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Industry Dynamics and Indeterminacy in an OLG Economy with Endogenous Occupational Choice

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Abstract

We model an oligopolistic industry that supplies intermediate goods in an overlapping generations economy. Agents can choose whether to provide labour or to become entrepreneurs and compete in the industry. The idea that entry is determined through occupational choice has major implications for the industry's dynamics. We find that the industry's convergence to the steady state equilibrium occurs through cyclical fluctuations. Furthermore, dynamic the path of convergence is not uniquely determined, as it is partially determined by self-fulfilling expectations. These results imply that differences in economic performance may not necessarily reflect differences in either structural characteristics or initial conditions.

Keywords: Dynamic general equilibrium, Firms' entry, Industry dynamics, Oligopoly

JEL Classification: D50, L11

1 Introduction

What are the structural characteristics that determine entry and the number of firms in industries? For many years, economists have sought to address this question through a variety of models and settings. As a result, contributions are numerous and include, among others, the novel theoretical analyses of Dixit and Stiglitz (1977); Dasgupta and Stiglitz (1980); Mankiw and Whinston (1986); Eaton and Ware (1987); Petrakis *et al.* (1997); Hunt (2004); Davidson and Mujherjee (2007); Etro (2008); and Davies and Eckel (2010).[1]

In this paper, our aim is to contribute to this strand of literature by arguing that structural characteristics may not be so informative on the equilibrium size of an industry. Using a dynamic model, we show that the only similarity of economies which are identical in terms of both structural parameters and initial conditions is their convergence to the same *long-run* value in terms of the industry's number of firms. Nevertheless, in the *transition* to this steady state the industry size may vary significantly despite the fact that, on the outset, economies are identical in every respect. The reason for this result has to do with two main factors. Firstly, the equilibrium is dynamically indeterminate; and, secondly, the shape of the industry's dynamics implies that convergence to the steady state is cyclical rather than monotonic.

We are motivated by an emerging literature of research papers that incorporate both endogenous entry and strategic interactions among oligopolists, into fully-fledged dynamic general equilibrium frameworks.[2] The papers by Ghironi and Melitz (2005), Etro and Colciago (2010), Colciago and Etro (2010) and Bilbiie *et al.* (2012) show that such frameworks can outperform Real Business Cycle (RBC) models in capturing stylised facts of key economic variables over the cycle. Bilbiie *et al.* (2008) analyse the efficiency properties of these frameworks and construct policies that restore the efficiency of market outcomes. Conceptually, our analysis is closer to the work of d'Aspremont *et al.* (1999) and Dos Santos Ferreira and Lloyd-Braga (2005, 2008) who find that dynamic equilibria can converge to periodic orbits characterised by limit (endogenous) cycles, in overlapping generations models with imperfect competition and endogenous entry.

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[1] See also Sutton (2007) and the references therein.
[2] See Etro (2009) and the references therein for a more detailed discussion on this strand of literature.

Similarly to the latter analyses, in our model the dynamics of the oligopolistic industry rest on the explicit distinction between the different stages of an agent's lifetime, made possible by the overlapping generations (OLG) setting that we employ. Particularly, the size of the industry varies over time because (i) contrary to labour, entrepreneurship requires some specific training that delays the agent's entrance in the industry for a latter stage of her lifetime and (ii) the number of agents that choose to become entrepreneurs and join the industry is determined through an occupational choice process. Essentially, in our model the more familiar zero profit condition is replaced by a condition according to which agents compare the utility associated with a particular choice of occupation. The OLG structure allows this condition to introduce rich dynamics with regards to the industry's structure.

Among the various deviations of our framework from the aforementioned body of research (d'Aspremont *et al.* 1999; Dos Santos Ferreira and Lloyd-Braga 2005, 2008), the endogenous occupational choice is the most significant one as it is responsible for an equilibrium where the number of oligopolistic firms is characterised by a second-order difference equation. Given this transition equation, we are able to derive explicitly the long-run (steady state) equilibrium and characterise it in terms of the economy's structural characteristics. Furthermore, the strong non-monotonicities that pervade the dynamics of the industry are sufficient to generate trajectories through which convergence to the steady state is cyclical.[3] These cycles do not rest on any temporary (exogenous) shocks. Rather, it is the structure of the economy that renders fluctuations an inherent characteristic of the industry's transitional dynamics.[4] Furthermore, the dynamic path along which the industry converges to its stationary equilibrium is not uniquely determined although the initial condition is given. This is because the dynamic equation that describes the industry's size implies that expectations about the number of firms operating in the future represent another important determinant of entry decisions today. Coupled with the presence of cyclical convergence, this outcome raises the possibility that economies that are identical both in terms of their structure and their initial endowment of entrepreneurial firms, may

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[3] In our model, such cycles do not represent a limiting solution as they do in d'Aspremont *et al.* (1999) and Dos Santos Ferreira and Lloyd-Braga (2005, 2008). Instead, they appear as damped oscillations through which the industry will eventually converge to a unique stationary solution.

[4] Another analysis which considers an occupational choice between entrepreneurship and labour, within a dynamic setting, is that of Cagetti and De Nardi (2006). However, in their paper they focus on issues related to wealth distribution in the presence of credit constraints while they abscond from an explicit analysis for the dynamics of the number of entrepreneurial firms.

differ significantly in terms of economic performance for some part of their transition towards the (common) steady state.

Given the above, our result echoes the main implications of the analysis by Mino *et al.* (2005). They also use an overlapping generations setting to show that occupational choice can be responsible for dynamic indeterminacy. However, there are notable differences between their setting and ours. Firstly, they do not endogenise the number of firms that operate in a particular sector; instead, they assume that both sectors in the economy (producing consumption and investment goods) are perfectly competitive. Secondly, the occupational choice entails a decision on whether to become a skilled worker or remain unskilled – with both types of labour being imperfect substitutes in production. Therefore, the aim and implications of our paper differ significantly.

We should also note that the OLG structure of our economy, also implies that the source, the nature, and the implications of dynamic patterns differ significantly in comparison to other papers of industry dynamics, such as those of Maskin and Tirole (1987; 1988a; 1988b); Hopenhayn 1992; Bergin and Bernhardt 2008; and Doraszelski and Satterthwhite (2010) among others. In these analyses, the authors construct partial equilibrium, infinite horizon models in which firms engage in dynamic competition. In our model, and given the structure of agents' lifetimes, the competition among entrepreneurs becomes essentially a static game. What generates the dynamics is occupational choice within a general equilibrium setting.

Despite the fact that our endeavour is to use a theoretical framework that offers qualitative implications, rather than quantitative ones, it should be noted that certain characteristics of the industry's equilibrium in our model are not alien to empirical facts. For example, there is considerable evidence to suggest that the dynamics of firms' movements in and out of industries are not particularly smooth. Entry and exit rates appear to be volatile, thus leading to fluctuations in the overall size of these industries (e.g., Portier 1995; Agarwal and Gort 1996; Campbell 1998; Lee and Mukoyama 2008). In terms of their frequency, there is also evidence on the presence of medium- and long-term oscillations in industrial activity (e.g., Keklik 2003; Baker and Agapiou 2006; Comin and Gertler 2006), in addition to the more commonly observed short-term movements related to the incidence of business cycles. Some existing theories have attributed such movements to the occurrence of idiosyncratic or aggregate shocks (e.g., Hopenhayn 1992; Bergin and Bernhardt 2008) and to the onset of innovations – drastic or not – that lead to a process of creative destruction (e.g., Aghion and Howitt 1992; Jovanovich and Tse 2006). Our explanation is based on the idea that entry decisions may display a transitory behaviour that allow such cycles to emerge naturally, thus it is closely related to papers that derive such oscillatory movements without the need to resort to exogenous (demand or technology) shocks (d'Aspremont *et al.* 1999; Dos Santos Ferreira and Lloyd-Braga 2005, 2008).

The remainder of our analysis is organised as follows. In Section 2 we lay down the basic set up of our economy. Section 3 derives the temporary equilibrium while Section 4 analyses and discusses the dynamic equilibrium and its implications. In Section 5 we conclude.

2 The Economic Environment

Time is discrete and indexed by $t = 0,1,2,...$. We consider an economy composed of a constant population of agents that belong to overlapping generations. Every period, a mass of $n > 1$ agents is born and each of them lives for two periods – youth and old age. During their youth, agents are endowed with a unit of time which they can devote (inelastically) to one of the two available occupational opportunities. One choice is to be employed by perfectly competitive firms who produce the economy's final good. In this case, they receive the competitive salary w_t for their labour services. Alternatively, they can devote their unit of time to some educational activity that will equip them with the ability to use entrepreneurial effort and produce units of a specific variety j of an intermediate good when they are old. Intermediate goods are used by the firms that produce and supply the final good. We assume that, once made, occupational choices are irreversible.

The lifetime utility function of an agent born in period by t is given by

$$u_j^t = c_{t,j}^t + \beta[c_{t+1,j}^t - \psi V(e_{t+1,j})], \tag{1}$$

where $\beta \in (0,1)$ is a discount factor, $c_{t,j}^t$ denotes the consumption of final goods during youth, $c_{t+1,j}^t$ denotes the consumption of final goods during old age, $e_{t+1,j}$ is entrepreneurial effort and $V(e_{t+1,j})$ is a continuous function that captures the disutility from effort and satisfies $V(0) = 0$ and $V' > 0$. The parameter ψ is a binomial indicator that takes the value $\psi = 0$ if the agent is a worker and $\psi = 1$ if the agent is an entrepreneur. As this notation is important for the clarity of the subsequent analysis, it is important to note that the time superscript indicates the period in which the agent is born whereas the time subscript indicates the period in which an activity actually occurs. The subscript j will be applicable only for entrepreneurs and, thus, will later be removed from variables that are relevant to workers.

We assume that the final good is the numéraire. The production of this good is undertaken by a large mass (normalised to one) of perfectly competitive firms. These firms combine labour from young agents, denoted L_t, and all the available varieties of intermediate goods, each of them denoted $x_{t,j}$, to produce y_t units of output according to

$$y_t = A_t \left[N_t^{-\frac{1}{\theta-1}} \left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} \right]^a L_t^{1-a}, \tag{2}$$

where $a \in (0,1)$. The parameter $\theta > 1$ is the elasticity of substitution between different varieties of intermediate goods and N_t gives the number of these different varieties (see Dixit and Stiglitz 1977).[5] Therefore, the latter variable is the number of entrants operating in the oligopolistic industry at time t. The variable A_t denotes total factor productivity, for which we assume that it grows at a constant rate $g > 0$ every period. Therefore,

$$A_t = (1+g)^t A_0, \tag{3}$$

where the initial value $A_0 > 0$ is given.

The production of intermediate goods takes place under Bertrand competition among entrepreneurs. Each of them uses her entrepreneurial effort and produces units of an intermediate good according to

$$x_{t,j} = \psi \gamma e_{t,j}, \quad \gamma > 0, \tag{4}$$

where ψ is the binomial indicator whose role we described earlier. Denoting the price of each intermediate good by $p_{t,j}$, the entrepreneur's revenue is given by $p_{t,j} x_{t,j}$. As we indicated above, the cost associated with the entrepreneurial activity is the effort/disutility cost characterised by the function $V(\cdot)$.

[5] The scale factor $N_t^{-1/(\theta-1)}$ implies that, in a symmetric equilibrium, $N_t^{-1/(\theta-1)} \left(\sum_{j=1}^{N_t} x_{t,j}^{(\theta-1)/\theta} \right)^{\theta/(\theta-1)} = N_t x_t$.

The process according to which agents choose their occupation involves the comparison of the lifetime utility that corresponds to being either a worker or an entrepreneur. This problem will be formally solved at a later stage of our analysis. Now we will identify the pattern of optimal consumption choices made by each agent, taking her occupational choice as given.

Suppose that there is a storage technology or, alternatively, a lending opportunity that provides a gross return of $1+r$ $(r \geq 0)$ units of output in period $t+1$ for every unit of output stored/lent in period t. Furthermore, denote the present value of an agent's lifetime income by i_t. Given these, we can write her lifetime budget constraint as

$$c_{t,j}^{t} + \frac{c_{t+1,j}^{t}}{1+r} = i_t . \tag{5}$$

Substituting (5) in (1), we can determine $\partial u_j^t / \partial c_{t+1,j}^t$ as follows:

$$\frac{\partial u_j^t}{\partial c_{t+1,j}^t} = \frac{-1}{1+r} + \beta . \tag{6}$$

As long as $\beta(1+r) < 1$, a condition which we henceforth assume to hold, agents will optimally want to consume all their income during their youth. Given that each worker earns w_t in labour income, we can determine that for those young agents whose choice is to provide labour we have $\psi = 0$, $c_t^{t,\text{worker}} = w_t$ and $c_{t+1}^{t,\text{worker}} = 0$. Therefore, we can use (1) to write the lifetime utility of a worker born in period t as

$$u^{t,\text{worker}} = w_t . \tag{7}$$

For entrepreneurs, however, the equilibrium characteristics are different. Although they would also prefer to consume during their youth, they can only earn income when old, i.e., when they produce and sell their intermediate products. Furthermore, it is impossible for them to borrow against their future income in order to consume in the first period of their lifetime. The reason for this is twofold. On the one hand, as we established earlier, the young workers alive in period t are not willing to lend any of their income. On the other hand, the old entrepreneurs alive in period t are also unwilling to lend because they will be dead by the time that repayment of the loan becomes possible. For these reasons, those who decide to be entrepreneurs have $\psi = 1$, $c_{t,j}^{t,\text{entrepreneur}} = 0$ and $c_{t+1,j}^{t,\text{entrepreneur}} = p_{t+1,j} x_{t+1,j}$. Using these results in (1), we get the lifetime utility of an entrepreneur born in period t as

$$u_j^{t,\text{entrepreneur}} = \beta[p_{t+1,j}x_{t+1,j} - V(e_{t+1,j})]\,. \tag{8}$$

With this expression we have completed the basic set up of our economy. In the sections that follow we derive the economy's temporary and dynamic equilibrium, with particular emphasis on the dynamics of the intermediate goods industry.

3 Temporary Equilibrium

For the producers of final goods, profit maximisation implies that each input earns its marginal product. In terms of labour income, we have

$$w_t = (1-a)A_t\left[N_t^{-\frac{1}{\theta-1}}\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right]^a L_t^{-a} = (1-a)\frac{y_t}{L_t}\,. \tag{9}$$

For intermediate goods we have

$$p_{t,j} = A_t L_t^{1-a} a N_t^{-\frac{a}{\theta-1}}\left[\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right]^{a-1}\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}-1} x_{t,j}^{\frac{\theta-1}{\theta}-1}\,. \tag{10}$$

Multiplying both sides of (10) by $x_{t,j}$ and summing over all j's, we can get

$$\sum_{j=1}^{N_t} p_{t,j}x_{t,j} = aA_t\left[N_t^{-\frac{1}{\theta-1}}\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right]^a L_t^{1-a}\,. \tag{11}$$

We can combine (10) and (11) and exercise some straightforward, but tedious, algebra to derive the demand function for an intermediate good. This is given by

$$x_{t,j} = \left(\frac{p_{t,j}}{P_t}\right)^{-\theta}\frac{X_t}{N_t}, \tag{12}$$

where

$$X_t = N_t^{-\frac{1}{\theta-1}}\left(\sum_{j=1}^{N_t} x_{t,j}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\,. \tag{13}$$

Furthermore, using $\sum_{j=1}^{N_t} p_{t,j}x_{t,j} = P_t X_t$, the price index is given by

$$P_t = \left(\frac{1}{N_t} \sum_{j=1}^{N_t} p_{t,j}^{1-\theta} \right)^{\frac{1}{1-\theta}}. \tag{14}$$

The result in (12) is nothing else than the familiar inverse demand function in models with a constant elasticity of substitution between different varieties of goods (Dixit and Stiglitz 1977). In other words, the share of product j in the overall demand for intermediate inputs is inversely related to its relative price. This effect is more pronounced with higher values of θ, i.e., if different varieties are less heterogeneous and, thus, more easily substitutable.

Now let us consider the equilibrium in the labour and final goods markets. With respect to the former, the demand for labour by firms (L_t) must be equal to the supply of labour by young agents. Recall that, in period t, out of the total population mass of n, some agents will decide to set up entrepreneurial firms and produce intermediate goods in period $t+1$. The number of these agents is N_{t+1}. Therefore, the labour market equilibrium is

$$L_t = n - N_{t+1}. \tag{15}$$

As for the goods market, let us denote the aggregate demand for final goods by d_t. According to our previous discussion, the demand for final goods is the sum of the demand by young workers and old entrepreneurs alive in period t, that is $d_t = (n - N_{t+1})c_t^{t,\text{worker}} + \sum_{j=1}^{N_t} c_{t,j}^{t-1,\text{entrepreneur}}$. Using previous results, this expression can be written as $d_t = L_t w_t + \sum_{j=1}^{N_t} p_{t,j} x_{t,j}$. However, from the expressions in (2), (9) and (11) it is clear that the unit constant returns technology implies that $y_t = L_t w_t + \sum_{j=1}^{N_t} p_{t,j} x_{t,j}$. Therefore, the final goods market clears since

$$y_t = d_t. \tag{16}$$

Now, we can use $\sum_{j=1}^{N_t} p_{t,j} x_{t,j} = P_t X_t$, $y_t - L_t w_t = \sum_{j=1}^{N_t} p_{t,j} x_{t,j}$, (9), (14) and (16) in (12) to write the demand function for the intermediate good as

$$x_{t,j} = \frac{p_{t,j}^{-\theta}}{\sum_{j=1}^{N_t} p_{t,j}^{1-\theta}} ad_t . \tag{17}$$

The result in (17) indicates the interactions in the pricing decisions made by competing oligopolists. It can be used to solve the utility maximisation problem of an entrepreneur who produces intermediate goods. To this purpose, it will be useful to specify a functional form for the effort cost component $V(e_{t+1,j})$. For this reason, and to ensure analytical tractability, we specify

$$V(e_{t+1,j}) = me_{t+1,j} , \quad m > 0 . \tag{18}$$

Writing (17) in terms of period $t+1$ and substituting it together with (4) and (18) in (8), allows us to write the utility function of the entrepreneur j as

$$u_j^{t,\text{entrepreneur}} = \beta\left(p_{t+1,j} - \frac{m}{\gamma}\right) \frac{p_{t+1,j}^{-\theta}}{\sum_{j=1}^{N_{t+1}} p_{t+1,j}^{1-\theta}} ad_{t+1} . \tag{19}$$

Given that entrepreneurs operate under Bertrand competition, their objective is to choose the price of their products in order to maximise their lifetime utility. In other words, their objective is

$$\max_{p_{t+1,j}} \left\{ \beta\left(p_{t+1,j} - \frac{m}{\gamma}\right) \frac{p_{t+1,j}^{-\theta}}{\sum_{j=1}^{N_{t+1}} p_{t+1,j}^{1-\theta}} ad_{t+1} \right\} . \tag{20}$$

After some straightforward algebra, it can be shown that the solution to this problem leads to a symmetric equilibrium for which

$$p_{t+1,j} = p_{t+1} \text{ and } x_{t+1,j} = x_{t+1} \ \forall j , \tag{21}$$

where the optimal price equals

$$p_{t+1} = \frac{m}{\gamma} \frac{[\theta(N_{t+1} - 1) + 1]}{(\theta - 1)(N_{t+1} - 1)} . \tag{22}$$

In addition, given (17) and (22), the equilibrium quantity of the intermediate good by each entrepreneur is

$$x_{t+1} = \frac{ad_{t+1}}{N_{t+1}} \frac{\gamma}{m} \frac{(\theta - 1)(N_{t+1} - 1)}{[\theta(N_{t+1} - 1) + 1]} . \tag{23}$$

The result in (22) resembles the familiar condition according to which the price is set as a markup over the marginal cost of production. In this case, each entrepreneur sets a markup over the marginal utility cost of producing the intermediate good, since one unit of production requires a utility cost of m/γ units of effort. Naturally, the markup is decreasing in the number of entrepreneurs because the latter implies a more intensely competitive environment. Additionally, the markup is also decreasing in θ because higher values of this parameter increase the degree of substitutability between different varieties of intermediate goods – yet another structural characteristic that enhances the degree of competition. From (23), we can see that the inverse demand function implies that the components that reduce the relative price of the input increase its share on aggregate demand.

The solutions above allow us to rewrite the utility of an entrepreneur, after substituting (16), (21) and (22) in (19), as follows:

$$u^{t,\text{entrepreneur}} = \frac{\beta a y_{t+1}}{\theta(N_{t+1}-1)+1}. \tag{24}$$

With this result at hand, we can now turn our attention to the occupational choice problem of an agent who is young in period t.

3.1 Occupational Choice

Our purpose in this section is to determine how many agents will decide to become entrepreneurs. Obviously, the equilibrium condition requires that an agent born in t should be indifferent between the two different occupational opportunities. Formally, a condition that needs to hold in equilibrium is

$$u^{t,\text{entrepreneur}} = u^{t,\text{worker}}, \tag{25}$$

or, after utilising (7), (9) and (24),

$$\frac{\beta a y_{t+1}}{\theta(N_{t+1}-1)+1} = (1-a)\frac{y_t}{L_t}. \tag{26}$$

We can manipulate algebraically the expression in (26) even further. First, we can use the symmetry condition (eq. 21) in (2) to get

$$y_t = A_t (N_t x_t)^a L_t^{1-a}. \tag{27}$$

Next, we substitute (16) in (23) to get

$$N_{t+1}x_{t+1}=ay_{t+1}\frac{\gamma}{m}\frac{(\theta-1)(N_{t+1}-1)}{[\theta(N_{t+1}-1)+1]}\Leftrightarrow N_t x_t=ay_t\frac{\gamma}{m}\frac{(\theta-1)(N_t-1)}{[\theta(N_t-1)+1]}. \tag{28}$$

Further substitution of (28) in (27) allows us to write

$$y_t = A_t^{\frac{1}{1-a}}\left\{\frac{a\gamma}{m}\frac{(\theta-1)(N_t-1)}{[\theta(N_t-1)+1]}\right\}^{\frac{a}{1-a}} L_t. \tag{29}$$

Finally, we can use (15), (28) and (29) in (26), and rearrange to get

$$\frac{n-N_{t+2}^E}{\theta(N_{t+1}-1)+1}=\frac{(1-a)}{a\beta(1+g)^{1/(1-a)}}\left\{\frac{[\theta(N_{t+1}-1)+1]}{(N_{t+1}-1)}\frac{(N_t-1)}{[\theta(N_t-1)+1]}\right\}^{a/(1-a)}, \tag{30}$$

where $1+g=\frac{A_{t+1}}{A_t}$ is derived by alluding to (3). Note that the superscript in N_{t+2}^E denotes the expectation formed on this variable.

The result in equation (30) is the most important in our set-up. It implies that the determination of the equilibrium number of oligopolistic firms in the intermediate goods industry is not a static one. Instead, there will be some transitional dynamics as the number of entrepreneurs converges to its long-run equilibrium. Particularly, we can see that the equilibrium number of entrepreneurs in any given period depends on both the predetermined number of entrepreneurs from the previous period and the expectation on the number of entrepreneurs that will be active during the next period. Note that the endogenous occupational choice is critical for these dynamics. It is exactly because of this choice that the determination of N_{t+1} is related to the previous period's demand conditions (and, thus, N_t) and the next period's labour market equilibrium (therefore N_{t+2}^E).

The intuition for these effects is as follows. If the existing number of entrepreneurial firms is large, then the overall amount of intermediate goods and, therefore, the marginal product of labour will be higher. This increases the equilibrium wage and thus, the relative benefit from the utility of being a worker when young, rather than setting up an entrepreneurial firm when old. Now suppose that, while forming their occupational choice, the current young expect that the future number of entrepreneurial firms in the intermediate goods industry will be high. For them, this implies that the amount of labour and, therefore, total demand in the next period will be relatively low. Thus, the relative utility benefit of being an entrepreneur when old, rather than a worker when young, is reduced.

Consequently, a reduced number of individuals, out of the current young, will opt for the choice of becoming entrepreneurs.

4 Dynamic Equilibrium

The remainder of our analysis will focus on the dynamics of the industry that produces intermediate goods. Given that agents are assumed to have perfect foresight, we consider equilibrium trajectories that satisfy $N_{t+2}^{E} = N_{t+2}$.

4.1 The Steady State

We can obtain the stationary equilibrium for the number of entrepreneurial firms, after substituting $N_{t+2}^{E} = N_{t+2}$ in (30) and using the steady state condition $N_{t+2} = N_{t+1} = N_t = \hat{N}$. This procedure will eventually allow us to derive

Proposition 1. *Suppose* $n > 1+\delta$ *where* $\delta = (1-a)/a\beta(1+g)^{1/(1-a)}$. *Then there exists a unique steady state equilibrium* $\hat{N} \in (1,n)$ *such that*

$$\hat{N} = \frac{n + (\theta - 1)(1-a)/a\beta(1+g)^{1/(1-a)}}{1 + \theta(1-a)/a\beta(1+g)^{1/(1-a)}}. \tag{31}$$

As long as the steady state solution is asymptotically stable, then for any predetermined $N_0 \in (1,n)$ the equilibrium number of entrepreneurs will eventually converge to $\hat{N}$ in the long-run. It is instructive to undertake some comparative statics to identify the effects of the economy's structural parameters on the steady state number of entrepreneurs competing in the intermediate goods industry. This is a task that can be easily undertaken through the use of equation (31). The results can be summarised in

Proposition 2. *The long-run equilibrium number of firms in the intermediate goods industry is:*

i. Increasing in the growth rate of total factor productivity (g) *and the relative weight attached to old age consumption (the discount component* β*);*

ii. *Decreasing in the relative share of labour income* $(1-a)$ *and the degree of substitutability between different varieties of intermediate products* (θ).

The economic interpretation for these results is as follows. A permanent increase in the growth rate causes future demand to become even higher compared to current demand because of the increase in the economy's resources. This effect boosts the relative utility benefit of becoming an entrepreneur, with corresponding implications for the occupational choices made by young agents. An increase in the relative share of labour income will motivate more agents to work for final goods firms, as the income earned from entrepreneurial activities becomes relatively low. The utility benefit of entrepreneurial activities is also impeded in an industry where goods are less heterogeneous. Finally, when individuals discount the utility from old age consumption less heavily, then they have a greater incentive to opt for the occupation from which old age consumption accrues – in this case, entrepreneurship.

With respect to output, once the oligopolistic industry converges to its steady state, the production of final goods will converge to a balanced growth path. Along this path, output will grow at the same rate as total factor productivity, i.e., at a rate g. However, during the transition to the balanced growth path, the dynamics of output will also be dictated by the transitional dynamics of the intermediate goods industry. The analysis of the latter is undertaken in the following section. It will become clear that certain characteristics of the dynamic equilibrium will imply that the structural parameters of the model are only informative for the limiting (long-run) situation with regards to the size of the industry. For a large part of the transition to this long-run equilibrium, even structurally identical economies may display large differences with regards to the number of entrants in the intermediate goods industry.

4.2 Transitional Dynamics

Let us use $N_{t+2}^E = N_{t+2}$ in (30) and solve the resulting expression for N_{t+2}. Eventually, we get

$$N_{t+2} = n - \frac{(1-a)}{a\beta(1+g)^{1/(1-a)}} \frac{[\theta(N_{t+1}-1)+1]^{1/(1-a)}}{(N_{t+1}-1)^{a/(1-a)}} \left[\frac{N_t - 1}{\theta(N_t - 1)+1} \right]^{a/(1-a)} = F(N_{t+1}, N_t). \quad (32)$$

As we can see, the dynamics of the intermediate goods industry are characterised by a non-linear, second-order difference equation in terms of the industry's size (i.e., the number of entrepreneurs who compete in the industry).

One way to analyse the transition equation in (32) is to define $Z_t = N_{t+1}$ and treat the dynamics as being generated by the following system of first-order difference equations:[6]

$$Z_{t+1} = F(Z_t, N_t) = n - \frac{(1-a)}{a\beta(1+g)^{1/(1-a)}} \frac{[\theta(Z_t - 1) + 1]^{1/(1-a)}}{(Z_t - 1)^{a/(1-a)}} \left[\frac{N_t - 1}{\theta(N_t - 1) + 1} \right]^{a/(1-a)}, \tag{33}$$

$$N_{t+1} = H(Z_t, N_t) = Z_t, \tag{34}$$

where $N_0, Z_0 \in (1, n)$ are taken as the initial conditions and the steady state satisfies $\hat{Z} = \hat{N}$. The Jacobian matrix of the dynamical system in (33)-(34) can then be used to check the stability of the steady state solution and to trace the transitional dynamics of the intermediate goods industry. Unfortunately, the complexity of the dynamic equation in (33), coupled with the also complicated steady state solution in (31), imply that obtaining conditions for stability and tracing the transitional dynamics analytically is an almost impossible task. For this reason, in what follows we will analyse the transition equation in (32) numerically, making sure to choose parameter values that render the solution in (31) stable, hence a meaningful one. We should reemphasise, however, that we undertake these numerical simulations solely because the dynamical system involved is too complicated to deal with analytically. The focus of our analysis is still purely qualitative, hence it is neither our intention nor do we claim any attempt to offer a quantitative match of key moments from stylised facts.

For the baseline parameter values, we choose $a = 0.5$, $g = 0.15$, $\beta = 0.95$ and $\theta = 1.2$, while the total population is set to $n = 1,000$.[7] The initial values are $N_0 = 200$ and $N_1 = 750$ – recalling that N_1 corresponds to Z_0 in (34). In Table 1 and Figure 1 we see the transitional dynamics from this simulation, illustrating that the industry converges to the steady state. Nevertheless, this convergence is clearly non-monotonic. Instead, convergence takes place through damped oscillations, or cycles, during which the number of entrepreneurs takes values above and below the stationary value as the industry approaches

[6] See Galor (2007).

[7] The choice of parameter values is made with the purpose of ensuring the stability of the equilibrium in (31). The numerical examples that follow indicate that the dynamical system can display a local bifurcation (see Galor 2007). In other words, low values for a, g and β, or a high value for θ, will eliminate the stability of the steady state solution that we derived in (31).

towards it. Of course, variations of these parameters do not only affect the steady state but also the speed of convergence. For example, we can see that as we vary the growth rate from 0.125 to 0.30 and keep all the other baseline parameters the same, the speed of convergence increases (Figures 2-4). Similar implications apply to variations of other structural parameters, in relation to the baseline case. For example, in Figures 5-7 we vary θ from 1.05 to 1.26 whereas in Figures 8-10 we vary a from 0.49 to 0.65.

In all these cases, we can observe how the transitional dynamics for the intermediate goods industry change as we vary the structural characteristics of the economy. One common theme in all the above examples, however, is that this convergence is certainly non-monotonic. Thus, we can use these observations to deduce an important characteristic for the transitional behaviour of the industry – a characteristic that we describe in

Proposition 3. *The number of firms in the intermediate goods industry converges to its long-term equilibrium through cycles.*

Recall that the number of entrepreneurs in any given period is affected by both the predetermined number of entrepreneurs from the previous period and the expectation on the number of entrepreneurs that will be active in the future. The manner and direction of these effects, both discussed at an earlier point of our analysis, render the result of Proposition 3 to be a quite intuitive one. For example, consider a situation where the existing number of entrepreneurs is low relative to the steady state. For the current young agents, the incentive to opt for entrepreneurial activities is enhanced because the marginal product of labour (and, therefore, the wage) is currently low. As a result, an increased fraction of the current young will choose to become entrepreneurs and compete in the intermediate goods industry when they become old. However, for this to happen they also need to expect that, next period, a lower fraction of the future generation's agents will decide to become entrepreneurs because this will increase labour and, therefore, aggregate demand during the period where entrepreneurs will reap the benefits of their activity. The mechanism that we described previously does verify this expectation, hence granting an even greater incentive to the current young for becoming entrepreneurs. Furthermore, it explains why the size of the intermediate goods industry converges to its long-run equilibrium through cycles.

Another issue of great significance has to do with the determinacy of the industry's dynamic equilibrium. As we have seen from the second order transition equation in (32), or the equivalent dynamical system in (33)-(34), the transitional dynamics are traced after we consider two initial values N_0 and Z_0 – the latter corresponding to N_1. Nevertheless, while N_0 is indeed predetermined, this is not the case for N_1. Instead, taking the value of N_0 as given, N_1 reflects an equilibrium formed on an expectation about N_2 and so on. In other words, the stability of the steady state equilibrium $\hat{N}$ implies that, for the same $N_0 \in (1, n)$, there are certainly more than one trajectories that are consistent with perfect foresight and through which the intermediate goods industry converges to its steady state. We can also see this outcome through a numerical example. In Table 2 and Figure 11 we use the same baseline parameter values from our original simulation, the same initial value $N_0 = 200$ but a different $Z_0 = N_1 = 400$. Obviously, convergence takes place to the same steady state value as the one in our baseline scenario. Yet the dynamic path, though consistent with perfect foresight, is a different one. We can summarise the preceding discussion through

Proposition 4. *The dynamic equilibrium is indeterminate in the sense that, for given* $N_0 \in (1, n)$*, there are multiple trajectories that converge to the unique steady state equilibrium* $\hat{N}$.

This indeterminacy, coupled with the fact that convergence is cyclical, has major implications for outcomes related to the equilibrium number of competing firms. Industries that are identical both in terms of structural parameters and predetermined conditions may display very different equilibrium characteristics for a large part of their transition towards the common steady state. In other words, differences between economies, in general, and competitive conditions in their industries, in particular, may not necessarily reflect any differences in structural characteristics and initial conditions. Indeed, in our example we consider two situations that entail identical structural characteristics and predetermined value N_0. Yet, due to the combination of dynamic indeterminacy and damped oscillations, outcomes are rather different for a large part of the transition (see Table 2). If we were to consider these scenarios as describing different economies, then we can see that whereas the number of competing firms is relatively high in one economy, the corresponding number is relatively low in the other. While the number of firms in the next period will decline in the former case, in the latter one it will actually increase.

These results apply despite the fact that these economies are identical on the outset. The reason for this outcome rests on the dynamic nature of our framework and the endogenous choice of occupation. As a result, the number of competing firms is characterised by second order difference equation that turns self-fulfilling expectations into a major component of equilibrium outcomes. As such, the informational strength of structural characteristics as determinants of the industry size is significantly reduced.

5 Conclusion

In this paper, our endeavour was to contribute to the emerging body of literature that studies the dynamic behaviour of oligopolistic industries in dynamic general equilibrium models. We showed that an overlapping generations setting, combined with the idea that entry decisions are made through an occupational choice process, can lead to very interesting implications concerning these dynamic patterns. We showed that convergence to the long-run equilibrium occurs through cycles in the transitional dynamics of the industry. This cyclical path of convergence is not uniquely determined – in fact, there are multiple trajectories leading to the long-run equilibrium that characterises the number of oligopolistic firms in the industry. The combination of these results implies that structural characteristics are only informative for the long-run outcomes concerning the number of competing firms in the intermediate goods industry. During the transition, however, economies that are identical in terms of both initial and structural characteristics, may display drastically different outcomes with regards to the equilibrium number of entrants in the industry.

A note of caution merits discussion here, given the fact that that our paper's transitional dynamics are characterised by periodic orbits that may resemble the type of fluctuations we observe in the data. We believe that a better interpretation of our results should entail a correspondence to low frequency waves in industry dynamics, rather than the high frequency fluctuations that are more suitably attributed to the occurrence of short-term economic fluctuations. For this reason, we need to clarify that our analysis in under no circumstances an attempt to invalidate other explanations for the cyclicality of industry dynamics, based on the idea of exogenous shocks – explanations that we actually view as being indubitably important. The main message form our work is that the cyclical behaviour of industries, in addition to being a response to changing economic conditions, may also reflect characteristics that render them inherently volatile. As we indicated at the very beginning of this paper, other authors have asserted the same through their research work, thus offering some momentum to this idea.

The model we presented is simple enough to guarantee a clear understanding of the mechanisms that are involved in the emergence of the basic results, without blurring either their transparency or their intuition. Of course, there is certainly a large scope for getting additional implications by modifying or enriching some of the model's founding characteristics. One obvious direction is to assume that the oligopolistic industry supplies firms with different varieties of capital goods while, at the same time, retaining the important characteristic of endogenous occupational choice. The ensuing process of capital accumulation could set in motion some very interesting implications concerning economic dynamics. We believe that this set up should certainly offer a potentially fruitful avenue for future research work.

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Appendix

Table 1

Baseline parameter values: $a = 0.5$; $g = 0.15$; $\beta = 0.95$; $\theta = 1.2$; $n = 1,000$
Initial condition: $N_0 = 200$, $N_1 (= Z_0) = 750$
Steady state: $\hat{N} \simeq 512$

t	N_t	t	N_t	t	N_t	t	N_t
0	200	30	455.1898989	60	498.7016576	90	508.6240316
1	750	31	565.2326367	61	523.7948116	91	514.3483867
2	286.0056558	32	460.4819683	62	499.9077824	92	508.8991842
3	726.4929716	33	560.1917613	63	522.6464763	93	514.0864501
4	307.4927766	34	465.2765466	64	501.0007379	94	509.1485208
5	706.0035607	35	555.6249973	65	521.6059032	95	513.8490903
6	326.868143	36	469.6205726	66	501.9911442	96	509.3744637
7	687.5293695	37	551.4875753	67	520.6629771	97	513.6340013
8	344.3598002	38	473.5565108	68	502.8886241	98	509.5792077
9	670.8528685	39	547.7389986	69	519.8085329	99	513.4390934
10	360.164213	40	477.1227901	70	503.7018972	100	509.7647417
11	655.7863931	41	544.3426237	71	519.0342663	101	513.262473
12	374.4531304	42	480.3541958	72	504.4388649	102	509.9328682
13	642.1658944	43	541.2652853	73	518.3326525	103	513.1024243
14	387.3780663	44	483.2822221	74	505.1066868	104	510.0852203
15	629.8466606	45	538.4769597	75	517.6968732	105	512.9573923
16	399.073548	46	485.9353879	76	505.7118506	106	510.2232781
17	618.7002149	47	535.9504624	77	517.1207505	107	512.8259682
18	409.659581	48	488.3395214	78	506.2602355	108	510.3483827
19	608.6119605	49	533.6611761	79	516.598686	109	512.706875
20	419.2435891	50	490.5180167	80	506.7571687	110	510.4617494
21	599.4793272	51	531.5868059	81	516.1256069	111	512.5989559
22	427.9219852	52	492.4920651	82	507.2074776	112	510.5644795
23	591.2102694	53	529.707158	83	515.6969168	113	512.5011624
24	435.7814734	54	494.2808636	84	507.6155368	114	510.657571
25	583.7220212	55	528.0039404	85	515.3084502	115	512.4125444
26	442.900148	56	495.901804	86	507.9853103	116	510.7419283
27	576.940046	57	526.4605827	87	514.956433	117	512.3322411
28	449.3484335	58	497.3706432	88	508.3203902	118	510.8183707
29	570.7971365	59	525.0620731	89	514.6374448	119	512.2594723

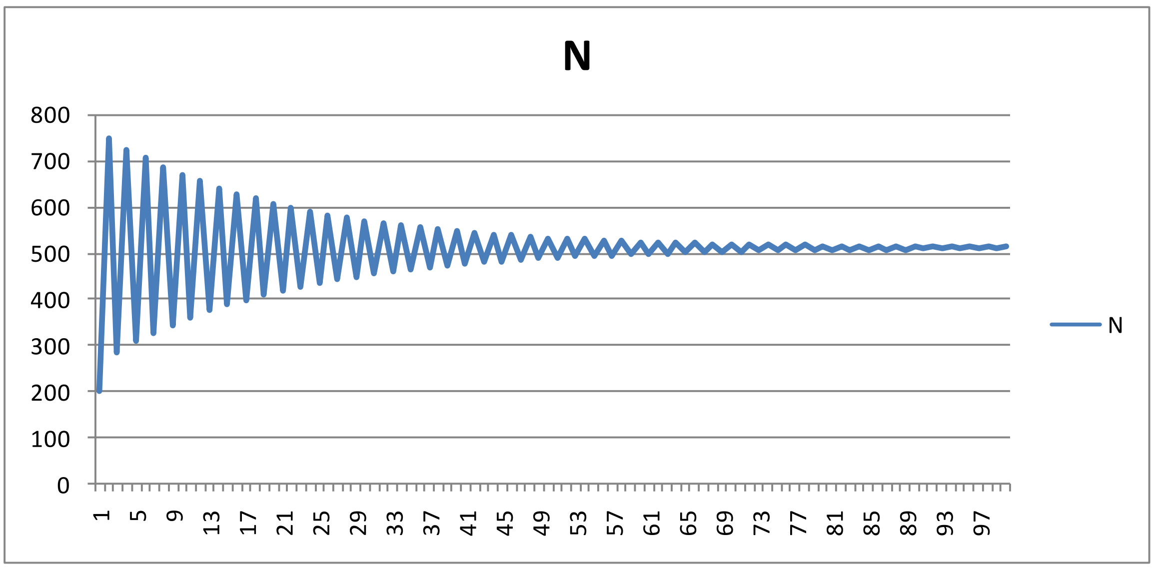

Figure 1: Baseline case

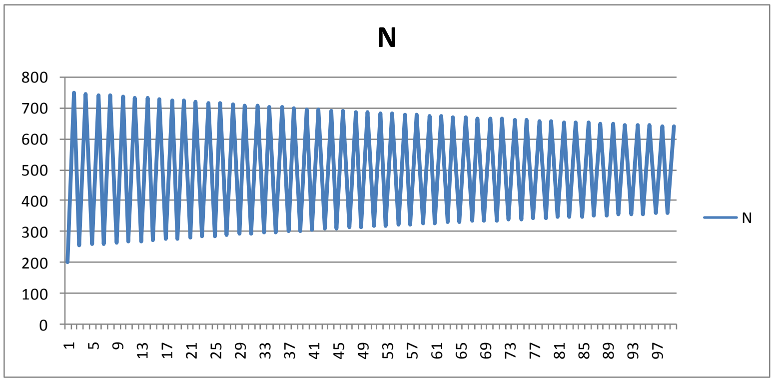

Figure 2: $g = 0.125$ **,** $\hat{N} \simeq 501$

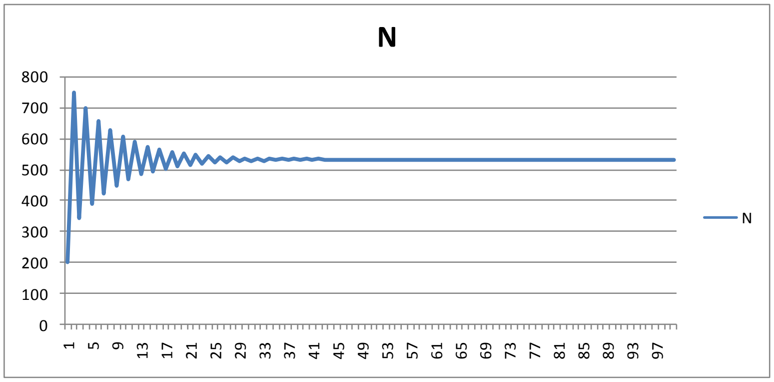

Figure 3: $g = 0.20$ **,** $\hat{N} \simeq 534$

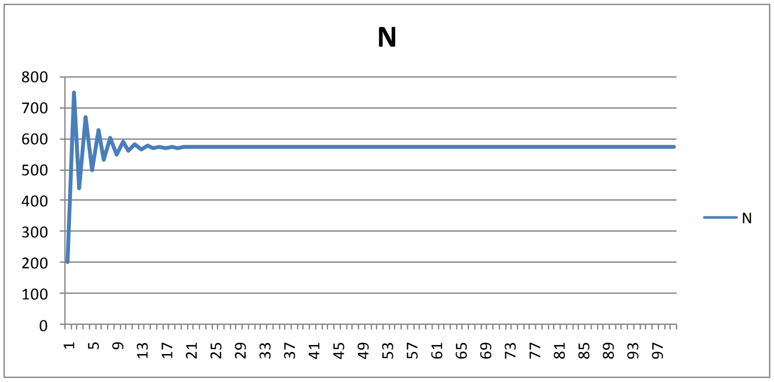

Figure 4: $g = 0.30$ **,** $\hat{N} \simeq 572$

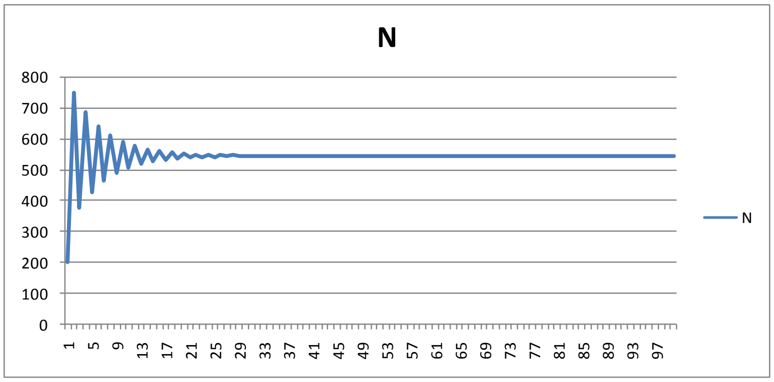

Figure 5: $\theta = 1.05$, $\hat{N} \simeq 545$

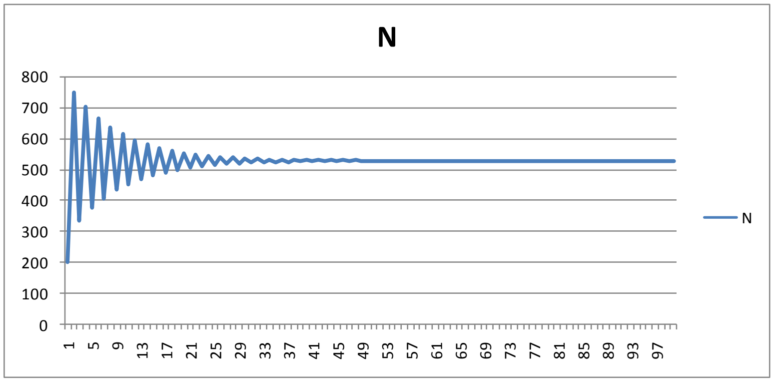

Figure 6: $\theta = 1.12$, $\hat{N} \simeq 529$

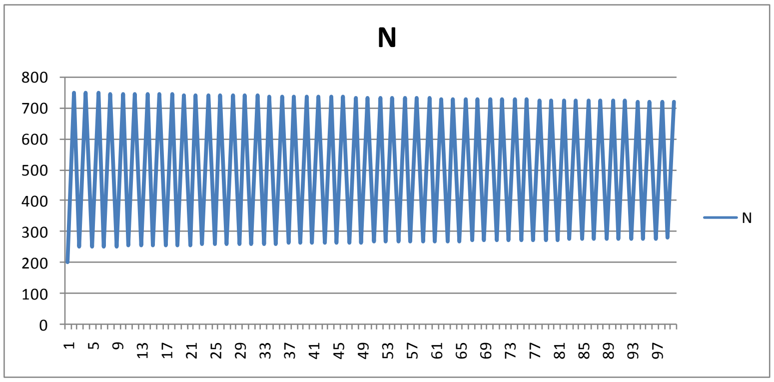

Figure 7: $\theta = 1.26$, $\hat{N} \simeq 499$

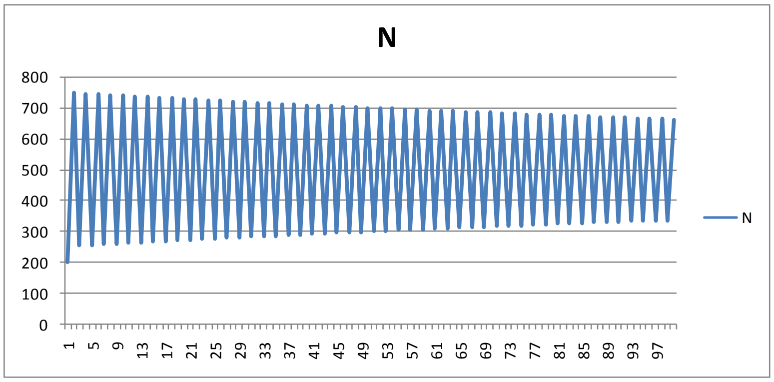

Figure 8: $a = 0.49$, $\hat{N} \simeq 500$

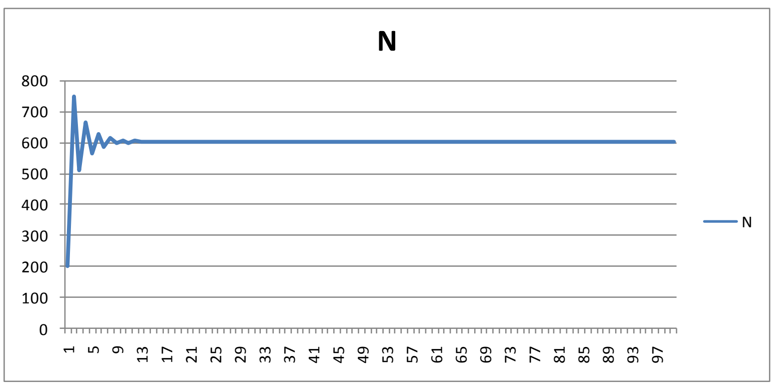

Figure 9: $a = 0.58$, $\hat{N} \simeq 604$

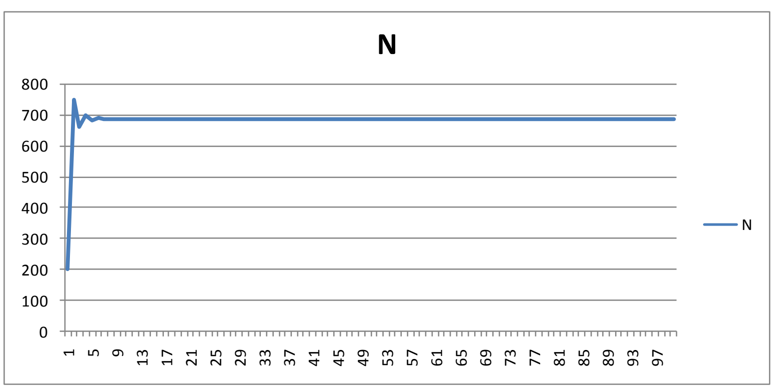

Figure 10: $a = 0.65$, $\hat{N} \simeq 687$

Table 2

Baseline case against scenario with $N_1(=Z_0)=400$

Steady state: $\hat{N} \simeq 512$

t	***N_t* *(Z_0=750)***	***N_t* *(Z_0=400)***	***t***	***N_t* *(Z_0=750)***	***N_t* *(Z_0=400)***
0	200	200	30	455.1898989	538.5293809
1	750	400	31	565.2326367	485.8855066
2	286.0056558	618.9059193	32	460.4819683	535.9979615
3	726.4929716	409.4606532	33	560.1917613	488.2943219
4	307.4927766	608.80153	34	465.2765466	533.7042158
5	706.0035607	419.0634704	35	555.6249973	490.4770592
6	326.868143	599.6509565	36	469.6205726	531.6258053
7	687.5293695	427.7588737	37	551.4875753	492.4549512
8	344.3598002	591.3656818	38	473.5565108	529.7424966
9	670.8528685	435.6337443	39	547.7389986	494.2472325
10	360.164213	583.8627677	40	477.1227901	528.0359621
11	655.7863931	442.7663368	41	544.3426237	495.8713288
12	374.4531304	577.0675241	42	480.3541958	526.489599
13	642.1658944	449.2272189	43	541.2652853	497.3430276
14	387.3780663	570.9126072	44	483.2822221	525.0883662
15	629.8466606	455.0800877	45	538.4769597	498.6766332
16	399.073548	565.3372384	46	485.9353879	523.8186372
17	618.7002149	460.3824824	47	535.9504624	499.885106
18	409.659581	560.2865225	48	488.3395214	522.668066
19	608.6119605	465.1864112	49	533.6611761	500.9801892
20	419.2435891	555.7108479	50	490.5180167	521.6254669
21	599.4793272	469.5389059	51	531.5868059	501.9725235
22	427.9219852	551.5653562	52	492.4920651	520.680705
23	591.2102694	473.4825151	53	529.707158	502.8717505
24	435.7814734	547.8094705	54	494.2808636	519.8245973
25	583.7220212	477.0557432	55	528.0039404	503.6866068
26	442.900148	544.4064751	56	495.901804	519.0488233
27	576.940046	480.2934441	57	526.4605827	504.4250091
28	449.3484335	541.3231395	58	497.3706432	518.3458435
29	570.7971365	483.2271736	59	525.0620731	505.094131

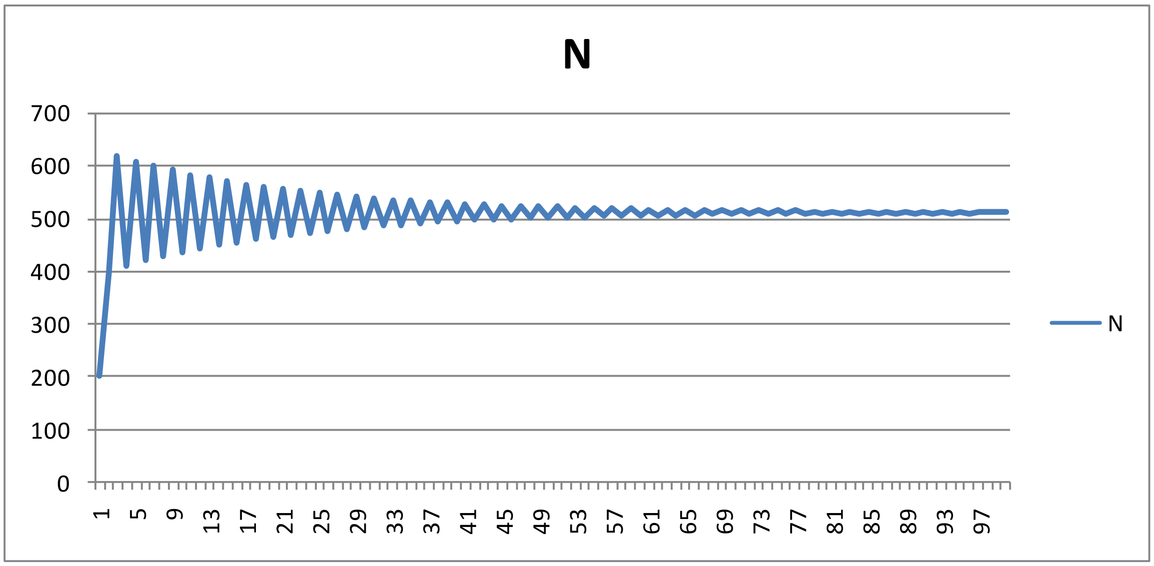
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Figure 11