Emission Taxes and the Adoption of Cleaner Technologies: The Case of Environmentally Conscious Consumers

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Abstract
We model a market with environmentally conscious consumers and a duopoly in which firms consider the adoption of a clean technology. We show that as pollution increases, consumers shift more resources to the environmental activities, thereby affecting negatively the demand faced by the duopoly. This effect generates incentives for firms to adopt the clean technology even in the absence of emissions taxes. When such taxes are considered, our results indicate that the benefit of adopting the clean technology is initially increasing and then decreasing in the emission tax. The range of values for which the emission tax increases this benefit becomes narrower when the consumers’ environmental awareness is stronger.

Keywords: Environmentally Conscious Consumers; Technology Choice; Environmental Taxation

JEL Classification: L13, Q55, Q58
1 Introduction

Recent years have witnessed an increased awareness for issues pertaining to the impact of economic activity on environmental degradation. As a result, both policy makers and the wider public have intensified their efforts and actions towards pollution reduction. On the one hand, policy makers have attempted to encourage firms’ investments in environmental R&D using a variety of instruments such as taxes on emissions, caps or R&D subsidies. On the other hand, environmentally conscious consumers have not only shifted their preferences towards goods with environmentally friendly attributes (e.g., recyclable packaging, organic produce, certification of environmentally friendly production techniques etc.) but they have also increased the resources they devote to general activities that mitigate the extent of environmental degradation.

There are many ways through which consumers can contribute resources to improve the environment. One example is the participation in carbon offsetting schemes. These schemes are supported by firms in a variety of industries, from aviation (British Airways, for example) to energy generation (Eon). Through these schemes, individuals contribute financially to the purchase of carbon credits to compensate for their own emissions. Another example is the individuals’ donations to certain NGOs who purchase permits from emissions trading systems on their behalf, thereby reducing the amount of available permits and therefore effective emissions. Examples of such NGOs are the Acid Retirement Fund and the Clean Air Conservancy Trust in the US or Sandbag in the UK. Finally, individuals can take part in environmental volunteering, which often involves not only the supply of unpaid work (with its associated opportunity cost) but an additional financial contribution.1

So far, the literature has contemplated the existence of environmentally conscious consumers in models of product differentiation, where consumers’ preferences for the environment motivate competing firms to choose the environmental attributes of their products (e.g., Bansal and Gangopadhyay 2003; Conrad 2005; Deltas et al. 2008; Andre et al. 2009).2 However, such frameworks fail to capture the essential features of the arrangements

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1 For example, Global Vision International is an organisation which runs a number of projects related to climate change and conservation all over the world. Volunteers pay a financial contribution and work on their chosen project for a certain amount of time.

2 These papers analyse the scope for public intervention even in the presence of environmentally conscious consumers in the context of horizontal differentiation (Conrad 2005); vertical differentiation (Bansal and
we described earlier (carbon-offsetting programmes, donations to charities, volunteering, etc.). In this type of arrangements, consumers effectively internalise (part of) the environmental damage and spend some of their resources to general environmentally friendly activities. The reason why the formal analysis of these issues is important is because by spending more on environmentally friendly activities, consumer resources are directed away from the consumption of goods produced by all competing firms. Thus, this may generate an aggregate demand effect which affects negatively the demand faced by all firms (rather than a shift in the relative demand between different goods as in models of vertical and horizontal differentiation). This different type of environmental consciousness therefore calls for an alternative frame of analysis in order to study its effects on firms’ behaviour and its implications for environmental policy.

In this paper, our aim is to fill the current void in the literature. We begin our analysis with the description of a market where consumers have preferences over the consumption of a homogeneous good and environmental quality. These consumers can also devote resources towards environmental improvements. We show that, in response to an increase in pollution, consumers shift their resources away from the consumption of goods and towards activities that mitigate the extent of environmental degradation. Subsequently, we analyse a Cournot duopoly in which this negative demand effect impinges on both firms’ decisions concerning output production and the cleanliness of the technologies they employ. The latter is characterised by the pollutants emitted per unit of production and its choice may entail positive technology spillovers across firms. In this context, we derive and discuss the implications of the negative demand effect of pollution for firms’ optimal choices. Furthermore, we show how the relative strength of this effect may impinge on the effectiveness of emission taxes as policy tools designed to motivate the adoption of cleaner production techniques by firms.

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Gangopadhyay 2003; Andre et al. 2009); and where two dimensions of differentiation, environmental and intrinsic characteristics of the goods, are considered (Deltas et al. 2008).

3 There is substantial evidence on the extent technological spillovers across firms and/or industries. See Griliches (1992), Buonanno et al. (2001) and Clarke et al. (2006).

4 In addition to the more mainstream instruments of environmental policy (i.e., emission taxes; subsidies to environmental innovation) the use of informational campaigns to raise environmental awareness as a policy instrument has become the focus of a number of studies (see Petrakis et al. 2005; Garcia-Gallego and Georgantzis 2009, 2011). Our decision to focus on emission taxes lies on the importance of (environmental) technology choice in our setting.
The remaining of the paper is structures as follows. In Section 2, we analyse a market in which environmentally conscious consumers devote resources towards environmental improvements and derive the implications of higher pollution for consumer demand. In Section 3, we use these implications in a Cournot duopoly model with endogenous technology choice and derive equilibria in both the absence and the presence of emission taxes. Section 4 shows how the negative demand effect from pollution affects the scope of emission taxes as incentive mechanisms for the adoption of cleaner production methods. Section 5 concludes.

2 A Market with Environmentally Conscious Consumers

Consider a market which consists of a unit mass of identical consumers and $N$ firms that produce and sell quantities of a homogeneous product. The price of this product is denoted $P$. Each consumer $i \in [0,1]$ is endowed with an (exogenous) income of $y_i > 0$ and her preferences are defined over the consumption of the homogeneous good, denoted $C_i$, and her environmental concerns, captured by the variable $E_i$, according to

$$u_i = \delta \ln(C_i) + (1-\delta) \ln(E_i),$$

where $\delta \in (0,1)$ weights the two arguments of the consumer’s utility.

We shall elaborate on the consumers’ preferences over the natural environment by assuming that these are composed of two components. Firstly, we incorporate the amenity value of improved environmental quality (common to all consumers) by postulating that pollution – a by-product of firms’ production activities – entails a utility cost for consumers. Secondly, we assume that each consumer is environmentally active in the sense that she is willing to devote resources to pro-environmental activities. Formally, we capture these two effects by assuming that

$$E_i = \varepsilon(M) + \chi_i,$$

where $\chi_i$ denotes the amount of a consumer’s endowment devoted to environmentally friendly activities, $M$ denotes pollution or total emissions, while the function $\varepsilon(M)$ satisfies $\varepsilon'(M) < 0$.\(^5\)

\(^5\) Effectively, we introduce a 'joy-of-giving' (or 'warm glow') argument to provide consumers with the motive to spend on environmentally friendly activities. Since the seminal work by Andreoni (1989, 1990) there has been
Each consumer’s problem is to choose $C_i$ and $x_i$ to maximise the utility function in (1) subject to (2) and her budget constraint

$$PC_i + x_i = y_i.$$ (3)

Naturally, when maximising her utility, the consumer takes $P$, $M$, and her exogenous income $y_i$ as given.

Assuming interior solutions, we can reformulate the problem by substituting (2) and (3) in (1) to write

$$x_i^* = \arg \max_{x_i} \delta \ln \left( \frac{y_i - x_i}{p} \right) + (1 - \delta) \ln [e(M) + x_i],$$ (4)

and

$$C_i^* = \frac{y_i - x_i^*}{p}. $$ (5)

The first-order condition for the problem is

$$\frac{\partial U_i}{\partial x_i} = \frac{-\delta}{y_i - x_i} + \frac{1-\delta}{e(M) + x_i}. $$ (6)

The second-order condition is

$$\frac{\partial^2 U_i}{\partial x_i^2} = \left( \frac{y_i - x_i}{(y_i - x_i)^2} - \frac{1-\delta}{[e(M) + x_i]^2} \right) < 0.$$ (7)

Therefore, we can obtain $x_i^*$ by setting $\partial U_i / \partial x_i = 0$ in (6). It is straightforward to establish that

$$x_i^* = (1-\delta)y_i - \delta e(M),$$ (8)

where we assume that the consumer’s endowment is sufficiently high to guarantee that $x_i^* > 0$. Substituting (8) in (5) we get

$$C_i^* = \delta \frac{y_i + e(M)}{p}. $$ (9)

Recall that consumers in the market are assumed to be identical – an assumption that applies to both their preferences and their endowments. Thus, we have $y_i = y \forall i$ and we

ample experimental evidence of this behaviour in public good games (see for example, Palfrey and Prisbey 1996, 1997; and Goeree et al. 2002). However, note that we can reformulate the problem to assume that consumers internalise the beneficial effects of their own actions for overall environmental quality, without causing any change to the qualitative characteristics of our subsequent results.
can use (9) to obtain the aggregate demand function for the homogeneous consumption
good according to

\[ C^* = \int_0^1 C_i \, di = \delta \frac{y + \epsilon(M)}{P}. \quad (10) \]

Equation (10) reveals a standard, negatively sloped demand function with respect to the
price (i.e., \( \partial C^* / \partial P < 0 \)). Interestingly, equation (10) also implies that pollution affects
aggregate demand. The next proposition formalises this claim.

**Proposition 1.** An increase in pollution will result, ceteris paribus, in a reduction of the consumption
good’s aggregate demand.

**Proof.** Using equation (10), we can see that \( \partial C^* / \partial M = P^{-1} \epsilon'(M) < 0 \). QED

The intuition for this result is straightforward. The increase in pollution will stimulate the consumers’ desire to devote resources towards environmentally oriented activities – an effect that is manifested in the increase of the marginal utility from such activities (see equation 6). The equilibrium can only be restored if this marginal utility falls back to its original level; thus, each consumer will optimally choose to increase her spending on \( x_i \). However, with a
given amount of income available, this shift has to materialise at the expense of consumption. Hence, the demand for the consumption good will ultimately decline as, for a
given price level, the demand curve shifts downwards (see Figure 1).\(^6\)

\(^6\) The assumption of a unit substitution elasticity between \( C_i \) and \( E_i \) is innocuous for our results. We can derive qualitatively identical results if we replace (1) with a more general CES utility function

\[ u_i = \left[ \delta C_i^{\theta / \gamma} + (1 - \delta) E_i^{\theta / \gamma} \right]^{\gamma / \theta}, \quad \theta > 0. \]

In this case, the aggregate demand function becomes

\[ C^* = \frac{y + \epsilon(M)}{P[1 + \delta^{-\theta}(1 - \delta)^{\theta} P^{\theta - 1}]} . \]
Figure 1

The preceding analysis has demonstrated a link between aggregate demand and pollution in an economy with environmentally conscious consumers. More importantly, the mechanism we have described implies that this link may be pertinent to various aspects of a firm’s decision making process. To see this, recall that pollution is a negative side-effect of firms’ production activities. Now, let us assume that each firm \( j = 1, \ldots, N \) produces and supplies a quantity \( q_j \) by accessing a technology that emits \( \mu_j \) units of pollutants per unit of production. In this case pollution will amount to

\[
M = \sum_{j=1}^{N} \mu_j q_j. \tag{11}
\]

The expression in (11), combined with the demand function in (10), reveals how and why the mechanism summarised in Proposition 1 can be a crucial characteristic of firms’ choices concerning production, technology adoption etc. As such, it may have significant implications for policies designed to induce the implementation of environmentally friendlier production methods by firms. Our purpose is to utilise the main point from the preceding discussion in order to provide a formal analysis of these implications. This is a task we undertake in the following sections of the paper.
3 Emission Taxes and Environmental Technology Choice

The preceding analysis has demonstrated a scenario that supports an aggregate demand function $C^* = \ell(P, M)$, where $\ell_p, \ell_M < 0$. Now, let us consider an industry whose firms face such a demand function. When doing so, we shall restrict our attention to a duopoly, henceforth $N = 2$. The equilibrium in the market where the good is sold requires that $C^* = \sum_{j=1}^{2} q_j$. Therefore, the aggregate demand function can be written as

$$\sum_{j=1}^{2} q_j = \ell(P, M).$$

Our demand equation in (12) has the same qualitative properties as (10), that is, it is decreasing in the price ($P$) and the level of pollution ($M$). To simplify matters, we shall follow the standard approach of working under a linear approximation of an aggregate demand function that possesses the same qualitative properties. Hence, the remaining analysis will be making use of

$$\ell(P, M) = a - \Gamma(M) - P, \quad a > 0,$$

where $\Gamma'(M) > 0$. A linear approximation is also employed to capture the negative effect of pollution on the demand for the good. That is

$$\Gamma(M) = \gamma M,$$

where $\gamma > 0$ quantifies the relative strength of this negative effect.

Since firms produce a homogeneous product, it is helpful to think of them as operating under Cournot competition. Therefore, our formal analysis will be undertaken on the basis of an inverse demand function which we can get after combining equations (11)-(14). That is

$$P = a - \sum_{j=1}^{2} (1 + \gamma \mu_j) q_j.$$

Each firm faces a constant marginal cost of production $m > 0$. Furthermore, it may be liable to a penalty (or tax) of $\tau \geq 0$ per unit of emissions. Given that a firm emits $\mu_j q_j$ units of pollution, its variable costs are

$$(m + \tau \mu_j) q_j.$$

All in all, firms’ variable profits can be written as follows:
\[ v_j = \left[ a - \sum_{j=1}^{2} (1 + \gamma \mu_j) q_j \right] q_j - (m + v \mu_j) q_j - \Phi_j. \]  

(17)

Firms can choose the type of technology they employ to manufacture their goods. In particular, we assume that each firm can choose between two alternative technologies (‘dirty’ or ‘clean’) which differ in their associated emissions per unit of output and adoption costs – the latter assumed to be fixed. We consider that there is a trade-off between the level of emissions and the adoption cost. That is, the dirty technology entails an emission rate \( \mu_j = \overline{\mu} \) and can be adopted at zero cost while the clean technology is associated with a lower emission rate \( \mu_j = \mu < \overline{\mu} \) but a higher adoption cost. In what follows, we shall be assuming that \( \overline{\mu} < 2 \mu \) holds. This restriction is sufficient, albeit not necessary, to ensure the stability of the equilibrium that we will derive after solving the system of best response functions later in our analysis.\(^7\)

We also introduce the possibility of positive spillovers associated with the design and implementation of the cleaner production method. That is, it is less costly for a firm to develop and adopt the clean technology if its competitor is also using this clean technology. Formally, firms face a fixed cost \( \Phi_j \) such that

\[ \Phi_j = \begin{cases} 0 & \text{if } \mu_j = \overline{\mu} \\ \varphi & \text{if } \mu_j = \mu, \mu_{j*} = \overline{\mu} \\ \varphi & \text{if } \mu_j = \mu, \mu_{j*} = \mu \end{cases} \]  

(18)

where \( \varphi < \overline{\varphi} \).

Given (17) and (18), each firm’s total profits, \( \pi_j \), are given by

\[ \pi_j = \left[ a - \sum_{j=1}^{2} (1 + \gamma \mu_j) q_j \right] q_j - (m + v \mu_j) q_j - \Phi_j. \]  

(19)

\(^7\) Notice that this notion of stability differs from the one applied to variables that display an explicit dynamic pattern. In this case, an equilibrium is said to be stable if, starting from any point in its neighbourhood, the adjustment process in which players take turns myopically playing a best response to each other’s current strategies converges to the equilibrium. Formally, using \( \pi_j \) to denote profits, the stability condition is

\[ \frac{\partial^2 \pi_j}{\partial q_j^2} > \frac{\partial^2 \pi_j}{\partial q_j \partial q_{j*}} \], where \( j = \{1,2\} \). See Martin (2001).
The objective of the firm is to maximise profits by the appropriate choices of \( q_j \) and \( \mu_j \).

We assume that firms will choose their technologies first. Once firms’ technology choices are observed, firms choose their output levels. Thus, the game has two stages: during the first stage, firms choose their technologies whereas during the second stage firms choose their output levels. We assume that firms choose simultaneously in each of these stages. As usual, we solve this game by backwards induction and use subgame perfection as our equilibrium concept.

### 3.1 The Second Stage: Output Choices

In this stage, firms set their output levels to maximise profits. The first order condition for maximisation is given by

\[
\frac{\partial \pi_j}{\partial q_j} = \left[ a - \sum_{j=1}^{2} (1 + \gamma \mu_j) q_j \right] - q_j (1 + \gamma \mu_j) - m - \tau \mu_j = 0. \tag{20}
\]

Notice that the second order condition for a maximum is fulfilled \( (\frac{\partial^2 \pi_j}{\partial q_j^2} = -2(1 + \gamma \mu_j) < 0) \). Thus, we can solve (19) for \( q_j \) to obtain the best response function for each firm, which is

\[
q_j^* = \frac{[a - m - \tau \mu_j - q_j \mu_j(1 + \gamma \mu_j)]}{2(1 + \gamma \mu_j)}. \tag{21}
\]

As expected, firms’ outputs are strategic substitutes since \( \frac{\partial q_j^*}{\partial q_{j^*}} < 0 \). An increase in the competitor’s output will put a downward pressure on the good’s price, thus reducing the firm’s marginal revenue. Given that the marginal cost of production is unchanged, the firm will find it profitable to reduce its production in order to restore the marginal revenue back to its original level. It is worth noting that the magnitude of this effect is reinforced by the presence of the parameter \( \gamma \). The intuition for this is that firm \( j’ \)’s output adds to the total level of emissions, triggering a shift of the consumers’ demand away from the good and, as a response, a lower level of output by firm \( j \). The same intuitive mechanism more or less applies when we try to explain the inverse relation between the firm’s production and its own emission rate. Notice, however, that this adverse effect is reinforced by the presence of the emission tax which, effectively, adds to the cost of production.
Solving the system of best response functions for \( j = \{1, 2\} \), we find the equilibrium levels of output, which are

\[
q_1^* = \frac{a - m - \tau (2\mu_1 - \mu_2)}{3(1 + \gamma \mu_1)},
\]

and

\[
q_2^* = \frac{a - m - \tau (2\mu_2 - \mu_1)}{3(1 + \gamma \mu_2)}.
\]

Given these results, we are now able to formalise our analysis on the equilibrium output responses, associated with different technology choices. The following proposition summarises the corresponding qualitative effects.

**Proposition 2.** A firm’s optimal production is, ceteris paribus, decreasing in its own emission rate but increasing in its competitor’s emission rate. The effect of the competitor’s technology choice (i.e., the competitor’s emission rate) on the firm’s output exists if and only if \( \tau > 0 \).

**Proof.** Using equations (21) and (22), it is straightforward to establish that \( \frac{\partial q_1}{\partial \mu_1}, \frac{\partial q_2}{\partial \mu_2} < 0 \ \forall \tau \geq 0 \) and \( \frac{\partial q_1}{\partial \mu_2}, \frac{\partial q_2}{\partial \mu_1} > 0 \ \text{iff} \ \tau > 0 \). □

A few points merit discussion here. On the one hand, the firm’s own emission rate has a negative effect on its own output through two different mechanisms. First, a higher emission rate means more pollution and therefore a higher shift in demand, leading to a reduction in output. Second, the own emission rate will positively affect the effective marginal cost of production of firms as long as emissions are taxed (\( \tau > 0 \)). The higher this cost, the lower the output will be. The competitor’s emission rate will affect positively the firm’s own output as long as emissions are taxed. Although this result may seem at odds with the effects we discussed earlier, it can be explained as follows. When a competitor chooses a more polluting technology, there are two conflicting effects on the firm’s output. The direct effect is the one we alluded to earlier during the discussion of the characteristics of the best response function – an effect which is negative. There is an indirect effect, however, which works in exactly the opposite direction. In particular, the competitor will combine her choice of a higher emission rate with a lower level of output – an effect partially attributed to the
presence of the parameter $\gamma$, but also amplified by the fact that the emission tax exacerbates the overall cost of production. It is this latter effect that renders the indirect impact to the competitor’s output dominant and, given that output levels are strategic substitutes, makes it profitable for the firm to increase its output. Moreover, this is precisely why taxation is crucial for the materialisation of technology choice interactions when firms determine their output levels.

Substituting (22) and (23) in (19), we can write each firm’s total profits as

$$\pi_1 = v_1 - \Phi_1 = \frac{[a - m - \tau(2\mu_1 - \mu_2)]^2}{9(1 + \gamma\mu_1)} - \Phi_1,$$

and

$$\pi_2 = v_2 - \Phi_2 = \frac{[a - m - \tau(2\mu_2 - \mu_1)]^2}{9(1 + \gamma\mu_2)} - \Phi_2.$$  

(24)

These expressions, together with (22) and (23), reveal that, in terms of equilibrium output and, therefore, variable profits, the choice of technology across firms is subject to strategic substitutability: other things being equal, a firm’s choice to implement a cleaner technology reduces the other firm’s variable profits, thus leaving fewer resources available for the implementation of the cleaner technology. Nevertheless, the technology choice entails fixed costs whose presence introduces a strategic complementarity according to (18): other things being equal, a firm’s decision to implement a less polluting technology makes it less costly for the other firm to do the same because of the positive spillover effect.

Given that the profits of each firm depend on their own and their competitor’s technology choices, four scenarios arise: one where both firms choose the cleaner technology, $(\mu, \mu)$; another where both firms choose the dirtier technology, $(\bar{\mu}, \bar{\mu})$; and two asymmetric ones, where firm 1 chooses the clean technology and firm 2 the dirty one, $(\mu, \bar{\mu})$, as well as the opposite case where $(\bar{\mu}, \mu)$. Thus, the variable profits in each of these scenarios can be written as follows:

$$v_{j1}^{\mu, \mu} = \frac{(a - m - 2\mu)^2}{9(1 + \gamma\mu)},$$

$$v_{j1}^{\mu, \bar{\mu}} = \frac{(a - m - \tau\mu)^2}{9(1 + \gamma\mu)},$$

(26)

$$v_{j1}^{\bar{\mu}, \mu} = \frac{(a - m - \tau\bar{\mu})^2}{9(1 + \gamma\bar{\mu})},$$

$$v_{j1}^{\bar{\mu}, \bar{\mu}} = \frac{(a - m - \tau\bar{\mu})^2}{9(1 + \gamma\bar{\mu})},$$

(27)
Note that in the above equations, we use the first superscript to identify firm j’s own technology choice and the second superscript to identify its competitor’s technology choice. Bringing together equations (26) to (29) and equation (18), we complete the matrix of payoffs in Table 1. In the next section we will identify under which conditions each of the above mentioned four scenarios may arise as equilibrium outcomes.

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Table 1. Payoff matrix

3.2 The First Stage: Technology Choices

In this stage, firms seek to maximise profits through the appropriate choice of technology. Our preceding analysis indicates that firms have an incentive to implement a cleaner technology even in the absence of taxation due to the effect of pollution on the aggregate demand of environmentally conscious consumers, captured by the parameter $γ$. We will start by analysing the case without environmental policy ($τ = 0$).

3.2.1 Technology Choice without Emission Taxes

Let us go back to equations (24) and (25) and set $τ = 0$ to get

\[
v_{j}^{μ,θ} = \frac{[a - m - τ(2μ - μ)]^2}{9(1 + γμ)},
\]

\[
v_{j}^{μ,θ} = \frac{[a - m - τ(2μ - μ)]^2}{9(1 + γμ)}.
\]
\[
\pi_1 = \frac{(a-m)^2}{9(1+\gamma \mu_1)} - \Phi_1, \\
\pi_2 = \frac{(a-m)^2}{9(1+\gamma \mu_2)} - \Phi_2.
\]

These equations reveal that the choice of a cleaner technology (i.e., \(\mu\) as opposed to \(\overline{\mu}\)) will be optimal if and only if pollution entails the type of aggregate demand effects that we identified in Section 2. If \(\gamma = 0\), and in the absence of environmental taxes, there is no incentive by neither firm to incur a cost for an activity that has no benefit whatsoever. However, insofar as \(\gamma > 0\), such benefit clearly exists: a firm may be willing to incur the fixed cost of environmental innovation, anticipating that this will induce environmentally conscious consumers to shift their resources towards the consumption of goods. In fact, such motive exists even in the absence of spillovers. Nevertheless, if spillovers exist, they will create a further incentive for firms to adopt the cleaner technology.

Let us discuss now the conditions under which each combination of strategies may arise as an equilibrium. Note that here, the variable profit is solely a function of the own emission rate. Thus, the net benefit in terms of variable profits of implementing a cleaner technology is the same irrespectively of the competitors’ choice, since \(\nu_j^{\mu \omega} - \nu_j^{\overline{\mu} \omega} = \nu_j^{\mu \overline{\omega}} - \nu_j^{\overline{\mu} \overline{\omega}}\). In fact, in both cases this difference yields

\[
\omega = \frac{(a-m)^2 \gamma (\overline{\mu} - \mu)}{9(1+\gamma \overline{\mu})(1+\mu \overline{\mu})}.
\]

It is evident from (32) that, in the absence of emission taxes, a benefit exists if and only if \(\gamma > 0\) – that is, only if pollution results in aggregate demand effects. Next, we need to compare this net benefit with the difference in fixed cost of adopting each technology. The result of this comparison will determine the equilibrium of the game. The results of this comparison are presented in the next proposition.

**Proposition 3.** Suppose that \(\tau = 0\). Then the following are equilibria:

i. \((\overline{\mu}, \overline{\mu})\) if \(\omega \leq \varphi\); 

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\(8\) As a tiebreaking rule, we assume that if a firm is indifferent between the two technologies, it will choose \(\overline{\mu}\).
ii. \((\overline{\mu}, \overline{\mu})\) and \((\underline{\mu}, \underline{\mu})\) if \(\varphi < \omega \leq \overline{\mu}\); 

iii. \((\underline{\mu}, \underline{\mu})\) if \(\omega > \overline{\mu}\).

**Proof.** See the Appendix.  □

Proposition 3 states that the only equilibria that arise are symmetric (either both firms choose the clean technology or both firms choose the dirty one). In particular, when the net benefit of choosing the clean technology is very small (too small to be profitable to adopt it even when there are spillovers), the only equilibrium is \((\overline{\mu}, \overline{\mu})\). Conversely, when this benefit is very high (high enough to make adoption profitable even without spillovers), the only equilibrium is \((\underline{\mu}, \underline{\mu})\). For intermediate levels, both \((\overline{\mu}, \overline{\mu})\) and \((\underline{\mu}, \underline{\mu})\) arise as equilibria since adopting the clean technology is only profitable if the competitor adopts it too; that is when there is a possibility of benefiting from spillovers. In this case, however, this outcome is not guaranteed as firms face a coordination problem due to the multiplicity of equilibria. For values satisfying \(\omega \in (\overline{\mu}, \overline{\mu})\) although the positive spillovers would make it mutually advantageous for both firms to choose the clean technology, the expectation that the competitor may not choose \(\underline{\mu}\) (hence, eliminating the benefits emanating from the spillover effect) may discourage firms from a choice of \(\underline{\mu}\).

In the next section, we analyse the first stage in the presence of emission taxes and try to identify the equilibrium outcomes that transpire in this scenario.

### 3.2.2 Technology Choice in the Presence of Emission Taxes

For subsequent purposes, we begin this part by defining

\[
\bar{\omega} = v^{*,\overline{\tau}} - v^{*,\overline{\tau}} = \frac{[a - m - \tau(2\mu - \overline{\mu})]^2}{9(1 + \gamma \overline{\mu})} - \frac{(a - m - \tau \overline{\mu})^2}{9(1 + \gamma \overline{\mu})},
\]  \(33\)

and

\(^9\)Effectively, the existence of spillovers introduces a strategic complementarity in the choice of technology. See Cooper and John (1988) for a detailed discussion on the implications of coordination failures in models with strategic complementarities.
The interpretation of (33) and (34) is similar to the corresponding one in equation (32). Here, however, the increase in variable profits from implementing the cleaner technology depends on whether the competitor chooses the dirty or the clean technology. We label with \( \omega \) this increase in the latter case and with \( \bar{\omega} \) in the former. If \( \bar{\omega} \) is higher than \( \bar{\varphi} \), a firm’s best response to its competitor choosing the dirty technology is to adopt the clean technology. Analogously, if \( \omega \) is higher than \( \varphi \), its best response to the clean technology is to adopt the clean technology too. Some tedious, but straightforward, algebra reveals that \( \bar{\omega} > \omega \) holds. In other words, the net benefit from implementing the less polluting technology is higher when the competitor actually implements the more polluting one. Recalling that the presence of the emission tax introduces some strategic substitutability in the optimal choice of technology (as opposed to the strategic complementarity emerging from the presence of spillover effects), the intuition behind this result is clear: when the competitor chooses the cleaner technology, and as long as the government taxes emissions, it becomes more competitive in the product market, as its marginal cost is lower than it would be if it chose the dirty technology. In turn, this affects negatively firm \( j \)'s output and therefore the (variable) profitability of adopting the cleaner technology.

With these in mind, we can use the two propositions that follow in order to summarise the equilibrium outcomes that materialise during the first stage of the competition (when \( \tau > 0 \)).

**Proposition 4.** Suppose that \( \tau > 0 \) and \( \bar{\omega} - \omega < \bar{\varphi} - \varphi \). Then, the following are equilibria:

i. \( (\mu, \bar{\mu}) \) if \( 0 \geq \omega - \varphi > \bar{\omega} - \bar{\varphi} \);

ii. \( (\mu, \bar{\mu}) \) and \( (\mu, \mu) \) if \( \omega - \varphi > 0 \geq \bar{\omega} - \bar{\varphi} \);

iii. \( (\mu, \mu) \) if \( \omega - \varphi > \bar{\omega} - \bar{\varphi} > 0 \).

*Proof.* See the Appendix. \( \square \)

**Proposition 5.** Suppose that \( \tau > 0 \) and \( \bar{\omega} - \omega \geq \bar{\varphi} - \varphi \). Then, the following are equilibria:
i. \((\bar{\mu}, \bar{\nu})\) if \(0 \geq \bar{\omega} - \bar{\nu} \geq \omega - \nu\);

ii. \((\bar{\mu}, \mu)\) and \((\mu, \bar{\nu})\) if \(\bar{\omega} - \bar{\nu} > 0 \geq \omega - \nu\);

iii. \((\mu, \mu)\) if \(\bar{\omega} - \bar{\nu} \geq \omega - \nu > 0\).

Proof. See the Appendix. □

The scenarios described in Proposition 4 and Proposition 5 differ with respect to the relative strength of the spillover effect \(\bar{\nu} - \nu\) which is larger in the scenario in Proposition 4. Nevertheless, there are some common outcomes in both scenarios. Particularly, when the net benefit of choosing the clean technology is very small or very high, the equilibria are analogous to the equilibria arising in the absence of taxes; that is, \((\bar{\mu}, \bar{\nu})\) in the former case and \((\mu, \mu)\) in the latter.

However, a major difference across scenarios arises for intermediate values of this benefit. In the first scenario, the relatively strong spillovers make the adoption of the clean technology profitable if and only if the competitor adopts it too. If the competitor uses the dirty technology, the benefit in terms of variable profits is not enough to make the adoption of the clean technology profitable. This is reflected in the equilibrium outcomes described in Proposition 4 (ii); in such a case, both \((\bar{\mu}, \bar{\nu})\) and \((\mu, \mu)\) are Nash equilibria. However, when the spillover effect is not so strong, it may be profitable for a firm to adopt the clean technology as a reply to the competitor’s adopting the dirty technology. In such a case, the competitor is in disadvantage in the output market due to the higher marginal cost. For the same reason, it is optimal for the competitor to adopt the dirty technology as a reply to the clean technology, as its less competitive position in the market and therefore lower output makes it less profitable to incur in the higher fixed costs of the clean technology. Thus, two asymmetric equilibria arise, \((\bar{\mu}, \mu)\) and \((\mu, \bar{\nu})\), as Proposition 5 (ii) shows. Consequently, firms face a coordination problem due to the multiplicity of equilibria in both scenarios for intermediate values of the benefit of adopting the clean technology.
4 Environmental Policy and Incentives for Pro-Environment Innovation

In this section, our purpose is to examine the effectiveness of environmental policy on increasing the incentive of firms in adopting the less polluting production method. Our previous analysis has made clear that the emission tax $\tau$ will affect this incentive through its impact on variable profits. In particular, it will do so through the effect it has on the net benefit of introducing the cleaner technology. That is, the increase in variable profits when a firm shifts from the dirty (i.e., $\mu$) to the clean technology (i.e., $\mu$).

The previous section has revealed that the increment in variable profits from introducing a cleaner technology varies depending on whether the competitor chooses to innovate ($\omega$) or not ($\bar{\omega}$). We also know that the higher $\omega$ and $\bar{\omega}$ are, the more likely that a firm will choose to adopt the clean technology as a reply to its competitor respectively adopting the clean technology or the dirty technology. Here we will use comparative statics to check whether $\tau$ has a positive, negative or non-monotonic effect on $\omega$ and $\bar{\omega}$. This will allow us to establish whether emissions taxation makes the adoption of the clean technology more likely to happen in equilibrium. To this aim, we shall examine $\omega$ and $\bar{\omega}$ separately, beginning with the former.

As it is evident from our previous analysis and discussion, we are interested on the effect of $\tau$ on the composite term $\omega$ in (34). Prior to undertaking the formal analysis, however, we need to impose an upper bound on the emission tax. This is necessary to ensure that variable profits are non-negative under any possible scenario concerning technology choice by the firm. A look at (34) reveals that the tax must satisfy $\tau \in \left(0, \frac{a - m}{2\mu} \right)$. Given this, the following proposition summarises our result.

**Proposition 6.** There exists $\tau^* \in \left(0, \frac{a - m}{2\mu} \right)$ such that
\[ \frac{\partial \omega}{\partial \tau} \begin{cases} > 0 & \text{for } \tau < \tau^* \\ = 0 & \text{for } \tau = \tau^* \\ < 0 & \text{for } \tau > \tau^* \end{cases} \]

Furthermore, it is \( \frac{\partial \tau^*}{\partial \gamma} < 0 \).

**Proof.** See the Appendix. □

One implication of Proposition 6 is that there is a tax rate that maximises the incentive to adopt a clean technology when the competitor is expected to act similarly. Additionally, we can see that when the negative aggregate demand effect from pollution is more pronounced (i.e., when \( \gamma \) is higher) then the tax rate that maximises the incentive for pro-environment R&D becomes lower.

In terms of intuition, the non-monotonic effect of the emission tax is due to the conflicting effects on variable profits. On the one hand, the emission tax motivates the firm to choose a cleaner technology in order to reduce its overall tax obligation. On the other hand, however, excessively high taxation makes the overall tax burden so high and the reduction in variable profits so strong, that it eliminates any incentive for the adoption of the cleaner technology. Naturally, the tax rate where these marginal benefits and costs are equal, is the one that will provide the highest incentive for pro-environment innovation.

An important element in our analysis comes from the impact of the negative demand effect of pollution, as this is captured by the parameter \( \gamma \). Given that this effect already provides an incentive for the adoption of a cleaner production method (see Section 3.2.1), the parameter \( \gamma \) exemplifies the distortive nature of the emission tax, meaning that the scope for environmental policy to increase the incentive for environmental innovation becomes limited for higher values of \( \gamma \). Consequently, it is possible that the same increase in taxation that would raise the incentive for the adoption of the cleaner technology when \( \gamma = 0 \), may actually decrease this incentive for \( \gamma > 0 \). In terms of Figure 2, this scenario is depicted for values of the emission tax that lie on the interval \( (\tau^*_{\gamma > 0}, \tau^*_{\gamma = 0}) \).

Next, we undertake a similar analysis for the case where the firm expects its competitor not to adopt the cleaner production technique – that is, we focus on the composite term \( \bar{\omega} \).
in (33). In order to ensure that variable profits remain non-negative in this case, we use equation (33) to identify a proper range of values for the emissions tax which turns out to be

\[ \tau \in \left(0, \frac{a-m}{\mu} \right) \].

The following proposition summarises our result.

**Proposition 7.** There exists \( \tau^* \in \left(0, \frac{a-m}{\mu} \right) \) such that

\[
\frac{\partial \omega}{\partial \omega} > 0 \quad \text{for} \quad \tau < \tau^*,
\]

\[
\frac{\partial \omega}{\partial \tau} = 0 \quad \text{for} \quad \tau = \tau^*,
\]

\[
< 0 \quad \text{for} \quad \tau > \tau^*.
\]

Furthermore, it is \( \frac{\partial \tau^*}{\partial \gamma} < 0 \).

**Proof.** See the Appendix. \( \square \)

As we can see, the possibility of non-monotonic effects from environmental policy emerges in this scenario as well – as does the impact of the preference parameter \( \gamma \) on the tax rate that maximises the incentive to use the clean technology. Consequently, the intuition for these results is exactly the same with the one discussed in the analysis of Proposition 6.

Given that both \( \omega \) and \( \bar{\omega} \) are inverted U-shapes in \( \tau \), it is clear that the likelihood that a firm adopts the clean technology in equilibrium is initially increasing in the tax rate but will eventually turn decreasing. The turning point will take place earlier, i.e. for a lower level of the tax rate, if consumers have some preference for the environment which makes them shift resources from consumption to environmental activities, as illustrated in Figure 2.

Before concluding this section, it is worth discussing the role of the tax rate in determining which combination of strategies arises in equilibrium. Thus, we need to consider jointly the effects that \( \tau \) has on both \( \omega \) and \( \bar{\omega} \). Figure 3 provides an illustration of \( \omega \) and \( \bar{\omega} \) for arbitrary levels of fixed costs and spillovers. One can see that five regions emerge with lead to different equilibrium outcomes. In the first region (between 0 and \( \tau_1 \)), the best reply to both \( \bar{\mu} \) and \( \mu \) is \( \bar{\mu} \). This implies that the arising equilibrium will be \( (\bar{\mu}, \bar{\gamma}) \). If the tax rate is set at a higher level (in the region between \( \tau_1 \) and \( \tau_2 \)), the best response to \( \bar{\mu} \) is \( \bar{\mu} \)
and to $\mu$ is $\mu$. Thus, it may occur that either $(\bar{\mu}, \bar{\mu})$ or $(\mu, \mu)$ arise in equilibrium. In contrast, if the tax rate was set in the third region (between $\tau_2$ and $\tau_3$), the only equilibrium would be $(\mu, \mu)$, as $\mu$ is the best reply to both $\bar{\mu}$ and $\mu$. Increasing the tax even further to be in the fourth region (between $\tau_3$ and $\tau_4$) reduces the profitability of adopting the clean technology and may induce asymmetric equilibria where only one of the firms adopts the clean technology, that is $(\mu, \bar{\mu})$ or $(\bar{\mu}, \mu)$, since the best reply to $\bar{\mu}$ is $\mu$ and vice versa.

Increasing the tax rate even further (to be higher than $\tau_4$) will lead to an equilibrium where the two firms adopt the polluting technology $(\bar{\mu}, \bar{\mu})$. Thus, setting the tax rate to be in the intermediate region (between $\tau_2$ and $\tau_3$) would be the only way in which the policy maker can warrant an outcome where both firms adopt the clean technology in equilibrium.\(^{10}\)

Some important implications for policy making can be drawn from the above results. The first and perhaps counterintuitive implication is that increasing the tax rate on emissions does not necessarily create further incentives for the adoption of clean technologies. In fact, a higher tax rate may actually reduce the incentives to adopt the clean technology and therefore reduce the likelihood of this adoption in equilibrium, particularly if the original tax rate on emission is already relatively high. Secondly, the policy maker should be particularly weary of such an effect in situations where consumers are environmentally conscious as in such situations, the benefit of adopting the clean technology turns decreasing in the emissions tax rate for lower levels of taxation. Thus, the policy maker should take into account the behaviour of consumers when designing its environmental policy. For example, if carbon offsetting schemes are introduced in a given industry and consumers are actively participating in them, it may be optimal for the government to reduce its level of emissions taxation in that industry, especially if this level is initially high; otherwise, the incentives of firms in that given industry to adopt clean methods of production may be damaged.

\(^{10}\) Note that Figure 3 is presented as an illustrative example only. We could in fact find that the second and the fourth regions are switched over, or even that they do not arise. This would depend on the specific combination of parameters in each case. What is generally true is that as $\tau$ increases, there is a transition from less adoption of clean technologies to more adoption of clean technologies and then again to less adoption of clean technologies.
4 Conclusions

We have analysed how the existence of environmentally conscious consumers affects firms’ adoption of cleaner manufacturing technologies. Although, in recent years the literature has introduced the presence of environmentally conscious consumers in models of product differentiation, such models are not suitable for the analysis of situations where consumers are involved in environmental activities such as participation in carbon offsetting schemes, environmental volunteering, donations etc.

In this paper we propose a framework of analysis for this alternative type of environmental conscious consumers. In particular, we have assumed that consumers’ utility is a function of both their level of consumption of a good, the quality of the environment and the warm glow derived from taking part in the environmental activities. As for the technology choice, we have assumed that firms have two technologies at their disposal which differ in their associated emissions per unit of output ratio and fixed costs.

We have shown that following an increase in pollution, consumers will channel resources away from the consumption to environmental activities, thereby reducing the demand for the good which firms face and the subsequent levels of output produced by firms in equilibrium. This reduction in demand due to consumers’ environmental conscience generates incentives for firms to adopt the clean technology even in the absence of emissions taxes or technology spillovers.

Our results also indicate that increasing the tax rate on emissions does not necessarily lead to the adoption of clean technologies. In fact, the benefit of adopting the clean technology follows an inverted U-shape in the tax rate, which implies that after a threshold value of the emission tax rate, further tax increases make less likely the adoption of the clean technology in equilibrium. This counterintuitive effect is more prevalent in situations where consumers are environmentally conscious.

Although our results have been derived in a streamlined duopolistic model, we conjecture that they will hold even under a more general setting due to the clear-cut manner of the mechanism that generates them; that is, the negative effect on the demand caused by shifting resources away from the consumption of the good. Consequently, the main policy lesson that can be extracted from our model is that any environmental policy aimed at improving the technological profile of firms should take into account the behaviour of environmentally conscious consumers where relevant (for example, in markets where environmental activities
such as the ones described above take place); otherwise, the policy may have undesirable effects on firms’ decisions to invest in cleaner technologies.

Appendix

Proof to Proposition 3

From Table 1 and equation (32), we know that that firm 1’s best response to firm 2 choosing \( \underline{\mu} \) is \( \overline{\mu} \) if \( \omega \leq (>) \varphi \). Likewise, its best response to firm 2 choosing \( \underline{\mu} \) is \( \overline{\mu} \) if \( \omega \leq (>) \varphi \). Recall that \( \varphi \leq \varphi \). Thus, if \( \omega \leq \varphi \), firm 1 has a dominant strategy which is \( \overline{\mu} \).

Moreover, if \( \omega > \varphi \), its dominant strategy is \( \underline{\mu} \) instead. As the game is symmetric, the same applies to firm 2. Hence, \( (\overline{\mu}, \overline{\mu}) \) and \( (\underline{\mu}, \underline{\mu}) \) are the equilibria in dominant strategies if \( \omega \leq \varphi \) and \( \omega > \varphi \) respectively.

Now assume that \( \varphi < \omega \leq \varphi \). In such a case, firm 1’s best response to firm 2 choosing \( \overline{\mu} \) is \( \overline{\mu} \) whereas its best response to firm 2’s choice of \( \underline{\mu} \) is \( \underline{\mu} \). Again, due to symmetry, the same applies to firm 2. Thus, if \( \varphi < \omega \leq \varphi \), two Nash equilibria arise: \( (\overline{\mu}, \overline{\mu}) \) and \( (\underline{\mu}, \underline{\mu}) \).

QED.

Proof to Proposition 4

Consider the scenario where \( \tau > 0 \) and \( \overline{\omega} - \omega < \overline{\varphi} - \varphi \) or \( \overline{\omega} - \varphi < \omega - \varphi \) after rewriting. From Table 1 and equations (33) and (34) we know that that firm 1’s best response to \( \overline{\mu} \) is \( \underline{\mu} \) if \( \overline{\omega} - \varphi > (\leq) 0 \) and to \( \underline{\mu} \) is \( \underline{\mu} \) if \( \overline{\omega} - \varphi > (\leq) 0 \). Due to symmetry, the same applies to firm 2. Given these conditions and \( \overline{\omega} - \omega < \overline{\varphi} - \varphi \), three cases may emerge:

(i) \( \overline{\omega} - \varphi \leq 0 \) and \( \omega - \varphi \leq 0 \);

(ii) \( \overline{\omega} - \varphi \leq 0 \) and \( \omega - \varphi > 0 \);

(iii) \( \overline{\omega} - \varphi > 0 \) and \( \omega - \varphi > 0 \).

In case (i) both firms have a dominant strategy in \( \overline{\mu} \) (it is their best response in both \( \overline{\mu} \) and \( \underline{\mu} \)). Thus, \( (\overline{\mu}, \overline{\mu}) \) is an equilibrium in dominant strategies. In case (iii) both firms have a dominant strategy in \( \underline{\mu} \) (it is their best response in both \( \overline{\mu} \) and \( \underline{\mu} \)). Therefore, \( (\underline{\mu}, \underline{\mu}) \) is an
equilibrium in dominant strategies. In case (ii), firm 1’s best response to \( \mu \) is \( \mu (\mu) \) if \( \omega - \varphi > 0 \) and \( \mu \) is \( \mu (\mu) \) if \( \omega - \varphi > 0 \). Of course, the same applies to firm 2 due to symmetry. Thus, two Nash equilibria emerge in this case; that is, \( (\mu, \mu) \) and \( (\mu, \mu) \).

To complete the proof, note that the conditions in (i), (ii), and (iii) can be written respectively as and \( 0 > \omega - \varphi > \omega - \varphi \), \( \omega - \varphi > 0 > \omega - \varphi \) and \( \omega - \varphi > \omega - \varphi > 0 \) since \( \omega - \varphi < \omega - \varphi \) applies in this scenario. QED.

**Proof to Proposition 5**

Consider the scenario where \( \tau > 0 \) and \( \omega - \varphi > \omega - \varphi \) or \( \omega - \varphi > \omega - \varphi \) after rewriting. From Table 1 and equations (33) and (34) we know that firm 1’s best response to \( \mu \) is \( \mu (\mu) \) if \( \omega - \varphi > 0 \). Due to symmetry, the same applies to firm 2. Given these conditions and \( \omega - \varphi > \omega - \varphi \), three cases may emerge:

1. \( \omega - \varphi > 0 \) and \( \omega - \varphi \leq 0 \);
2. \( \omega - \varphi > 0 \) and \( \omega - \varphi \leq 0 \);
3. \( \omega - \varphi > 0 \) and \( \omega - \varphi > 0 \).

In case (i) both firms have a dominant strategy in \( \mu \) (it is their best response in both \( \mu \) and \( \mu \)). Thus, in this case \( (\mu, \mu) \) is an equilibrium in dominant strategies. In case (iii) both firms have a dominant strategy in \( \mu \) (it is their best response in both \( \mu \) and \( \mu \)). Therefore, \( (\mu, \mu) \) is an equilibrium in dominant strategies. In case (ii), firm 1’s best response to \( \mu \) is \( \mu \) and its best response to \( \mu \) is \( \mu \). Of course, the same applies to firm 2 due to symmetry. Thus, two Nash equilibria emerge in this case; that is, \( (\mu, \mu) \) and \( (\mu, \mu) \).

To complete the proof, note that the conditions in (i), (ii), and (iii) can be written respectively as and \( 0 > \omega - \varphi > \omega - \varphi \), \( \omega - \varphi > 0 > \omega - \varphi \) and \( \omega - \varphi > \omega - \varphi > 0 \) since \( \omega - \varphi < \omega - \varphi \) applies in this scenario. QED.

**Proof to Proposition 6**

Using the expression in (34), we can calculate the first derivative as
\[
\frac{\partial \omega}{\partial \tau} = \frac{2}{9} \left\{ -\frac{(a-m-\tau \mu)\mu}{(1+\gamma \mu)} + \frac{[a-m-\tau(2\mu - \mu)](2\mu - \mu)}{9(1+\gamma \mu)} \right\}.
\] (A1)

Obviously, the sign of (A1) will be dictated by the sign of the expression incised brackets.

After some tedious algebra, we can reduce this expression to
\[
J(\tau) = (\mu - \mu) \{ 2(a-m-\tau \mu) + \gamma \mu [a-m-\tau(4\mu - \mu)] \}.
\] (A2)

We can use (A2) to check that \( J(0) = (\mu - \mu)(2+\gamma \mu)(a-m) > 0 \) and
\[
J\left( \frac{a-m}{2\mu - \mu} \right) = (\mu - \mu)(a-m) \left[ 2 \left( 1 - \frac{2\mu}{2\mu - \mu} \right) + \mu \left( 1 - \frac{4\mu - \mu}{2\mu - \mu} \right) \right] < 0.
\]
Now, we can use (A1) to calculate the second derivative. This is
\[
\frac{\partial^2 \omega}{\partial \tau^2} = \frac{2}{9} (\mu - \mu) \left[ -4\mu - \gamma \mu (4\mu - \mu) \right] < 0.
\] (A3)

Thus, we verify that there is a unique \( \tau^* \in \left( 0, \frac{a-m}{2\mu - \mu} \right) \) that maximises \( \omega \) and can be found by setting \( J(\tau^*) = 0 \) in (A2). Eventually, this leads to
\[
\tau^* = \frac{(a-m)(2+\gamma \mu)}{4\mu + \gamma \mu (4\mu - \mu)}.
\] (A4)

To complete the proof, we use (A4) to calculate
\[
\frac{\partial \tau^*}{\partial \gamma} = \frac{-2\mu(a-m)(2\mu - \mu)}{[4\mu + \gamma \mu (4\mu - \mu)]^2} < 0.
\] (A5)

QED.

**Proof to Proposition 7**

The expression in (33) allow us to derive
\[
\frac{\partial \omega}{\partial \tau} = \frac{2}{9} \left\{ \frac{(a-m-\tau \mu)\mu}{(1+\gamma \mu)} - \frac{[a-m-\tau(2\mu - \mu)](2\mu - \mu)}{9(1+\gamma \mu)} \right\}.
\] (A6)

The sign of (A6) depends on the sign of the expression incised brackets. This expression can be reduced to
\[
J(\tau) = (\mu - \mu) \{ 2(a-m-\tau \mu) + \gamma \mu [a-m-\tau(4\mu - \mu)] \}.
\] (A7)
From (A7) we can see that \( f(0) = (\mu - \mu)(2 + \gamma \mu)(a - m) > 0 \) and
\[
J\left( \frac{a - m}{\mu} \right) = -\frac{(\mu - \mu)(a - m)(2\mu - \mu)(1 + \gamma \mu)}{\mu} < 0.
\]
Next, we compute the second derivative from (A6) which is
\[
\frac{\partial^2 \bar{\omega}}{\partial \tau^2} = \frac{2}{9} (\mu - \mu)[-4\mu - \gamma \mu(4\mu - \mu)] < 0. \tag{A8}
\]
The previous analysis confirms that there is a unique \( \tau^* \in \left( 0, \frac{a - m}{\mu} \right) \) that maximises \( \bar{\omega} \) and can be found by setting \( f(\tau^*) = 0 \) in (A7). We can calculate \( \tau^* \) as
\[
\tau^* = \frac{(a - m)(2 + \gamma \mu)}{4\mu + \gamma \mu(4\mu - \mu)}. \tag{A9}
\]
To complete the proof, we use (A9) to calculate
\[
\frac{\partial \tau^*}{\partial \gamma} = \frac{-2\mu(a - m)(2\mu - \mu)}{(4\mu + \gamma \mu(4\mu - \mu))^2} < 0. \tag{A10}
\]
QED.

References


