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Evolutionary Dynamics of Nationalism and Migration

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Abstract

I present a dynamic evolutionary game model to address the relation between nationalism against immigrants and assimilation of the latter into the host country culture. I assume a country composed of two different large polymorphic populations, one of native citizens and the other of immigrants. A native citizen may behave nationalistically or may welcome immigrants. Immigrants may have an interest in learning the host country language or not. Evolution is modelled using replicator dynamics (RD). I also account for the presence of an enclave of immigrants in the host country. In the RD, the latter represents the immigrants' own population effect, which contribution to fitness is controlled using a parameter ρ, $0 \leq \rho \leq 1$, that represents the enclave size. In line with the empirical literature on migration, the existence of an enclave of immigrants makes assimilation less likely to occur. For large values of ρ, complete assimilation may not occur even if immigrants and natives share very close cultures and norms. Government policy regarding nationalism is modelled both exogenously and endogenously. A single or multiple asymptotically stable states exist for all cases studied but one in which the dynamics is similar to that found in the predator-prey model of Lotka-Volterra for competing species.
Keywords: Econophysics, replicator dynamics, migration, nationalism, enclave.

1 - Introduction

In the last years the development of Econophysics and Sociophysics has brought a different perspective for addressing traditional problems in main stream Economics and Sociology [1-6]. In this paper, a socio-economics problem is modelled using non-linear dynamics in a set up with many similarities with problems in statistical physics. I use an evolutionary game model to study how nationalism and assimilation evolve in a country composed of two different large polymorphic populations, one of native citizens and the other of immigrants, the latter coming from the same origin country. In the population of natives some individuals show a nationalistic behaviour while others simply welcome immigrants. The population of immigrants has individuals who have an interest in becoming culturally

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assimilated through learning the language of the host country while other immigrants simply do not have such an interest and do not learn it at all. Based on [7], language learning is used in this paper as a proxy for the acquisition of the host country's culture by immigrants.

In [7] nationalism is classified as corporate nationalism, hegemonic nationalism or primordial nationalism, all having in common their emergence within the native society due to fears of "denationalization" of the host country culture as a result of the presence of immigrants. In my model, some natives exhibit a nationalistic behaviour resulting from their xenophobia (feeling of hating the immigrants). Their reaction to this hate are their nationalistic attitudes against the immigrants, translated in my model through a tentative of boycotting them when immigrants try to use essential public services in the host country. Depending on the attitude of policy makers toward the prevention or not of nationalism as well as the interaction between natives and immigrants, nationalism may spread over time or even disappear.

Evolutionary game theory (EGT) has been previously used to study analytically other social and economic problems such as the evolution of crime in [8], the Lithuanian banking system and corporate governance in [9] and the effect of economic agents' behaviour on the long-run performance of the economy in [10], among many other examples. A theoretical background on deterministic EGT can be found in [11-15]. The use of EGT to address such social issues has the key aspect of relaxing the main stream Economics paradigm that all agents are rational and also allows to study the problem dynamically, thus understanding which and how asymptotically stable states (also called evolutionary equilibria as defined in [11]) can be achieved over time.

I start the analysis with a benchmark model in which immigrants live dispersed across the country and the government policy toward preventing nationalism is exogenous, thus a policy dictated by the government. I then extend the benchmark model to study two specific cases: a first one in which a significant number of immigrants live in an enclave and meetings among them are more likely to occur and a second case where the country's policy toward nationalism becomes endogenous through a parliament perfectly representing the native population behaviour toward immigrants.

The importance of investigating the influence of an enclave in the process of immigrants' assimilation follows from the empirical literature on migration [16-17], according to which there is a negative correlation between immigrants' fluency in the host country language and the level of immigrant concentration living in the same area (an enclave of immigrants). Also, enclaves act as language traps in the sense that they attract the immigrants with low proficiency in the host country language and help to sustain their low proficiency in the long run [18]. My results show that such a negative correlation between assimilation and enclave size persists even in the framework of an evolutionary game model in which rationality is partially or even completely dropped from the assumptions.

The results also show that, when the policy toward nationalism is exogenous and immigrants live dispersed in the country, both populations become isomorphic in the long run for all but one case, while polymorphism may be an evolutionary equilibrium (EE) when immigrants live in an enclave. When the government's policy toward nationalism is made endogenous, nationalism can still survive but the EE that prevails in the long run is then dependent on the initial conditions of both populations and on the basins of attraction in the state space.

The remainder of the paper is organized as follows: in section 2, I set up, solve and discuss the benchmark model, section 3 introduces the existence of an enclave in the benchmark model and section 4 makes the government policy endogenous. Section 5 concludes.

2 - Setting a benchmark model

I assume a country in which natives work in essential public services such as utility companies or city councils and, over time, immigrants have to make use of such services leading to pairwise meetings between randomly selected individuals from each population. Immigrants leave dispersed and do not meet among themselves (later I relax this assumption). In each pairwise meeting, an individual drawn at random from the population of native citizens may behave either in a nationalistic way (play pure strategy N - nationalist) or welcoming immigrants (strategy W - welcome). The individual drawn from the population of immigrants may be an individual fluent in the host country language (strategy L - learner) or may not have learned it (strategy NL - non-learner). Learner immigrants are able to have complete access to any public service, independently of the behaviour of the native employee they meet. When they travel from their house to the public service office, they are also able to completely interact with the host country society in the sense of being able to communicate, read the newspapers, signs, advertisements and have a complete understanding of what is happening in the streets. They obtain a payoff or utility $P > 0$ and $K > 0$ from the public service and the social interaction, respectively. On the other hand, they incur an effort (or a cost) of learning the language $c > 0$. These are average payoffs and average cost, thus assumed identical to every individual in the population.

Non-learner immigrants do not incur this cost of learning but also do not obtain any payoff from socially interacting and, whenever meeting with a nationalist employee, on average, they are not able to have complete access to the public service obtaining an expected payoff αP, $\alpha \in [0, 1)$. The parameter α, controlled by the government, measures the ability nationalists have to boycott immigrants when the latter seek for public services. In this benchmark model α is exogenous, being dictated by the government, and may not reflect the proportion of nationalists in the population of natives. The lower α is, more efficient the boycott against immigrants becomes. In the extreme case of a completely successful boycott, non-learners are fully prevented from getting any service, $\alpha = 0$.

Independently of the nationalists' behaviour against immigrants, government policies to reduce or eliminate this kind of barrier in the access of non-learner immigrants to the public services may exist, such as the availability of forms in different languages as well as information about the citizens' rights at the public service office or even a bilingual ombudsman service where immigrants can register complaints. Such policies translates into a higher value of α.

Within the native population, nationalist individuals face a trade-off due to the combination of the degree of xenophobia they feel and their capacity to boycott immigrants. While their sentiment of xenophobia brings them an average disutility X; $0 < X < P$, caused by being forced to interact with immigrants at their workplace, on the other hand they also obtain some pleasure when they are able to restrict the immigrants' access to essential services. This pleasure gives them an average payoff $(1-\alpha)P$, which is the payoff level they are able to extract from non-learners when the latter have partial access to the services they request. The payoff nationalists obtain, $(1-\alpha)P$, depends on two aspects: the first aspect is the payoff P that immigrants get from being completely able to use the public services. For given α, the higher this payoff is, the higher the utility an immigrant gets from using the service and the greater the pleasure a nationalist obtains if she is able to boycott a non-learner making the public service the most unavailable possible. The second aspect is the parameter α. This parameter depends on exogenous policies set by the government to try to enforce the accessibility of the public services to everyone. For given P, the higher this parameter is, the greater the accessibility of non-learners to the public services is, making the boycott carried out by the nationalists less successful and, hence, bringing the latter less utility from boycotting the immigrants. Overall, nationalists get a positive payoff whenever the amount of harm they are able to inflict on immigrants more than offsets the burden of having to face them. Natives who welcome immigrants are indifferent between dealing with learners or non-learners and they do not earn or lose any utility from meeting them, thus getting a payoff equal to zero.

Summarizing the assumptions, the total payoff $u^n(\bullet,\bullet)$ a native individual gets from every possible meeting is $u^n(N,NL) = (1-\alpha)P - X$, $u^n(N,L) = -X$ and $u^n(W,NL) = u^n(W,L) = 0$. And the total payoffs for an immigrant are $u^m(W,L) = u^m(N,L) = P + K - c$, $u^m(W,NL) = P$ and $u^m(N,NL) = \alpha P$. The dynamics governing the evolution of both populations is the standard replicator dynamics and the normalized payoff matrix for the game played between two individuals, each drawn from a different population is:

$$\begin{array}{c} \\ N \\ W \end{array} \begin{array}{c} \begin{array}{cc} L & NL \end{array} \\ \begin{pmatrix} 0,0 & n_1, m_2 \\ n_2, m_1 & 0,0 \end{pmatrix} \end{array}$$

where:

$$\begin{aligned} n_1 &= u^n(N,NL) - u^n(W,NL) = (1-\alpha)P - X \\ n_2 &= u^n(W,L) - u^n(N,L) = X \\ m_1 &= u^m(W,L) - u^m(W,NL) = K - c \\ m_2 &= u^m(N,NL) - u^m(N,L) = c - (1-\alpha)P - K \end{aligned}$$

Similar to [19], I consider generic games such that $n_i \neq 0 \wedge m_i \neq 0$; $i = 1,2$. Defining p as the proportion of nationalists in the population of natives while q is the proportion of learners in the population of immigrants, defining x (y) as a mixed strategy played by an individual in the population of natives (immigrants), from the matrix above, the state of the population of natives $s^n = (p; 1-p)$ is governed over time by:

$$\dot{p} = [u(N,y) - u(x,y)]\,p \tag{1}$$

$$\dot{p} = [n_1(1-q) - n_2 q]\,p(1-p) \tag{2}$$

where $u(N,y)$ is the expected payoff of a native who chooses to adopt the (pure) strategy nationalist playing against a completely mixed strategy y, that is, the state of the population of immigrants. $u(x,y)$ is the native population's average payoff, i.e., the expected payoff of an individual randomly selected from the population of natives where the mixed strategy x represents the state of the population of natives.

The state of the population of immigrants $s^m = (q; 1-q)$ is governed by:

$$\dot{q} = [u(x,L) - u(x,y)]\,q \tag{3}$$

$$\dot{q} = [m_1(1-p) - m_2 p]\,q(1-q) \tag{4}$$

where $u(x,L)$ is the expected payoff of an immigrant who adopts the (pure) strategy learner playing against a completely mixed strategy x, that is, the state of the population of natives. $u(x,y)$ in the context of the equation above is the immigrant population's average payoff.

Looking at (1) for the case of natives, one can see the replicator dynamics selection mechanism. Whenever $u(N,y) < u(x,y)$, i.e., whenever the choice to adopt the nationalistic behaviour does worse than the average behaviour exhibited by natives, the proportion of natives who choose to be nationalist gradually decreases over time. With respect to the immigrants, from (3), whenever $u(x,L) < u(x,y)$, the proportion of immigrants who choose to learn gradually decreases over time.

The intuition behind (1) and (3) is that a native chooses to adopt a pure strategy/behaviour after her personality is formed and then she holds this behaviour over life. For the case of immigrants, the Economics literature on migration shows that networking plays a significant role in immigration [20]. In the host country, if immigrants psychologically oriented (programmed) to learn tend to be more successful than non-learners, they send this information to the origin country, attracting over time to the host country a higher rate of new learning oriented immigrants in detriment of new non-learners. Thus strategies/behaviours exhibiting a more successful outcome induce a higher adoption rate among newly born natives or newly arrived immigrants.

The game state space is the unit square $s^n \times s^m = [0,1]^2 \subset \Re^2$ and, for the sake of simplicity, I will represent a given state by $\theta = (p, q)$. Substituting n_1, n_2, m_1 and m_2 in (2) and (4):

$$\dot{p} = \{[(1-\alpha)P - X](1-q) - Xq\} p(1-p) \tag{5}$$

$$\dot{q} = \{(K-c)(1-p) - [c - (1-\alpha)P - K] p\} q(1-q) \tag{6}$$

From (5) and (6), we may have the following cases:

$$(1-\alpha)P > X \quad \Rightarrow \quad n_1 > 0;\ n_2 > 0 \tag{7}$$

$$(1-\alpha)P < X \quad \Rightarrow \quad n_1 < 0;\ n_2 > 0 \tag{8}$$

$$c \in [0; K-\epsilon] \quad \Rightarrow \quad m_1 > 0;\ m_2 < 0 \tag{9}$$

$$c \in [K+\epsilon; K+(1-\alpha)P-\epsilon] \quad \Rightarrow \quad m_1 < 0;\ m_2 < 0 \tag{10}$$

$$c \in [K+(1-a)P+\epsilon; +\infty) \quad \Rightarrow \quad m_1 < 0;\ m_2 > 0 \tag{11}$$

where $\epsilon \to 0$. Equation (7) refers to a country where nationalist behaviour is weakly or even not prevented by the government (low α such that $n_1 > 0$) and nationalists are able to create more successful boycotts against non-assimilated immigrants, leading a nationalist individual to get on average a positive payoff from the game. Equation (8) refers to the opposite situation, in which the disutility due to xenophobia more than offsets the utility due to the boycott because there is some institutional policy giving enough protection for the access of any kind of individual to almost any public service (α is sufficiently high for $n_1 < 0$, i.e., nationalism strongly prevented).

With respect to the cost of learning, c may fall in one of 3 intervals as in (9-11), which I will call respectively low, intermediate and high cost of learning. Eq. (9) refers to a situation in which both host and origin countries share the same or a very closely related language. In this case, the cost of language acquisition is low. The other extreme is given in (11), in which immigrants have a very different cultural background when compared to the natives and language acquisition is very costly. I now study the stationarity and stability of the system given by (5-6):

Lemma 1: The game state space has the following asymptotically stable states (evolutionary equilibria) θ^{EE}: $(0,0)$ when $n_1 < 0 \wedge m_1 < 0$, i.e., nationalism is strongly prevented by the government and the cost of leaning is either intermediate or high; $(1,0)$ for $n_1 > 0 \wedge m_2 > 0$, i.e., weakly prevented nationalism and high cost of learning; $(0,1)$ for $n_2 > 0 \wedge m_1 > 0$, i.e., either weakly or strongly prevented nationalism and low cost of learning. Also, $\left(\overline{p} = \frac{c-k}{(1-\alpha)P}, \overline{q} = \frac{(1-\alpha)P-X}{(1-\alpha)P}\right) \notin \theta^{EE}$; it is a neutrally stable state when $n_1 > 0 \wedge m_i < 0;\ i = 1, 2$, i.e., weakly prevented nationalism and intermediate cost of learning.

Proof. (5) and (6) define a non-linear planar system which has the following stationary points $\{(p, q) \in [0, 1]^2 :\ \dot{p} = 0 \wedge \dot{q} = 0\}$: $(0, 0)$; $(1, 0)$; $(0, 1)$; $(1, 1)$ and $\left(\overline{p} = \frac{c-k}{(1-\alpha)P}, \overline{q} = \frac{(1-\alpha)P-X}{(1-\alpha)P}\right)$, the latter when $n_1 > 0 \wedge m_i < 0;\ i = 1, 2$.

Rewriting (2) and (4) as:

$$\begin{aligned} \dot{p} &= F^1(p, q) \\ \dot{q} &= F^2(p, q) \end{aligned}$$

I study the local stability in the neighbourhood of the isolated stationary points in the linearised planar system by obtaining the eigenvalues of the Jacobian matrix Ω evaluated at the stationary points. I define the trace of the Jacobian matrix:

$$\begin{aligned} tr(\Omega) &= \frac{\partial F^1(p, q)}{\partial p} + \frac{\partial F^2(p, q)}{\partial q} \\ &= (1 - 2p)\left[n_1 - (n_1 + n_2)q\right] + (1 - 2q)\left[m_1 - (m_1 + m_2)p\right] \end{aligned} \tag{12}$$

and the determinant of the Jacobian matrix:

$$\begin{aligned} det(\Omega) &= \frac{\partial F^1(p, q)}{\partial p} \cdot \frac{\partial F^2(p, q)}{\partial q} - \frac{\partial F^1(p, q)}{\partial q} \cdot \frac{\partial F^2(p, q)}{\partial p} \\ &= (1 - 2p)\left[n_1 - (n_1 + n_2)q\right](1 - 2q)\left[m_1 - (m_1 + m_2)p\right] - \\ &\quad -(n_1 + n_2)p(1 - p)(m_1 + m_2)q(1 - q) \end{aligned} \tag{13}$$

The eigenvalues of Ω are given by the roots of the characteristic polynomial $\lambda^2 - tr(\Omega)\lambda + det(\Omega) = 0$. Hence a stationary point is locally asymptotically stable if $\lambda_i < 0;\ i = \{1, 2\} \Rightarrow tr(\Omega) < 0 \wedge det(\Omega) > 0$. The stationary point is a saddle if $\lambda_i < 0 \wedge \lambda_j > 0;\ i, j = \{1, 2\};\ i \neq j \Rightarrow\ det(\Omega) < 0$. And it is unstable if $\lambda_i > 0;\ i = \{1, 2\} \Rightarrow tr(\Omega) > 0 \wedge det(\Omega) > 0$. Analysing the local stability at each stationary point using (12) and (13), at point $(0, 0)$: $tr[\Omega(0, 0)] = n_1 + m_1$; $det[\Omega(0, 0)] = n_1 m_1$. For asymptotic stability, requires $m_1 < 0 \wedge n_1 < 0$. At point $(1, 1)$: $tr[\Omega(1, 1)] = n_2 + m_2$; $det[\Omega(1, 1)] = n_2 m_2$. Asymptotic stability requires $n_2 < 0$, which is never satisfied, hence, never an EE. At point $(1, 0)$: $tr[\Omega(1, 0)] = -n_1 - m_2$; $det[\Omega(1, 0)] = n_1 m_2$. For asymptotic stability, requires $n_1 > 0 \wedge m_2 > 0$. At point $(0, 1)$: $tr[\Omega(0, 1)] = -n_2 - m_1$; $det[\Omega(0, 1)] = n_2 m_1$. For asymptotic stability, requires $n_2 > 0 \wedge m_1 > 0$.

When $(\overline{p}, \overline{q}) \in [0, 1]^2$, the Jacobian matrix evaluated at this point has $tr[\Omega(\overline{p}, \overline{q})] = 0$; $det[\Omega(\overline{p}, \overline{q})] = -(\overline{p} - \overline{p}^2)(\overline{q} - \overline{q}^2)(n_1 + n_2)(m_1 + m_2) > 0$. The dynamic system is non-hyperbolic at this stationary point given the eigenvalues are pure imaginary. I employ the following Liapunov function to study its stability:

$$\begin{aligned} V(p,q) &= m_1 \ln p + m_2 \ln(1-p) - n_1 \ln q - n_2 \ln(1-q) - c; \\ & c \text{ such that } V(\overline{p},\overline{q}) = 0 \end{aligned}$$

$V(p,q)$ is a non-strict Liapunov function for $(\overline{p},\overline{q})$, hence $(\overline{p},\overline{q})$ is Liapunov (neutrally) stable. To see this, $V(p,q)$ has a strict minimum at $(\overline{p},\overline{q})$:

$$\begin{aligned} \frac{\partial V(p,q)}{\partial p} &= \frac{m_1}{p} - \frac{m_2}{1-p} = 0 \Rightarrow p = \overline{p} \\ \frac{\partial V(p,q)}{\partial q} &= \frac{-n_1}{q} + \frac{n_2}{1-q} = 0 \Rightarrow q = \overline{q} \end{aligned}$$

2^{nd} order conditions of the Hessian at $(\overline{p},\overline{q})$:

$$\begin{aligned} \frac{\partial^2 V(p,q)}{\partial p^2} &= \frac{-m_1}{p^2} + \frac{-m_2}{(1-p)^2} > 0 \\ \frac{\partial^2 V(p,q)}{\partial p^2}\frac{\partial^2 V(p,q)}{\partial q^2} &= \left(\frac{-m_1}{p^2} + \frac{-m_2}{(1-p)^2}\right)\left(\frac{n_1}{q^2} + \frac{n_2}{(1-q)^2}\right) > 0 \end{aligned}$$

Hence, $V(\overline{p},\overline{q}) = 0 \wedge V(p,q) > 0 \Leftrightarrow (p,q) \neq (\overline{p},\overline{q})$.

Let B be an open ball about $(\overline{p},\overline{q})$ in the plane. For neutral stability of $(\overline{p},\overline{q})$, all we require is $\frac{dV(p,q)}{dt} \leq 0$ for all $(p,q) \in \{B - (\overline{p},\overline{q})\}$ and $\frac{dV(p,q)}{dt} = 0$ for $(\overline{p},\overline{q})$:

$$\begin{aligned} \frac{dV(p,q)}{dt} &= \frac{m_1(1-p) - m_2 p}{p(1-p)}[n_1(1-q) - n_2 q]p(1-p) + \\ &+ \frac{-n_1(1-q) + n_2 q}{q(1-q)}[m_1(1-p) - m_2 p]q(1-q) = 0; \\ & \forall p; \forall q \in B \end{aligned}$$

■

Discussing the results in lemma 1, from (2), when the central government implements strong measures against nationalistic behaviour (high α such that $n_1 < 0$) we have $n_1 < 0$; $n_2 > 0$; $\dot{p} < 0$ and the dynamics is quite trivial. For any interior initial conditions of both populations, nationalism is monotonically decreasing over time and ends up disappearing independently of the EE achieved. When this is the case, unless the cost of learning is low, over time immigrants will not become culturally assimilated. From lemma 1, we have $\theta^{EE} = (0,0)$ for $c > K$ $(m_1 < 0)$ in which case there is no assimilation or we have $\theta^{EE} = (0,1)$ for $c < K$ $(m_1 > 0)$.

On the other hand, when the central government does not implement enough measures to prevent the nationalistic behaviour against immigrants (low α such that $n_1 > 0$), we may have three possible cases. From lemma 1, when the cost of learning the host country language is low, $c < K$, over time both populations become isomorphic with all natives welcoming immigrants and the latter becoming completely assimilated, that is, $\theta^{EE} = (0,1)$ is the only asymptotically stable state in the state space.

In this case, $n_i > 0$, $m_1 > 0$ and $m_2 < 0$. From (4), $\dot{q} > 0$ and in the long run non-learners disappear from the population of immigrants.

From (2), one can see that, differently from the case where α is sufficiently high and nationalism is monotonically decreasing over time, now nationalism may initially increase, $\dot{p} > 0$. But the decrease in the proportion of non-learners over time leads to $\dot{p} < 0$ at some point. Hence over time non-learners and nationalists decrease their share in their respective populations until becoming extinct and in the limit both populations become isomorphic. Now nationalism is driven to extinction in the long run as a consequence of assimilation and not due to the government policy as before (high α). These two different patterns are shown in figure 1.

This is the context in a host country which language has a very small distance from the language spoken in the origin country. Countries which languages are close tend to share the same norms and values, which is a facilitator or even an incentive to attract immigrants who end up learning the language once they start to live in the host society. On the side of the natives, nationalistic behaviour does not pay over time given the proximity between the cultures of both origin and host countries, which makes it possible, independently of the level of xenophobia and nationalistic boycott against immigrants, for any individual to have access to the services because they all become learners. In other words, given assimilation takes place over time, calls for nationalistic behaviour in the population of natives lose strength in the long run and nationalistic behaviour becomes extinct.

On the other extreme, in countries with high costs of learning the language, $c > (1-\alpha)P + K$, lemma 1 shows that natives and immigrants evolve to $\theta^{EE} = (1, 0)$, which characterizes a state with isomorphic populations in which immigrants are not assimilated and natives behave nationalistically. In this case, multiculturalism is an equilibrium which may create political turmoil in the country in the long run.

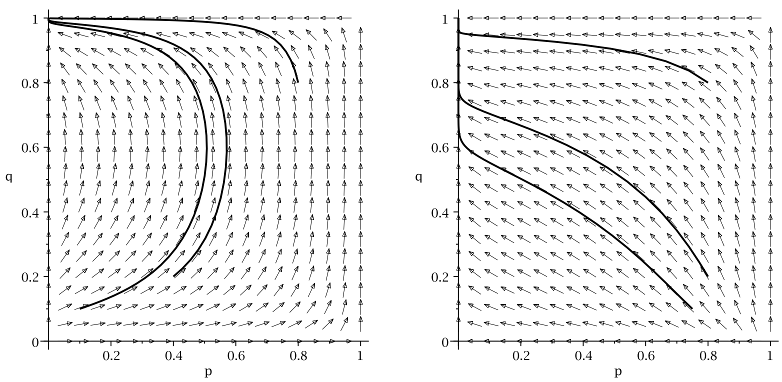

Figure 1: Evolution for low cost of learning; left: nationalism weakly prevented; right: nationalism strongly prevented.

Thus, one can see that complete assimilation does not depend on the policy adopted by the government with respect to nationalism prevention and is actually related to the degree of cultural distance. But, on the other hand, the pattern of multiculturalism a country develops over time is closely related to the way policy makers deal with nationalism. The multiculturalism pattern found for $\theta^{EE} = (1,0)$ is very different from the one when $\theta^{EE} = (0,0)$. The peaceful multiculturalism found at state $\theta^{EE} = (0,0)$, in which immigrants keep their culture and are welcomed by natives, is one in line with policies found in countries which try to promote the well being of the immigrants.

The final possible case we can have when nationalism is weakly prevented occurs when the cost of learning is neither low nor high, $c \in [K + \epsilon; K + (1 - \alpha)P - \epsilon]$. From lemma 1, in such a case, fully polymorphic populations will always exist in the country over time, in which the proportions of nationalists and learners will oscillate over generations. This case is similar to the Buyer-Seller game of [21] which was later used by [8] in their model to study the dynamics of crime. We have a dynamics similar to the predator-prey model of Lotka-Volterra in which both species of animals compete against each other but not against themselves. In the context of my model, an EE does not exist. In the interior of the unit square $[0,1]^2$ we have closed orbits which spiral anti-clockwise orbiting the only (neutrally) stable stationary state. The coordinates (p, q) of such a state represent the average behaviour of the populations over a long interval of time [8] as can be seen on the right hand side of figure 2.

If we assume an initial state in which natives who welcome immigrants and non-learner immigrants are numerous within their populations, e.g.: initial conditions given by $\theta = (0.1, 0.1)$, nationalism starts to rise among the native population, achieves a very high proportion rising at the same time the incentives to favour assimilated immigrants. When the proportion of nationalists starts to outnumber that of natives adopting a welcoming behaviour, the proportion of non-learners starts to decrease sharply in the population of immigrants. This pattern in the immigrant population acts over time on the native population reducing the incentives for nationalistic behaviour. Natives who welcome immigrants start to outnumber the proportion of nationalists and as a consequence, over time, the proportion of non-learners grows. The cycle then re-starts. This evolutionary pattern as well as the numerical output corresponding to one complete cycle starting with a proportion of 10% learners and 10% nationalists in their respective populations is presented in figure 2 and table 1. For this numerical simulation, I used a constant step size equal to $\Delta t = 0.02$ and the parameters $c = 0.55$, $K = 0.2$, $X = 0.6$ and $(1 - \alpha)P = 1.5$. The entries in the following table were obtained using the classic fourth order four-stage Runge-Kutta method with the same parameters and step size.

Number of steps	Nationalists	Learners	Number of steps	Nationalists	Learners
initial conditions	10.00%	10.00%	step 600	27.96%	97.51%
step 100	33.86%	9.17%	step 700	11.21%	97.15%
step 200	67.95%	19.06%	step 800	4.06%	95.43%
step 300	82.02%	54.10%	step 900	1.51%	91.80%
step 400	75.70%	86.68%	step 1,200	0.29%	59.13%
step 500	54.05%	95.89%	step 1,600	9.96%	10.01%

Table 1: Proportion of nationalists and learners for weakly prevented nationalism and intermediate cost of learning.

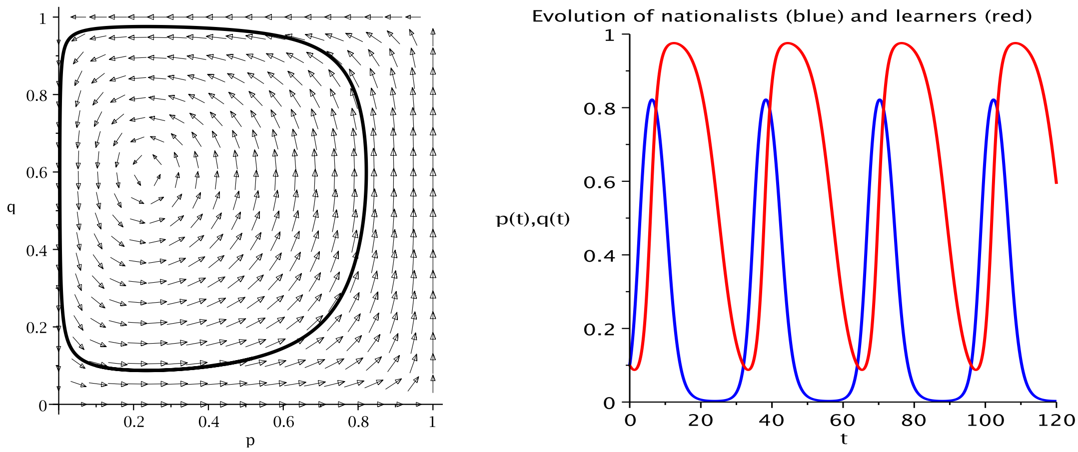

Figure 2: Evolutionary pattern for the case of weakly prevented nationalism and intermediate cost of learning.

3 - Effect of immigrants' enclaves

In this section, I assume the existence of an enclave of immigrants in the host country. In this case, immigrants are more likely to interact among themselves and the evolution of their population depends not only on the evolution of the population of natives but also on the so called own population effect in the EGT literature. I rely on two simple assumptions: when an enclave exists, an individual drawn from the population of immigrants meets with (plays the game against) an individual drawn from the population of natives with probability 1 and with an individual drawn from the population of immigrants with probability ρ, $0 \leq \rho \leq 1$. When $\rho = 0$, we have the case without an enclave as in section 2. ρ measures the degree of importance the enclave has or, alternatively, the weight that the own population effect contributes to fitness (and selection) in the replicator dynamics. A high ρ corresponds to a large enclave size and a high level of interaction among immigrants, leading the own population effect to have an important contribution to the evolution of the immigrant population.

The second assumption is with respect to the payoffs immigrants get from the own population effect. I assume that the benefit from the interaction between immigrants accrues mainly to the non-learners. A straightforward example of the kind of gain the non-learners obtain from this interaction is their increased ability to acquire new information about the host country. This becomes possible only when they network with their own people due to their lack of knowledge of the host country language. The acquisition of information about the host country society is higher when they meet with a learner than when they meet another non-learner. The fact that learners are able to have a better understanding about what is going on in the host country makes them on average more updated about the host society than the non-learners and this is reflected in the amount and quality of information they can provide to non-learners. On the other hand, compared to non-learners, learners benefit residually from this own population effect due to the fact that they are perfectly able to acquire a high amount of information by themselves. I normalize to zero the payoff learners get from the own population effect. Relying only on these two assumptions and without assuming any rationality, I show that the dynamics of the model leads to one of the core results in the empirical literature on migration, according to which enclaves make assimilation less likely. The matrix M' corresponding to the payoffs due to the immigrants' own population effect is given by:

$$M' = \begin{array}{c} \\ L \\ NL \end{array} \begin{array}{c} \begin{array}{cc} L & NL \end{array} \\ \begin{pmatrix} 0 & m'_1 \\ m'_2 & 0 \end{pmatrix} \end{array}$$

where $m'_1 = m'_{12} - m'_{22} = -\gamma K$, $m'_2 = m'_{21} - m'_{11} = \beta K$ and:

$$\begin{aligned} m'_{11} &= u^m(L, L) = 0 \\ m'_{12} &= u^m(L, NL) = 0 \\ m'_{21} &= u^m(NL, L) = \beta K \\ m'_{22} &= u^m(NL, NL) = \gamma K \end{aligned}$$

with $1 > \beta > \gamma > 0$ and K is the utility learners obtain from their complete ability to socially interact with the host country society (see the payoff matrix in section 2). The dynamics governing the state of both populations is now given by equation (5), once the population of natives continues to have no own population effect, and by:

$$\dot{q} = \{u(x, L) - u(x, y) + \rho\,[u(y, L) - u(y, y)]\}\, q$$

where $u(y, L) = m'_1(1 - q)$ and $u(y, y) = m'_1 q(1 - q) + m'_2(1 - q)q$, leading to:

$$\dot{q} = [m_1(1 - p) - m_2 p + \rho m'_1(1 - q) - \rho m'_2 q]\,(1 - q)q \tag{14}$$

$$\dot{q} = \{(K - c)(1 - p) - [c - (1 - \alpha)P - K]\,p - \rho K\,[\gamma(1 - q) + \beta q]\}\, q(1 - q) \tag{15}$$

I start analysing the specific equilibria for the case when an enclave exists and nationalism is strongly prevented by the government. When this is the case, depending on the cost of learning c, the game state space $[0,1]^2$ has a minimum of four (the corners of the unit square) and a maximum of six stationary states which are candidates for an EE. These states are $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, as before, and $\left(\overline{p_2}=0,\ \overline{q_2}=\frac{K(1-\rho\gamma)-c}{\rho K(\beta-\gamma)}\right)$ for $c \in [K(1-\beta\rho)+\epsilon; K(1-\gamma\rho)-\epsilon]$; $\left(\overline{p_3}=1,\ \overline{q_3}=\frac{K(1-\rho\gamma)+(1-\alpha)P-c}{\rho K(\beta-\gamma)}\right)$ for $c \in [K(1-\beta\rho)+(1-\alpha)P+\epsilon; K(1-\gamma\rho)+(1-\alpha)P-\epsilon]$.

Lemma 2: As in the case when an enclave does not exist, the stationary state $\theta=(1,1)$, in which all natives behave nationalistically and all immigrants are learners is never an EE.

Proof. equations (2) and (14) define a new non-linear planar system whose Jacobian matrix is the same as in lemma 1, except for the entry Ω_{22} which now becomes:

$$\frac{\partial F^2(p,q)}{\partial q} = (1-2q)\left[m_1(1-p)-m_2p+\rho m_1'(1-q)-\rho m_2' q\right]-(q-q^2)\rho(m_1'+m_2')$$

At the stationary state $\theta=(1,1)$, $tr[\Omega(1,1)]=n_2+m_2+\rho m_2'$ and $det[\Omega(1,1)]=n_2(m_2+\rho m_2')$. Thus if $det(\Omega)>0 \Rightarrow tr(\Omega)>0$ and both distinct eigenvalues are positive leading to an unstable node. On the other hand, if $det(\Omega)<0$ we have two real distinct eigenvalues, one positive and the other negative, for which the corresponding critical point is a saddle point.

■

Lemma 3: When nationalism is strongly prevented ($n_1<0$) and an enclave exists, the possible evolutionary equilibria are:

$$\begin{array}{lll} (0,1) & \text{for} & c \in [0; K(1-\rho\beta)-\epsilon] \\ (0,\overline{q_2}) & \text{for} & c \in [K(1-\rho\beta)+\epsilon; K(1-\rho\gamma)-\epsilon] \\ (0,0) & \text{for} & c \in [K(1-\rho\gamma)+\epsilon; \infty) \end{array}$$

Proof. at the stationary state $(0,1)$, $tr(\Omega)=-n_2-(m_1-\rho m_2')$ and $det(\Omega)=n_2(m_1-\rho m_2')$. Given $n_2>0$, $det(\Omega)>0$ only if $(m_1-\rho m_2')>0$, implying also $tr(\Omega)<0$. Hence $(0,1)$ is asymptotically stable when $c\in[0,K(1-\rho\beta)-\epsilon]$. At $(0,0)$, $det(\Omega)=n_1(m_1+\rho m_1')$ and $tr(\Omega)=n_1+m_1+\rho m_1'$. For asymptotic stability, we need $n_1<0$, i.e., nationalism strongly prevented and $c>K(1-\rho\gamma)$. This state is never an EE for the case of weakly prevented nationalism. At $(1,0)$, $det(\Omega)=n_1(m_2-\rho m_1')$ and $tr(\Omega)=-n_1-(m_2-\rho m_1')$. Asymptotic stability requires $n_1>0 \Rightarrow (1-\alpha)P>X$, i.e., weakly prevented nationalism and $c>(1-\alpha)P+K(1-\rho\gamma)$. Hence this state is never an EE in the case with nationalism strongly prevented. At $(0,\overline{q_2})$, $tr(\Omega)=[n_1(1-\overline{q_2})-n_2\overline{q_2}]-(\overline{q_2}-\overline{q_2}^2)\rho(m_1'+m_2')<0$, given $n_1<0$ for the case with strongly prevented nationalism. Also $det(\Omega)=-[n_1(1-\overline{q_2})-n_2\overline{q_2}][(\overline{q_2}-\overline{q_2}^2)\rho(m_1'+m_2')]>0$. Thus $(0,\overline{q_2})$ is an asymptotically stable node. For $(1,\overline{q_3})$, given $n_1<0$, we have $det(\Omega)=[n_1(1-\overline{q_3})-n_2\overline{q_3}][(\overline{q_3}-\overline{q_3}^2)\rho(m_1'+m_2')]<0$, hence $(1,\overline{q_3})$ is always a saddle.

■

Proposition 1: The existence of an enclave of immigrants in the host country makes assimilation less likely to occur.

Proof. comparing the results of section 2 and this section, when an enclave does not exist, an EE with (complete) assimilation of immigrants requires $c < K$. When an enclave exists, an EE in which there is some degree of assimilation requires $c < K(1 - \rho\gamma)$. Given $\rho > 0$, $\gamma > 0$, $K(1 - \rho\gamma) < K$ necessarily holds.

■

The result in proposition 1 is in line with the empirical literature on migration and assimilation. [16] states that only individuals with very low switching language costs find it profitable to become assimilated when they live in an enclave. Similar conclusion is found in [17,22] for the Canadian and the US cases, respectively. [18] conclude that enclaves act as language traps because they attract the immigrants with low proficiency in the host country language and sustain their low proficiency, once living in the enclave reduces their likelihood of learning the foreign language. In this paper, using a dynamic model without any assumption regarding rationality, when an enclave exists, replicator dynamics is more likely to select non-learner oriented individuals among the population of immigrants. Assimilation will take place in the long run only for a narrower set of values of the cost to learn the host country language.

With an enclave, there exists even the possibility that an EE involving complete assimilation of immigrants does not exist. From lemma 3, complete assimilation of immigrants living in an enclave is an equilibrium only when $c < S = K(1 - \rho\beta)$. In the extreme case when the immigrants' enclave makes random matches among immigrants very likely to occur ($\rho \to 1$), and non-learner immigrants are able to acquire a very high amount of information about the host country society, i.e., their utility from networking βK approaches K ($\beta \to 1$), $S \to 0$ and complete assimilation is not possible given that $\theta = (0, 1)$ is asymptotically stable for $c = \emptyset$. In this case, the expected payoff a non-learner immigrant obtains within the enclave when networking with a learner is very close to K, the expected payoff learners obtain from their social networking within the host society.

Proposition 2: The existence of an enclave of immigrants in the host society allows for an EE with a polymorphic population of immigrants in which learners and non-learners coexist.

Proof. from lemma 3, this is the case when $K(1-\rho\beta) < c < K(1-\rho\gamma)$. The width of this interval, as well as the minimum learning cost below which all immigrants become assimilated and the maximum learning cost above which no immigrant becomes assimilated over time, depends on the enclave's importance, ρ, and on the parameters β and γ.

■

Comparing the cases with and without an enclave, starting from $c = K$ and decreasing the cost of learning, without an enclave, all non-learner immigrants would disappear over generations because the learning oriented immigrants would have an advantage in the host country and would be naturally selected over time (higher fitness given the extra payoff K, only obtained by learners, more than offsets the cost of learning c). With an enclave things change. In order to have some assimilation, natural selection governed by replicator dynamics requires a lower cost of learning. Also, if this cost of learning is below $K(1-\rho\gamma)$ but still above $K(1-\rho\beta)$, natural selection does not lead to complete assimilation over time, allowing for the co-existence of learners and non-learners. In the population of immigrants, some individuals end up learning given the extra utility K more than offsets the incurred cost of learning c. On the other hand, non-learners do not face the cost c but still get an extra expected utility $\rho K[\beta q+\gamma(1-q)]$ due to the existence of the enclave. At the evolutionary equilibrium $\theta^{EE}=(0,\overline{q_2})$, the expected fitness of a learner and a non-learner individual are equal, i.e., $K-c=\rho K[\beta\overline{q_2}+\gamma(1-\overline{q_2})]$ and a proportion $\overline{q_2}$ of learners and $1-\overline{q_2}$ of non-learners co-exist in an equilibrium robust to shocks given asymptotic stability. Without an enclave, polymorphism could not be an equilibrium because non-learning oriented immigrants could not benefit from such networking utility. Over generations the immigrant population then had to evolve either to complete assimilation or no assimilation at all.

I now start the analysis of the case in which nationalism is weakly prevented and an enclave exists in the host country. When this is the case, depending on the cost of learning, the game state space $[0,1]^2$ has a minimum of four (the corners of the square) and a maximum of seven stationary states which are candidates for an EE. These stationary states are the same six states defined before plus $\left(\overline{p_1}=\frac{c-k(1-\gamma\rho(1-\overline{q_1})-\rho\beta\overline{q_1})}{(1-\alpha)P},\ \overline{q_1}=\frac{(1-\alpha)P-X}{(1-\alpha)P}\right)$, the latter when $c\in[K(1-\gamma\rho(1-\overline{q_1})-\rho\beta\overline{q_1})+\epsilon;K(1-\gamma\rho(1-\overline{q_1})-\rho\beta\overline{q_1})+(1-\alpha)P-\epsilon]$ and $(1-\alpha)P>X$.

Lemma 4: When nationalism is weakly prevented ($n_1>0$) and an enclave exists, the possible evolutionary equilibria are:

$$
\begin{array}{lll}
(0,1) & \text{for} & c\in[0;K(1-\rho\beta)-\epsilon] \\
(0,\overline{q_2}) & \text{for} & c\in[K(1-\rho\beta)+\epsilon;K(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})-\epsilon] \\
(\overline{p_1},\overline{q_1}) & \text{for} & c\in[K(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})+\epsilon;(1-\alpha)P+K(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})-\epsilon] \\
(1,\overline{q_3}) & \text{for} & c\in[(1-\alpha)P+K(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})+\epsilon;(1-\alpha)P+K(1-\rho\gamma)-\epsilon] \\
(1;0) & \text{for} & c\in[(1-\alpha)P+K(1-\rho\gamma)+\epsilon;+\infty)
\end{array}
$$

It is possible to have an EE in which both populations continue to be polymorphic.

Proof. from the proof of lemma 3, $(0,1)$ is asymptotically stable when $c\in[0,k(1-\rho\beta)-\epsilon]$, $(0,0)$ is never an EE for the case of weakly prevented nationalism and $(1,0)$ is asymptotically stable for $c>(1-\alpha)P+K(1-\rho\gamma)$. Additionally, at the stationary state $(\overline{p_1},\overline{q_1})$, we have $tr(\Omega)=$

$-(m_1'+m_2')\rho(\overline{q_1}-\overline{q_1}^2)$, which is always negative and $det(\Omega) = -(\overline{p_1}-\overline{p_1}^2)(\overline{q_1}-\overline{q_1}^2)(m_1+m_2)(n_1+n_2)$, which is always positive. Hence for any range of the cost of learning for which $(\overline{p_1}, \overline{q_1})$ is a stationary state, it is asymptotically stable and an EE in which both populations continue to be polymorphic. In the particular case when $[tr(\Omega)]^2 - 4det(\Omega) < 0$, $\theta^{EE} = (\overline{p_1}, \overline{q_1})$ is a stable focus. At $(0, \overline{q_2})$, $tr(\Omega) = [n_1(1-\overline{q_2})-n_2\overline{q_2}]-(\overline{q_2}-\overline{q_2}^2)\rho(m_1'+m_2')$ and $det(\Omega) = -[n_1(1-\overline{q_2})-n_2\overline{q_2}][(\overline{q_2}-\overline{q_2}^2)\rho(m_1'+m_2')]$. In order to have $tr(\Omega) < 0$ and $det(\Omega) > 0$, we need $c < K(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})$. Hence $(0, \overline{q_2})$ is asymptotically stable for $K(1-\rho\beta) < c < K(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})$. For $(1, \overline{q_3})$, we have $tr(\Omega) = -[n_1(1-\overline{q_3})-n_2\overline{q_3}]-[(\overline{q_3}-\overline{q_3}^2)\rho(m_1'+m_2')]$ and $det(\Omega) = [n_1(1-\overline{q_3})-n_2\overline{q_3}][(\overline{q_3}-\overline{q_3}^2)\rho(m_1'+m_2')]$. For being an asymptotically stable critical point we require $c > (1-\alpha)P + K(1-\rho\gamma(1-\overline{q_1})-\rho\beta\overline{q_1})$.
■

Comparing the results in lemma 4 with those in lemma 3, the most important difference is that when nationalism is weakly prevented, both populations can end-up in a polymorphic state $\theta^{EE} = (\overline{p_1}, \overline{q_1})$. For an intermediate level of the cost of learning the host country language, in a country where the population of immigrants departs from an initial state containing both learners and non-learners and the population of natives has both nationalists and individuals welcoming immigrants, none of the behaviours will become extinct over generations. Both populations will reach an equilibrium in which all the behaviours continue to coexist. Also, as when an enclave does not exist, when the cost of learning is high enough, multiculturalism is an equilibrium which may lead to political instability, $\theta^{EE} = (1, 0)$.

Before ending this section, I present two short numerical simulations. In the first simulation, I show how the EE changes when c varies. I used the following parameters: $(1-\alpha)P = 0.1$; $K = 0.2$; $X = 0.08$, $\rho = 1$, $\gamma = 0.2$ and $\beta = 0.85$. The corresponding equation governing the share of nationalists in the population of natives is:

$$\dot{p} = (0.02 - 0.1q)p(1-p)$$

The equation governing the share of learners has the form:

$$\dot{q} = [(0.16 - c) + 0.1p - 0.13q]\, q(1-q)$$

I used the following values for c in the simulation carried out: 0.020; 0.200; 0.235 and the corresponding evolutionary equilibria were, respectively, $\theta^{EE} = (0, 1)$, corner; $\theta^{EE} = (0.66, 0.2)$, center, with both populations evolving to a polymorphic EE; $\theta^{EE} = (1, 0.192)$, edge of the unit square with a isomorphic population of nationalists and a polymorphic population of immigrants. Figure 3 presents the corresponding vector fields leading to each of the above evolutionary equilibria. The second numerical simulation reproduces the same case presented in figure 2 but with the addition of the enclave effect. I again used a constant step size equal to $\Delta t = 0.02$ and the parameters $c = 0.55$, $K = 0.2$,

$X = 0.6$ and $(1-\alpha)P = 1.5$. For the own-population effect matrix, I used $\beta = 0.85$, $\gamma = 0.2$ and $\rho = 0.7$ for the strength of the matrix contribution to selection. Results are presented in figure 4.

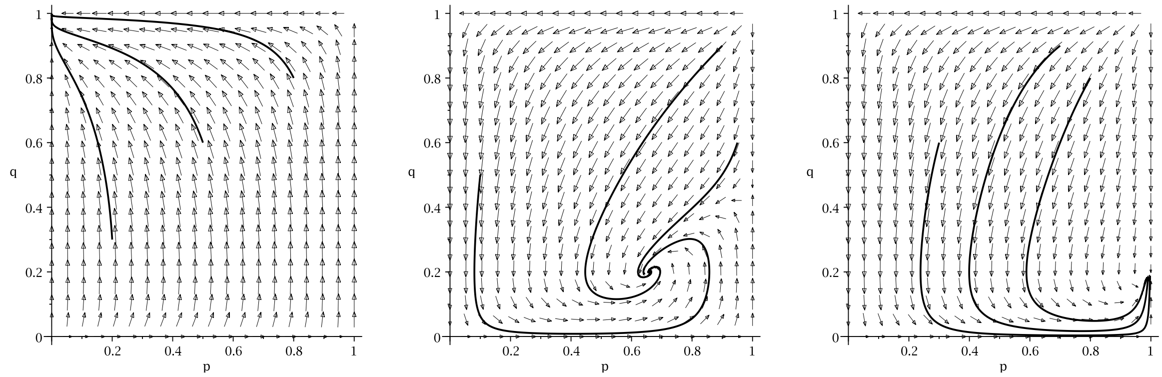

Figure 3: From left to right, vector fields for $c = 0.020$, 0.200 and 0.235.

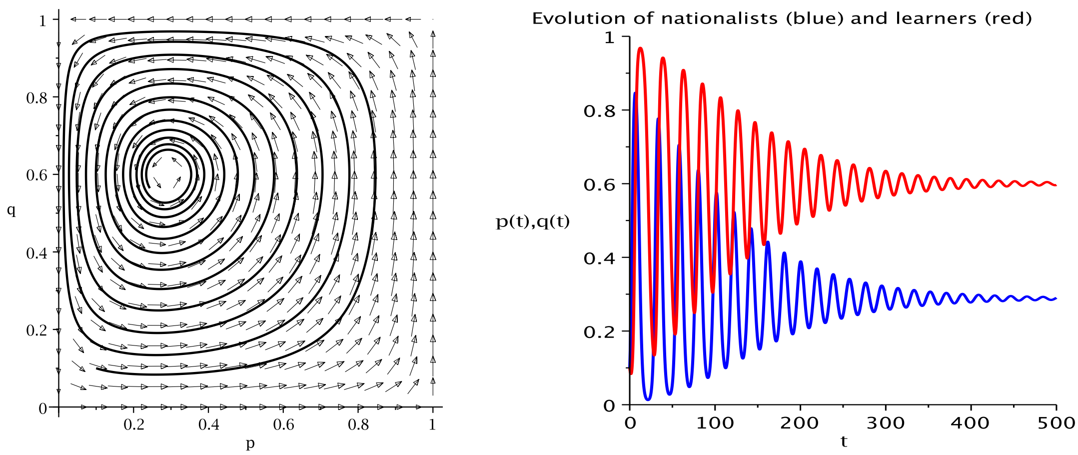

Figure 4: Evolutionary pattern for the case of weakly prevented nationalism and intermediate cost of learning when an enclave strongly influences selection.

4 - A robustness test: endogenous government policy

The natural question one poses is how robust the results of the benchmark model in section 2 are given that α, the government policy toward nationalism, is completely exogenous. Particularly, had α not been dictated by the government, would nationalism still be able to survive in the long run? In this section, I explore the endogeneization of the government policy using a rule according to which the ideology of the native population is perfectly represented by the country's government. I assume that the strength of nationalism prevention depends endogenously on the proportion of nationalists in the population of natives given that the native citizens are the ones electing the government. I assume that parameter α accounting for how accessible the public services are to non-learners is now endogenous. The parameter depends on the ideology of the elected government. The idea of government here is associated with a bi-party parliament composed of a nationalist party and a pro-immigration party, where the proportions of nationalist deputies and deputies who welcome immigrants are identical to the ones present in the population of natives. Hence a population of natives composed only of nationalists elects a 100% nationalist parliament so that the policy approved by the parliament leads to $\alpha = 0$. On the other extreme, when nationalism becomes extinct, the parliament is completely composed of deputies who welcome immigrants and $\alpha = 1$. One straightforward way of making α endogenous is to assume $\alpha = 1 - p$ and $(1 - \alpha) = p$. This has the effect of making some of the entries in the payoff matrix of section 2 endogenous. As in section 2, I focus on the case where $P > X > 0$ and there is room for nationalism to spread out and become an EE in the long run.

Substituting $(1 - \alpha) = p$ into (5) and (6), I can rewrite them as:

$$\dot{p} = [pP(1-q) - X]\, p(1-p) \tag{16}$$

$$\dot{q} = \left[p^2 P + m_1\right] q(1-q) \tag{17}$$

Lemma 5: When the degree of nationalism prevention becomes endogenous, $\alpha = 1 - p$, the dynamic system composed of (16) and (17) has one of the possible topologies presented in table 2, given the cost of learning c.

Proof. the state space has a maximum of six stationary points, $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, $\left(\frac{X}{P}, 0\right)$ and $(\widetilde{p}, \widetilde{q})$, where $\widetilde{p} = \sqrt{\frac{c-K}{P}}$ and $\widetilde{q} = \frac{\sqrt{(c-K)P} - X}{\sqrt{(c-K)P}}$. For $P > X$, $(\widetilde{p}, \widetilde{q}) \in [0,1]^2 \Leftrightarrow K + \frac{X^2}{P} < c < K + P$. The Jacobian matrix Ω evaluated at a given stationary point $\theta = (p, q)$ has the following entries:

$$\Omega(p,q) = \begin{pmatrix} [pP(1-q) - X](1-2p) + p(1-p)P(1-q) & -p^2(1-p)P \\ 2pPq(1-q) & (1-2q)\left[p^2 P + m_1\right] \end{pmatrix}$$

At $(0,0)$, $tr(\Omega) = -X + m_1$ and $det(\Omega) = -Xm_1$. Hence $(0,0)$ is asymptotically stable and an EE for $c > K$, i.e., $m_1 = K - c < 0$ and a saddle node for $c < K$ $(m_1 > 0)$. At $(1,1)$, $tr(\Omega) = X - (P + m_1)$ and $det(\Omega) = -X(P + m_1)$. This node is a saddle for $c < P + K$ and an unstable node for $c > P + K$. At $(0,1)$, $tr(\Omega) = -(X + m_1)$ and $det(\Omega) = Xm_1$, consequently $(0,1)$ is asymptotically stable for $c < K$ and a saddle for $c > K$. At $(1,0)$, $tr(\Omega) = X + m_1$ and $det(\Omega) = -(P - X)(P + m_1)$. When $P + m_1 > 0 \Rightarrow c < P + K$, $(1,0)$ is a saddle and when $P + m_1 < 0 \Rightarrow c > P + K$, the maximum value for the trace of the Jacobian is $X - P - \epsilon$, which is necessarily negative, implying asymptotic stability for the state $(1,0)$. At $\left(\frac{X}{P}, 0\right)$, $tr(\Omega) = X\left(1 - \frac{X}{P}\right) + \left(\frac{X^2}{P} + m_1\right)$ and $det(\Omega) = \frac{X}{P}(P - X)\left(\frac{X^2}{P} + m_1\right)$.

For $c > K + \frac{X^2}{P}$ this state is a saddle and it is an unstable node for $c < K + \frac{X^2}{P}$. Finally, at $(\widetilde{p}, \widetilde{q})$, $tr(\Omega) = \widetilde{p}(1-\widetilde{p})P(1-\widetilde{q}) > 0$ and $det(\Omega) = 2\widetilde{p}^3 P^2 \widetilde{q}(1-\widetilde{p})(1-\widetilde{q}) > 0$, implying that $(\widetilde{p}, \widetilde{q})$ is unstable, being an unstable focus for $\widetilde{p} > \frac{1-\widetilde{q}}{1+7\widetilde{q}}$ and an unstable node otherwise.

■

Case with $P > X$						
Value of c	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$	$\left(\frac{X}{P},0\right)$	$(\widetilde{p},\widetilde{q})$
$[0, K-\epsilon]$	saddle	EE	saddle	saddle	unstable	$\notin [0,1]^2$
$[K+\epsilon, K+\frac{X^2}{P}-\epsilon]$	EE	saddle	saddle	saddle	unstable	$\notin [0,1]^2$
$[K+\frac{X^2}{P}+\epsilon, K+P-\epsilon]$	EE	saddle	saddle	saddle	saddle	unstable/focus
$[K+P+\epsilon, +\infty)$	EE	saddle	EE	unstable	saddle	$\notin [0,1]^2$

Table 2: Possible topologies when the government policy is endogenous.

In table 2, one can see that for $0 < c < K$, $\theta^{EE} = (0,1)$ continues to hold as in the benchmark model in section 2.

Proposition 3: When the government policy is endogenous, there is no neutrally stable state in the state space.

Proof. in the benchmark model, for $(1-\alpha)P > X$ and $K < c < (1-\alpha)P + K$, no EE existed. Instead, there was a neutrally stable state and the population of natives (immigrants) would oscillate over generations with respect to its share of nationalists (learners). When the government policy becomes endogenous, for the same set of values of c, the neutrally stable state vanishes from the state space and there is one single EE characterized by the extinction of both learners and nationalists in the long run. This is shown in figure 5 for the particular case in which the state space contains six stationary points. In such a case, $\left(\frac{X}{P}, 0\right)$ and $(\widetilde{p}, \widetilde{q})$ define a separatrix in the interior of the state space. It can be seen in the figure that, in any country which society departs from an initial condition located to the left of the separatrix, nationalism is weak enough (low proportion) such that it is monotonically decreasing over time up to extinction. On the other hand, similarly to the cycle case in the benchmark model, an initial condition to the right of the separatrix implies that nationalism is strong enough to react to a low proportion of learners and nationalism initially increases over time. This increase leads to an increase in the proportion of learners among the immigrants and consequently to a fall in the proportion of nationalists. But, differently from the benchmark case, when nationalism decreases below a certain level, it enters a “trap region” in the vector field inside which p is low enough to allow for $\dot{p} > 0$ and a recovery in nationalism is not possible any more. Welcoming type natives then increase monotonically up to reaching $\theta^{EE} = (0,0)$.

■

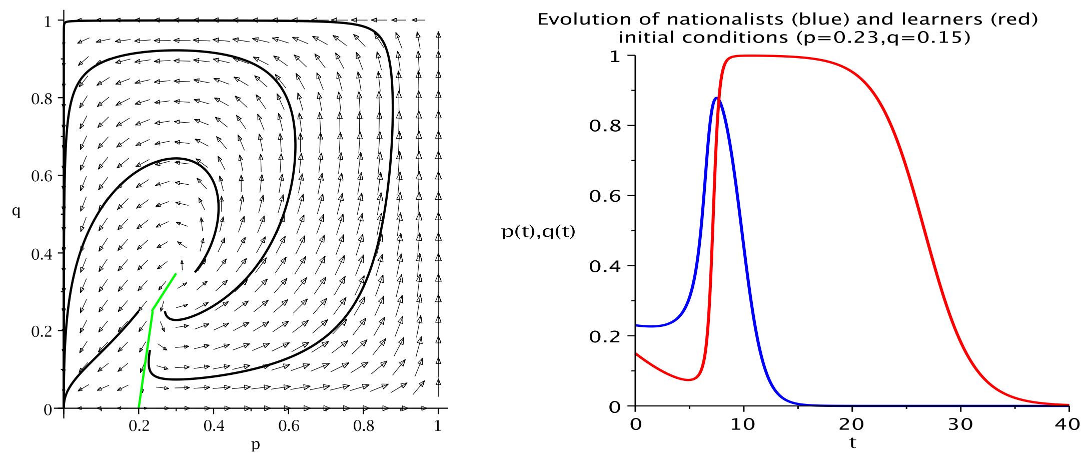

Figure 5: Evolutionary equilibrium for $K + \frac{X^2}{P} < c < K + P$. The separatrix is shown in green.

Proposition 4: When the government policy perfectly reflects the diffusion of nationalistic behaviour among the natives, nationalism is less likely to become an EE than in the case when the policy toward nationalism can be dictated by the government. The likelihood of an equilibrium in which nationalism prevails increases the lower the ratio $\frac{X}{P}$ is. As in the benchmark model, in the absence of an enclave, evolutionary equilibria always correspond to isomorphic populations.

Proof. in order to have an EE in which nationalism prevails in the endogenous model, it is required that the immigrants' social norms/culture are even more different from the host country's norms than in the benchmark model, $c > P + K$ instead of $c > (1 - \alpha)P + K$, $\forall \alpha > 0$. But even when this is the case, the evolutionary equilibrium $\theta^{EE} = (1, 0)$ now co-exists with $\theta^{EE} = (0, 0)$, and the one which is achieved depends on which basin of attraction the society's initial conditions are located in. Moreover, the likelihood of ending up with a nationalistic native society depends on the size of the basin of attraction leading the dynamics to flow towards $\theta^{EE} = (1, 0)$. This basin of attraction is larger the smaller the proportion $\frac{X}{P}$ is, given the separatrix dividing both basins of attraction is defined at its extremes by the stationary points $\theta = (1, 1)$ and $\theta = (\frac{X}{P}, 0)$ as can be seen in figure 6. The ratio $\frac{X}{P}$ can be understood as the ratio between the cost (disutility) of xenophobia and the maximum possible reward for being nationalist (recall that P is the maximum possible payoff a nationalist gets from boycotting a non-learner). The lower this cost is compared to the reward, the higher the attractiveness of nationalism is, making it more likely to succeed in attracting young native individuals. When $X << P$, $Prob\left\{\theta^{EE} = (1, 0)\right\} \to 1$ and when $X \to P$, $Prob\left\{\theta^{EE} = (1, 0)\right\} \to 0$, where $Prob\left\{\theta^{EE} = (1, 0)\right\}$ refers to the probability of the population dynamics ending up at the EE given by $\theta^{EE} = (1, 0)$.

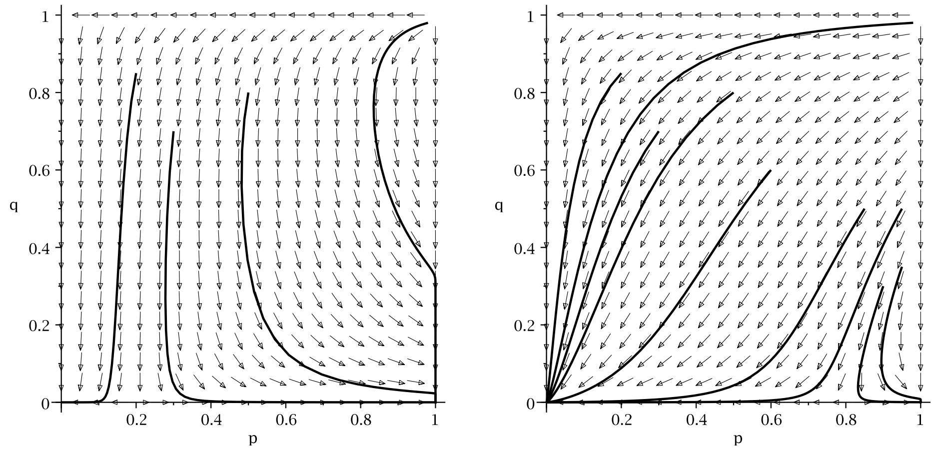

Figure 6: From left to right, large and small basin of attraction leading to $\theta^{EE} = (1, 0)$. Simulations carried out with $\frac{X}{P}$ equal to 0.2 and 0.8, respectively.

The last part of the proposition, regarding isomorphism, can be observed directly from table 2 given that the only possible evolutionary equilibria are $\theta^{EE} = \{(0, 0), (1, 0), (0, 1)\}$.

■

5 - Summary

In this paper I analysed the evolutionary pattern of the behaviours of immigrants and natives living in a country where both cultural barriers and nationalism co-exist at the initial conditions. Two central ideas in the model are the use of language learning as a proxy for cultural assimilation and that immigrants have to meet natives whenever the individuals in the former group look for public services, in which employment is restricted to native citizens.

When there is no enclave, independently of the degree of nationalism prevention, a isomorphic population of learner immigrants together with the extinction of nationalism is an EE for a low cost of learning the host country language. On the other hand, when the effort for becoming assimilated is high, multiculturalism is always an EE because immigrants evolve to a isomorphic population in which learners become extinct over time. In this case, the pattern of multiculturalism is directly linked to the government policy given that strong nationalism prevention brings nationalism to extinction while weak nationalism prevention leads to the extinction of native citizens welcoming immigrants.

For the case when the effort of learning falls in an intermediate level and nationalism is weakly prevented, no EE exists and the dynamics is similar to the Lotka-Volterra predator-prey model for competition between two animal species with no-overcrowding. Over generations, both populations of immigrants and natives continue to be polymorphic in such a way that the shares of their sub-populations keep changing over time. When the government policy becomes endogenous, such a pattern of evolution vanishes and results in an EE where both nationalists and learners are brought to extinction in the long run.

Extending the benchmark model to allow for the existence of an enclave in the host country leads to the possibility of having EEs with polymorphic populations. In the case when the government policy becomes endogenous, although nationalism can still be part of an EE, the latter necessarily co-exists with another EE in the state space in which nationalism becomes extinct. The likelihood of nationalism prevailing in the long run increases the lower the ratio between the cost and reward for being nationalist is.

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