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Contagion and risk-sharing on the inter-bank market

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Abstract

Increasing inter-bank lending has an ambiguous impact on financial stability. Using a computational model with endogenous bank behavior and interest rates we identify the conditions under which inter-bank lending promotes stability through risk sharing or provides a channel through which failures may spread. In response to large economy-wide shocks, more inter-bank lending relationships worsen systemic events. For smaller shocks the opposite effect is observed. As such no inter-bank market structure maximizes stability under all conditions. In contrast, deposit insurance costs are always reduced under greater numbers of inter-bank lending relationships. A range of regulations are considered to increase system stability.

Keywords: Systemic risk, Inter-bank lending, Contagion, Regulation, Network

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1 Introduction

The financial regulation of banks has primarily focused on ensuring that individual institutions have sufficient funds to protect themselves from the risk of their own investments. The events of 2007 and 2008 demonstrated the shortcomings of this approach. Problems in a small number of banks spread throughout the financial system resulting in the collapse of institutions which, according to regulatory requirements, were adequately protected. The inter-bank market was supposed to provide stability by allowing banks to access liquidity and share risk. Instead, it served as a mechanism by which problems could spread between institutions. In this paper we examine how the structure of the inter-bank lending market effects the stability of the financial system[1]. We consider a partial equilibrium model of a closed economy in which heterogeneous banks interact. Banks receive money from non-financial sector depositors and in turn lend those funds to borrowers to invest in risky projects. Banks interact with each other through an inter-bank market, obtaining funds but exposing themselves and other banks to counter-party risk and contagion. In equilibrium banks determine their balance sheet structure based on the loans and deposits they attract through their choice of lending and borrowing rates together with the inter-bank market rate. The inter-bank interest rate itself, is determined endogenously to be the rate at which supply and demand of funds are equal.

The structure of the inter-bank market is found to have a significant effect on the ability of the system to resist contagion in response to system-wide macroeconomic shocks. The optimal structure, however, is dependent on the magnitude of the shock faced. For small shocks a highly connected market provides a risk-sharing effect, reducing the probability of a contagious failure. In contrast, for larger systemic shocks, rather than reducing risk, inter-bank connections act to propagate the effects of failures making more highly connected markets the most vulnerable. Regardless of shock size, the cost to the deposit insurer is minimized for the most connected markets as more of the cost of failures is borne by surviving banks. The effect of regulatory changes are investigated. A higher

[1]These are not the only linkages which can propagate distress. For instance Allen and Carletti (2006), Markose et al. (2010) and Mendoza and Quadrini (2010) demonstrate alternative mechanisms.

equity ratio is found to decrease the market's susceptibility to contagion by reducing the number of banks who cause a second bank to fail. A higher reserve ratio, however, worsens contagious events as more banks need to use the interbank market to meet their liquidity needs. Constraining the size of inter-bank loans, is found to reduce the number of bankruptcies for large shocks. Care, however, must be taken, if this regulation is too tight it inhibits the efficiency of the economy. Finally if banks condition their confidence of being repaid on recent bankruptcies the economy becomes less stable. Whilst if banks condition their lending on the financial position of the borrower stability increases.

The paper is structured as follows: the next section will review related literature. Section 3 will describe the model. Section 4 will consider the systems susceptibility to contagion. Section 5 examines the effect of regulation whilst Section 6 demonstrates parameter stability and extends the model. Section 7 concludes.

2 Literature review

The inter-bank lending market allows financial institutions to lend funds or borrow money to meet liquidity or investment requirements. In their influential work, Allen and Gale (2001) show that in equilibrium banks will optimally insure themselves against liquidity risk by holding deposits in other banks. This protection, however, makes them vulnerable to counter-party risk. As such the failure of a single bank may spread if its creditors are unable to recover lent funds. This may potentially cause severe contagious events (Gai and Kapadia, 2010), resulting in a loss of equity (Eisenberg and Noe, 2001) and may justify government or regulatory intervention (Kahn and Santos, 2010).

The majority of trading in the inter-bank market happens over-the-counter (OTC), directly between pairs of banks, as opposed to through a central counter-party. Banks borrow funds and repay them over a length of time which can range from overnight, up to periods of several years. At any point a particular bank may be involved in multiple lending or borrowing relationships and as such may be connected to multiple counter-parties. Across all banks these linkages form a structure which may be described by a weighted, directed graph in which nodes are financial institutions and edges are lending relationships of a specific value (e.g. Iori et al., 2008).

If a single bank fails, initially only those banks to which it owes money suffer directly, the remainder of the system is unaffected[2]. The direct impact, however, may cause one or more of the banks counter-parties to fail, harming further institutions within the system. The structure of inter-bank markets, the number, size and distribution of linkages, has a large effect on the markets susceptibility to systemic events (Haldane and May, 2011). Muller (2006) and Upper and Worms (2004) show that in the Swiss and German banking systems there is significant potential for contagion. Highly centralized markets, those with large hub banks like the UK (Becher et al., 2008), are particularly susceptible. In contrast Angelini et al. (1996), Boss et al. (2004) and Furfine (2003) find that there is relatively little danger of systemic events, only a small number of banks could cause others to fail.

The difference in conclusions is driven in part by differences in the inter-bank markets, e.g. trade volume (Angelini et al., 1996). However, theoretical models present a similarly ambiguous picture (e.g. Leitner, 2005). Vivier-Lirimont (2006) finds that increasing the number of inter-bank connections worsens contagion. This is partially supported by Brusco and Castiglionesi (2007) who show that increasing cross-holdings increase the extent of contagion but reduces the effect on individual institutions. In contrast Giesecke and Weber (2006), in line with Allen and Gale (2001), find that more connections reduce contagion. Nier et al. (2007) show computationally that a small increase in connectivity increases systemic risk but beyond a certain point the degree of systemic risk decreases. In contrast, Lorenz and Battiston (2008) and Battiston et al. (2009) find the opposite relationship, the scale of bankruptcies is minimized for intermediate levels of connectivity.

The models above give apparently contradictory results regarding the effect of the inter-bank market. Some show increasing connectivity as providing stabilization, others as increasing the potential for contagion whilst a few give non-monotone relationships. The mixed results are due to the interaction of the two effects of inter-bank relationships discussed by Allen and Gale (2001), risk sharing versus contagious vulnerability. Whilst sparser networks limit the ability of shocks to spread, reducing contagion, they also reduce

[2] For the present we ignore issues regarding market confidence and beliefs. In reality, a bank that is not directly effected may alter their portfolio to limit the possibility of losses (Lagunoff and Schreft, 2001).

the risk sharing capacity of the market and so increase the risk of individual banks failing. The model presented in the next section will examine this interaction and the behavior of the financial system as a whole. Previous analytical papers have derived the optimal behavior of banks in various settings, however, they analytical tractability constrains the structure of the markets which can be examined. In contrast, whilst simulation studies are not constrained in this manner they frequently specify behavior and characteristics exogenously. For instance whilst Iori et al. (2006) is able to analyze the effects of connectivity, to do so they set sizes of banks and do not require supply to equal demand in the inter-bank market. This paper consider complex networks of inter-bank connections in a partial equilibrium setting in which banks choice to lend or borrow on the inter-bank market and the inter-bank interest rate are determined together endogenously. At the same time banks determine their own portfolio choice subject to the deposits and lending opportunities the bank is able to attract through its choice of lending and borrowing rates. As such we determine an equilibrium of bank behavior within the economy.

3 Model

We consider a model of a closed economy containing N banks, M depositors and Q borrowers. Depositors, banks and borrowers each occupy locations on the circumference of a unit circle. This circle represents a dimension, not necessarily physical, on which the agents differ. Banks are equidistantly spaced with bank 1 being located at the top of the circle and the remaining banks arrayed in index order clockwise around the circumference. The same arrangement is used for the non-bank depositors and borrowers with the agent with index 1 being at the top of the circle. The distance between a bank and another economic agent affects the banks ability to attract that agent as a potential borrower or depositor.

The model operates in discrete time and repeats for an infinite number of time steps. The actions and investments of each bank in each time step effect their financial position in future periods. We consider each time step to represent a period of one year. The following sub-sections describe the behavior of the banks, borrowers and depositors during each period (state and choice variables may be seen in Table 1).

3.1 Depositors

Each depositor, j, is a non-bank entity which holds an endogenously determined quantity of depositable funds (d_j). The depositors place these funds in the bank which maximizes their expected return:

$$\arg\max_{i \in N} d_j(r_i^{deposit} - g(i,j)) \tag{1}$$

Where $g(i,j)$ is the distance[3] between i and j and $r_i^{deposit}$ is bank i's deposit interest rate. If no i exists such that Equation 1 is positive the depositor retains its funds and earns no interest. Banks do not refuse any deposits. Full deposits insurance is provided by an agent outside of the system who guarantees that depositors will be repaid in the event of bank failure. Depositors are, therefore, not concerned with the risk of bank default and so select the bank offering the highest return[4].

3.2 Borrowers

Each time period, each non-bank borrower, q, has a single limited liability investment opportunity, l_q^t. Each opportunity requires an initial investment of l^S currency at time t and provides a payoff to the borrower at time $t+2$ of μl^S with probability $\theta_{l_q^t}$. With probability $1-\theta_{l_q^t}$ the investment provides zero payoff. Values of μ and l^S are fixed across loans whilst $\theta_{l_q^t}$ is drawn from a uniform distribution (see Table 2). In order to invest in the opportunity borrowers are required to borrow the full amount from a bank. Each borrower, q, approaches the single bank which maximizes the borrowers expected return:

[3]In line with the previous hotelling literature (e.g. Salop, 1979) we model transaction costs as linear in the distance between two actors. Alternative functions were tested and had little qualitative effect.

[4]Depositors are modeled as being highly active in their management of deposits, however, in reality deposits tend to be sticky. Individuals are slow to respond to changes in interest rates, frequently maintaining their deposits in institutions paying suboptimal rates, rather than switching. Experiments were performed in which deposits were moved with a fixed probability. Values of switching greater than 4% produced no significant difference in results.

$$\arg\max_{i \in N} \theta_{l_q^t} l^S(\mu - (1 + r_i^{loan})^2) - g(i, q) \tag{2}$$

Where r_i^{loan} is bank i's per period lending interest rate. If no i exists such that Equation 2 is positive the opportunity goes unfunded. If bank i funds an investment opportunity, l_q^t, with probability $\theta_{l_q^t}$ the bank receives $l^S(1 + r_i^{loan})^2$ at time $t + 2$ whilst with probability $1 - \theta_{l_q^t}$ the bank receives nothing.

3.3 Banks

Each bank, i, has a balance sheet comprising equity (E_i), deposits (D_i), cash reserves (R_i), loans to the non-bank sector (L_i) and loans to the other banks (I_i)[5]. Each time step, each bank, i, attempts to maximize its expected return, $E(r_i)$ given by:

$$(\sum_{k_i^t=1}^{K_i^t} \theta_{k_i^t} l^S(1 + r_i^{loan})^2 - 1) + I_i^t((1 + r^{interbank})^2 f(I_i^t) - 1) - D_i r_i^{Deposit} \tag{3}$$

Where K_i^t is the set of loans funded by bank i in period t, $\theta_{k_i^t}$ is the probability of loan k_i^t being repaid[6] and $f(I_i^t)$ is a function giving an estimate of the probability of inter-bank lending being repaid:

$$f(I_i^t) = \begin{cases} \theta_i^{interbank}, & \text{if } I_i^t > 0 \\ 1, & \text{if } I_i^t \leq 0 \end{cases} \tag{4}$$

Here $\theta_i^{interbank}$ is bank, i's estimate of the probability of being repaid. The failure to repay inter-bank lending results in the bankruptcy of the defaulting bank. Consequently, in calculating their expected return banks assume that they repay their own inter-bank borrowing with probability 1. This maximization is subject to the following constraints:

$$L_i + R_i + I_i = E_i + D_i \tag{5}$$

[5]Positive values correspond to lending, negative to borrowing.

[6]Each bank commits to a lending rate prior to being approached by borrowers with loan opportunities. They are not permitted to change this rate dependent on the risk of a project. Experiments were performed in which all projects had the same probability of success allowing banks to set interest rates dependent on risk. There was no qualitative change in the results.

$$D_i = \sum_{j=1}^{M} S(i,j) d_j \text{ where } S(i,j) = \begin{cases} 1, & \text{if } i = \arg\max_{i \in N} d_j (r_i^{deposit} - g(i,j)) \\ 0, & \text{Otherwise} \end{cases} \tag{6}$$

$$R_i \geq max(\alpha_g, \alpha_i) D_i \tag{7}$$

$$E_i \geq max(\beta_g, \beta_i)(L_i + max(I_i, 0)) \tag{8}$$

$$L_i = \|K_i^t\| + \|K_i^{t-1}\| \tag{9}$$

The first constraint states that each bank's balance sheet must balance; i.e. assets are equal to liabilities. The second constraint specifies that the bank's holding of deposits is equal to the sum of deposits placed in that bank. The bank may neither refuse deposits nor gain access to additional deposits outside of those contributed by the depositors it has attracted. The third constraint governs the level of liquid cash reserves which the bank holds. It is the maximum of the banks preferred level, α_i and a minimum level imposed by regulation α_g. The fourth constrain specifies the maximum equity to risky assets ratio. Where β_i is the bank's preferred equity ratio and β_g is a minimum value imposed by regulation. The second *max* operator means only positive values, i.e. inter-bank lending and not inter-bank borrowing are considered. Note, reserves are risk-less and so are not included in this ratio. In this model inter-bank lending and non-bank lending are equally weighted in the risk calculation. The fifth constraint states that the amount invested in loans is equal to the total funds invested in individual projects (we define $\|.\|$ to be the sum of the values of loans in the included set). Since loans last for two periods, this includes all projects funded at times t and $t-1$.

We consider this maximization problem to proceed in two stages. First, at the start of each time period each bank publicly declares its deposit, $r_i^{deposit}$, and lending, r_i^{loan} interest rates. Depositors and borrowers respond to these rates, placing deposits and submitting lending request to the appropriate banks. Banks then determine the allocation of assets and liabilities on their balance sheets to maximize the expected return. Money is distributed from deposits and inter-bank borrowing to fund loans to non-bank borrowers, inter-bank lending and to save as cash reserves. The banks' equity is the result of its previous investment decisions up to the current time period. This together with the above constraints mean that in any given period at the point returns are maximized the level of Equity, Deposits and Reserve are all known. Additionally the bank still has positions in loans to non-banks and inter-bank lending from the previous time step which it may not change. The maximization problem it therefore the distribution of the remaining funds between new inter-bank lending and borrowing and new loans to non-banks. In making this decision bank i determines the composition of K_i^t the set of funded investment opportunities. The loans are selected from P_i^t, the set of investment opportunities presented to bank i by borrowers at time t, i.e. $K_i^t \subseteq P_i^t$. Bank's invest in zero or more loans in decreasing order of expected return until the expected return falls below the inter-bank lending rate or the bank runs out of funds. If the bank runs out of suitable loan opportunities whilst it still has available funds the bank may lend to other institutions subject to the expected return of the loan being positive. Alternatively if a bank has excess loan opportunities it may borrow money from other banks to fund these investments. As such the banks position in the inter-bank market, whether it is a lender or borrower, is determined endogenously by its portfolio optimization. The next section will go on to describe how inter-bank relationships between lenders and borrowers are established.

3.4 Inter-bank market

Inter-bank lending occurs through an over-the-counter market. This means that transactions are bilateral, when a bank lends money it lends to one (or more) specific counterparties. The inter-bank rate is dependent on the lending and borrowing preferences of individual banks which, as shown above, are themselves dependent on the inter-bank rate. There is no closed form solution for the equilibrium, so in order to find the interest rate it is necessary to use an iterative numerical approach. To simplify the initial analysis we assume all transactions in each period occur at the single market clearing inter-bank interest rate[7]. In section 6 we relax this assumption.

[7]During non-crisis periods, both in reality and this model, the rate at which banks fail is very low and in a steady state there should be little difference in the offered inter-bank rates between banks.

Within the model, whether a bank is a lender or borrower on the inter-bank market is determined endogenously by the allocation of funds within their portfolio each period. The pattern of inter-bank connections between lenders and borrowers is determined exogenously and constructed as follows. Initially the population of banks is partitioned into three sets by their desired positions: lenders, borrowers and those with no position. Each member of the set of lenders is considered in turn in decreasing order of the magnitude of funds offered. Let the set of borrowers to which lender i lends money be C_i. For each borrower, b, in the population, with probability λ, b is added to C_i. If, after this, the total amount of funds requested by members of C_i is less than the amount i wishes to lend, further borrowers are added to C_i in decreasing order of magnitude of requested funds until this is no longer the case. This method ensures the minimum number of banks are added to the set allowing the generation of minimally connected markets. The lender, i, lends money to each member of C_i in proportion to their requested funds.

For $\lambda = 1$ all borrowers are connected to all lenders. Whilst for $\lambda = 0$, the requirement for the total funds requested in C_i to be at least that demanded by bank i leads to the formation of multiple disconnected sub-graphs, where borrowers are connected to the minimum number of lenders to satisfy their demand. In this scheme the parameter λ describes the degree of voluntary diversification in the inter-bank market, i.e. how many lending relationships banks form beyond the minimum. As λ increases the density of inter-bank connections increases allowing a range of structures to be investigated. By ensuring a minimum level of requested funds in set C_i we model an efficient financial market where lenders and borrowers are able to find each other. If this minimum were not imposed there would be a degree of credit rationing where banks would be constrained in the amount they could lend or borrow by the identities of their potential partners. Evidence suggests e.g. Iori et al. (2008) that in developed markets for large financial institutions during non-crisis periods the market is efficient and the chosen representation is appropriate.

In the next section we will show that this method produces networks which match many features observed in reality. Other approaches were considered, however, they produced results similar to those generated with this mechanism for the same number of connections[8]. It should be noted that the mechanism used here generates only one particular class of network. There is evidence that in reality there is a wider and possibly richer range of inter-bank market structures. For instance Becher et al. (2008) show a hierarchical structure in the UK market. Cossin and Schellhorn (2007) and Georg (2011) examine the effect of different types of market structures on inter-bank stability whilst the endogenous formation of networks has also been considered (Babus, 2007). The mechanism presented here, however, produces networks which are simple and reflect many of the features observed in reality[9]. Future work will consider alternative classes of networks.

The two period nature of investments is important in capturing the structure of the inter-bank market. In any period each bank may be either an inter-bank lender or a borrower, they may not be both. Consequently if investments and the inter-bank borrowing funding it, lasted only a single period the network would be bipartite. This would limit the potential for contagion to the failed banks direct creditors. Two period loans allow a bank to be both a lender and borrower in subsequent periods, allowing failures to spread and richer, more realistic contagious events. Here inter-bank lending has the same period as lending to borrowers. This does not have to be the case, in reality inter-bank lending is frequently of shorter duration. This assumption simplifies the model, removing the requirement for banks to predict future liquidity needs. This may have the effect of reducing the possibility for contagion, particularly that associated with liquidity shortages.

3.5 Model Operation

This section details the order of events within each time period. At the start of period t, interest is paid by banks to depositors on the deposits established during period $t-1$ (Equation 1). After interest is paid, loan success is evaluated for loans established in period $t-2$ and banks repaid by borrowers. The inter-bank lending from time $t-2$ which funded these investments is then repaid. If after interest payments and loan success have

[8] We also considered λ as an endogenous variable set by each bank. It was found that there was no significant change in results.

[9] Allen and Babus (2009) provide an overview of the range of networks investigated in the literature.

been evaluated the bank has negative equity, or if a bank has insufficient cash reserves to repay its inter-bank debts, it is declared bankrupt. In the event of a bank failure sufficient assets are retained, if available, to cover the value of deposits, any remaining liquid assets are used to repay creditors in proportion to the size of their debt. If a creditor bank is not fully repaid it suffers a loss in equity which may potentially cause it to go bankrupt. If this occurs any inter-bank borrowing on its balance sheet is resolved in the same manner. As such the failure of one bank may spread to counter-parties and beyond. A bankrupt bank is removed from the financial system and takes no further actions.

If a bank fails to which a bank or non-bank borrower owes money, the borrower is still required to repay its loan at the appropriate due date. This is consistent with an administrator ensuring creditors of a bank meet their requirements. Any funds arising from such repayments are considered to either be absorbed by the administrators of the failed bank or to go to the deposit insurer (Equation 4). After bankruptcies are resolved depositors place their deposits in banks. Banks allocate their funds and the inter-bank rate is calculated along with the lending relationships. Whilst allocating their funds banks are required to pay dividends to shareholders in the non-bank sector. The dividend rate is given by F_t:

$$F^t = \frac{N}{\sum_{i=1}^{N} E_i^t} \tag{10}$$

Where bank i pays $F_t E_i$ to the non-bank sector[10]. The effect of these payments is to fix the total value of bank equity. In essence the value of the economy is normalized so the effects of growth do not need to be considered. Without this process the many iterations required for the learning mechanism described below would lead to the overall equity of the system growing towards infinity and preventing a stable solution.

3.6 Parameters and Learning

Banks' allocation of funds is determined by several endogenous parameters. These are: reserve ratio (α_i), equity ratio (β_i), lending interest rate (r_i^{loan}), deposits interest rate

[10] If F_t is greater than 1 no dividends are paid.

($r_i^{deposit}$) and their estimate of being repaid in the inter-bank market ($\theta_i^{interbank}$). There is no closed form solution for assigning optimal values to these parameters within this model. Instead the values of these parameters are randomly assigned and then optimized by a genetic algorithm[11]. Here we maximize the profitability of banks, i.e. we find those parameters which lead to higher equity. Details of the genetic algorithm are presented in the appendix. Importantly bankrupt banks which are selected in the GA are reintroduced to their previous location on the circle with $E = 1$, $R = 1$ and no other assets or liabilities. This process ensures that the parameter space is explored whilst bankrupt banks are replaced and the population of banks converges to optimal parameters.

4 Results

In order to evaluate the behavior of the model it is necessary to first consider the steady state. All experiments use the parameters presented in Table 2 unless otherwise stated. An analysis of robustness to parameters and assumptions is provided in Section 6. The first two parameters are chosen based on real world values. US banking regulation defines a minimum reserve requirement of 10% and a minimum capital requirement for a bank to be adequately capitalized of 8%. In making this calculation we count both inter-bank loans and loans to non-banks as having a risk weighting of 1 whilst reserves are risk-less. All depositors have the same amount of funds, d_j, to deposit and this value is constant over time[12]. At the start of the simulation $E_i = 1$, $R_i = 1$ for all banks. All other assets and liabilities for all agents are set to zero. Whilst all banks start the same size and the distribution of banks, borrowers and depositors are symmetric, the random variation in initial parameters along with the stochastic payoffs from risky projects permits heterogeneity to develop within the model. For instance a bank may fail due to an investment not repaying whilst a similar bank may prosper because a similar loan did. This means the model is path dependent, therefore, 500 repetitions, with different random seeds were conducted for each of 11 different values of λ to generated distributions

[11] See Arifovic (1996) and Noe et al. (2003) for examples of GA's used in economics.

[12] Heterogeneous distributions and time varying quantities were considered but for a large range of specification the results were qualitatively similar.

of results. Each simulation was run for 100000 time steps to allow the model to reach a steady state. To test convergence the average values of market parameters during periods $80000 - 89999$ and $90000 - 99999$ were calculated and a T-Test performed to ensure the parameters were stable. At this point market statistics were recorded. The 100000 time step run period provides sufficient time for parameters to be optimized and a steady state to be achieved, however, it should not be interpreted in terms of historical time (100000 years). The evolutionary approach used within this model by its nature incorporates a large amount of undirected variation and experimentation. In reality, however, banks would shortcut this process through learning and deduction meaning an optimization period this long would be unnecessary. For a similar reason this paper only considers the immediate effect of bankruptcies within the system and does not look at those several periods in the future; a time period in which real banks could adapt their behavior but in which the evolutionary process cannot.

4.1 Steady state analysis

In this section we present statistics describing the state of the converged model. Key ratios and quantities are shown to be of the same magnitudes as those observed empirically. We do not match exactly the balance sheets of a particular country. To do so would require a more complex model with many more parameters and asset types. Correct magnitudes are sufficient such that conclusions drawn from the model hold for a range of financial systems.

Table 3 shows the average asset and liability holdings of all banks within the model, together with the balance sheets of all American commercial banks in 2006. Pre-crisis data was chosen to compare to pre-shock model data. Model balance sheet terms are matched to their closest equivalents. Terms on the real balance sheet which have no model equivalents are omitted. In this, and all subsequent tables, inter-bank loans are the total funds lent within the system. The sum of all positions would be 0 as inter-bank lending is equal to inter-bank borrowing within this closed economy[13].

[13]During this period American banks were net borrowers, the figure for Borrowing (including both national and international relationships) is therefore a better estimate than that of Inter-bank Lending.

Crucially the level of inter-bank lending within the model is close to the value calculated within the real economy. Inter-bank loans are the mechanism by which failures spread. If the level of loans in the model were of a different magnitude to that seen in reality then bank failures would either spread much more easily or much less frequently than in reality.

The level of deposits are also very similar in both cases. The amount of loans in the model is slightly below that observed in reality, however, the loans term in the real data also includes treasury securities which, whilst lending, are risk less. The only significant difference comes in the level of cash reserves which are approximately twice as great in the model as reality. This is because within the model we do not separate demand deposits (where reserve are required) and timed deposits (where they are not). As a result some deposits in the real balance sheets require no reserves. As was previously mentioned, however, real banks have access to highly liquid assets such as treasury bills which could be sold to provide almost instant liquidity[14]. Importantly, however, bank's preferred equity and reserve ratios (Table 4) are both less than the values specified by regulations i.e. 8% and 10%. This means the regulated values are used in all cases and the banks are maximally leveraged. The banks therefore, behave in a similar manner to those in reality.

The loan and deposit rates within the model of 2.9% and 0.9% (Table 4) are empirically plausible real interest rate (there is no inflation). The inter-bank rate of 2.3% is high compared to historical values, however, within this model there is no other source of funds so this rate reflects demand for funds to lend to borrower rather than risk. The model does a good job of matching the magnitudes and key ratios observed in empirical data suggesting that it may be used to identify relationships and draw conclusions about stability for a range of financial system.

4.2 Market Structure

The structure of the inter-bank market is determined by a combination of endogenous bank behavior and exogenous structure. The number of lenders and borrowers, their size and

[14] Imposing a lower reserve ratio, such that the model reserves matched those from American data did not qualitatively change the results.

distribution, is determined endogenously by the supply and demand of funds and loan opportunities whilst the matching of lenders and borrowers is determined exogenously. This section will describe and characterize the endogenously determined features of the inter-bank market and compare them to empirical observations.

Table 4 shows that in line with the empirical results of Muller (2006) that borrowers tend to be larger (have higher equity) than lenders. This is because within the model large banks are constrained by the amount of funds they are able to raise through deposits. These banks have high equity and so in order to be maximally leveraged they must borrow on the inter-bank market. In contrast small banks do not need to borrow as often. They are constrained by their level of equity and would be unable to invest borrowed funds in risky projects. This implies that banks would tend to lend or borrow from banks of a different size to them-self who are not constrained in the same manner. Empirically this is demonstrated by Cocco et al. (2009) who examines the distribution of loans between banks, finding that the most common links are between large and small banks whilst the least common are between pairs of small banks. Table 5 shows a similar relationship in the model when the population is partitioned around the median wealth. It is important to emphasize that these features of the market were not specified in the model, rather they were endogenously determined as the optimal behavior of banks within the model.

4.3 Individual Bankruptcy

To examine the stabilizing and contagion spreading effects discussed above we first consider the failure of a single bank and its impact on the financial system. Similar analysis has been conducted in other studies both analytically and empirically for a range of inter-bank markets with mixed results[15]. Elsinger et al. (2006) show that systemic failures from the collapse of a single bank only occur in about 1% of cases. Further, only a small proportion of banks are able to cause systemic crisis (Boss et al., 2004) or are susceptible should a partner institution fail (Angelini et al., 1996). The effect of contagion when it occurs, however, can be very large (Gai and Kapadia, 2010). For instance Humphrey

[15] For example: Boss et al. (2004), Upper and Worms (2004), Nier et al. (2007), Gai and Kapadia (2010) Vivier-Lirimont (2006) and Allen and Gale (2001).

(1986) show the collapse of a large U.S. bank could bankrupt 37% of banks in the market.

The converged economies presented previously serve as a basis for this analysis. The state of the market, the bank positions and inter-bank loans, are recorded and a single bank made bankrupt by setting its equity and reserves to zero. The effect of this bankruptcy on the rest of the economy is analyzed before the state of the market is reset. This is repeated for each bank in turn.

Table 6 shows that as the market becomes more connected the effect of a bankruptcy, as measured by the average number of subsequent failures, is reduced, in line with previous findings (Allen and Gale, 2001; Giesecke and Weber, 2006; Freixas et al., 2000). This is because fewer banks are able to cause another to fail (decreasing Probability in Table 6). As Brusco and Castiglionesi (2007) argue, whilst more banks may be touched by contagion, if the market is more heavily connected the probability that any of them will fail is reduced. The higher level of connectivity provides diversification of credit risk for the banks. When a bank fails the impact is more spread and the effect on each individual is reduced.

Higher connectivity generally reduces the average size of contagious events, however, Table 6 shows that markets with low-intermediate levels of connectivity exhibit the largest contagious events. These markets are sufficiently poorly connected that if one bank fails the shock is strong enough to drive others to also fail. At the same time the market is sufficiently well connected that a single bankruptcy can potentially affect many other banks. The combination of large shocks and wide spread make these markets particularly vulnerable if the wrong bank fails. Whist in less well connected markets there are more banks which can cause contagion, the low level of connectivity means the spread of bankruptcies is inhibited and the average size of failures is reduced.

An alternative measure of contagion is the maximum number of bankruptcies a failure may cause. The sizes of the largest failures are of the same magnitude as those seen in reality, e.g. 15% in Germany (Upper and Worms, 2004) and 37% in the U.S. (Humphrey, 1986) and follows a similar pattern to that of average contagion size. The average equity of failing banks (not shown) is approximately 0.9 for all market structures which is less than the market average of one, indicating that smaller banks are more vulnerable to contagious failure.

4.4 Systemic Shocks

The results presented in the previous section are important in understanding the vulnerability of the financial system to a single failure. In reality, however, the failure of a bank is often not an isolated event. Instead a failure may be caused by a shock which affects the whole financial system. Macroeconomic events may affect multiple institutions simultaneously, weakening balance sheets and potentially causing several unconnected banks to fail at the same time (Gorton, 1988).

Few studies have examined the effect of the inter-bank market during a systemic shock. It is not clear whether contagion in the inter-bank market will be significant or if it will be secondary to the financial shock itself (Giesecke and Weber, 2006). At the same time it is unclear how the risk-bearing and contagion spreading effects interact as equity is eroded. A market in which each bank is connected to a greater number of counter-parties may allow system liquidity to be better utilized reducing the impact. Alternatively, in better connected markets the weakest banks may be more likely to be effected and fail.

A systemic shock is applied to the converged state by changing the success probability for projects which finish in that time step from θ_k^{t-2} to θ^{shock}. All projects ending at other times are unchanged. We perform experiments for a range of shock severity's, θ^{shock} and market connectivity's, λ.

Figure 1 presents results showing the average number of bankruptcies across market architectures and shock severity's. For the smallest shocks very few banks fail as the losses from the non-performing loans are absorbed by the funding banks. As θ^{shock} decreases fewer projects are completed successfully, leading to higher losses for banks and more failures. Market connectivity has a non-linear effect on this relationship. For small shocks a more highly connected market reduces bankruptcies, limiting the spread of contagion by spreading the impact of failures. In contrast for larger shocks the pattern is reversed, more sparsely connected markets are less susceptible to contagion. For intermediate shock sizes, moderately connected markets may be the most vulnerable, for example $\theta^{shock} = 0.89$.

The results show that there is no optimal degree of connectivity which minimizes the effect of shocks in all cases.

A systemic shock reduces the equity of all banks. For small shocks equity is only slightly damaged such that in highly connected markets if a bank fails the impact is sufficiently well spread that the bankruptcy rarely cause a counter-party to fail. The diversifying effect of increased inter-bank connectivity reduces risk. As connectivity decreases the size of inter-bank linkages on average increase and failures becomes more likely. Larger systemic shocks result in heavily reduced bank equities and so smaller counter-party losses may cause bankruptcy. Consequently banks in more connected markets start to be at risk from the failure of their counter-parties. For the largest systemic shocks bank equities are damaged to such an extent that regardless of connectivity the failure of any counter-party is sufficient to cause a lender to fail. Instead of spreading the impact the higher connectivity results in more banks being affected and failing. At the same time the diversification effect from many inter-bank connections is weakened as the failure of banks becomes increasingly correlated. In less well-connected markets banks fail but the scope of contagion is reduced as each bank failure effects a smaller subset of the population. For $\theta^{shock} = 0.89$ the point at which the likelihood of a bank failing and spreading a shock is maximized at intermediate levels of connectivity. At this level of shock, more connected markets spread impacts sufficiently well that relatively few banks fail whilst in less connected markets the spread of the shock is limited.

The results support a range of empirical and analytical findings. They agree with Giesecke and Weber (2006) that for small shocks connections reduce contagion whilst they also support the finding of Vivier-Lirimont (2006) that more connected markets result in more banks in the contagion process. Similarly they support the results of Georg (2011) and Iori et al. (2006) that contagion may be more significant when the market is more connected. For $\theta^{shock} = 0.89$ the results have a similar pattern to that identified by Nier et al. (2007) and Gai and Kapadia (2010), an increase in connectivity first leads to an increase in failures but beyond a certain point a decrease in contagion is observed.

The pattern of failures found in this paper differs from that shown by Lorenz and

Battiston (2008) and Battiston et al. (2009). Both of these papers find a U shaped distribution of failures as opposed to the increasing, decreasing and hump shaped distributions shown above. This difference is driven by the presence of as inter-temporal feedback mechanism, referred to by Battiston et al. (2009) as the *financial accelerator*. Under this mechanism a deterioration in a company's financial state is perceived by its creditors who in response tighten their terms of credit. This tightening may further worsen the company's state leading to yet worse terms. With higher levels of connectivity banks become increasingly sensitive to the states of more counter-parties and so the deterioration of a single bank may spread throughout the system leading to a general constriction of credit and an exacerbation of shocks. This mechanism may dominate the diversification effect of a better connected inter-bank market reversing the relationship observed in this model. The financial accelerator does not have an equivalent within our model as we focus on short term (within period) effects. Consequently we do not see any evidence of a U shaped distribution of bankruptcies. Without the financial accelerator, however, the behavior of the two models is similar. As Battiston et al. (2009) notes when this mechanism is removed contagion is decreasing in connectivity as seen here for small shocks.

To examine the importance of contagion we separate the failures in the banking system into two groups (Figure 1) in a similar manner to Martinez-Jaramillo et al. (2010); the casualties of the initial shock and those caused by the failure of counter-parties. In line with Elsinger et al. (2006), for all but the smallest shock over half of the bankruptcies are caused by contagion. The systemic shock plays a major role in weakening the banks' equity, however, it is the failure of counter-parties which induces bankruptcy in the majority of cases and so is a significant aspect of systemic risk.

The number and size of banks which fail in the face of a systemic crisis is only one measure of the severity of the impact. If a bank fails the deposit insurer has to step in to compensate depositors. The insurer may therefore be concerned with the cost of repaying deposits rather than the number of bank failures in judging the optimal inter-bank market structure. Figure 2 shows that regardless of the size of the shock as connectivity decreases the cost to the insurer increase. This is because the more connected a market is the more of the cost of failures are born by the surviving banks. When a bank fails in a weakly connected market it has a large impact on a relatively small number of creditors. The impact heavily damages their balance sheets resulting in a large loss in equity and little left to pay depositors. In contrast, in a strongly connected market the failure of each bank affects many more counter-parties. This may result in more bankruptcies, however, the smaller impacts mean that those banks which fail may still have some assets on their balance sheet and be able to partially repay depositors. The surviving banks effectively bear some of the cost in reduced equity. For the insurer increased connectivity is beneficial as it reduces costs, even if it potentially increases the number of failures[16].

5 Regulation

The previous section highlighted the effects of the market structure on contagion under both individual and systemic shocks. Here we consider mechanisms for limiting the impact of these events and their wider effect on the market.

5.1 Equity and Reserve ratio

A key proposal put forward in Basel III requires banks to hold a higher percentage of capital relative to their risky assets. Doing this reduces leverage and so potentially decreases the risk of failure due to poor investments. A second proposal is to tighten banks minimum reserve ratios. This would force banks to hold a higher proportion of liquid reserves providing them with increased protection against liquidity shocks. The effect of changes in the equity and reserve ratio's on individual bank failures has already received much attention[17], we therefore, focus on their effect under systemic shocks. We consider both of these mechanisms independently. Each ratio is increased by 2% to give equity and reserve ratios of 10% and 12% respectively. 500 further runs are conducted for each case. Both of these changes are found to have small negative effects on the efficiency of

[16] There may be additional social costs due to damage to the payment system if many banks fail.

[17] Iori et al. (2006), Nier et al. (2007) and Gai and Kapadia (2010) all find that increasing the amount of reserves reduces the number of bankruptcies.

the financial system. The average value of loans to borrowers reduces by 2.1% to 743.7 for the change in reserve ratio and 2.5% to 741.3 for the change in equity ratio.

Figure 3 shows that increasing the equity ratio results in a large reduction in failures in nearly all cases. The decreased level of leverage reduces the impact of the macro-economic shock. At the same time a reduction in inter-bank lending by approximately 50% limits the effect of failing banks on their counter-parties[18]. In contrast increasing the reserve ratio results in a small increase in susceptibility. Forcing banks to hold more of their deposits as cash reserves, without constraining the level of risky assets, makes those banks with larger numbers of investment opportunities borrow more to fund them. As a result interbank lending increases by approximately 12% and despite the increased level of reserves, failures are able to spread more easily. This finding highlights the importance of considering the effects of regulatory changes in equilibrium. Whilst an increase in reserve requirements may seem like a simple method to reduce contagion, if banks behave optimally under the regulatory regime the opposite effect is observed [19].

5.2 Borrowing Constraints

An alternative to constraining the total lending or borrowing is instead to constrain the maximum funds a bank may lend to a single counter-party. This forces banks to diversify their inter-bank lending, making them less susceptible to the failure of a single debtor. Here we implement this regulation by limiting the maximum a particular lender may lend to a particular borrower to be no more than a multiple η of the borrowers equity.

Table 7 presents the results of 500 simulation for four different borrowing constraints. We focus on one value of λ (in this case $\lambda = 0.5$) as the effect of lending constraints are dependent on the number of banks to which a lender lends[20]. For $\eta = 20$ there is no

[18]It appears that for very small shocks there is a slight increase in failures. This, however, is misleading, the equity of the failed banks is unchanged from the base case, whilst for all other sizes of shock there is a significant difference. The additional failures in this case are very small institutions which are unable to meet the regulatory requirement.

[19]For a much larger increases in reserve requirements a reduction in bankruptcies may be achieved, however, this leads to a decrease in the efficiency of the financial system and a marked drop in the funding of projects.

[20]For different values of λ similar patterns may be observed, however, the level at which η has an effect if different. For a more (less) connected market the average amount lent between a pair of connected banks is lower (higher) and so η must be smaller (larger) to have the same effect.

significant change in any of the market statistics. As η is decreased the constraint becomes binding. For $\eta = 10$ the number of systemic bankruptcies is significantly reduced for larger shocks. This is driven by a reduction in inter-bank lending which reduces the strength of connections between banks. This reduction is accompanied by a small (1%) reduction in lending to non-banks as funds are less efficiently allocated. As η decreases further the reduction in inter-bank lending and lending to households becomes more marked. For $\eta = 5$ the reduction in bankruptcies for large shocks is greater, however, for small shocks their is a small increase in bankruptcies. This occurs because the regulation effectively changes the interbank network. Constraints on the maximum size of loans particularly effects lending to those borrowers with the lowest equity. The size of a specific banks loan to one of these banks decreases and in turn the size of loans to bigger, less constrained, banks increase[21]. The effect of this is to qualitatively change the shape of the network, reducing the number of connections which may realistically be able to spread contagion. As a result the connectivity of the network is effectively decreased. Earlier results showed that under small shocks less connected networks tend to be more susceptible to systemic shocks whilst under larger shocks they are more resilient - matching the pattern we observe above. If $\eta = 2$ there is a further reduction in bankruptcies, however, the inter-bank market is heavily impaired. Funds are no longer efficiently allocated and the value of loans to non-bank borrowers is reduced significantly.

6 Model Sensitivity

In forming the model above assumptions were made which, whilst increasing transparency, simplified important aspects of real world behavior. Here we relax several of these assumptions to move the model closer to reality whilst also permitting a greater degree of heterogeneity within the system.

[21]If the total amount of lending from a bank is fixed, when the amount it lends to small banks is decreased the amount lent to larger banks will increase.

6.1 Parameter sensitivity

The results presented above are based on one parameter combination. Here we demonstrate the robustness of the results and behavior to changes in these values. Table 2 details the models seven key parameters. Of these seven, changes to α_g and β_g have already been considered as regulatory actions. Further simulations were run in which the remaining five parameter values were changed and the key affects reported[22].

Varying the payoff from investments, μ, affects the loan, deposit and inter-bank interest rates. Greater returns from investments allow banks to charge borrowers higher interest rates which in turn allows banks to pay higher rates for funds from both depositors and on the inter-bank market. The model is robust to a wide range of values. $\mu = 1.1$ was chosen as it produced deposit and loans rates comparable to reality.

The parameters controlling the probability of a successful investment, θ, the investment size l^S and the number of borrowers, Q, are closely linked. Together they control the supply of potentially fundable loan requests. A decrease in borrowers results in fewer loan requests per time-period, a decrease in l^S decreases the cash value of loans requested, whilst a decrease in θ reduces the expected return of projects making fewer profitably fundable[23]. Results are robust across a wide range of parameter values ($0.9 < \theta < 0.999$, $0.01 < l^S < 1$, $Q > 20N$), if any of these values are too low there may be insufficient profitable investment proposals resulting in unallocated funds and little inter-bank lending. If l^S is too high the market may be unstable as the chance failure of even a single loan will bankrupt most banks. $Q = 4500$, $l^S = 0.1$ and $\theta = 0.99$ provided sufficient supply of funding requests whilst maintaining computational tractability. Increasing Q beyond this point slows execution without changing the results.

While θ and Q describe the supply of investment projects, N, the number of banks, controls the demand. Qualitatively similar results were found for a wide range of values ($N > 40$). $N = 100$ was chosen as it is the same magnitude as the number of banks in many of the worlds inter-bank markets. M the number of depositors has little effect on

[22] Tables of results demonstrating the relations are available from the author upon request.

[23] Note this parameter also interacts with μ. The larger the value of μ the lower θ may be whilst maintaining a profitable project.

the behavior of the model as depositors simply pay deposits into the banks. The number of depositors was set equal to the number of borrowers, however, for $500 < M < 1000000$ there was little quantitative effect.

6.2 Bank confidence

One key feature of the recent financial crisis was the loss of liquidity on the inter-bank markets. Banks observed the failures of other institutions and became reluctant to lend resulting in a shortage of liquidity and an exacerbation of the crisis. In the model above one bankruptcy may cause other banks to fail. Banks, however, do not take this into account, they do not become more reluctant to lend even though the probability of being repaid is potentially reduced. To capture this effect equation 4 is changed:

$$f(I_i^t) = \begin{cases} \theta_i^{interbank} - \kappa_i B^t, & \text{if } I_i^t > 0 \\ 1, & \text{if } I_i^t \leq 0 \end{cases} \tag{11}$$

Where B^t is the number of bank failures in the current time step t and κ_i is a parameter controlling the size of bank i's reaction to bankruptcies. A larger value of κ_i means that bank i reacts more strongly to a bankruptcy with a greater loss of confidence and so a greater reduction in the banks estimate of the likelihood of being repaid. The value of κ_i is assigned randomly at the start of the simulation and is optimized in the same way as other variables. It is important to distinguish between $\theta_i^{interbank}$ and $\kappa_i B^t$. The first is the banks long run estimate of the probability of being repaid if it lends in the inter-bank market. In the absence of bankruptcies, for instance if inter-bank lending were externally guaranteed, this value would be optimized to 1[24]. The second value modifies the banks probability of being repaid immediately after failures. This allows banks to reflect that whilst the long run repayment probability may be $\theta_i^{interbank}$ the probability of repayment of loans established in period t in which B^t banks failed may be somewhat lower.

Allowing banks to react to failures results in slightly fewer loans to borrowers and

[24]This has been confirmed by simulating the model, although with guaranteed inter-bank lending the results are of little interest.

a large reduction in loans between banks. Both quantities also have a higher standard deviation as the market becomes susceptible to panics caused by the failure of a bank (Table 8). These panics are accompanied by higher interest rates. Under larger systemic shocks the reduction in interbank lending, however, does reduce the number of bankruptcies. Less inter-bank lending means fewer banks fail due to contagion. Unfortunately this reduction is accompanied by a much larger fall in the amount of loans to borrowers. Banks react to the failure of counter-parties by stopping lending on the inter-bank market. As a consequence funds are less efficiently allocated and the economy as a whole suffers.

6.3 Credit Worthiness

In the base model it was assumed that there existed a single inter-bank interest rate. It was argued that this was a reasonable assumption if banks have limited information about each others states, the probability of systemic events is low, and the market is efficient. In a crisis, however, banks vary their inter-bank rates depending on the counter-party. More credit worthy banks, those thought less likely to fail, pay lower rates.

Each time period each bank is assigned a risk premium, ζ_i drawn from $|N(0, 1/E_i)|$, which is the markets valuation of the fair compensation to lenders for the risk of it failing. This simplifies a potentially complex effect. In reality the risk premium is dependent on a banks own state and the risk attitudes of other market participants. This mechanism, however, matches the empirical findings of Akram and Christophersen (2010) that larger banks receive more favorable inter-bank interest rates. It also agrees with the earlier observation that larger banks are less likely to fail (Section 4.3). The premium, as a percentage, is added to the inter-bank rate which bank i pays when it borrows. When a bank lends money it calculates its lending preferences using the base inter-bank rate. The recipients premium is not included as the additional value received is considered to be fair compensation for the increased risk. As such the bank does not have a preference between potential borrowers.

The addition of a risk premium reduces inter-bank lending and increases stability. There is a very small reduction in funds allocated to non-bank borrowers and a small increase in the variance of this value, although much less than under the confidence scenario. Interest rates whist in some cases significantly different are economically very similar (Table 8). The system as a whole, however, is more resilient, in response to systemic shocks the number of failures is reduced whilst the amount of lending to borrowers is increased compared to the base scenario. This agrees with Park (1991), who shows that the availability of solvency information regarding individual banks reduces the severity of panics. The introduction of the risk premium makes it relatively more expensive for smaller and potentially more vulnerable banks to borrow. As a consequence the potential for systemic risk is reduced. This occurs in a consistent and stable manner resulting in the increased stability and lending to non-bank borrowers during crisis.

7 Conclusion

Within this paper we have presented a partial equilibrium model of a closed economy in which heterogeneous banks interact with borrowers and depositors and with each other through an inter-bank market. The behavior of banks and the determination of the inter-bank interest rate are determined endogenous. It is shown that the endogenous features of bank behavior and the inter-bank market closely match those observed in reality. Whilst the structure of the inter-bank lending market is seen to have a major effect on the stability of the financial system. Previous work has shown two interacting relationships, an increasing and decreasing likelihood of failures with increasing market connectivity. The model presented here demonstrates regimes under which each is dominant. For systemic shocks the optimal inter-bank market connectivity varies with shock size. Under small shocks higher connectivity helps to resist contagion but for larger shocks it has the opposite effect. As a consequence there is no single best market architecture able to limit contagion from systemic shocks. There is, however, an optimal structure for reducing the costs of shocks. The more connected a market is, the more the costs of failures are internalized reducing the cost to an insurer. In response to a single bankruptcy more inter-bank connections generally reduce the expected number of failures. Despite this relationship it is found that intermediately connected markets potentially suffer the largest contagious effects. These markets share risk less well than those better connected yet are potentially susceptible to the failure of a single bank spreading and affecting the whole market making them particularly vulnerable to the failure of the largest banks.

The effect of regulatory actions were examined. Increases in the equity ratio were found to reduce contagion. Increases in the reserve ratio, however, had the opposite effect as banks use the interbank market more to meet their liquidity needs creating stronger inter-bank linkages. Constraints on the amount a lender may lend to a particular borrower were also considered. For larger shocks this regulation tended to reduce contagion but for smaller shocks the effect was increased. If this constraint was very tight, bankruptcies were uniformly reduced but so was lending to non-bank borrowers. It was shown that if banks react to the failure of their peers the economy is destabilized and funds are allocated less efficiently. In contrast if banks condition their lending rates on the credit worthiness of their counter-parties risk is reduced and the market is less susceptible to contagion.

The inter-bank market structures considered in this paper were imposed exogenously, banks had no choice about their counter-parties. In future this constraint could be relaxed, allowing lenders to select and decline potential borrowers and to offer different interest rates based on the counter-parties financial position. Even without making the network endogenous there are other market structures which should be investigated, e.g. hierarchical networks as seen in the UK inter-bank market. The role of the central bank was also not considered. Allen et al. (2009) have shown how a central bank may limit volatility through open market operations whilst Georg (2011) examines the ability of a central bank to stabilize a financial network. Central bank intervention, in the form of bail outs or injections of liquidity would be potentially interesting extensions.

Appendix

Each time step two tournaments are conducted as set in the algorithm below.

Algorithm 1 tournament()

```
for i = 1 to bankCount do
  if banks[i].isBust then
    bustList.add(banks[i])
  end if
end for
if size(bustList) > 0 then
  bankOne = bustList(randomInteger(bustList.size))
else
  bankOne = banks(randomInteger(bankCount))
end if
repeat
  bankTwo ← selectIndividual()
until bankOne ≠ bankTwo
if bankOne.equity ≤ bankTwo.equity then
  bankOne.reserveRatio ← mutate(banksTwo.reserveRatio)
  bankOne.equityToAssetRatio ← mutate(banksTwo.equityToAssetsRatio)
  bankOne.lendingRate ← mutate(banksTwo.lendingRate)
  bankOne.depositRate ← mutate(banksTwo.depositRate)
  bankOne.chanceOfRepayment ← mutate(banksTwo.chanceOfRepayment)
  if banksOne.isBust then
    banksOne.reserves ← 1.0
  end if
end if
```

Algorithm 2 selectIndividual()

```
equityTotal ← 0
for i = 1 to bankCount do
  equityTotal ← equityTotal + 1 + banks[i].equity
end for
val ← randomFloat(0,1) × equityTotal;
for i = 1 to bankCount do
  val ← val - (1+banks[i].equity)
  if val < 0 then
    Return i
  end if
end for
```

Algorithm 3 mutate(parameter)

```
repeat
  newValue ← parameter + randomFloat(-0.001,0001)
until newValue ≤ 1.0 and newValue ≥ 0
Return newValue
```

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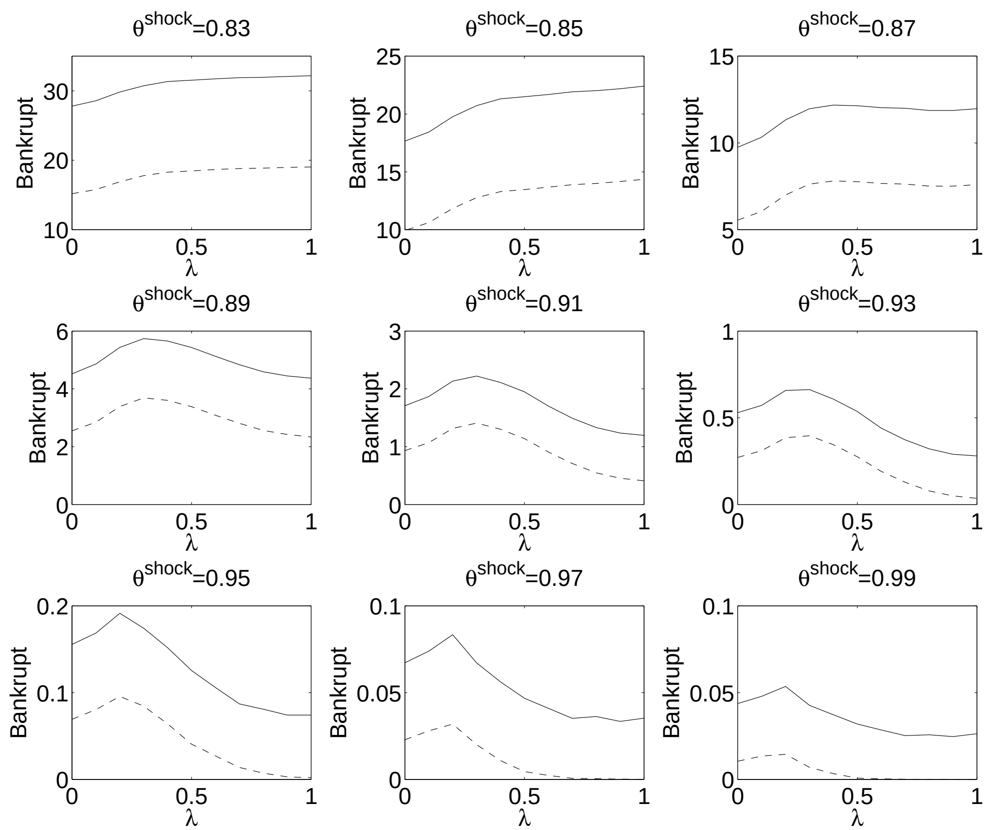

Figure 1: Total number of bankruptcies occurring on shock period (solid line) and the number of bankruptcies which were caused by contagion (dashed line), for different values of shock size (θ^{shock}) and diversification (λ). Note the scale on the Y axis changes to illustrate the effect of λ. All shocks conducted at period 100000 and averaged over 500 repetitions.

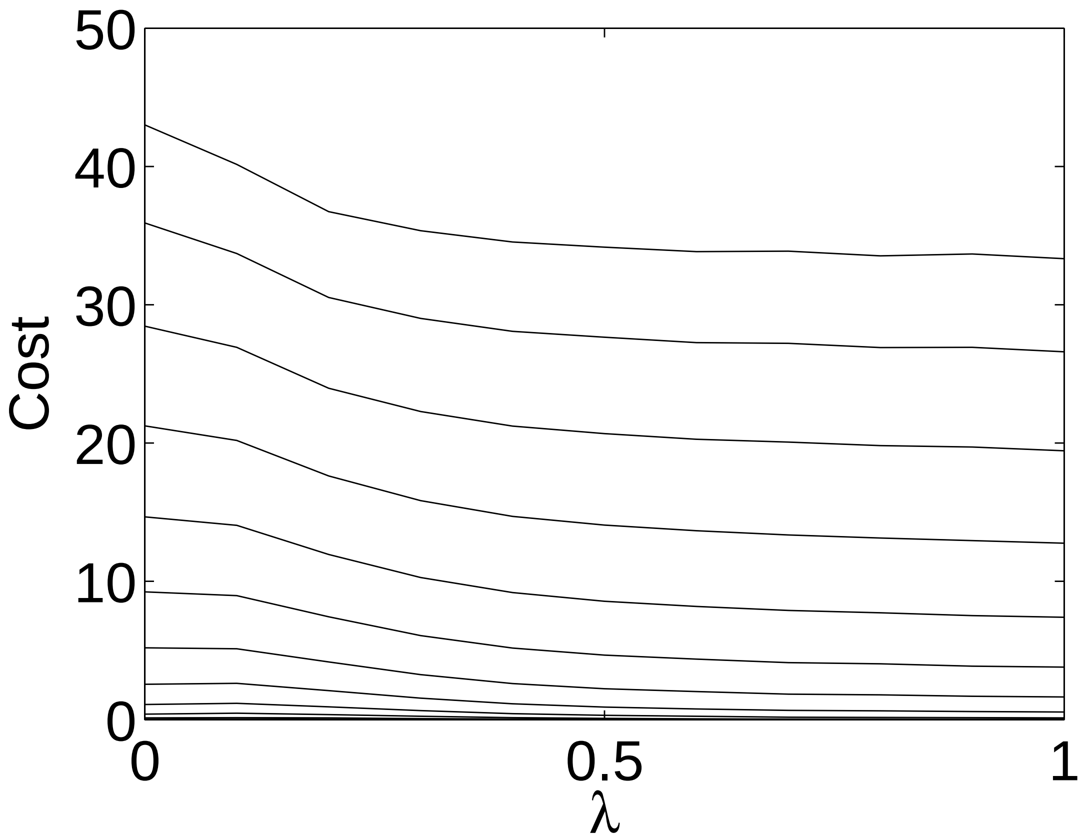

Figure 2: Total cost of repaying depositors of failed banks for different values of θ^{shock} and λ. The top line corresponds to the largest shock ($\theta^{shock} = 0.77$) whilst the bottom line corresponds to the smallest ($\theta^{shock} = 0.97$) the lines between are for shocks of decreasing size. All shocks conducted on period 100000 and averaged over 500 repetitions.

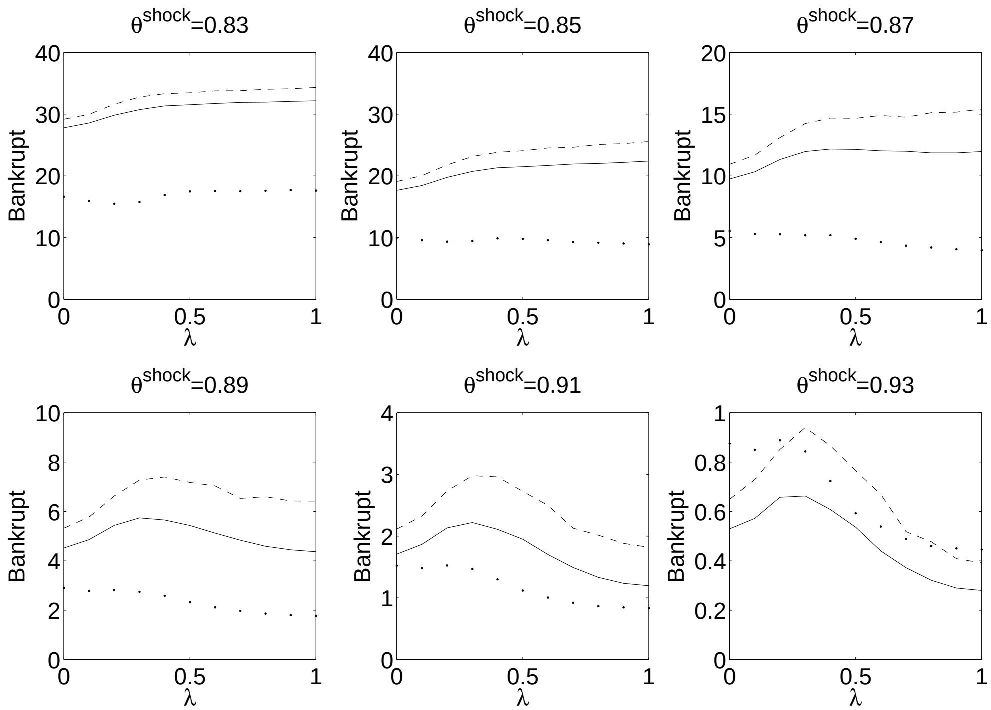

Figure 3: Total number of bankruptcies occurring on shock period for the base model (solid line), increased reserve ratio (dashed line) and increased equity ratio (dotted line), for different values of θ^{shock} and λ. Note the changing scale on the Y axis to illustrate changes with λ. All shocks conducted at period 100000 and averaged over 500 repetitions in each case.

Agent	Choice	State
Depositors (j)	i	d_j, j
Borrowers (q)	i	$\theta_{l_q^t}$, q
Banks (i)	α_i, β_i, r_i^{loan}, $r_i^{deposit}$, $r_i^{interbank}$, K_i^t, I_i^t	E_i, α_g, β_g, K_i^{t-1}, I_i^{t-1}, i

Table 1: State and choice variables for each agent type. Note for banks, the composition of K_i^t, together with α_i, β_i and the interest rates are sufficient to determine the remaining aggregate balance sheet quantities. The table shows only those state variables which are determined by the environment or previous decisions of the agent. By convention it does not include as state variables those which may be regarded as choices of other agents in the same time-step e.g. The choice of lending rate by banks is not listed as a state variable of borrowers. The inclusion of i, j and q as state variables are sufficient to define agents location on the unit circle.

Parameter	Meaning	Value
α_g	Reserve Requirement	0.10
β_g	Capital Requirement	0.08
N	Banks	100
M	Depositors	4500
Q	Borrowers	4500
θ_k^t	Project success probability	$U(0.99, 1.0)$
μ	Project payoff	1.1
l^S	Project size	0.1
d_j	Deposit size	$\frac{750}{M}$

Table 2: Parameters used for all simulations (unless otherwise stated).

Model Type	Value	SD	Empirical Type	Normalized	Real
Loans	759.1	(6.1)	Loans	995.7	8281.9
Inter-bank Loans	289.7	(21.5)	Inter-bank Borrowing	235.5	1958.8
Reserves	75.0	(0.4)	Cash Assets	36.2	301.0
Unused capital	15.4	(6.0)			
Deposits	749.5	(2.0)	Deposits	757.8	6303.2
Equity	100.0	(0.1)	Residual	100.0	831.8

Table 3: Assets and liabilities of model data along with data for commercial banks in the USA (billions of Dollars), December 2006, source: H.8 statement, Board of Governors of the Federal Reserve System. The left hand side of the table presents the model data whilst the right hand side presents empirical data normalized such that the Residual is equal to the model Equity. Unused capital is capital placed in reserves above that which the banks reserve ratio specifies due to the bank being unable to find a profitable way to allocate the funds. The level of inter-bank lending in the model is the sum of all positive positions. By definition the sum of all positions, positive and negative is 0. Items in the H.8 statement with no equivalent in the model are omitted

Term	Value	SD	Term	Value	SD
Loan Rate	0.029	(0.002)	Inter-bank Rate	0.023	(0.002)
Deposit Rate	0.009	(0.000)	Dividend Rate	0.12	(0.007)
Lenders	52.3	(7.7)	Average Lender Equity	0.96	(0.02)
Borrowers	91.0	(3.2)	Average Borrower Equity	1.00	(0.01)
Both	47.8	(7.1)	Average Both Equity	0.96	(0.02)
Bankrupt	0.03	(0.20)	α_i	0.05	(0.03)
$\theta_i^{interbank}$	0.96	(0.01)	β_i	0.04	(0.02)

Table 4: Aggregate model statistics at period 100000 averaged over 500 runs. Standard deviations in parenthesis. Values calculated prior to inflation effect. 'Both' in the table refers to those banks in the system who were lenders in one period and borrowers in the next (or vise versa).

λ	Connections		Large to Large		Large to Small		Small to Small	
0.0	387.7	(51.1)	89.3	(31.3)	213.1	(58.5)	85.3	(23.9)
0.1	523.5	(69.6)	114.8	(34.7)	288.1	(68.2)	120.7	(28.1)
0.2	758.8	(109.5)	164.0	(44.7)	421.1	(91.5)	173.7	(37.0)
0.3	1039.1	(156.3)	220.9	(57.7)	576.1	(113.5)	242.1	(47.1)
0.4	1342.2	(203.9)	280.1	(72.4)	746.5	(153.8)	315.6	(62.8)
0.5	1619.1	(250.9)	338.1	(85.0)	900.0	(178.9)	381.0	(74.7)
0.6	1924.5	(304.0)	396.9	(104.4)	1069.4	(210.1)	458.2	(92.2)
0.7	2233.1	(351.5)	458.6	(110.5)	1239.7	(251.5)	534.9	(101.3)
0.8	2543.4	(400.9)	525.3	(139.6)	1415.9	(290.8)	602.2	(117.0)
0.9	2825.6	(451.1)	573.9	(145.2)	1570.9	(322.3)	680.9	(133.4)
1.0	3114.8	(499.4)	641.5	(165.8)	1733.2	(358.2)	740.0	(147.0)

Table 5: Statistics describing the structure of the inter-bank market network for variation in λ. Statistics collected at day 100000 and averaged over 500 runs. Standard deviations in parenthesis. The last three columns give the number of lending relationships between large banks (above median size) and small banks (below median size).

λ	Failures		Probability		Size		Largest	
0.0	14.64	(4.32)	0.71	(0.05)	20.63	(5.68)	28.55	(4.36)
0.1	18.99	(4.34)	0.67	(0.05)	28.23	(5.74)	34.85	(4.19)
0.2	20.06	(3.59)	0.58	(0.05)	34.73	(4.84)	39.14	(4.17)
0.3	16.40	(2.83)	0.44	(0.05)	36.88	(4.26)	40.82	(3.97)
0.4	11.45	(2.42)	0.31	(0.05)	36.28	(4.50)	41.13	(3.85)
0.5	7.25	(2.09)	0.21	(0.05)	33.92	(5.52)	40.79	(4.03)
0.6	4.24	(1.69)	0.14	(0.04)	30.58	(7.03)	40.11	(4.56)
0.7	2.33	(1.25)	0.08	(0.03)	26.91	(9.19)	38.61	(7.57)
0.8	1.24	(0.93)	0.05	(0.03)	22.57	(12.54)	33.58	(14.18)
0.9	0.59	(0.67)	0.03	(0.02)	16.48	(15.17)	23.79	(18.98)
1.0	0.26	(0.44)	0.01	(0.02)	10.48	(15.16)	13.70	(18.28)

Table 6: Statistics showing the effects of single bankruptcies on the economy for variation in λ. 'Failures' is the average number of banks which fail as a consequence of a single bank being made bankrupt (excluding the initial bank). 'Probability' is the chance that contagion will occur. 'Size' is the average number of banks which go bankrupt conditional on contagion occurring. 'Largest' is the average size of the largest contagious event. Data collected using market states saved at period 100000 and averaged over 500 runs.

Shock Size	η ∞		20		10		5		2	
0.83	31.5	(6.0)	31.5	(6.1)	28.2	(7.5)**	20.5	(4.9)**	10.7	(9.4)**
0.87	12.1	(6.4)	12.1	(6.4)	11.1	(6.0)*	10.1	(5.0)**	4.4	(6.4)**
0.91	1.9	(2.8)	1.9	(2.8)	2.1	(3.0)	3.1	(3.9)**	1.2	(3.1)**
0.95	0.1	(0.6)	0.1	(0.5)	0.2	(0.9)	0.3	(1.4)**	0.2	(0.9)
Loans	759.1	(6.1)	759.1	(6.0)	755.5	(9.6)**	745.7	(8.1)**	740.6	(19.4)**
I-B Loans	289.7	(21.5)	290.3	(21.9)	276.9	(34.9)**	259.1	(40.7)**	52.6	(105.7)**

Table 7: Statistics showing the effects of systemic shocks on the economy for different borrowing constraints for $\lambda = 0.5$. All shocks conducted at period 100000 and averaged over 500 repetitions in each case. $\eta = \infty$ corresponds to the base case where there is no constraint. The market statistics at the bottom are pre-crash values. 1% significance indicated by **, 5% significance by *.

	Inter-bank Confidence			Credit Worthiness		
	Lending	Interbank	Deposit	Lending	Interbank	Deposit
Interest rates	0.039 (0.004)**	0.018 (0.004)**	0.032 (0.014)**	0.028 (0.001)**	0.008 (0.001)**	0.017 (0.001)**
	Bankrupt	Loans	I-B Loans	Bankrupt	Loans	I-B Loans
Steady state	0.27 (1.15)**	754.2 (31.52)**	153.6 (95.75)**	0.06 (0.30)	757.0 (8.60)**	126.4 (20.5)**
0.83	21.34 (10.44)**	524.2 (57.7)**	28.6 (24.3)**	18.10 (5.84)**	612.72 (31.61)**	69.01 (21.63)**
0.87	7.64 (6.29)**	646.1 (42.0)**	64.7 (52.2)**	5.44 (3.68)**	689.98 (18.74)**	104.22 (24.64)**
0.91	1.75 (2.65)	702.7 (42.0)**	114.7 (84.5)**	1.00 (1.46)**	729.05 (12.08)**	119.47 (21.18)**
0.95	0.44 (1.36)**	740.9 (34.5)	145.7 (94.0)**	0.17 (0.56)	748.41 (9.56)**	123.87 (20.38)**

Table 8: Statistics showing the steady state and the effect of systemic shocks for two different model cases. Values averaged across λ, results, however, are qualitatively similar for all values of λ. Data collected at period 100000 for 500 repetitions in each case. 1% significance indicated by **, 5% significance by *.