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Vertical integration and product differentiation¹

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We study horizontal product differentiation as a strategic decision of downstream firms facing a threat of vertical integration and market foreclosure by an upstream monopolist. We model product differentiation either as pure market segmentation or as generating positive value to consumers. Because of the threat of vertical integration, the downstream firms prefer more differentiation when the latter merely yields the anticompetitive effects of market segmentation, while they may prefer less differentiation when the latter would generate additional social value. Therefore, instead of market foreclosure, we indicate market segmentation or under-investment in socially valuable activities, such as product innovation, design, and informative advertising, as possible social costs of a lenient antitrust policy towards vertical mergers.

Key Words: Vertical integration; product differentiation; market foreclosure; market segmentation.

JEL Codes: D43, L13, L42

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1. INTRODUCTION

One of the most controversial issues in competition policy is the evaluation of the welfare consequences of vertical mergers. Although vertical integration can yield a variety of pro-competitive efficiency gains, from the elimination of double marginalization to the solution of incentive problems caused by incomplete contracts, it has been subjected to the major criticism of vertical foreclosure, whereby a vertically integrated firm may restrict its supply (demand) to downstream (upstream) competitors, rise the rivals' costs and extend its market power in the industry.⁴ This has stimulated a vast literature on the effect of vertical mergers on the competitive structure of upstream and downstream markets and welfare, which typically concentrates on production activities and contractual relationships of the firms. Less attention has been paid to non-production activities, particularly of the downstream firms. (Notable exceptions are discussed in the next section.)

However, as increasingly argued by both US and EU antitrust authorities, firms' strategic activities such as product innovation, positioning and design, may channel important effects of vertical mergers on competition and welfare. For instance, discussing the case of *Silicon Graphics'* acquisition of *Alias* and *Wavefront*, Christine Varney⁵ (1995a) argues that "*....the combined entity would not need to bar other software developers completely, but could redirect them away from direct competition by, for example, encouraging the development of products that are complement to, rather than direct substitutes for, Alias and Wavefront software*".⁶ These activities, in turn, may affect the incentives for vertical integration, and therefore the vertical

⁴The possibility of vertical foreclosure has now been demonstrated in a wide variety of vertical integration models: Salinger (1988), Ordover, Saloner and Salop (1990), Hart and Tirole (1990), Riordan and Salop (1995), Riordan (1998), Choi and Yi (2000), Chen (2001), Chen and Riordan (2004), among many others. Facilitating strategic coordination on collusive outcomes is a second source of anticompetitive effects antitrust authorities (e.g., the US 1984 Merger Guidelines) and, more recently, the theoretical literature (Nocke and White, 2007) have ascribed to vertical integration. Rey and Tirole (2007) and Riordan (2008) provide excellent surveys of this literature.

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⁶More in general, in 1995 both the US Department of Justice (DOJ) and Federal Trade Commission (FTC) strengthened their focus on innovation by formalizing the application of the antitrust laws to innovative markets. Varney (1995b) motivated this development as necessary for the antitrust authorities to "understand all the dimensions of competition among firms", which is vital to "advancing consumer welfare". As for vertical mergers, the DOJ Antitrust Division underlined that, under certain conditions, vertical mergers may chill innovation (Sunshine, 1994). Similarly, in the 2008 EU Guidelines on the assessment of non-horizontal mergers, we read: "...the Commission prevents mergers that would be likely to deprive customers of these benefits by significantly increasing the market power of firms. An increase of market power in this context refers to the ability of one or more firms to profitably increase prices, reduce output, choice or quality of goods and services, diminish innovation, or otherwise negatively influence parameters of competition".

structure of an industry.

In this paper, we show that once final producers' strategic decisions on their product characteristics are taken into account, vertical foreclosure may not be a concern, but different kinds of social costs of vertical integration, generally overlooked in the literature, may arise. More specifically, instead of vertical foreclosure we identify *i*) market segmentation through horizontal product differentiation and *ii*) under-investment in valuable activities, like product innovation, design, and informative advertising, conducive of socially valuable differentiation, as new social costs of vertical integration.

We consider a simple model where, without vertical integration, an upstream monopolist would charge a linear price on the sole input needed by two downstream firms in order to produce the final product. In the downstream market, firms compete in quantities. In this setting, partial vertical integration (i.e., the upstream monopolist vertically integrates with one downstream firm) eliminates double marginalization in one segment of the final product market. Moreover, by setting the input price, the integrated firm affects the marginal production cost of its downstream competitor, and it may choose to foreclose the downstream market.

The vertical structure of the industry (i.e., partial vertical integration or vertical separation) is endogenously determined as the outcome of a competitive integration game played before the market stage. Vertical integration occurs in equilibrium if and only if it is profitable (i.e., the gain from integrating exceeds a fixed integration cost), in which case the downstream firms' competition for integrating with the upstream firm allows the latter to appropriate more than the full surplus from integration. As a consequence, vertical integration is a threat to the downstream firms at the initial stage of the model, when they decide whether to horizontally differentiate their products.

The threat of vertical integration encourages the downstream firms to differentiate their products in a way that either makes vertical integration unprofitable, thereby deterring integration, or minimizes the amount of downstream profits the upstream monopolist will be able to reap through integration. One of the contributions of this paper is to show that the social welfare consequences of this effect of vertical integration crucially depends on the *nature* of product differentiation. Our model combines two versions of the representative consumer linear-quadratic model

of horizontal differentiation which lend themselves to alternative interpretations of the *nature* of differentiation. In the first version, consumers do not exhibit any preference for product differentiation: a decrease in product substitutability does not directly generate additional value to consumers and hence does not enlarge the final product market.⁷ This version formalizes product differentiation as pure market segmentation, which is likely to be the case of horizontal differentiation in mature industries. In the second version, consumers have a strict preference for differentiation: a decrease in product substitutability directly increases utility and enlarges the final product market.⁸ We think of this version as a reduced form of socially valuable activities, such as product innovation, design, and informative advertisement, which are likely to be embodied in horizontal differentiation in innovative industries.

Irrespective of its nature, product differentiation eliminates vertical foreclosure under vertical integration. However, the effect of product differentiation on the profitability of vertical integration and on social welfare crucially depends on the nature of differentiation. When product differentiation merely consists of market segmentation, it unambiguously reduces both the profitability of vertical integration and the equilibrium social welfare (in any vertical structure of the industry). As a consequence, when feasible (i.e., when the integration costs are not too small), vertical integration deterrence always incentivizes the downstream firms to increase product differentiation, which reduces social welfare (relative to the benchmark case where the threat of integration is not effective either because of prohibitive integration costs or because vertical integration is banned by the antitrust authorities). Moreover, vertical integration strengthens the downstream firms' incentive to differentiate products also when differentiation can not prevent integration (i.e., when the integration costs are small).⁹ The analysis of this case shows that the social cost of vertical integration in terms of anticompetitive market segmentation can be so strong to offset the social benefit of vertical integration from the elimination of

⁷We use a specification of the Shubik and Levitan (1980) model suggested, in a different context, by De Fraja and Norman (1993).

⁸This is the standard version of the linear-quadratic model of horizontal differentiation originally introduced by Bowley (1924), and subsequently popularized by Spence (1976), Dixit (1979) and Sing and Vives (1984)).

⁹The reason is that both downstream firms share the incentive to eliminate vertical foreclosure and soften the competitive pressure the integrated firm will exert on the independent firm at the market stage, since higher independent firm's profits also help the integrating downstream firm extract higher profits from integration.

double marginalization in one segment of the downstream market.

When product differentiation generates additional value to consumers, it unambiguously increases the equilibrium social welfare in any vertical structure of the industry. However, its effect on the profitability of vertical integration becomes non-monotonic. When feasible (i.e., when the integration costs are not too small), vertical integration deterrence may either weakens or strengthens the downstream firms' incentive for differentiation, depending on the level of the integration costs and the differentiation opportunities. We identify circumstances where vertical integration deterrence unambiguously leads to less differentiation (high integration costs and strong differentiation opportunities), causing a decrease in social welfare. In these circumstances, the social cost of vertical integration takes the form of under-investment in socially valuable differentiation.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the model. In Section 4, we solve the market stage (final stage) under the two alternative vertical structures of the industry, vertical separation and partial vertical integration, for given degrees of product differentiation. Section 5 solves the vertical integration game (second stage), and characterizes the equilibrium vertical structure of the industry as a function of the nature and the degree of product differentiation, and the level of exogenous integration costs. Section 6 studies the downstream firms' incentive to differentiate products (first stage), and the effects of vertical integration on product differentiation. Section 7 analyses the welfare effects of vertical integration. Section 8 provides some concluding remarks. All proofs are collected in the appendix.

2. RELATED LITERATURE

Interesting contributions in the recent literature have considered the effects of vertical relations on process and product innovations. Stefanidis (1997) shows that upstream firms may foreclose the downstream market (through exclusive supply contracts) in order to decrease the rivals' incentive to invest in process innovations. Banarjee and Lin (2003) study a rising-the-rival-cost effect which strengthens the downstream firms' incentive to invest in process innovations. Brocas (2003) and Buehler and Shmutzler (2008) analyze endogenous vertical integration and process innovations. Brocas (2003) considers process innovations discovered by upstream

firms and licensed to downstream firms. She shows that vertical integration and process innovation mutually reinforce when switching between alternative technologies is not too costly. Buehler and Schmutzler (2008) consider downstream process innovations. They unveil an intimidation effect which increases the incentive to innovate of a vertically integrated firm, and show that downstream process innovations make complete vertical separation more unlikely. Economides (1999) considers endogenous upstream and downstream product quality in a successive monopoly model, showing that vertical integration can improve overall quality. Milliou (2004), Miliou and Petrakis (2011) and Allian, Chambolle and Rey (2011), study the role of informational spillovers on downstream innovations potentially conveyed by vertically integrated firms.

Several works in the vertical integration literature have analyzed the impact of horizontal product differentiation on the incentives for vertical integration and foreclosure, e.g. Ordover, Saloner and Salop (1990), Colangelo (1995), Hackner (2001), Chen (2001), Economides (2004). Whilst the countervailing effect of horizontal differentiation on vertical foreclosure has been noticed in these works, none of them have considered different forms (models) of differentiation and hence the role played by the nature of product differentiation on the incentive for vertical integration. Moreover product differentiation is exogenous in these studies.

A related literature has focused on the relationship between upstream input specialization and downstream product differentiation under alternative vertical structures. Choi and Yi (2000) show that input specialization can be the means to enforce vertical foreclosure by a vertically integrated firm. Pepall and Norman (2001) consider exogenous input specialization and product differentiation, and show that the latter is crucial to assess the relative profitability of alternative vertical structures. Partial vertical integration never entails vertical foreclosure in their model. Belleflamme and Toulemonde (2003) consider endogenous input specialization and product differentiation in a successive oligopoly, highlighting mutually reinforcing incentives for less specialization and less differentiation.

Matsushima (2004) and Matsushima (2009) are the papers closest to ours. Matsushima (2004) develops a successive duopoly model where both the upstream and the downstream markets are modeled à la Hotelling. The transportation costs in the upstream market formalize the idea of input conversion costs, which increase

with the distance between the input version offered by an upstream firm (upstream firm's location) and the most efficient version to produce a downstream firm's variety of final product (downstream firm's location). The paper shows that higher input conversion costs may lead to less product differentiation and higher welfare, and briefly discusses the effect of vertical integration on product differentiation when upstream locations are exogenous. Matsushima (2009) extends the analysis of vertical integration when upstream locations are endogenous. In Matsushima's setting, vertical integration improves productive efficiency by reducing conversion costs. It however enhances downstream product differentiation, which increases final consumers' transportation costs and may reduce social welfare.

This result and our finding that vertical integration can decrease social welfare by inducing too much differentiation complement each other, as they arise from very different mechanisms and effects. In Matsushima (2004 and 2009), vertical integration enhances product differentiation because it solves an hold-up problem related to the management of the input supply channel: without integration, an increase in downstream differentiation would increase the bargaining power of (more) specialized input suppliers, which undermines the downstream firms' incentive to differentiate products. This effect crucially depends on specialized inputs. In our model, on the contrary, the essential input is generic, and the positive effect of vertical integration on product differentiation arises from the downstream firms' incentive to deter vertical integration or limit the amount of downstream profit the upstream firm can reap through integration. This effect requires competition for vertical integration, which is ruled out in Matsushima's analysis by the assumption that a vertical merger can only happen between pre-assigned pairs of firms.¹⁰ Furthermore, we show that vertical integration may exert entirely different welfare effects when product differentiation directly generates additional value to consumers. In this case, the possibility of vertical integration may reduce social welfare by decreasing downstream product differentiation.

¹⁰Another difference between Matsushima (2004 and 2009) and our paper is that he considers an inelastic demand function in the downstream market, which implies that the anticompetitive effect of horizontal differentiation on prices does not impose a deadweight loss on society.

3. THE MODEL

Technology and preferences. We consider an industry with upstream and downstream markets. In the upstream market, a monopolist (firm U) produces the sole input needed by two downstream firms (firms D_1 and D_2) to produce the final product. For simplicity, we assume that the upstream monopolist produces the essential input at zero-cost, and both downstream firms require one unit of input to produce one unit of final product. Each downstream firm produces a single variety of final product. The two varieties can be homogeneous or horizontally differentiated according to a product differentiation decision taken by the downstream firms before the production stage. Specifically, the product differentiation decision sets the degree of substitutability of the two varieties of final product, $\gamma \in [0, 1]$, as perceived by a representative consumer with preferences:

$$U(q_1, q_2, x) = a(q_1 + q_2) - \frac{1}{2} [\delta_m(q_1^2 + q_2^2) + 2\gamma q_1 q_2] + x. \quad (1)$$

In equation (1), x denotes the consumption of a numeraire good, q_1 and q_2 are, respectively, the consumption of variety one and variety two of final product, and

$$\delta_m = \begin{cases} 2 - \gamma & \text{for } m = A \\ 1 & \text{for } m = B \end{cases}$$

is an *indicator function* that selects between the two alternative models of product differentiation, model A and model B , we discuss in more details below. Maximizing utility (1), subject to the budget constraint $p_1 q_1 + p_2 q_2 + x = R$, leads to the inverse demand system

$$p_i = a - \delta_m q_i - \gamma q_j \quad (i, j = 1, 2; i \neq j), \quad (2)$$

where R is the representative consumer's income in terms of the numeraire good, and p_i denotes the price of variety i of final product.

Preference for differentiation and market size effect. In both models of horizontal differentiation selected by the indicator function δ_m , the two varieties of final product are independent in demand for $\gamma = 0$ and perfect substitutes for $\gamma = 1$.

In model A (i.e., for $\delta_m = 2 - \gamma$), product differentiation does not directly generate additional value to consumers, and hence it does not affect the total size of the downstream market.¹¹ More precisely, utility and the demand price of both

¹¹Although consumers do not exhibit any preference for differentiation in this model, they still exhibit a strict preference for variety: for any given $q > 0$, $U(q, q, x) > U(2q, 0, x) = U(0, 2q, x)$.

varieties of final product are independent of γ if consumption is equalized between the two varieties ($q_1 = q_2 > 0$). If consumption is relatively specialized in one of the two varieties ($q_i > q_j \geq 0$), then stronger differentiation (i.e., a decrease in γ) reduces utility and decreases (increases) the demand price of the most (less) consumed variety.¹² In any case, the aggregate demand of final product, as derived from (2),

$$q_1 + q_2 = a - \frac{1}{2}(p_1 + p_2),$$

is independent of γ . Absent any preference for differentiation and market size effect, model *A* formalizes product differentiation as pure market segmentation.

In model *B* (i.e., for $\delta_m = 1$), product differentiation generates additional social value to consumers and enlarges the downstream market. For any positive quantities q_1 and q_2 , utility and the demand price of both varieties of final product strictly increase with differentiation (i.e., they strictly decrease with γ). This generates a market size effect of differentiation, whereby the aggregate demand of final product,

$$q_1 + q_2 = \frac{2a - (p_1 + p_2)}{1 + \gamma},$$

strictly increases with differentiation. Since differentiation creates additional social value, we adopt model *B* to formalize the idea that horizontal differentiation can embody valuable activities to consumers, such as product innovation and informative advertising.

Timing. The model consists of three stages, illustrated separately below. In stage 1, the downstream firms set the degree of product differentiation. In stage 2, the vertical structure of the industry is determined as the outcome of a competitive integration game.¹³ If vertical integration occurs, the upstream monopolist

¹²Formally, from equation (1) with $\delta_m = 2 - \gamma$, we calculate:

$$\frac{\partial U}{\partial \gamma} = \frac{1}{2}(q_1 - q_2)^2,$$

which is positive for $q_1 \neq q_2$, zero for $q_1 = q_2$. Similarly, from equation (2) with $\delta_m = 2 - \gamma$, we derive:

$$\frac{\partial p_i}{\partial \gamma} = q_i - q_j,$$

which is positive (negative) for $q_i > q_j$ (resp., $q_i < q_j$), and zero for $q_i = q_j$.

¹³The relative timing of the differentiation and integration stages reflects the idea that product design and advertising can be less reversible than integration. When product differentiation is innovative (model *B*), it likely requires specific skills and innovative knowledge difficult to codify and transmit. This may impede the assessment of the innovation market value, and hence the integration of the innovating firm, before the innovation is fully developed. A similar argument applies when product differentiation is not innovative (model *A*), provided that the upstream firm does not have sufficient expertise of the downstream market to assess the effectiveness of, say, a

vertically integrates with one of the two downstream firms, otherwise we are left with the vertically separated structure of the industry mentioned before.¹⁴ In stage 3, the price of the essential input is set by the upstream monopolist under vertical separation, or by the vertically integrated firm under vertical integration, and Cournot competition takes place in the downstream market. Production and profits are finally determined.

Product differentiation decision. Our focus is on the effect exerted by a prospective threat of vertical integration on the downstream firms' incentive to differentiate products. We therefore abstract from any strategic aspect related to competition at the differentiation stage, and we measure the incentive for product differentiation by the joint-profit gain the downstream firms would obtain by reducing the degree of product substitutability, γ , from 1 (the perfect substitutes extreme) to a given lower value $\hat{\gamma} \in [0, 1)$. This clearly corresponds to the highest fixed differentiation cost they would be willing to pay in order to achieve the degree of differentiation $1 - \hat{\gamma}$. We denote it by $\hat{k}(\hat{\gamma})$. A change in the incentive for differentiation intuitively translates into a consistent change in the actual degree of differentiation in a broad class of differentiation games and technologies.

Integration game. We model the vertical integration game as a first-price auction. The upstream monopolist asks the downstream firms for simultaneous and independent price offers in order to integrate with one of them. On the basis of the offers received, the upstream monopolist then decides whether to vertically integrate with the downstream firm asking for the lowest price, thus paying the lowest bid. In the case of tie, we assume that each downstream firm has fifty percent probability of merging with the upstream firm, should the latter decide to accept the offer. We further assume that vertical integration involves a fixed integration cost, denoted by E .¹⁵

publicity campaign before the latter has been fully designed and launched, and the downstream firm's investment in the campaign is sunk.

¹⁴Like many other works on vertical integration (e.g., Hart and Tirole (1990)), we assume away horizontal mergers, likewise the complete integration of the industry in a single monopoly. Both assumptions can be motivated by the antitrust authorities banning horizontal or vertical mergers leading to the full monopolization of the downstream market or of the entire industry. Prohibitive re-organization costs may also justify the assumption that vertical integration cannot involve both downstream firms.

¹⁵See Hart and Tirole (1990) for an extensive discussion of the organizational, incentive, and legal costs of vertical integration which can be summarized in a fixed integration cost term.

Market stage. The outcome of the integration game sets the vertical structure of the industry. If vertical integration does not occur, the upstream monopolist supplies the essential input to the downstream firms charging a linear price w_u .¹⁶ The input price acts as the marginal cost of production for both downstream firms, which finally compete á la Cournot in the downstream market. If vertical integration occurs, the downstream market is populated by a vertically integrated firm (firm I), and an independent firm (firm N). The integrated firm can use the essential input at zero cost, and optimally sets the price of the input supplied to the rival, w_I .¹⁷ Finally, the two firms compete á la Cournot in the downstream market.

Our solution concept is subgame perfect equilibrium. We therefore solve the model by backward induction starting from the market stage.

4. THE MARKET STAGE

Consider first the market equilibrium under vertical separation. Given the input price, w_u , Cournot competition leads to a symmetric equilibrium in the downstream market, where the downstream firms produce

$$q^{D1}(w_u) = q^{D2}(w_u) \equiv q^D(w_u) = \frac{a - w_u}{2\delta_m + \gamma},$$

and earn profits

$$\pi^{D1}(w_u) = \pi^{D2}(w_u) \equiv \pi^D(w_u) = \delta_m \left[\frac{a - w_u}{2\delta_m + \gamma} \right]^2.$$

The total demand for the essential input is $2q^D(w_u)$, so that the upstream monopolist sets w_u to maximize

$$\pi^U(w_u) = w_u \frac{2(a - w_u)}{2\delta_m + \gamma}.$$

This leads to the equilibrium price of the essential input

$$w_u^* = \frac{a}{2}, \tag{3}$$

¹⁶Linear pricing of the essential input is a common assumption in the vertical integration literature, e.g., Salinger (1988), Ordover, et al. (1990), Colangelo (1995), Economides (1998), Hackner (2003), Arya et al. (2008).

¹⁷We model the integrated firm as a single firm. However, our main results extend to the case where the integrated firm behaves as a multi-division firm which can credibly (from the viewpoint of the independent firm) set an optimal transfer price of the essential input from the upstream to the downstream division. The proof of this extension is available on request.

and the equilibrium quantities and profits:¹⁸

$$\begin{aligned} q_m^D &= \frac{1}{2\delta_m + \gamma} \left(\frac{a}{2} \right), & \pi_m^D &= \delta_m \left(\frac{1}{2\delta_m + \gamma} \right)^2 \left(\frac{a}{2} \right)^2, \\ \pi_m^U &= \frac{2}{2\delta_m + \gamma} \left(\frac{a}{2} \right)^2. \end{aligned} \quad (4)$$

We turn now to the market equilibrium under vertical integration. The integrated firm produces its variety of final product at zero-cost, and charges the linear price w_I on the input supplied to the independent firm. For a given $w_I > 0$, Cournot competition yields an asymmetric equilibrium in the downstream market, where the independent firm (N) and the integrated firm (I) respectively produce:

$$q^N(w_I) = \frac{(2\delta_m - \gamma)a - 2\delta_m w_I}{4\delta_m^2 - \gamma^2}, \quad q^I(w_I) = \frac{(2\delta_m - \gamma)a + \gamma w_I}{4\delta_m^2 - \gamma^2}.$$

The corresponding profits are

$$\pi^N(w_I) = \delta_m \left[\frac{(2\delta_m - \gamma)a - 2\delta_m w_I}{4\delta_m^2 - \gamma^2} \right]^2$$

for the independent firm, and

$$\pi^I(w_I) = \delta_m \left[\frac{(2\delta_m - \gamma)a + \gamma w_I}{4\delta_m^2 - \gamma^2} \right]^2 + w_I \frac{(2\delta_m - \gamma)a - 2\delta_m w_I}{4\delta_m^2 - \gamma^2}$$

for the integrated firm, where the second term of the integrated firm's profit accounts for the sales of the essential input to the rival. The integrated firm sets w_I to maximize $\pi^I(w_I)$, obtaining:

$$w_I^* = \frac{(2\delta_m - \gamma)(4\delta_m^2 - \gamma^2 + 2\delta_m \gamma)}{8\delta_m^3 - 3\delta_m \gamma^2} \left(\frac{a}{2} \right). \quad (5)$$

Notice that, since $\delta_m \geq 1$,¹⁹ w_I^* in (5) is strictly positive for any $\gamma \in [0, 1]$, as initially claimed. The equilibrium quantities and profits are finally given by:

$$\begin{aligned} q_m^N &= \frac{4(\delta_m - \gamma)}{8\delta_m^2 - 3\gamma^2} \left(\frac{a}{2} \right), & \pi_m^N &= \delta_m \left(\frac{4(\delta_m - \gamma)}{8\delta_m^2 - 3\gamma^2} \right)^2 \left(\frac{a}{2} \right)^2, \\ q_m^I &= \frac{(2\delta_m - \gamma)(4\delta_m + \gamma)}{\delta_m(8\delta_m^2 - 3\gamma^2)} \left(\frac{a}{2} \right), & \pi_m^I &= \frac{(2\delta_m - \gamma)(6\delta_m - \gamma)}{\delta_m(8\delta_m^2 - 3\gamma^2)} \left(\frac{a}{2} \right)^2. \end{aligned} \quad (6)$$

The following lemma collects three crucial results for the previous stages of the model, which hold irrespective of the nature of product differentiation.

¹⁸The subscript $m = A, B$ underlines that the equilibrium quantities and profits depend on the model of product differentiation selected by the indicator function δ_m .

¹⁹In model A: $\delta_A > 1$ for $\gamma \in [0, 1]$, $\delta_A = 1$ for $\gamma = 1$. In model B, $\delta_B = 1$ by definition.

LEMMA 1. *Irrespective of the specification of δ_m :*

i) The independent firm's profit in the vertical integration equilibrium, π_m^N , and the symmetric profit of any downstream firm in the vertical separation equilibrium, π_m^D , monotonically increase with product differentiation (i.e., they monotonically decrease with γ).

ii) Unless the two varieties of final product are independent in demand, the independent firm's profit in the vertical integration equilibrium is lower than the symmetric profit of any downstream firm in the vertical separation equilibrium: $\pi_m^N < \pi_m^D$ for $\gamma \in (0, 1]$, $\pi_m^N = \pi_m^D$ for $\gamma = 0$.

iii) Vertical integration leads to market foreclosure only when the two varieties of final product are perfect substitutes: $\pi_m^N = 0$ for $\gamma = 1$, $\pi_m^N > 0$ for $\gamma \in [0, 1)$.

Part (i) of lemma 1 evidences the anticompetitive effect of product differentiation: by segmenting the downstream market, product differentiation softens the intensity of competition faced by the downstream firms. This increases the equilibrium profits of symmetric competitors (the two downstream firms under vertical separation), as well as the equilibrium profit of a high cost competitor (the independent firm under vertical integration).

The intuition of part (ii) relies on the tougher competitive pressure faced in the downstream market by the independent firm under vertical integration – that is, competition from the integrated firm, which produces the final good at zero cost and sets the marginal cost of the rival – relative to the competitive pressure faced by any of the two downstream firms under vertical separation – that is, competition from a symmetric rival.

Part (iii) shows the effect of product differentiation on the possibility of market foreclosure under vertical integration. The integrated firm has a strategic incentive to raise the input price charged to the independent firm, since its price, output, and profit in the downstream market increase with the rival's marginal cost (as first argued by Salop and Scheffman (1983)). On the other hand, its sales of the essential input decrease. Intuitively, the strategic incentive to raise the rival's cost strengthens with the degree of product substitutability (it actually vanishes if products are independent). Part (iii) of lemma 1 has an interesting implication for the integration stage. By guaranteeing a positive profit to the independent firm, product differentiation also allows the downstream firm that vertically integrates to reap a

positive profit from integration even if the upstream firm has full bargaining power in the integration game.

We conclude this section characterizing the effect of product differentiation on the equilibrium total surplus (i.e., the sum of the upstream and downstream profits and the consumer surplus) generated in the industry in the two alternative models of differentiation.²⁰

LEMMA 2. *Irrespective of the vertical structure of the industry (i.e., either under vertical integration or under vertical separation), the equilibrium total surplus generated in the industry: i) monotonically decreases with product differentiation in model A; ii) monotonically increases with product differentiation in model B.*

In both the vertical separation and the vertical integration equilibria, the anticompetitive effects of product differentiation unambiguously cause a reduction in the total surplus if differentiation only consists of market segmentation (model A). In contrast, they are unambiguously dominated by the additional social value product differentiation creates when it embodies valuable activities to consumers (model B).

5. THE VERTICAL INTEGRATION STAGE

Vertical integration is profitable if the integrated firm's profit, net of the fixed cost of integration, exceed the joint profit the two firms involved in the merger (i.e., the upstream monopolist and one of the two downstream firms) would gain under vertical separation. Denoting with $S_m = \pi_m^I - (\pi_m^U + \pi_m^D)$ the surplus from vertical integration before the integration cost, the profitability condition is:

$$S_m - E > 0. \quad (7)$$

If vertical integration occurs at a takeover price P , the upstream monopolist's net gain from integration is:

$$\pi_m^I - P - E - \pi_m^U = (S_m - E) + (\pi_m^D - P). \quad (8)$$

With these preliminaries in place, the next lemma characterizes the equilibrium outcome of the integration game in terms of the profitability condition (7).

²⁰In appendix 2 we provide a detailed analysis of the effects exerted by product differentiation on consumer surplus and industry profits in the vertical integration and vertical separation equilibria of models A and B. These effects compound to the clear-cut results stated in lemma 2.

LEMMA 3. *If vertical integration is profitable (i.e., if condition (7) holds), then vertical integration occurs at the takeover price $P^* = \pi_m^N$, so that the integrating downstream firm and the independent firm finally collect the same profit, π_m^N . If vertical integration is not profitable (i.e., if condition (7) is violated), then vertical integration does not occur, so that both downstream firms finally collect profit π_m^D .*

When vertical integration is profitable, competition between the downstream firms to be integrated causes the downstream firm that finally integrates to reap only its outside option under vertical integration, that is, the equilibrium profit of the independent firm. The upstream monopolist, by contrast, collects more than the net surplus from integration (this is apparent from equation (8) once recalled that, by part (ii) of lemma 1, $\pi_m^D > \pi_m^N$). On the contrary, when vertical integration is not profitable, the downstream firms can refrain from competing to be integrated, and avoid integration.²¹

Our next task is to determine the vertical structure of the industry after the integration stage as a function of the degree of product differentiation and the fixed integration cost. By lemma 3, this amounts to evaluate the sign of the net surplus from integration, $S_m(\gamma) - E$, along the product substitutability range $\gamma \in [0, 1]$. We first show that the gross surplus from integration, $S_m(\gamma)$, monotonically decreases with differentiation in model A, whereas it is U-shaped in the degree of differentiation in model B.

PROPOSITION 1. *The gross surplus from integration, $S_m(\gamma)$, is: i) positive and monotonically increasing in γ in model A; ii) positive and U-shaped in γ , with global maximum at $\gamma = 0$, in model B.*

In our setting, a positive surplus from vertical integration springs from two sources: the avoidance of double marginalization in one segment of the downstream market, and the competitive advantage of the integrated firm over the independent firm in the downstream market competition. In both differentiation models, the second source clearly weakens with product differentiation. The crucial difference between the two models lies in the effect of product differentiation on the first source

²¹To be precise, when condition (7) is violated the game admits two equilibrium outcomes. In the proof of the lemma, collected in appendix 1, we motivate why the equilibrium outcome adopted in the statement can be seen as the most natural one when vertical integration is not profitable.

of surplus. In model A , product differentiation does not enlarge the downstream market. On the contrary, it decreases the willingness to pay for the integrated firm's variety of final product, since consumption is relatively specialized in that variety. This weakens the first source of surplus, since double marginalization is avoided in a smaller segment of the downstream market. Then, by weakening both sources, product differentiation always reduces the gross surplus from integration. In model B , product differentiation enlarges the total size, and the integrated firm's segment, of the downstream market. This strengthens the first source of surplus from integration, since double marginalization is avoided in a larger market. As a result, the gross surplus from integration is high either when differentiation is strong (via the first source) or when differentiation is weak (via the second source).

Lemma 3 and proposition 1 lead to the following characterization of the equilibrium vertical structure of the industry after the integration stage.

COROLLARY 1. *In model A , vertical integration occurs: i) for any value of γ if the integration cost is small: $E \in [0, S_A(0)]$; ii) for sufficiently high values of γ if the integration cost is intermediate or high: $E \in (S_A(0), S_A(1)]$; iii) for no values of γ if the integration cost is prohibitive: $E > S_A(1)$.*

COROLLARY 2. *In model B , vertical integration occurs: i) for any value of γ if the integration cost is small: $E \in [0, S_B(\tilde{\gamma}_B)]$, where $S_B(\tilde{\gamma}_B) = \underset{\gamma}{\text{Min}} S_B(\gamma)$; ii) for high or for low, but not for intermediate, values of γ if the integration cost is intermediate: $E \in (S_B(\tilde{\gamma}_B), S_B(1)]$; iii) for low values of γ if the integration cost is high: $E \in (S_B(1), S_B(0)]$; iv) for no values of γ if the integration cost is prohibitive: $E > S_B(0)$.*

Figures 1 and 2 illustrate Corollaries 1 and 2, respectively. In Figure 1, E_A^s , E_A^{ih} , and E_A^p stand for small, intermediate or high, and prohibitive integration costs in model A . Similarly, in Figure 2, E_B^s , E_B^i , E_B^h , and E_B^p stand for small, intermediate, high, and prohibitive integration costs in model B .²²

²²The two figures are drawn under the assumption that the parameter a is the same in the two models. This implies the same gross surplus from integration for $\gamma = 1$. However, any comparison of the integration cost categories of Corollaries 1 and 2 is arbitrary, since the height of demand, as set by the parameter a , may well vary across the different industries formalized by the two models.

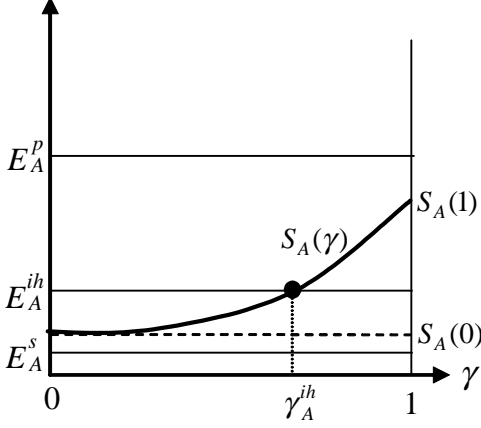


Figure 1

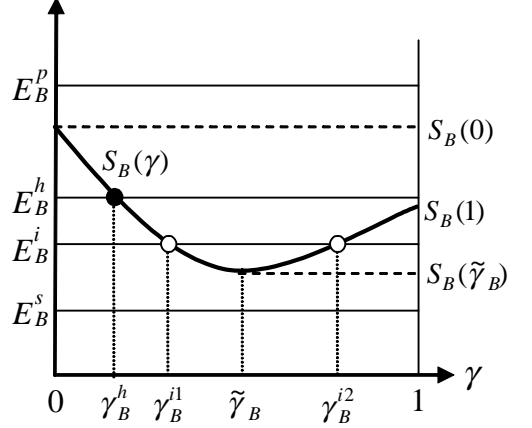


Figure 2

6. VERTICAL INTEGRATION AND PRODUCT DIFFERENTIATION

At the differentiation stage, the two downstream firms share identical profit expectations under any future evolution of the game: the independent firm's profit $\pi_m^N(\gamma)$ under vertical integration (by lemma 3), and the symmetric downstream profit $\pi_m^D(\gamma)$ under vertical separation. Let $\pi_m(\gamma)$ generically denotes their profit expectation. For any $\hat{\gamma} \in [0, 1)$, we measure the incentive for product differentiation by the joint-profit gain the downstream firms would obtain by decreasing γ from 1 to $\hat{\gamma}$:

$$\hat{k}_m(\hat{\gamma}) = 2[\pi_m(\hat{\gamma}) - \pi_m(1)]. \quad (11)$$

We adopt the case of prohibitive integration costs – where vertical integration never occurs and hence the integration stage is irrelevant for the incentive to differentiate products – as a benchmark to assess how the threat of vertical integration affects the incentive for product differentiation in all other cases listed in Corollaries 1 and 2 for models *A* and *B*, respectively. In the benchmark case, the incentive for differentiation equals:

$$\hat{k}_m^p(\hat{\gamma}) = 2[\pi_m^D(\hat{\gamma}) - \pi_m^D(1)], \text{ for any } \hat{\gamma} \in [0, 1). \quad (12)$$

Consider first model *A*. With small integration costs, vertical integration occurs for any γ (Corollary 1), and the downstream market is foreclosed for $\gamma = 1$ (part

(iii) of lemma 1). Then, the incentive for differentiation equals:

$$\widehat{k}_A^s(\widehat{\gamma}) = 2\pi_A^N(\widehat{\gamma}) \quad \text{for any } \widehat{\gamma} \in [0, 1). \quad (13)$$

With intermediate or high integration costs, on the contrary, there exists a threshold level of product substitutability, $\gamma_A^{ih} \in (0, 1)$, such that vertical integration only occurs for $\gamma \in (\gamma_A^{ih}, 1]$ (see Figure 1).²³ Again, vertical integration and foreclosure occur for $\gamma = 1$, so that the incentive for differentiation equals:

$$\widehat{k}_A^{ih}(\widehat{\gamma}) = \begin{cases} 2\pi_A^D(\widehat{\gamma}) & \text{for } \widehat{\gamma} \in [0, \gamma_A^{ih}] \\ 2\pi_A^N(\widehat{\gamma}) & \text{for } \widehat{\gamma} \in (\gamma_A^{ih}, 1). \end{cases} \quad (14)$$

We show that, in model A, the incentive for product differentiation is almost always strengthened by the threat of vertical integration.

PROPOSITION 2. *In model A:*

i) *For any $\widehat{\gamma} \in [0, 0.98]$, the incentive for product differentiation is stronger when the vertical integration threat is effective (small, intermediate or high integration costs) than when the vertical integration threat is ineffective (prohibitive integration costs).*²⁴

ii) *When the integration cost is intermediate or high and $\widehat{\gamma} \in [0, \gamma_A^{ih}]$, the possibility of deterring vertical integration through product differentiation further strengthens the incentive for differentiation (relative to all other cases).*

Figure 3 illustrates proposition 2. The *dashed* and the *thin-solid lines* represent the incentive for differentiation with prohibitive and with small integration costs, respectively. The incentive for differentiation with intermediate or high integration costs follows the *thin-solid line* from $\widehat{\gamma} = 1$ to $\widehat{\gamma} = \gamma_A^{ih}$, and the *thick-solid line* from $\widehat{\gamma} = \gamma_A^{ih}$ to $\widehat{\gamma} = 0$.

²³ γ_A^{ih} increases with E_A^{ih} , spanning the entire range of γ .

²⁴ As shown in the proof of the proposition, the condition in the statement is sufficient but not necessary for the incentive for differentiation to be stronger in the case of intermediate or high integration costs than in the benchmark case of prohibitive integration costs.

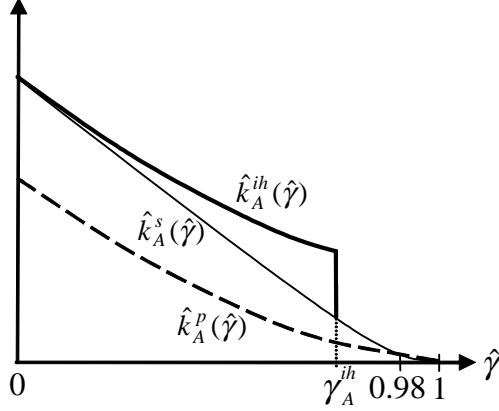


Figure 3

When the integration cost is small and vertical integration cannot be avoided, the incentive for differentiation is strengthened by the downstream firms' interest to avoid market foreclosure, soften the competitive pressure of a more efficient competitor, the integrated firm, and decrease the latter's incentive to rise the rival's cost.²⁵ These effects only require that product differentiation is not so mild to leave the independent firm's profit negligible, which is the only situation where the incentive to soften the competitive pressure of a symmetric competitor – the motive for product differentiation in the benchmark case – results stronger. With higher integration costs, the strategic incentive to deter vertical integration plays a crucial role in strengthening the downstream firms' incentive for product differentiation when the latter allows them to prevent integration.

Consider now model *B*. With small integration costs, vertical integration again occurs for any γ (Corollary 2), and $\gamma = 1$ leads to market foreclosure (part (iii) of lemma 1). Therefore:

$$\hat{k}_B^s(\hat{\gamma}) = 2\pi_B^N(\hat{\gamma}) \quad \text{for any } \hat{\gamma} \in [0, 1]. \quad (15)$$

With intermediate integration costs, there exist two critical levels of product substitutability, $0 < \gamma_B^{i1} < \gamma_B^{i2} < 1$, such that vertical integration does not occur only for $\gamma \in [\gamma_B^{i1}, \gamma_B^{i2}]$ (see Figure 2). Since $\gamma = 1$ still leads to vertical integration

²⁵This is in the interest of both downstream firms since, by lemma 3, also the downstream firm involved in the merger will finally collect the independent firm's profit π_m^N .

and foreclosure, the incentive for differentiation equals:

$$\widehat{k}_B^i(\widehat{\gamma}) = \begin{cases} 2\pi_B^N(\widehat{\gamma}) & \text{for } \widehat{\gamma} \in [0, \gamma_B^{i1}) \cup (\gamma_B^{i2}, 1) \\ 2\pi_B^D(\widehat{\gamma}) & \text{for } \widehat{\gamma} \in [\gamma_B^{i1}, \gamma_B^{i2}] \end{cases} \quad (16)$$

Finally, with high integration costs, there exists a critical value of product substitutability, $\gamma_B^h \in (0, 1)$, such that vertical integration occurs only for $\gamma \in [0, \gamma_B^h)$ (see Figure 2). Since vertical integration and foreclosure do not occur for $\gamma = 1$, the incentive for differentiation equals:

$$\widehat{k}_B^h(\widehat{\gamma}) = \begin{cases} 2[\pi_B^N(\widehat{\gamma}) - \pi_B^D(1)] & \text{for } \widehat{\gamma} \in [0, \gamma_B^h) \\ 2[\pi_B^D(\widehat{\gamma}) - \pi_B^D(1)] & \text{for } \widehat{\gamma} \in [\gamma_B^h, 1]. \end{cases} \quad (17)$$

Like in model *A*, also in model *B* the perspective of vertical integration generally strengthens the incentive for differentiation when the integration cost is small and vertical integration is unavoidable. The interpretation is the same as in model *A*. In model *B*, however, the strategic incentive of deterring integration reduces the downstream firms' incentive to achieve high degrees of differentiation when the integration cost is high, and creates an incentive to limit differentiation to intermediate degrees when the integration cost is intermediate.

PROPOSITION 3. *In model B:*

- i) For any $\widehat{\gamma} \in [0, 0.81)$, the incentive for product differentiation is stronger with small and intermediate than with prohibitive integration costs.*
- ii) With intermediate integration costs and $\widehat{\gamma} \in [\gamma_B^{i1}, \gamma_B^{i2}]$, the possibility of deterring integration increases the incentive for differentiation relative to all other cases.*
- iii) With high integration costs and $\widehat{\gamma} \in [0, \gamma_B^h)$, the possibility of deterring vertical integration by not differentiating products reduces the incentive for differentiation relative to all other cases.*

Figures 4 and 5 illustrate proposition 3. In Figure 4, the *dashed line* represent the incentive for differentiation in the benchmark case of prohibitive integration costs, the *thin-solid line* the incentive for differentiation in the case of small integration costs. The incentive for differentiation with intermediate integration costs follows the *thin-solid line* in the two disjoint intervals $[0, \gamma_B^{i1})$ and $(\gamma_B^{i2}, 1)$, and the *thick-solid line* in the intermediate interval $[\gamma_B^{i1}, \gamma_B^{i2}]$. Figure 5 illustrates the case of high integration costs: the incentive for differentiation follows the *dashed line* of the benchmark case for $\widehat{\gamma} \in [\gamma_B^h, 1)$, and the *thick-dashed line* for $\widehat{\gamma} \in [0, \gamma_B^h)$.

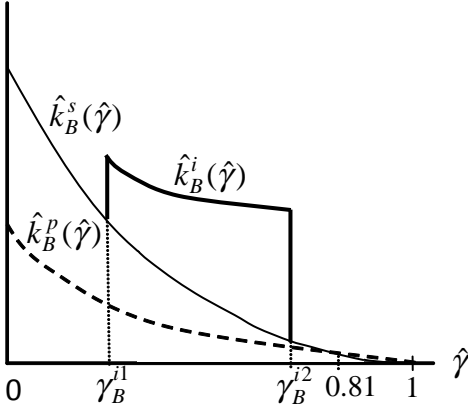


Figure 4

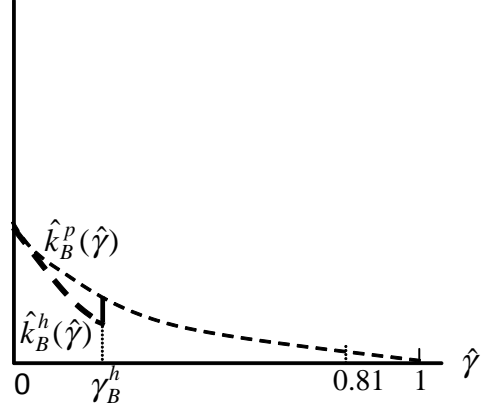


Figure 5

Summarizing, both the nature of product differentiation and its effect on the upstream monopolist's incentive for vertical integration are crucial to determine how a prospective threat of vertical integration affects downstream product differentiation. If the integration cost is small and vertical integration is unavoidable, then, irrespective of the nature of product differentiation, the downstream firms' incentive to avoid market foreclosure and soften the tougher competitive pressure of the integrated firm generally strengthens their incentive for differentiation (parts (i) in propositions 2 and 3). Higher integration costs create the strategic incentive of deterring integration. In model *A*, where differentiation does not enlarge the final product market, the incentive for integration monotonically decreases with differentiation, so that the strategic incentive to deter integration always strengthens the incentive to differentiate products (part (ii) of proposition 2). In model *B*, on the contrary, the incentive for integration is strongest when products are strongly differentiated, since differentiation enlarges the downstream market. With intermediate integration costs, vertical integration occurs either for weak or for strong degrees of differentiation, so that the incentive to deter integration fosters the downstream firms' incentive to target intermediate degrees of differentiation (part (ii) of proposition 3). With high integration costs, vertical integration occurs only when products are strongly differentiated, so that the incentive to deter integration decreases the downstream firms' incentive to achieve high degrees of differentiation (part (iii) of proposition 3).

7. NEW SOCIAL COSTS OF VERTICAL INTEGRATION

Our analysis reveals new social costs of vertical integration through its effects on product differentiation. In model *A*, where product differentiation merely yields the anticompetitive effects of market segmentation without generating additional social value, a prospective threat of vertical integration almost always fosters the downstream firms' incentive to differentiate products (proposition 2). If we reinterpret the benchmark case of prohibitive integration costs — where the threat of integration is ineffective — as the case of a severe antitrust policy which prohibits vertical integration, we can argue that a lenient antitrust policy towards vertical integration in mature industries — where product differentiation is likely to be only anticompetitive — is likely to impose a welfare cost on society in terms of anticompetitive market segmentation. Thus, instead of market foreclosure, the anticompetitive effect of vertical integration here takes the form of market segmentation. This social cost is the only welfare effect a threat of vertical integration exerts in model *A* when product differentiation prevents integration.

PROPOSITION 4. In model A, vertical integration deterrence through product differentiation always decreases social welfare.

The proof is straightforward. Part (ii) of proposition 2 assures that the possibility of preventing vertical integration strengthens the incentive for differentiation relative to the benchmark case where the integration threat is ineffective. Assume that this stronger incentive actually translates into an increase in the degree of differentiation which deters integration. Then, irrespective of any possible extra cost of differentiation, social welfare decreases. Indeed, integration deterrence implies that the industry remains vertically separated, and part (i) of lemma 2 assures that, given the vertical structure of the industry, more differentiation reduces total surplus in model *A*.

The case of small integration costs — where vertical integration occurs irrespective of product differentiation — is of particular interest to ascertain whether the social cost of vertical integration in terms of anticompetitive market segmentation can be so strong to offset the social benefit of vertical integration, namely the elimination of double marginalization in one segment of the downstream market. To fix ideas, let us assume the following simple differentiation technology and choice:

ASSUMPTION 1. *By paying a fixed differentiation cost, k , the downstream firms can cooperatively reduce γ from 1 to a given value $\hat{\gamma} < 1$. Products remain perfect substitutes otherwise.*²⁶

Let $\hat{\gamma}$ range in the interval $[0, 0.98]$, so that, by part (i) of proposition 2, the incentive for differentiation is stronger with small than with prohibitive integration costs: $\hat{k}_A^p(\hat{\gamma}) < \hat{k}_A^s(\hat{\gamma})$. Then, if $k \in [\hat{k}_A^p(\hat{\gamma}), \hat{k}_A^s(\hat{\gamma})]$, products would be differentiated in the case of small integration costs but not in the benchmark case, and the difference in welfare between the two cases would equal:

$$\Delta W_A(\hat{\gamma}) \big|_p^s = TS_A^{vi}(\hat{\gamma}) - TS_A^{vs}(1) - k - E_A^s. \quad (18)$$

In equation (18), $TS_A^{vi}(\hat{\gamma})$ measures the industry total surplus with small integration costs – where vertical integration occurs, vi , and $\gamma = \hat{\gamma}$ – while $TS_A^{vs}(1)$ is the industry total surplus with prohibitive integration costs – where the industry is vertically separated, vs , and $\gamma = 1$. E_A^s denotes the small integration cost: $E_A^s \in [0, S_A(0)]$, by Corollary 1.

Figure 6 plots $\Delta W_A(\hat{\gamma}) \big|_p^s$ in the two extreme cases where: 1) the integration and the differentiation costs are both at their respective lowest values $E_A^s = 0$ and $k = \hat{k}_A^p(\hat{\gamma})$, the *thin curve*; 2) the integration and the differentiation costs are both at their respective highest values $E_A^s = S_A(0)$ and $k = \hat{k}_A^s(\hat{\gamma})$, the *thick curve*.²⁷ Since $\Delta W_A(\hat{\gamma}) \big|_p^s$ steadily decreases with $k + E_A^s$, all intermediate cases can be easily traced out in the band between the two lines. The grey area emphasizes the region where $\Delta W_A(\hat{\gamma}) \big|_p^s < 0$: provided that the integration and differentiation costs are not too small, sufficiently high degrees of product differentiation make social welfare lower with than without integration and differentiation. It is worth noting that the effect of vertical integration on market segmentation is crucial for this result: for any *given* degree of differentiation, no integration cost level in the range under consideration would suffice to make social welfare lower with than without vertical integration.

²⁶ A discrete investment choice may not be too inappropriate to model, say, the decision to undertake a publicity campaign. Nevertheless, the fact that the overall welfare effect of vertical integration results negative in a sizeable region of all relevant parameters ($\hat{\gamma}$, E and k), suggests that this result should generalize to smoother differentiation technologies.

²⁷ More details on the analytical derivation of these two boundaries are provided in appendix 1.

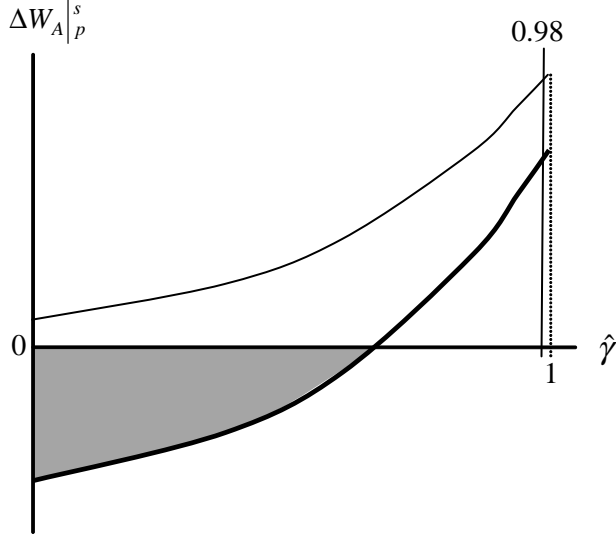


Figure 6

In model B , product differentiation directly generates social value to consumers. In both vertical structures of the industry, this social benefit dominates the anti-competitive effect of market segmentation, so that the total surplus monotonically increases with differentiation (part (ii) of lemma 2). However, the threat of integration may either weakens or strengthens the incentive for differentiation. Specifically, with high integration costs, vertical integration can be prevented by not-differentiating products, which reduces the downstream firms' incentive for differentiation relative to the benchmark case (part (iii) of proposition 3). In this case, the new social cost of vertical integration consists of less socially valuable product differentiation. Integration deterrence would preserve the vertical structure of the industry, so that less differentiation would decrease industry surplus relative to the benchmark case. The welfare comparison must however account for possible cost savings due to less differentiation. Resorting to Assumption 1, we find that the social cost of less differentiation always dominates the social benefit of lower differentiation costs.

PROPOSITION 5. *In model B , vertical integration deterrence through less differentiation decreases social welfare.*

With intermediate integration costs, the possibility of preventing integration strengthens the incentive to target intermediate degrees of differentiation (part (ii)

of proposition 3). Relative to the benchmark case, this may translate either into weaker or into stronger differentiation.²⁸ Proposition 5 still applies if integration is deterred by less differentiation. Integration deterrence through more differentiation, on the contrary, generally increases social welfare.²⁹

Finally, vertical integration always increases social welfare when the integration costs are small in model B . To see this, notice first that, for any *given* degree of differentiation, the elimination of double marginalization makes vertical integration welfare enhancing when the integration cost is small.³⁰ Resorting to assumption 1, consider next the situations where vertical integration alters the degree of differentiation relative to the benchmark case. Assume first that $\hat{\gamma} \in [0, 0.81)$, so that, by part (i) of proposition 3, the downstream firms' incentive for differentiation is stronger with small than with prohibitive integration costs: $\hat{k}_B^p(\hat{\gamma}) < \hat{k}_B^s(\hat{\gamma})$. Then, if $k \in [\hat{k}_B^p(\hat{\gamma}), \hat{k}_B^s(\hat{\gamma})]$, products would be differentiated in the case of small integration costs but not in the benchmark case. The difference in social welfare between the two cases would be given by the analogue of equation (18) for model B :

$$\Delta W_B(\hat{\gamma}) \big|_p^s = TS_B^{vi}(\hat{\gamma}) - TS_B^{vs}(1) - k - E_B^s, \quad (19)$$

where $E_B^s \in [0, S_B(\tilde{\gamma}_B)]$ (by Corollary 2). The *thick curve* in Figure 7 shows that $\Delta W_B(\hat{\gamma}) \big|_p^s$ is strictly positive on $[0, 0.81]$ even if the integration and the differentiation costs are both set at the upper extremes of their respective ranges, $E_B^s = S_B(\tilde{\gamma}_B)$ and $k = \hat{k}_B^s(\hat{\gamma})$. The same is *a fortiori* true for lower values of E_B^s and k .

²⁸For instance, weaker differentiation would result if the downstream firms could choose the degree of differentiation up to a maximum level lying outside the interval where integration is deterred, and the differentiation cost is not too steep in the degree of differentiation.

²⁹More precisely, under assumption 1, vertical integration deterrence through stronger differentiation increases social welfare except in the special case where high differentiation costs are required to achieve mild degrees of differentiation which are nevertheless sufficient to deter integration. In this case, the incentive to deter integration generates overinvestment in differentiation, and lower welfare, relative to the benchmark case: although differentiation adds little value to society, its strategic value (integration deterrence) induces the downstream firms to invest in differentiation despite the high differentiation cost. The proof of this case is available on request.

³⁰This is formally shown in appendix 1, where we also provide the analytical derivation of the results illustrated in Figure 7 below.

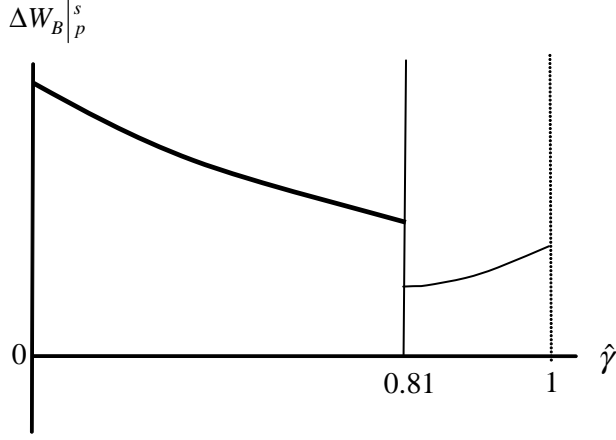


Figure 7

Assume now that $\hat{\gamma} \in (0.81, 1]$, so that the downstream firms' incentive for differentiation is stronger with prohibitive than with small integration costs: $\hat{k}_B^p(\hat{\gamma}) > \hat{k}_B^s(\hat{\gamma})$. Then, if $k \in [\hat{k}_B^s(\hat{\gamma}), \hat{k}_B^p(\hat{\gamma})]$, products would be differentiated in the benchmark case but not in the case of small integration costs, and the difference in welfare between the two cases would equal:

$$\Delta W_B(\hat{\gamma})|_p^s = TS_B^{vi}(1) - TS_B^{vs}(\hat{\gamma}) + k - E_B^s. \quad (20)$$

The *thin curve* in Figure 7 shows that $\Delta W_B(\hat{\gamma})|_p^s$ is positive on $(0.81, 1]$ even if the integration cost is set at its upper extreme, $E_B^s = S_B(\tilde{\gamma}_B)$, and the differentiation cost is set at its lower extreme, $k = \hat{k}_B^s(\hat{\gamma})$. The same is *a fortiori* true for lower values of E_B^s and higher values of k .

8. CONCLUSIONS

Market foreclosure through vertical integration is a major concern to antitrust authorities. In this paper, we have shown that when final goods producers can strategically choose the characteristics of their products, vertical foreclosure may not be a concern, but vertical integration can yield other kinds of social costs: anticompetitive market segmentation or under-investment in socially valuable differentiation. Specifically, whilst horizontal product differentiation eliminates market foreclosure, it affects the profitability of vertical integration and social welfare in a way that crucially depends on the *nature* of differentiation. When horizontal

differentiation merely consists of market segmentation, which is arguably the case in mature industries, it decreases both the profitability of vertical integration and social welfare. This incentivizes the downstream firms to increase product differentiation either to deter integration or to limit the amount of downstream profits the upstream firm can reap through integration, which imposes on society the welfare cost of anticompetitive market segmentation. When horizontal differentiation creates positive value to consumers, which is arguably the case in innovative industries, it increases social welfare but exerts ambiguous effects on the profitability of vertical integration. This may either increase or decrease the downstream firms' incentive to differentiate products, with ambiguous effects on social welfare. Social welfare surely decreases when vertical integration is deterred through less differentiation (which requires high integration costs and strong differentiation opportunities), whereas it surely increases if vertical integration can never be prevented (small integration costs). Therefore, besides evidencing new social costs of vertical integration, our findings also suggest a more cautious policy attitude towards vertical mergers in innovative industries than in mature industries. Finally, the notion of consumers' preference for horizontal differentiation and the unified framework we have developed to combine alternative models of differentiation may be of interest for applications to other industrial economics and policy issues.

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Appendix 1

Proof of lemma 1. *Part (i).* Differentiating π_m^D (in equation (4)) with respect to γ , yields:

$$\frac{\partial \pi_m^D}{\partial \gamma} = -\frac{\pi_m^D}{\delta_m(2\delta_m + \gamma)} \left(-\gamma \frac{\partial \delta_m}{\partial \gamma} + 2\delta_m \left(1 + \frac{\partial \delta_m}{\partial \gamma} \right) \right) < 0,$$

where the inequality follows from: $\pi_m^D > 0$ (apparent in (4)), $\delta_m \geq 1$, and $\frac{\partial \delta_m}{\partial \gamma} = -1$ (model A) or $\frac{\partial \delta_m}{\partial \gamma} = 0$ (model B).

Differentiating π_m^N (in equation (6)) with respect to γ , yields:

$$\frac{\partial \pi_m^N}{\partial \gamma} = \pi_m^N \frac{\frac{\partial \delta_m}{\partial \gamma} (3\gamma^3 - 9\gamma^2 \delta_m + 24\gamma \delta_m^2 - 8\delta_m^3) + 2\delta_m (6\gamma \delta_m - 3\gamma^2 - 8\delta_m^2)}{\delta_m(\delta_m - \gamma)(8\delta_m^2 - 3\gamma^2)}.$$

In model A ($\delta_m = 2 - \gamma$), the expression above specializes as:

$$\frac{\partial \pi_A^N}{\partial \gamma} = -\frac{64(1 - \gamma)(64 - 48\gamma - 6\gamma^2 + 10\gamma^3)}{2(5\gamma^2 - 32\gamma + 32)^3} \left(\frac{a}{2} \right)^2,$$

which is strictly negative for $\gamma \in [0, 1)$ and zero for $\gamma = 1$.

In model B ($\delta_m = 1$), we have:

$$\frac{\partial \pi_B^N}{\partial \gamma} = -\frac{32(1 - \gamma)(8 - 6\gamma + 3\gamma^2)}{(8 - 3\gamma^2)^3} \left(\frac{a}{2} \right)^2,$$

which is strictly negative for $\gamma \in [0, 1)$ and zero for $\gamma = 1$.

Part (ii). From equations (4) and (6), $\pi_m^D \geq \pi_m^N$ is equivalent to:

$$\gamma^2 + 4\delta_m \gamma \geq 0.$$

$\delta_m \geq 1$ guarantees that the inequality is strict for any $\gamma \in (0, 1]$. Equality clearly holds for $\gamma = 0$.

Part (iii). It follows immediately from the expression of π_m^N in equation (6), once noticed that $\gamma \in [0, 1]$ and $\delta_m \geq 1$.

Proof of lemma 2. In all cases except under vertical integration in model A, the statement follows directly from Results 1 and 2 proved in appendix 2.³¹ For the case of vertical integration in model A, we calculate:

$$TS_A^{vi} = CS_A^{vi} + \Pi_A^{vi} = \frac{339\gamma^4 - 3064\gamma^3 + 9200\gamma^2 - 11264\gamma + 4864}{2(2 - \gamma)(5\gamma^2 - 32\gamma + 32)^2} \left(\frac{a}{2} \right)^2$$

³¹ These results show that both consumer surplus and industry profit: 1) monotonically decrease with differentiation in the vertical separation equilibrium of model A; 2) monotonically increase with differentiation in model B irrespective of the vertical structure of the industry.

Differentiating in γ yields:

$$\frac{\partial TS_A^{vi}}{\partial \gamma} = \frac{1695\gamma^6 - 19792\gamma^5 + 92704\gamma^4 - 224704\gamma^3 + 297728\gamma^2 - 204800\gamma + 57344}{2(2-\gamma)^2(5\gamma^2 - 32\gamma + 32)^3} > 0,$$

where the inequality follows from the fact that both polynomials at the numerator and at the denominator of the expression above are strictly positive on $[0, 1]$ (calculations with *Mathematica*).

Proof of proposition 1. Using (4) and (6), we calculate:

$$S_m \equiv \pi_m^I - (\pi_m^U + \pi_m^D) = \left(\frac{\gamma^4 + 2\gamma^3\delta_m - \gamma^2\delta_m^2 + 8\delta_m^4}{\delta_m(8\delta_m^2 - 3\gamma^2)(2\delta_m + \gamma)^2} \right) \left(\frac{a}{2} \right)^2.$$

Part (i). In model A ($\delta_m = 2 - \gamma$), the expression above specializes as:

$$S_A = 2 \frac{3\gamma^4 - 28\gamma^3 + 94\gamma^2 - 128\gamma + 64}{(4 - \gamma)^2(-5\gamma^3 + 42\gamma^2 - 96\gamma + 64)} \left(\frac{a}{2} \right)^2.$$

Differentiating in γ yields:

$$\frac{\partial S_A}{\partial \gamma} = 2 \frac{-15\gamma^7 + 220\gamma^6 - 1290\gamma^5 + 3792\gamma^4 - 5504\gamma^3 + 3072\gamma^2}{(4 - \gamma)^3(5\gamma^3 - 42\gamma^2 + 96\gamma - 64)^2}$$

The sign of the derivative above equals the sign of the polynomial at the numerator (the denominator is strictly positive on $[0, 1]$). The latter has only one real root in $[0, 1]$ at $\gamma = 0$, and it is positive for $\gamma = 1$. Therefore, the derivative is positive on $(0, 1]$. Finally, $S_A(0) = 0.125 \left(\frac{a}{2} \right)^2 > 0$, which implies that S_A is positive on $[0, 1]$.

Part (ii). In model B ($\delta_m = 1$), we have:

$$S_B = \frac{8 - \gamma^2 + 2\gamma^3 + \gamma^4}{(8 - 3\gamma^2)(2 + \gamma)^2} \left(\frac{a}{2} \right)^2.$$

Differentiating in γ yields:

$$\frac{\partial S_B}{\partial \gamma} = 2 \frac{-3\gamma^5 - \gamma^4 + 40\gamma^3 + 96\gamma^2 + 32\gamma - 64}{(8 - 3\gamma^2)^2(2 + \gamma)^3} \left(\frac{a}{2} \right)^2.$$

The sign of the derivative above equals the sign of the polynomial at the numerator (the denominator is strictly positive on $[0, 1]$). The latter has only one real root in $[0, 1]$, $\tilde{\gamma}_B \simeq 0.61037$ (calculations with *Mathematica*), and it is negative for $\gamma = 0$, positive for $\gamma = 1$. Therefore, the derivative is negative on $[0, \tilde{\gamma}_B)$, positive on $(\tilde{\gamma}_B, 1]$. Finally:

$$S_B(0) = \frac{1}{4} \left(\frac{a}{2} \right)^2 > S_B(1) = \frac{10}{45} \left(\frac{a}{2} \right)^2 > S_B(\tilde{\gamma}_1) \simeq 0.1753 \left(\frac{a}{2} \right)^2 > 0.$$

Proof of lemma 3. From equation (8), the upstream monopolist's payoff is negative for $P > \pi_m^D + (S_m - E)$, and hence he will never accept such price offers. Assume that condition (7) holds, which implies $\pi_m^D < \pi_m^D + (S_m - E)$. In this case, any downstream firm, say firm i , will find it optimal to:

- *Submit the offer $P_i = \pi_m^D + (S_m - E)$ if the rival (firm j) either does not submit any offer, or if it submits an offer $P_j > \pi_m^D + (S_m - E)$ which would be rejected by the upstream firm.* Firm i 's offer would be accepted, guaranteeing firm i a payoff of $\pi_m^D + (S_m - E)$. This payoff is higher than the payoff offered by all possible alternatives, namely: 1) making any other offer $P_i < \pi_m^D + (S_m - E)$, which would be accepted by the upstream firm; 2) not making any offer or making an unacceptable offer, $P_i > \pi_m^D + (S_m - E)$, which would lead to vertical separation and a payoff of $\pi_m^D < \pi_m^D + (S - E)$ for firm i .
- *Undercut by an arbitrary small amount any rival's offer $\pi_m^N < P_j \leq \pi_m^D + (S_m - E)$,³² which would be accepted by the upstream firm.* Bidding above the rival would leave firm i in the position of the independent firm under vertical integration, with payoff π_m^N . Matching the rival's offer would give firm i equal probability of being independent or integrated, with expected profit $\frac{1}{2}(P_j + \pi_m^N)$. Undercutting the rival's offer by an arbitrarily small amount $\epsilon > 0$, would guarantee firm i to be integrated at the price and profit $P_j - \epsilon > \frac{1}{2}(P_j + \pi_m^N) > \pi_m^N$.
- *Submit, indifferently, any offer $P_i \geq \pi_m^N$ if the rival bids $P_j = \pi_m^N$.* Firm i 's profit would equal π_m^N in any case, either because it will be the independent firm under vertical integration (for $P_i > \pi_m^N$) or because it will have equal probability to be independent or integrated at price π_m^N (for $P_i = \pi_m^N$).

It follows that $(P_i = \pi_m^N, P_j = \pi_m^N)$ are the unique Nash equilibrium offers. Since the upstream firm's payoff is positive at the takeover price $P^* = \pi_m^N$, he accepts the offer and vertically integrates with one of the two downstream firms randomly selected.

Violation of condition (7) implies $\pi_m^D + (S_m - E) < \pi_m^D$. In this case, there are two possible equilibrium outcomes. The pair of offers (π_m^N, π_m^N) still identify a Nash equilibrium. However, we select the alternative Nash equilibrium outcome: each downstream firm submits an offer which is unacceptable by the upstream monopolist (any offer $P > \pi_m^D + (S_m - E)$ will do), and latter rejects. Clearly, no downstream firm has an incentive to deviate from its equilibrium strategy by making an acceptable offer ($P \leq \pi_m^D + (S_m - E)$), since the deviant firm would be integrated at a price and profit strictly lower than its equilibrium payoff, π_m^D . Therefore, vertical integration does not occur.

From the viewpoint of the downstream firms (the only active players at the bidding stage of the integration game), the (π_m^N, π_m^N) equilibrium is strictly Pareto dominated by the equilibrium adopted in the text. Pareto dominance is therefore a first, but not the only, reason to consider the equilibrium adopted in the statement of the lemma more likely. Indeed, since the equilibrium strategy of each player in the adopted equilibrium (i.e., submit an unacceptable offer) weakly dominates the

³²Recall that, by part (ii) of lemma 1, $\pi_m^N < \pi_m^D$ for any $\gamma \in (0, 1]$.

player's equilibrium strategy in the (π_m^N, π_m^N) equilibrium, the second equilibrium would not survive to other selection criteria, like trembling hand perfection and risk dominance.

Proof of proposition 2. *Part (i).* From equations (12), (4), and $\delta_m = 2 - \gamma$, we get:

$$\widehat{k}_A^p(\widehat{\gamma}) = \frac{4 - 2\widehat{\gamma} - 2\widehat{\gamma}^2}{9(4 - \widehat{\gamma})^2} \left(\frac{a}{2}\right)^2.$$

Similarly, from equation (13), (6), and $\delta_m = 2 - \gamma$, we have:

$$\widehat{k}_A^s(\widehat{\gamma}) = \frac{128(1 - \widehat{\gamma})^2(2 - \widehat{\gamma})}{(5\widehat{\gamma}^2 - 32\widehat{\gamma} + 32)^2} \left(\frac{a}{2}\right)^2.$$

From the two expressions above, $\widehat{k}_A^s(\widehat{\gamma}) > \widehat{k}_A^p(\widehat{\gamma})$ is equivalent to:

$$25\widehat{\gamma}^6 - 871\widehat{\gamma}^5 + 7886\widehat{\gamma}^4 - 30592\widehat{\gamma}^3 + 57344\widehat{\gamma}^2 - 50176\widehat{\gamma} + 16384 > 0.$$

The polynomial on the LHS of the inequality is positive for $\widehat{\gamma} = 0$, and it has only two real roots in the relevant range $[0, 1]$: $\widehat{\gamma} \simeq 0.983$ and $\widehat{\gamma} = 1$. This implies that the inequality holds for $\widehat{\gamma} \in [0, 0.983)$, and it reverses for $\widehat{\gamma} \in (0.983, 1)$.

As for intermediate or high integration costs, we prove in part (ii) below that $\widehat{k}_A^{ih}(\widehat{\gamma}) > \widehat{k}_A^p(\widehat{\gamma})$ for $\widehat{\gamma} \in [0, \gamma_A^{ih}]$. In the interval $\widehat{\gamma} \in (\gamma_A^{ih}, 1)$, $\widehat{k}_A^{ih}(\widehat{\gamma}) = \widehat{k}_A^s(\widehat{\gamma})$ (by equations (13) and (14)), so that the analysis above applies with the following qualification. If $\gamma_A^{ih} < 0.983$, then: $\widehat{k}_A^{ih}(\widehat{\gamma}) > \widehat{k}_A^p(\widehat{\gamma})$ for $\widehat{\gamma} \in (\gamma_A^{ih}, 0.983)$, $\widehat{k}_A^{ih}(\widehat{\gamma}) \leq \widehat{k}_A^p(\widehat{\gamma})$ for $\widehat{\gamma} \in [0.983, 1)$. If $\gamma_A^{ih} > 0.983$, then $\widehat{k}_A^{ih}(\widehat{\gamma}) < \widehat{k}_A^p(\widehat{\gamma})$ only in the smaller interval $(\gamma_A^{ih}, 1)$.

Part (ii). If $\widehat{\gamma} \in [0, \gamma_A^{ih}]$, equation (14) gives:

$$\widehat{k}_A^{ih}(\widehat{\gamma}) = 2\pi_A^D(\widehat{\gamma}).$$

Using equation (12), we calculate:

$$\widehat{k}_A^{ih}(\widehat{\gamma}) - \widehat{k}_A^p(\widehat{\gamma}) = 2\pi_A^D(\widehat{\gamma}) - 2[\pi_A^D(\widehat{\gamma}) - \pi_A^D(1)] = 2\pi_A^D(1) > 0.$$

Similarly, using equation (13), we calculate:

$$\widehat{k}_A^{ih}(\widehat{\gamma}) - \widehat{k}_A^s(\widehat{\gamma}) = 2\pi_A^D(\widehat{\gamma}) - 2\pi_A^N(\widehat{\gamma}),$$

which, by part (ii) of lemma 1, is strictly positive for $\widehat{\gamma} \in (0, 1]$.

Proof of proposition 3. *Part (i).* Equations (12), (4), and $\delta_m = 1$ yield:

$$\widehat{k}_B^p(\widehat{\gamma}) = \frac{10 - 8\widehat{\gamma} - 2\widehat{\gamma}^2}{9(2 + \widehat{\gamma})^2} \left(\frac{a}{2}\right)^2.$$

Similarly, equations (15), (6), and $\delta_m = 1$ imply:

$$\widehat{k}_B^s(\widehat{\gamma}) = \frac{32(1 - \widehat{\gamma})^2}{(8 - 3\widehat{\gamma}^2)^2} \left(\frac{a}{2}\right)^2.$$

From the two expressions above, $\widehat{k}_B^s(\widehat{\gamma}) > \widehat{k}_B^p(\widehat{\gamma})$ is equivalent to:

$$9\widehat{\gamma}^6 + 36\widehat{\gamma}^5 + 51\widehat{\gamma}^4 + 96\widehat{\gamma}^3 - 128\widehat{\gamma}^2 - 320\widehat{\gamma} + 256 > 0.$$

The polynomial on the LHS of the inequality is positive for $\widehat{\gamma} = 0$, and it has only two real roots on $[0, 1]$: $\widehat{\gamma} \simeq 0.816$ and $\widehat{\gamma} = 1$. Therefore the inequality holds for $\widehat{\gamma} \in [0, 0.816)$, it reverses for $\widehat{\gamma} \in (0.816, 1)$.

As for the case of intermediate integration costs, we prove in part (ii) below that $\widehat{k}_B^i(\widehat{\gamma}) > \widehat{k}_B^p(\widehat{\gamma})$ for $\widehat{\gamma} \in [\gamma_B^{i1}, \gamma_B^{i2}]$. For $\widehat{\gamma} \in [0, \gamma_B^{i1}) \cup (\gamma_1^{i2}, 1)$, $\widehat{k}_B^i(\widehat{\gamma}) = \widehat{k}_B^s(\widehat{\gamma})$, and the analysis above applies with the following qualifications: 1) since $\gamma_B^{i1} < \widetilde{\gamma}_B < 0.816$ (see the proof of proposition 1), then $\widehat{k}_B^i(\widehat{\gamma}) > \widehat{k}_B^p(\widehat{\gamma})$ for $\widehat{\gamma} \in [0, \gamma_B^{i1})$; 2) if $\gamma_1^{i2} < 0.816$, then $\widehat{k}_B^i(\widehat{\gamma}) > \widehat{k}_B^p(\widehat{\gamma})$ for $\widehat{\gamma} \in (\gamma_B^{i1}, 0.816)$, $\widehat{k}_B^i(\widehat{\gamma}) \leq \widehat{k}_B^p(\widehat{\gamma})$ for $\widehat{\gamma} \in [0.816, 1)$; 3) if $\gamma_1^{i2} > 0.816$, then $\widehat{k}_B^i(\widehat{\gamma}) \leq \widehat{k}_B^p(\widehat{\gamma})$ only in the smaller interval $(\gamma_1^{i2}, 1)$.

Part (ii). For $\widehat{\gamma} \in [\gamma_1^{i1}, \gamma_1^{i2}]$, equations (12) and (16) imply:

$$\widehat{k}_B^i(\widehat{\gamma}) - \widehat{k}_B^p(\widehat{\gamma}) = 2\pi_B^D(\widehat{\gamma}) - 2[\pi_B^D(\widehat{\gamma}) - \pi_B^D(1)] = 2\pi_B^D(1) > 0.$$

Similarly, equations (15) and (16) yield:

$$\widehat{k}_B^i(\widehat{\gamma}) - \widehat{k}_B^s(\widehat{\gamma}) = 2\pi_B^D(\widehat{\gamma}) - 2\pi_B^N(\widehat{\gamma}),$$

which, by part (ii) of lemma 1, is strictly positive for $\widehat{\gamma} \in (0, 1]$.

Part (iii). For $\widehat{\gamma} \in [0, \gamma_1^h]$, equations (12) and (17) imply:

$$\widehat{k}_B^h(\widehat{\gamma}) - \widehat{k}_B^p(\widehat{\gamma}) = 2\pi_B^N(\widehat{\gamma}) - 2\pi_B^D(\widehat{\gamma}),$$

which, by part (ii) of lemma 1, is strictly negative for $\widehat{\gamma} \in (0, 1]$.

Similarly, from equations (15) and (17), we have:

$$\widehat{k}_B^h(\widehat{\gamma}) - \widehat{k}_B^s(\widehat{\gamma}) = -2\pi_B^D(1) < 0.$$

For $\widehat{\gamma} \in (\gamma_1^h, 1)$, $\widehat{k}_B^h(\widehat{\gamma}) = \widehat{k}_B^p(\widehat{\gamma})$ (by equations (12) and (17)).

Proof of proposition 5. Assume that the integration cost is high, $E \in (S_B(1), S_B(0)]$ (by Corollary 2), and $\widehat{\gamma} \in [0, \gamma_B^h)$. Then, part (iii) of proposition 3 assures that the incentive for differentiation is weaker in the case of high integration costs than in the benchmark case:

$$k_B^h(\widehat{\gamma}) < k_B^p(\widehat{\gamma}).$$

Assume that the fixed differentiation cost, k , satisfies $k_B^h(\widehat{\gamma}) \leq k \leq k_B^p(\widehat{\gamma})$. Then, products will not be differentiated ($\gamma = 1$) and vertical integration will be prevented

in the case of high integration costs, while products will be differentiated ($\gamma = \hat{\gamma}$) in the benchmark case. Total surplus will be lower in the former case (by part (ii) of lemma 2), but society will bear the differentiation cost k . Formally, the difference in social welfare between two cases will equal:

$$\Delta W_B|_p^h = TS_B^{vs}(1) - TS_B^{vs}(\hat{\gamma}) + k.$$

We show that $\Delta W_B|_p^h$ is negative when $k = k_B^p(\hat{\gamma}) = 2[\pi_{m_1}^D(\hat{\gamma}) - \pi_{m_1}^D(1)]$ (see equation (12)). This clearly suffices to prove that $\Delta W_B|_p^h$ is negative for any k in the relevant range. Recall that:

$$TS_B^{vs}(\gamma) = CS_B^{vs}(\gamma) + \Pi_B^{vs}(\gamma) = CS_B^{vs}(\gamma) + \pi_B^U(\gamma) + 2\pi_B^D(\gamma),$$

where $CS_B^{vs}(\gamma)$ and $\Pi_B^{vs}(\gamma)$ denotes the consumer surplus and the industry profit under vertical separation, respectively. For $k = 2[\pi_B^D(\hat{\gamma}) - \pi_B^D(1)]$, we calculate:

$$\begin{aligned} \Delta W_B|_p^h &= CS_B^{vs}(1) + \pi_B^U(1) + 2\pi_B^D(1) - \\ &\quad - CS_B^{vs}(\hat{\gamma}) - \pi_B^U(\hat{\gamma}) - 2\pi_B^D(\hat{\gamma}) + 2[\pi_B^D(\hat{\gamma}) - \pi_B^D(1)] \\ &= CS_B^{vs}(1) - CS_B^{vs}(\hat{\gamma}) + \pi_B^U(1) - \pi_B^U(\hat{\gamma}) < 0, \end{aligned}$$

where the inequality follows from the fact that, in the vertical separation equilibrium of model B , both the consumer surplus (Result 1 in appendix 2) and the upstream profit (equation (4) with $\delta_m = 1$) monotonically increase with differentiation.

Welfare comparison of small and prohibitive integration costs in model A (Figure 6). Total surplus, in the vertical integration and the vertical separation equilibria of model A , is calculated using the corresponding expressions for industry profits and consumer surplus derived in appendix 2. We can then expand equation (18) as:

$$\Delta W_A(\hat{\gamma})|_p^s = \frac{500\hat{\gamma}^5 - 4349\hat{\gamma}^4 + 12104\hat{\gamma}^3 - 11920\hat{\gamma}^2 + 1024\hat{\gamma} + 2816}{18(2 - \hat{\gamma})(5\hat{\gamma}^2 - 32\hat{\gamma} + 32)^2} \left(\frac{a}{2}\right)^2 - k - E_A^s.$$

Setting $E_A^s = 0$ and $k = \hat{k}_A^p(\hat{\gamma}) = \frac{2(2-\gamma^2-\gamma)}{9(\gamma-4)^2} \left(\frac{a}{2}\right)^2$, we derive the expression of the *thin curve* in Figure 6. Similarly, setting $E_A^s = S_A(0) = 0.125 \left(\frac{a}{2}\right)^2$ and $k = \hat{k}_A^s(\hat{\gamma}) = 2 \frac{64(2-\hat{\gamma})(1-\hat{\gamma})^2}{(5\hat{\gamma}^2 - 32\hat{\gamma} + 32)^2} \left(\frac{a}{2}\right)^2$ we obtain the expression of the *thick curve* in Figure 6 (without loss of generality, $\frac{a}{2}$ is normalized to 1 for both cases).

Welfare comparison of small and prohibitive integration costs in model B (Figure 7). Consider first the welfare comparison between small and prohibitive integration costs for a *given* degree of product differentiation, γ . The difference in welfare between the two cases would be given by:

$$\Delta W_B(\gamma)|_p^s = TS_B^{vi}(\gamma) - TS_B^{vs}(\gamma) - E_B^s.$$

Total surplus, in the vertical integration and the vertical separation equilibria of model B , is calculated from the corresponding expressions for industry profits and consumer surplus derived in appendix 2. Setting the integration cost at the upper extreme of the small integration cost range of model B , $E_B^s = S_B(\tilde{\gamma}_B) \simeq 0.1753 \left(\frac{a}{2}\right)^2$, we calculate:

$$\Delta W_B(\gamma) \big|_p^s = \frac{230.25 - 25.754\gamma - 147.12\gamma^2 + 3.3152\gamma^3 + 30.207\gamma^4 + 5.3784\gamma^5 - 0.1554\gamma^6}{2(16 + 8\gamma - 6\gamma^2 - 3\gamma^3)^2} \left(\frac{a}{2}\right)^2,$$

which is strictly positive for any $\gamma \in [0, 1]$.

Consider next the case where $\hat{\gamma} \in [0, 0.81]$ and products are differentiated only with small integration costs. Setting $E_B^s = S_B(\tilde{\gamma}_B) \simeq 0.1753 \left(\frac{a}{2}\right)^2$ and $k = \hat{k}_B^s(\hat{\gamma}) = \frac{32(1-\hat{\gamma})^2}{(8-3\hat{\gamma}^2)^2} \left(\frac{a}{2}\right)^2$ in equation (19), we calculate the expressions of the *thick curve* in Figure 7:

$$\Delta W_B(\hat{\gamma}) \big|_p^s = \left(\frac{880 - 576\hat{\gamma} - 516\hat{\gamma}^2 + 540\hat{\gamma}^3 - 153\hat{\gamma}^4}{18(3\hat{\gamma}^2 - 8)^2} - 0.1753 \right) \left(\frac{a}{2}\right)^2.$$

Turn finally to the case where $\hat{\gamma} \in (0.81, 1]$ and products are differentiated only with prohibitive integration costs. Setting $E_B^s = S_B(\tilde{\gamma}_B)$ and $k = \hat{k}_B^s(\hat{\gamma})$ in equation (20), we calculate the expression of the *thin curve* in Figure 7:

$$\Delta W_B(\hat{\gamma}) \big|_p^s = \left(\frac{128 + 128\hat{\gamma} + 96\hat{\gamma}^2 - 160\hat{\gamma}^3 - 98\hat{\gamma}^4 + 54\hat{\gamma}^5 + 27\hat{\gamma}^6}{2(16 + 8\hat{\gamma} - 6\hat{\gamma}^2 - 3\hat{\gamma}^3)^2} - 0.1753 \right) \left(\frac{a}{2}\right)^2.$$

($\frac{a}{2}$ is again normalized to 1 in both cases).

Appendix 2

In this section, we characterize the effects of product differentiation on consumer surplus and industry profit in the vertical separation and the vertical integration equilibria of model A and model B .

Result 1 (Consumer Surplus). *i) In model A , the consumer surplus monotonically decreases with product differentiation in the vertical separation equilibrium, whereas, in the vertical integration equilibrium, it first increases and then decreases as the product differentiation rises from the perfect substitutes ($\gamma = 1$) to the independent goods ($\gamma = 0$) extremes. ii) In model B , the consumer surplus monotonically increases with product differentiation irrespective of the vertical structure of the industry.*

Proof. The net consumer surplus can be expressed in terms of the equilibrium quantities, \hat{q}_1 and \hat{q}_2 , as:³³

$$CS_m = \frac{1}{2} [\delta_m(\hat{q}_1^2 + \hat{q}_2^2) + 2\gamma\hat{q}_1\hat{q}_2].$$

Using the equilibrium quantities in (4), for vertical separation (*vs*), and in (6), for vertical integration (*vi*), we obtain:

$$CS_m^{vs} = \frac{\delta_m + \gamma}{(2\delta_m + \gamma)^2} \left(\frac{a}{2}\right)^2, \quad CS_m^{vi} = \frac{9\gamma^4 + 12\gamma^3\delta_m - 76\gamma^2\delta_m^2 + 80\delta_m^4}{2\delta_m(8\delta_m^2 - 3\gamma^2)^2} \left(\frac{a}{2}\right)^2.$$

Part (i). In model *A*, the expressions above specialize as:

$$CS_A^{vs} = \frac{2}{(4 - \gamma)^2} \left(\frac{a}{2}\right)^2, \quad CS_A^{vi} = \frac{\gamma^4 - 312\gamma^3 + 1616\gamma^2 - 2560\gamma + 1280}{2(2 - \gamma)(5\gamma^2 - 32\gamma + 32)^2} \left(\frac{a}{2}\right)^2.$$

CS_A^{vs} clearly increases with γ on $[0, 1]$. As for CS_A^{vi} , we calculate:

$$\frac{\partial CS_A^{vi}}{\partial \gamma} = \frac{5\gamma^6 - 3088\gamma^5 + 27136\gamma^4 - 95040\gamma^3 + 161024\gamma^2 - 131072\gamma + 40960}{2(2 - \gamma)^2(5\gamma^2 - 32\gamma + 32)^3} \left(\frac{a}{2}\right)^2.$$

The sign of $\frac{\partial CS_A^{vi}}{\partial \gamma}$ equals the sign of the polynomial at the numerator (the polynomial at the denominator is strictly positive on $[0, 1]$). The latter has a unique real root in $[0, 1]$ for $\gamma = 0.94873$ (calculations with *Mathematica*). Since $\frac{\partial CS_A^{vi}}{\partial \gamma}$ is continuous, positive for $\gamma = 0$ and negative for $\gamma = 1$, it must be positive for $\gamma \in [0, 0.94873]$, negative for $\gamma \in (0.94873, 1]$.

Part (ii). In model *B*, we have:

$$CS_B^{vs} = \frac{1 + \gamma}{(2 + \gamma)^2} \left(\frac{a}{2}\right)^2, \quad CS_B^{vi} = \frac{9\gamma^4 + 12\gamma^3 - 76\gamma^2 + 80}{2(8 - 3\gamma^2)^2} \left(\frac{a}{2}\right)^2.$$

Differentiating in γ yields:

$$\begin{aligned} \frac{\partial CS_B^{vs}}{\partial \gamma} &= -\frac{\gamma}{(2 + \gamma)^3} \left(\frac{a}{2}\right)^2 < 0 \\ \frac{\partial CS_B^{vi}}{\partial \gamma} &= -\frac{2\gamma(8 - 6\gamma + 3\gamma^2)(8 - 3\gamma)}{(8 - 3\gamma^2)^3} \left(\frac{a}{2}\right)^2 < 0. \end{aligned}$$

³³The representative consumer's optimization problem is:

$$\begin{aligned} \text{Max } U &= a(q_1 + q_2) - \frac{1}{2} [\delta_m(q_1^2 + q_2^2) + 2\gamma q_1 q_2] + x \\ \text{s.t. } p_1 q_1 + p_2 q_2 + x &= R. \end{aligned}$$

From the first order conditions $p_i = a - \delta_m q_i - \gamma q_j$ ($i, j = 1, 2$; $i \neq j$) and the budget constraint, we have:

$$x = R - a(\hat{q}_1 + \hat{q}_2) + \delta_m(\hat{q}_1^2 + \hat{q}_2^2) + 2\gamma\hat{q}_1\hat{q}_2,$$

where \hat{q}_i denotes the consumer's optimal demand of good i at given prices (and hence, the equilibrium quantity of good i if prices are at their equilibrium values). Substituting for x into the utility function, we get:

$$\hat{U} = R + \frac{1}{2} [\delta_m(\hat{q}_1^2 + \hat{q}_2^2) + 2\gamma\hat{q}_1\hat{q}_2],$$

so that the net consumer surplus finally results as:

$$CS = \hat{U} - R = \frac{1}{2} [\delta_m(\hat{q}_1^2 + \hat{q}_2^2) + 2\gamma\hat{q}_1\hat{q}_2].$$

■

Result 2 (Industry Profit). *Irrespective of the vertical structure of the industry, industry profit: i) monotonically decreases with product differentiation in model A; ii) monotonically increases with product differentiation in model B.*

Proof. From (4) and (6), industry profits in the vertical separation (vs) and the vertical integration (vi) equilibria are, respectively:

$$\begin{aligned}\Pi_m^{vs} &= \pi_m^U + 2\pi_m^D = \frac{2(3\delta_m + \gamma)}{(2\delta_m + \gamma)^2} \left(\frac{a}{2}\right)^2 \\ \Pi_m^{vi} &= \pi_m^I + \pi_m^N = \frac{(112\delta_m^4 - 96\gamma\delta_m^3 - 12\gamma^2\delta_m^2 + 24\gamma^3\delta_m - 3\gamma^4)}{\delta_m(8\delta_m^2 - 3\gamma^2)^2} \left(\frac{a}{2}\right)^2.\end{aligned}$$

Part (i). In model A, the expressions above specialize as:

$$\begin{aligned}\Pi_A^{vs} &= \pi_A^U + 2\pi_A^D = \frac{4(3 - \gamma)}{(4 - \gamma)^2} \left(\frac{a}{2}\right)^2 \\ \Pi_{m1}^{vi} &= \pi_A^I + \pi_A^N = \frac{169\gamma^4 - 1376\gamma^3 + 3792\gamma^2 - 4352\gamma + 1792}{(2 - \gamma)(5\gamma^2 - 32\gamma + 32)^2} \left(\frac{a}{2}\right)^2.\end{aligned}$$

Differentiating in γ yields:

$$\begin{aligned}\frac{\partial \Pi_A^{vs}}{\partial \gamma} &= \frac{4(2 - \gamma)}{(4 - \gamma)^3} \left(\frac{a}{2}\right)^2 > 0 \\ \frac{\partial \Pi_A^{vi}}{\partial \gamma} &= \frac{845\gamma^6 - 8352\gamma^5 + 32784\gamma^4 - 64832\gamma^3 + 68352\gamma^2 - 36864\gamma + 8192}{(2 - \gamma)^2(32 - 32\gamma + 5\gamma^2)^3} \left(\frac{a}{2}\right)^2 > 0.\end{aligned}$$

The positive sign of $\frac{\partial \Pi_A^{vi}}{\partial \gamma}$ follows from the fact that the polynomial at the denominator is strictly positive on $\gamma \in [0, 1]$, while the polynomial at the numerator is positive for any real value of γ (it does not have real roots and it is positive for $\gamma = 0$ – calculations with *Mathematica*).

Part (ii). In model B, we have:

$$\begin{aligned}\Pi_B^{vs} &= \frac{2(3 + \gamma)}{(2 + \gamma)^2} \left(\frac{a}{2}\right)^2 \\ \Pi_B^{vi} &= \frac{(112 - 96\gamma - 12\gamma^2 + 24\gamma^3 - 3\gamma^4)}{(8 - 3\gamma^2)^2} \left(\frac{a}{2}\right)^2\end{aligned}$$

Differentiating in γ yields:

$$\begin{aligned}\frac{\partial \Pi_B^{vs}}{\partial \gamma} &= -\frac{2(4 + \gamma)}{(2 + \gamma)^3} \left(\frac{a}{2}\right)^2 < 0 \\ \frac{\partial \Pi_B^{vi}}{\partial \gamma} &= -\frac{24(1 - \gamma)(32 - 16\gamma - 4\gamma^2 + 3\gamma^3)}{(8 - 3\gamma^2)^3} \left(\frac{a}{2}\right)^2 \leq 0,\end{aligned}$$

where $\frac{\partial \Pi_B^{vi}}{\partial \gamma}$ is strictly negative for $\gamma \in [0, 1)$ and zero for $\gamma = 1$. ■