundamental part of the stock market prices, should be added for a country where financial sector is relatively well developed. Such addition can further reduce frictions of output.

The basic concept applied here is that of Mankiw and Reis (2003) *stability price index*, *SPI*, which consists of setting weights in the price index targeted by central bankers which minimize output fluctuations (for further development of this idea see Reis, 2005). In this paper Malaysia (as an example of exporting oil economy) and Pakistan (oil-importing country, and one of the Asia largest exporters of a number of agricultural products) are considered. Malaysia has significantly developed financial sector (with market capitalization at the level of over 170% in 2007, while in Pakistan the capitalization of the official financial market (with thriving unofficial, grey and black financial markets) market capitalization was at the level of 49%.

2. MANKIW-REIS MODEL: OUTLINE

Following Mankiw and Reis (2003), the *SPI* is obtained through solving the following optimization problem for each time *t*:

$$\min_{\{\omega_k\}} Var(y_t) \ , \tag{1}$$

subject to:

$$\breve{p}_t = \sum_{k=1}^{K} \omega_k p_{t,k} \ , \tag{2}$$

$$\sum_{k=1}^{K} \omega_k = 1 \ , \tag{3}$$

$$p_{t,k} = \lambda_k p^*_{t,k} + (1- \lambda_k) E(p^*_{t,k}) \ , \tag{4}$$

$$p_t = \sum_{k=1}^{K} \delta_k p_{t,k} \ , \tag{5}$$

$$p^*_{t,k} = p_t + \beta_k y_t + \varepsilon_{t,k} \ , \tag{6}$$

where $\breve{p}_t$ is the resulting *SPI* with optimal weights ω_k, $p_{t,k}$ are the sectoral prices for sector k, $p^*_{t,k}$ is the equilibrium price in sector k, p_t is the official aggregate consumers' price index (*CPI*) with known weights δ_k, y_t is the output gap (the difference of the logs of actual and potential output), β_k measures the responsiveness of sector equilibrium price to the business cycle, and $\varepsilon_{t,k}$ are the shocks to the k^{th} sector with variance-covariance matrix $\Sigma = \{\sigma_{i,j}\}$, $i,j = 1,\ldots,K$. All price variables are in logs. $E(p^*_{t,k})$ is the sectoral price expected in time $t-1$ for time t and the parameter λ_k measures the sluggishness of prices in sector k. Subject to the restrictions above, the central bank selects target weights ω_k in its price index by minimizing (1) with the restrictions (2) - (6). This minimization results in the set of the optimal weights ω_k in a target price index given β_k, δ_k, Σ and λ_k.

3. DATA AND ESTIMATION

The *SPI*'s are evaluated using annual data for Malaysia (1982-2009) and Pakistan, (1982-2010) with the economies disaggregated into 4 sectors (that is, *K*=4): energy, foodstuffs, other goods and services and the fundamental component of stock market. Data on sectoral prices, sectoral weights δ_k and output are taken from official publications and are available on request.[1] Output gap for both countries has been estimated as deviations of real *GDP* from a Hodrick-Prescott smoother. The stock market price index for Malaysia it is the Kuala Lumpur Composite Index and for Pakistan it is the weighted average from share prices of all joint stock companies listed at Karachi Stock Exchange (both in logarithms). As the fundamental components of the stock market price index the trend part of the Hodrick-Prescott (*HP*) decomposition have been used.

The parameters β_k, δ_k, Σ and λ_k. have been set as follows. First, original data have been filtered from shocks by estimating the vector autoregressive model containing p_k's p_t and y_t and taking residuals. For the sluggishness parameter λ_k, (and unlike in the Mankiw and Reis, 2003), we are allowing for some uncertainty, and investigating the sensitivity of our results for its misspecification by sampling 10,000 draws from the intervals [0.9 , 1] for the energy and foodstuffs sectors, where prices are assumed to be highly flexible, and [0.45 , 0.55] for other sectors. This enables us to infer on the possible sensitivity of the results due to the uncertainty about the true levels of λ_k's. For each simulated value of λ_k the parameters β_k have been estimated from (6) by the generalized method of moments, with lagged $p_{t,k}$'s and y_t used as instruments, after substituting $p_{t,k}/\lambda_k$ for $p^*_{t,k}$. The procedure used here was *LGMM* by Ogaki (1993), available at http://seminar.econ.keio.ac.jp/ogaki/english-profile.html. Finally, the matrix Σ has been estimated as a variance-covariance matrix of all $p_{t,k}$'s. The values of β_k, σ_k^2, λ_k, δ_k and Σ have next been substituted into the analytical formula for the variance of output gap, given in Appendix. Variance of the output gap has been numerically minimized with respect to ω_k subject to the constraints (2) - (6).[2] The additional restriction here is that the weights cannot be negative $(\omega_k \geq 0)$.

4. RESULTS

Table 1 reports the average optimal weights ω_k's from 10,000 optimization experiments. Table 2 contains variances of output gap computed numerically for the *CPI* inflation, core inflation indices (*CI*) which exclude the stock market component and, additionally, energy, foodstuffs and both energy and foodstuff respectively, and *SPI*. The formula for computing the variance is the same as for *SPI* used in optimization (that is using the formula in Appendix), but with zero restrictions imposed on weights for the sectors excluded. In both

[1] Main sources: http://www.statistics.gov.my for Malaysia and http://www.sbp.org.pk/publications/index2.asp for Pakistan. For Malaysia the data on energy prices are combined with that on housing prices and given identical weights in the *CPI*.

[2] The optimization algorithm applied here is Newton-Raphston method with step halving. Computations have been performed using GAUSS 12 and optimization package CO.

tables the corresponding sensitivity standard errors (*s.s.e.*) that is the standard errors reflecting the variability of the estimates due to sampling of the λ_k's, are reported.

Table 1: *CPI* and optimal weights and the estimates of the optimal output variance

	Foodstuff	Energy	Other	Finance	Output variance
Malaysia					
CPI weights	0.369	0.224	0.407	0.000	0.17724
SPI weights	0.581	0.035	0.000	0.384	0.00007
s.s.e.	0.024	0.007	0.000	0.024	
Pakistan					
CPI weights	0.499	0.053	0.445	0.000	0.00010
SPI weights	0.350	0.619	0.031	0.000	0.00004
s.s.e.	0.142	0.066	0.077	0.000	

In fact more optimization experiments were conducted, for the cases where (1) shocks were assumed to be orthogonal rather than correlated, (2) there were no constraints for the weights imposed and (3) the unsmoothed stock market index, or its bubble part were used alternatively as the stock market components. All these results have been markedly inferior to these reported above, in terms of achieving output variance reduction.

The results indicate that resetting the weights in the price index to that of the *SPI*'s would lead to a substantial gain in terms of reducing output volatility for both countries. However, such resetting should be in opposite directions in both countries. For oil-exporting Malaysia the weights related to energy sector should be substantially reduced, and the weights for foodstuffs increased. For Pakistan, however, in order to achieve reduction in output variability, weights for the energy sector should be increased and weights for the foodstuffs sector reduced. Nevertheless, these changes in weights cannot be as extreme in the core inflation indices. In most cases setting the energy and foodstuffs (and both) weights to zero and increasing the other weights respectively, does not lead to a reduction of output variance. The exception here is *CI* excluding foodstuffs for Pakistan, which gives a slight reduction in output variance in relation to *CPI*.

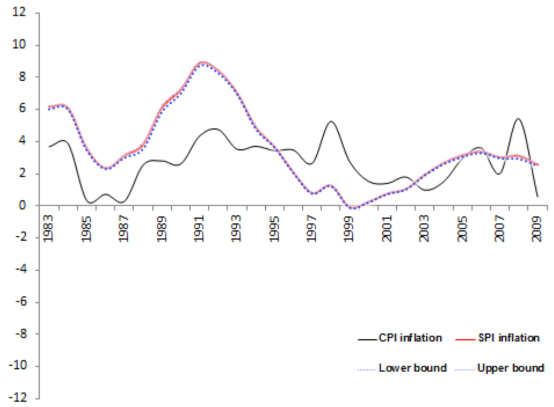
Figures 1 and 2 compare the annual inflation indices based on *CPI* and *SPI* respectively. For the *SPI* inflation the confidence bounds (± 2×*s.s.e*) are given which reflect the uncertainty related to settings of the sluggishness parameter. For both countries the inflation rate measured by the *CPI* and *SPI* do not differ drastically. For Malaysia, *SPI* inflation is slightly less volatile than *CPI* inflation, while for Pakistan it is the opposite. For Malaysia, between 1983 and 1996, the *SPI* inflation is markedly above that of the *CPI*, which indicates space for more active contractionary monetary policy in this period. For Pakistan, in some periods, the bounds around the average *SPI* inflation are much wider than for Malaysia. This shows substantial uncertainty regarding the estimated sectoral weights.

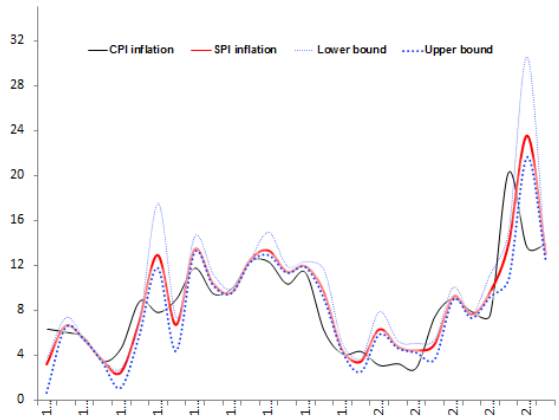
4. CONCLUSIONS

The 'exclusion' core inflation indices which put zero weights on some sectors of the economy are not the optimal targets in monetary policy, as they do not lead to minimizing of output variance. However, they point to the right direction. Our results indicate that in natural recourse economies prices for the exporting industry (in our case oil for Malaysia and foodstuff for Pakistan) should be weighted down (albeit not necessarily to zero) in the price index used for targeting inflation. At the same time, in the developing, oil importing and foodstuff exporting, countries like Pakistan, more weight should be given to the energy sector and less to the foodstuffs.

Headline and stability inflation

Figure 1: Malaysia **Figure 2: Pakistan**





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Appendix

Variance of output gap in the four-sector Mankiw-Reis model

Solving the system (2) - (6) for y_t and computing the unconditional variance of the solution, and regarding $p_{t,k} - E(p^*_{t,k})$ as the unexpected sectoral shock, gives :

$\mathrm{var}(y_t) = K / L$, where:

$K = A+B+C+D+E+F+G+H+I+J$, and

$$A = \sigma_{11}^2(\lambda_1\omega_1 + \delta_1\lambda_1\lambda_4 - \lambda_1\lambda_4\omega_1 + \delta_1\lambda_1\lambda_3\omega_3 - \delta_1\lambda_1\lambda_4\omega_3 + \delta_1\lambda_1\lambda_2\omega_2 - \delta_1\lambda_1\lambda_4\omega_2 - \delta_2\lambda_1\lambda_2\omega_1 - \delta_3\lambda_1\lambda_3\omega_1 + \delta_2\lambda_1\lambda_4\omega_1 + \delta_3\lambda_1\lambda_4\omega_1)^2 \quad ,$$

$$B = \sigma_{22}^2(\lambda_2\omega_2 + \delta_2\lambda_2\lambda_4 - \lambda_2\lambda_4\omega_2 + \delta_2\lambda_2\lambda_3\omega_3 + \delta_2\lambda_1\lambda_2\omega_1 - \delta_2\lambda_2\lambda_4\omega_3 + \delta_3\lambda_2\lambda_3\omega_2 - \delta_2\lambda_2\lambda_4\omega_1 - \delta_1\lambda_1\lambda_2\omega_2 - \delta_3\lambda_2\lambda_3\omega_2 + \delta_1\lambda_2\lambda_4\omega_2 + \delta_3\lambda_2\lambda_4\omega_2)^2 \quad ,$$

$$C = \sigma_{33}^2(\lambda_3\omega_3 + \delta_3\lambda_3\lambda_4 - \lambda_3\lambda_4\omega_3 + \delta_3\lambda_1\lambda_3\omega_1 - \delta_3\lambda_3\lambda_4\omega_1 - \delta_3\lambda_3\lambda_4\omega_2 - \delta_1\lambda_1\lambda_3\omega_3 - \delta_2\lambda_2\lambda_3\omega_3 + \delta_1\lambda_3\lambda_4\omega_3 + \delta_2\lambda_3\lambda_4\omega_3)^2 \quad ,$$

$$D = \sigma_{44}^2(\lambda_4 + \lambda_3\lambda_4\omega_3 + \lambda_1\lambda_4\omega_1 + \lambda_2\lambda_4\omega_2 - \delta_1\lambda_1\lambda_4 - \delta_2\lambda_2\lambda_4 - \delta_3\lambda_3\lambda_4 - \lambda_4\omega_1 - \lambda_4\omega_2 - \lambda_4\omega_3 - \delta_1\lambda_3\lambda_4\omega_3 - \delta_2\lambda_3\lambda_4\omega_3 - \delta_2\lambda_1\lambda_4\omega_1 - \delta_3\lambda_1\lambda_4\omega_1 - \delta_1\lambda_2\lambda_4\omega_2 - \delta_3\lambda_2\lambda_4\omega_2 + \delta_1\lambda_1\lambda_4\omega_2 + \delta_1\lambda_1\lambda_4\omega_3 + \delta_2\lambda_2\lambda_4\omega_1 + \delta_2\lambda_2\lambda_4\omega_3 + \delta_3\lambda_3\lambda_4\omega_2 + \delta_3\lambda_3\lambda_4\omega_1)^2 \quad ,$$

$$E = 2\sigma_{12}(\lambda_1\omega_1 + \delta_1\lambda_1\lambda_4 - \lambda_1\lambda_4\omega_1 + \delta_1\lambda_1\lambda_3\omega_3 - \delta_1\lambda_1\lambda_4\omega_3 + \delta_1\lambda_1\lambda_2\omega_2 - \delta_1\lambda_1\lambda_4\omega_2 - \delta_2\lambda_1\lambda_2\omega_1 - \delta_3\lambda_1\lambda_3\omega_1 + \delta_2\lambda_1\lambda_4\omega_1 + \delta_3\lambda_1\lambda_4\omega_1)(\lambda_2\omega_2 + \delta_2\lambda_2\lambda_4 - \lambda_2\lambda_4\omega_2 + \delta_2\lambda_2\lambda_3\omega_3 + \delta_2\lambda_1\lambda_2\omega_1 - \delta_2\lambda_2\lambda_4\omega_3 + \delta_3\lambda_2\lambda_3\omega_2 - \delta_2\lambda_2\lambda_4\omega_1 - \delta_1\lambda_1\lambda_2\omega_2 - \delta_3\lambda_2\lambda_3\omega_2 + \delta_1\lambda_2\lambda_4\omega_2 + \delta_3\lambda_2\lambda_4\omega_2) \quad ,$$

$$F = 2\sigma_{13}(\lambda_1\omega_1 + \delta_1\lambda_1\lambda_4 - \lambda_1\lambda_4\omega_1 + \delta_1\lambda_1\lambda_3\omega_3 - \delta_1\lambda_1\lambda_4\omega_3 + \delta_1\lambda_1\lambda_2\omega_2 - \delta_1\lambda_1\lambda_4\omega_2 - \delta_2\lambda_1\lambda_2\omega_1 - \delta_3\lambda_1\lambda_3\omega_1 + \delta_2\lambda_1\lambda_4\omega_1 + \delta_3\lambda_1\lambda_4\omega_1)(\lambda_3\omega_3 + \delta_3\lambda_3\lambda_4 - \lambda_3\lambda_4\omega_3 + \delta_3\lambda_1\lambda_3\omega_1 - \delta_3\lambda_3\lambda_4\omega_1 - \delta_3\lambda_3\lambda_4\omega_2 - \delta_1\lambda_1\lambda_3\omega_3 - \delta_2\lambda_2\lambda_3\omega_3 + \delta_1\lambda_3\lambda_4\omega_3 + \delta_2\lambda_3\lambda_4\omega_3) \quad ,$$

$$G = 2\sigma_{14}(\lambda_1\omega_1 + \delta_1\lambda_1\lambda_4 - \lambda_1\lambda_4\omega_1 + \delta_1\lambda_1\lambda_3\omega_3 - \delta_1\lambda_1\lambda_4\omega_3 + \delta_1\lambda_1\lambda_2\omega_2 - \delta_1\lambda_1\lambda_4\omega_2 - \delta_2\lambda_1\lambda_2\omega_1 - \delta_3\lambda_1\lambda_3\omega_1 + \delta_2\lambda_1\lambda_4\omega_1 + \delta_3\lambda_1\lambda_4\omega_1)(\lambda_4 + \lambda_3\lambda_4\omega_3 + \lambda_1\lambda_4\omega_1 + \lambda_2\lambda_4\omega_2 - \delta_1\lambda_1\lambda_4 - \delta_2\lambda_2\lambda_4 - \delta_3\lambda_3\lambda_4 - \lambda_4\omega_1 - \lambda_4\omega_2 - \lambda_4\omega_3 - \delta_1\lambda_3\lambda_4\omega_3 - \delta_2\lambda_3\lambda_4\omega_3 - \delta_2\lambda_1\lambda_4\omega_1 - \delta_3\lambda_1\lambda_4\omega_1 - \delta_1\lambda_2\lambda_4\omega_2 - \delta_3\lambda_2\lambda_4\omega_2 + \delta_1\lambda_1\lambda_4\omega_2 + \delta_1\lambda_1\lambda_4\omega_3 + \delta_2\lambda_2\lambda_4\omega_1 + \delta_2\lambda_2\lambda_4\omega_3 + \delta_3\lambda_3\lambda_4\omega_2 + \delta_3\lambda_3\lambda_4\omega_1) \quad ,$$

$$H = 2\sigma_{23}(\lambda_2\omega_2 + \delta_2\lambda_2\lambda_4 - \lambda_2\lambda_4\omega_2 + \delta_2\lambda_2\lambda_3\omega_3 + \delta_2\lambda_1\lambda_2\omega_1 - \delta_2\lambda_2\lambda_4\omega_3 + \delta_3\lambda_2\lambda_3\omega_2 - \delta_2\lambda_2\lambda_4\omega_1 - \delta_1\lambda_1\lambda_2\omega_2 - \delta_3\lambda_2\lambda_3\omega_2 + \delta_1\lambda_2\lambda_4\omega_2 + \delta_3\lambda_2\lambda_4\omega_2)(\lambda_3\omega_3 + \delta_3\lambda_3\lambda_4 - \lambda_3\lambda_4\omega_3 + \delta_3\lambda_1\lambda_3\omega_1 - \delta_3\lambda_3\lambda_4\omega_1 - \delta_3\lambda_3\lambda_4\omega_2 - \delta_1\lambda_1\lambda_3\omega_3 - \delta_2\lambda_2\lambda_3\omega_3 + \delta_1\lambda_3\lambda_4\omega_3 + \delta_2\lambda_3\lambda_4\omega_3) \quad ,$$

$$I = 2\sigma_{24}(\lambda_2\omega_2 + \delta_2\lambda_2\lambda_4 - \lambda_2\lambda_4\omega_2 + \delta_2\lambda_2\lambda_3\omega_3 + \delta_2\lambda_1\lambda_2\omega_1 - \delta_2\lambda_2\lambda_4\omega_3 + \delta_3\lambda_2\lambda_3\omega_2 - \delta_2\lambda_2\lambda_4\omega_1 - \delta_1\lambda_1\lambda_2\omega_2 - \delta_3\lambda_2\lambda_3\omega_2 + \delta_1\lambda_2\lambda_4\omega_2 + \delta_3\lambda_2\lambda_4\omega_2)(\lambda_4 + \lambda_3\lambda_4\omega_3 + \lambda_1\lambda_4\omega_1 + \lambda_2\lambda_4\omega_2 - \delta_1\lambda_1\lambda_4 - \delta_2\lambda_2\lambda_4 - \delta_3\lambda_3\lambda_4 - \lambda_4\omega_1 - \lambda_4\omega_2 - \lambda_4\omega_3 - \delta_1\lambda_3\lambda_4\omega_3 - \delta_2\lambda_3\lambda_4\omega_3 - \delta_2\lambda_1\lambda_4\omega_1 - \delta_3\lambda_1\lambda_4\omega_1 - \delta_1\lambda_2\lambda_4\omega_2 - \delta_3\lambda_2\lambda_4\omega_2 + \delta_1\lambda_1\lambda_4\omega_2 + \delta_1\lambda_1\lambda_4\omega_3 + \delta_2\lambda_2\lambda_4\omega_1 + \delta_2\lambda_2\lambda_4\omega_3 + \delta_3\lambda_3\lambda_4\omega_2 + \delta_3\lambda_3\lambda_4\omega_1) \quad ,$$

$$J = 2\sigma_{34}(\lambda_3\omega_3 + \delta_3\lambda_3\lambda_4 - \lambda_3\lambda_4\omega_3 + \delta_3\lambda_1\lambda_3\omega_1 - \delta_3\lambda_3\lambda_4\omega_1 - \delta_3\lambda_3\lambda_4\omega_2 - \delta_1\lambda_1\lambda_3\omega_3 - \delta_2\lambda_2\lambda_3\omega_3 + \delta_1\lambda_3\lambda_4\omega_3 + \delta_2\lambda_3\lambda_4\omega_3)(\lambda_4 + \lambda_3\lambda_4\omega_3 + \lambda_1\lambda_4\omega_1 + \lambda_2\lambda_4\omega_2 - \delta_1\lambda_1\lambda_4 - \delta_2\lambda_2\lambda_4 - \delta_3\lambda_3\lambda_4 - \lambda_4\omega_1 - \lambda_4\omega_2 - \lambda_4\omega_3 - \delta_1\lambda_3\lambda_4\omega_3 - \delta_2\lambda_3\lambda_4\omega_3 - \delta_2\lambda_1\lambda_4\omega_1 - \delta_3\lambda_1\lambda_4\omega_1 - \delta_1\lambda_2\lambda_4\omega_2 - \delta_3\lambda_2\lambda_4\omega_2 + \delta_1\lambda_1\lambda_4\omega_2 + \delta_1\lambda_1\lambda_4\omega_3 + \delta_2\lambda_2\lambda_4\omega_1 + \delta_2\lambda_2\lambda_4\omega_3 + \delta_3\lambda_3\lambda_4\omega_2 + \delta_3\lambda_3\lambda_4\omega_1) \quad ,$$

$$L = [(\beta_4\lambda_4 + \beta_1\lambda_1\omega_1 + \beta_2\lambda_2\omega_2 + \beta_3\lambda_3\omega_3 - \beta_4\lambda_4\omega_1 - \beta_4\lambda_4\omega_1 - \beta_4\lambda_4\omega_2 - \beta_4\lambda_4\omega_3 + \beta_1\delta_1\lambda_1\lambda_3\omega_3 - \beta_4\delta_1\lambda_3\lambda_4\omega_3 + \beta_2\delta_2\lambda_2\lambda_3\omega_3 + \beta_4\omega_3\lambda_3\lambda_4 + \beta_4\lambda_1\lambda_4\omega_1 + \beta_4\lambda_2\lambda_4\omega_2 - \beta_4\delta_2\lambda_3\lambda_4\omega_3 + \beta_2\delta_2\lambda_1\lambda_2\omega_1 + \beta_3\delta_3\lambda_1\lambda_3\omega_1 - \beta_4\delta_2\lambda_1\lambda_4\omega_1 - \beta_4\delta_3\lambda_1\lambda_4\omega_1 + \beta_3\delta_3\lambda_2\lambda_3\omega_2 - \beta_4\delta_2\lambda_3\lambda_4\omega_3 + \beta_2\delta_2\lambda_1\lambda_2\omega_1 + \beta_3\delta_3\lambda_1\lambda_3\omega_1 - \beta_4\delta_4\lambda_1\lambda_4\omega_1 - \beta_4\delta_3\lambda_1\lambda_4\omega_1 + \beta_3\delta_3\lambda_2\lambda_3\omega_2 - \beta_4\delta_1\lambda_2\lambda_4\omega_2 + \beta_1\delta_1\lambda_1\lambda_4 - \beta_1\delta_1\lambda_1\lambda_4\omega_3 - \beta_2\delta_2\lambda_2\lambda_4\omega_3 + \beta_1\delta_1\lambda_1\lambda_2\omega_2 + \beta_2\delta_2\lambda_2\lambda_4 + \beta_3\delta_3\lambda_3\lambda_4 + \beta_4\delta_1\lambda_1\lambda_4\omega_2 - \beta_4\delta_3\lambda_2\lambda_4\omega_2 - \beta_2\delta_2\lambda_2\lambda_4\omega_1 - \beta_3\delta_3\lambda_3\lambda_4\omega_1 - \beta_1\delta_1\lambda_1\lambda_4\omega_2 - \beta_3\delta_3\lambda_3\lambda_4\omega_2 - \beta_3\delta_2\lambda_2\lambda_3\omega_3 + \beta_4\delta_3\lambda_3\lambda_4\omega_1 - \beta_4\delta_3\lambda_3\lambda_4 + \beta_4\delta_3\lambda_3\lambda_4\omega_2 - \beta_2\delta_3\lambda_2\lambda_3\omega_2 - \beta_2\lambda_2\lambda_4\omega_2 - \beta_4\delta_1\lambda_1\lambda_4 - \beta_2\delta_1\lambda_1\lambda_2\omega_2 + \beta_4\delta_1\lambda_1\lambda_4\omega_3 - \beta_3\delta_1\lambda_1\lambda_3\omega_3 + \beta_4\delta_2\lambda_2\lambda_4\omega_1 - \beta_4\delta_2\lambda_2\lambda_4 - \beta_1\delta_2\lambda_1\lambda_2\omega_1 + \beta_3\delta_2\lambda_3\lambda_4\omega_1 + \beta_4\delta_2\lambda_2\lambda_4\omega_3 - \beta_1\delta_3\lambda_1\lambda_3\omega_1 + \beta_2\delta_1\lambda_2\lambda_4\omega_2 + \beta_3\delta_1\lambda_3\lambda_4\omega_3 + \beta_1\delta_2\lambda_1\lambda_4\omega_1 + \beta_3\delta_2\lambda_3\lambda_4\omega_3 + \beta_2\delta_3\lambda_2\lambda_4\omega_2 + \beta_1\delta_3\lambda_1\lambda_4\omega_1 - \beta_1\lambda_1\lambda_4\omega_1 - \beta_3\lambda_3\lambda_4\omega_3)^2] \quad .$$