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Assortative Matching in Partnerships and Over-Education*

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Abstract
This paper argues that assortative matching can explain over-education. Education determines individuals’ income and, due to the presence of assortative matching, the quality of the partner, who can be a colleague or a spouse. Thus an individual acquires some education to improve the expected partner’s quality. But since everybody does that, the expected partner’s quality does not increases and over-education emerges. Public intervention can solve over-education through a progressive income tax.

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[Very Preliminary]

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1 Introduction

In recent decades, the level of educational attainment in developed countries has surpassed the skill requirements of available jobs\(^1\). This is known as “over-education”. There is a large empirical literature measuring over-education\(^2\), while this paper aims to contribute to a theoretical understanding of it.

We propose an explanation for the existence of over-education based on the idea that acquiring education has two main effects. First, it improves job conditions: income, job quality, and so on. Second, it influences the quality of the future colleagues and spouses.

School and university are among the places where people create their own social networks, make friends and spend a considerable part of their youth. At school, individuals can meet their future colleagues. For instance, school or university mates can apply to the same company, decide to work in partnership or find themselves working in the same firm. Also, many people meet their spouse at school\(^3\). Colleagues and spouses who met at school share similar education levels\(^4\). We refer to this positive correlation as “assortative matching”\(^5\). Assortative matching reflects similarities in innate ability, since this is similar in individuals who share the same school experience. Our idea is that the presence of assortative matching may cause over-education.

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\(^1\) Vaisey (2006) shows evidence that a substantial and growing number of American workers are over-qualified for their jobs along the period 1972-2002. The principal time-trend is positive and linear, and appears to be the result of the widening gap between a large expansion in educational attainment and only modest increases in job educational requirements over the past three decades. Budria and Moro-Egido (2007) find same evidence in European countries and a negative differential in salary between over-qualified individuals and their well-matched counterparts.

\(^2\) For discussions, see Hartog, 2000 and McGuinness, 2006.

\(^3\) Stevens (1991) analysed the reasons of why spouses tend to have similar educational levels. In the sample considered, more than 50% of spouses attended the same school, college or university.


\(^5\) The expression “assortative matching” has been coined by Gary Becker (1973), and it alludes to a relationship (either positive or negative) between characteristics of spouses. We refer to the similarities in the levels of education specifically, and we apply the relationship not only to spouses, but also to colleagues.
We build up a model where individuals differ in ability. They study and are matched in the working period with a partner, who can be a colleague or a spouse. The partner’s ability positively affects the individual’s utility. This may be due to a variety of reasons. An individual can benefit from a colleague by informal apprenticeship, appraising or good influence, and from a spouse by sharing interests and income. Individuals maximise their expected utility by choosing their education levels and taking into account their matching.

This can be random or assortative. Random matching takes place when partners meet each other by chance. Assortative matching occurs if an individual meets the partner at school or university, or in any situation where the educational level influences the chance of a meeting. Whether matching is assortative depends on the institutions and tradition of a society: for example, the more the educational system requires that students spend time together, the more likely the matching will be assortative.

Our results suggest that assortative matching makes the education acquired inefficient from a social point of view. In particular, individuals would reach a lower level of education in a socially optimal solution. Thus we define over-education as the difference between the actual level of education and the socially optimal level of education.

What determines these results? Assortative matching gives an incentive to study more in order to increase the partner’s quality. However, every individual with the same level of ability acquires the same quantity of education and hence is matched with a partner of the same type. This approach is in the flavour of Akerlof (1976), where workers signal their ability through their work speed. In order to look more able, workers of a given ability work faster than they would if they were not observed. In our model, individuals observe the partner’s education level as a signal of ability, and in order to look more able they acquire more education than they would if assortative matching were not present.

The paper considers next whether public intervention can make individuals reach the socially efficient level of education by introducing a progressive income tax. This intervention can correct over-education by imposing a higher fiscal burden the higher the individual’s income. These results may justify income progressive taxation on efficiency grounds and not to answer to redistributive
arguments.

To our knowledge, over-education has not been largely developed from a theoretical perspective, with few notable exceptions. Frank (1978) investigates the differentials in wages between men and women as a consequence of female over-qualification. This is caused by family location decisions, since a family is more likely to move close to better jobs for the husband, sacrificing the wife’s opportunities. Hence the role differences between men and women are essential for his results, and over-education is generated by a job search process. Compared to this work, we do not consider differences in wages among sexes, job search nor the different role in society between men and women.

Our results are consistent with Lommerud (1989), where over-education occurs as individuals care about social status, determined by the relative income. Like in our paper, he corrects over-education through a progressive income taxation. This can weaken the incentive to undertake education, hence subsidies might be necessary to restore this incentive.

Konrad and Lommerud (2000) explain over-education through a household bargaining model where young individuals individually choose their level of education and, once married, they sacrifice their returns to education in favour of an optimal level of family public goods (i.e., to spend time with children, partner, and so on). Over-education emerges because the educational decisions affect the threat point (i.e., the reservation utility given by being single) of spouses. To over-invest in education is inefficient in order to optimise the quantity of the family public good, but leads to an increase in the threat point so as to be in an advantaged position in the household bargaining.

This paper shares with studies by Peters and Siow (2001), Baker and Jacobsen (2005), Iyigun and Walsh (2005), Chiappori et al. (2006) and Nosaka (2007) the link between education and assortative matching. However, in these contributes this link does not explain over-education, and they consider assortative matching only between spouses.

The remainder of the paper is organized as follows: Section 2 describes the model. Section 3 shows the results. Section 4 illustrates government intervention. Section 5 concludes.
2 The model

There is a continuum of individuals\(^6\) normalised to 1. Individuals differ in ability, denoted by \(\theta \in [\underline{\theta}, \overline{\theta}]\) and distributed according to density \(f(\theta)\) with cumulative distribution function \(F(\theta)\). We refer to ability as every innate characteristic that contributes to income potential. Individuals choose their level of education. We denote as \(e \geq 0\) the quantity of education acquired by an individual. Education is costly for individuals. We denote the utility cost of education as \(e^2\), where \(c > 0\).

After deciding their education, individuals work and are matched with a partner. We denote as \(e\theta\) the income of an individual with education \(e\) and ability \(\theta\). The partner can be seen as a colleague or a spouse. An individual benefits from the partner’s quality\(^7\). This is represented by \(\alpha \theta_p\), where \(\alpha \in [0, 1]\) is the relative importance of the partner’s quality in determining the individual’s utility, while \(\theta_p \in [\underline{\theta}, \overline{\theta}]\) denote the partner’s ability. Thus an individual’s utility is determined by\(^8\):

\[
U(e, \theta, \theta_p) = e\theta + \alpha \theta_p - \frac{c}{2} e^2. \tag{1}
\]

We analyse the matching technology and then the educational problem.

2.1 Matching

Matching can be of two types: random or assortative. A random matching occurs when partners meet each other by chance. This happens anytime a meeting takes place in situations that are completely unrelated to the acquired

\(^6\)We do not consider differences in sex. This implies that men and women behave symmetrically, and excludes the case (more credible in reality) that educational decisions change according to sex (due to a different role in society and household, childbearing and so forth). However, the message of the paper does not change by considering differences in sex and these would only complicate the analysis.

\(^7\)In teamwork, individuals find the performance of their duties easier if those they cooperate with are able, competent and dedicated. In individual jobs, a good environment improves job performance through suggestions or discussions. In love life, individuals share the advantages of a more able spouse: a better income, work flexibility (which reflects more availability in the love life), a more interesting conversation and more open mindedness.

\(^8\)We assume a linear additive utility in order to keep the analysis tractable. Different formulations would complicate the algebra without adding any insight.
education. For example, a match between a lawyer and a botanist sharing the passion for football and playing in the same team is totally casual. Two individuals meeting at the supermarket can have completely different educational backgrounds.

Assortative matching occurs when an individual meets the partner at school, university or in any situation where the educational level influences the chance of a meeting. For example when individuals attend the same social environment given by previous school friendships, or when a certain activity is related to the studies attended, like individuals with a degree in arts meeting in a museum or in an exhibition, and so on. In all these cases, the partners’ education is positively related. For the sake of simplicity, we assume that with assortative matching, a perfect positive correlation exists in partners’ levels of education. In other words, the partner of an individual who acquires education $e$ has the same level of education $e$. Considering an imperfect correlation would not alter our results.

Let $\beta \in [0,1]$ denote the exogenous probability that the matching is assortative. This is independent of the individual’s ability $\theta$. The value of $\beta$ depends on the customs and the educational system of the society we are considering. For instance, the more an educational system requires that students spend years at school for obtaining a certain qualification, the more the probability of assortative matching$^9$. Another example is the role of school tracking, that is the separation of pupils by academic ability into groups for all subjects within a school (Gamoran, 1992). An educational system that postpones school tracking keeps a more heterogeneous group of pupils together for a long time, by decreasing the probability of assortative matching$^{10}$.

$^9$Blossfeld and Timm (2003) analyse the relationship between educational system and marital assortative matching in many western countries. Their results show that the more time individuals spend at school, the greater the chance of marrying a partner with similar education (i.e., the higher $\beta$).

$^{10}$Holmlund (2007) studies the relationship the effects of a school reform on marital assortative matching. She examines an educational reform, implemented in Sweden in the 1950s and 60s, which postponed tracking and extended compulsory education from seven to nine years. Her results show that this might have resulted in a reduction in assortative matching.
2.2 Educational choice

When individuals decide the quantity of education to acquire, the future matching affects their decisions. According to equation (1), they prefer to be matched with a high-quality partner, as this increases their benefit. With random matching, since there is no correlation in partners’ education, individuals have no information about the partner’s characteristics during the educational decisions. Thus the partner’s expected quality is determined by the average individual type, $\bar{\theta}_p = \int \theta_p f(\theta_p) d\theta_p$, and hence random matching does not influence the educational choice.

With assortative matching instead, individuals can observe the education of some of their potential partners (for example, their school friends) during their educational period. Thus they may want to acquire more education in order to improve the probability of being matched with a better partner. Consequently, it is possible to influence the expected partner’s type through the educational decisions.

In particular, individuals can correctly infer the partner’s ability through their education. This is shown by supposing $E(\theta_p)$ being the education of a partner with ability $\theta_p$, and also that$^{11} E'(\theta_p) > 0$. The fact that in equilibrium, education is a strictly increasing function of ability allows the individual to recognise the partner’s ability through her education. From a technical perspective, this happens because an increasing function can be inverted$^{12}$. Given the assumption that in assortative matching partners have the same level of education, then an individual with ability $\theta$ acquiring an amount of education $e$ will be matched with a partner whose education is $e = E(\theta_p)$. Hence the individual can infer the partner’s ability $\theta_p$ as the inverse image of $E(\theta_p)$, so

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$^{11}$In practice, we are arguing that the belief in equilibrium is that education is an increasing and monotonic function of ability. In other words, individuals believe that the abler ones study more. The equilibrium that emerges is “separating” (i.e., the level of education will be different for each level of ability). This does not exclude the existence of other equilibria that are determined by different beliefs. For instance, if the belief is that the level of education is constant irrespective of the individuals’ ability, then a pooling equilibrium must emerge. However, the belief we focus on looks more consistent to what happens in the reality.

$^{12}$Clearly we need to verify that in equilibrium this condition holds.
\( \theta_p = E^{-1}(e) \). If this holds, we can rewrite equation (1) as:

\[
e \theta + \alpha \left( (1 - \beta) \frac{\beta}{2} \int \theta_p f(\theta_p) d\theta_p + \beta E^{-1}(e) \right) - \frac{c}{2} e^2.
\]

(2)

In equilibrium we consider, all type \( e \) individuals make identical choices, and so (2) is the expected utility in each individual type \( e \). The first part of (2) is the total benefit given by the individual’s income, the second part is the total benefit given by the partner’s quality, and the third part is the total cost of education. The second part of (2) can be in turn decomposed into two parts: (i) \( \alpha(1 - \beta) \int \theta_p f(\theta_p) d\theta_p \), and (ii) \( \alpha \beta E^{-1}(e) \), which represent the expected benefit given by the partner with random and assortative matching, respectively.

Equation (2) shows that, in the presence of assortative matching, the educational choice \( e \) influences not only the future income (\( e \theta \)) but also the partner’s expected quality (\( \beta E^{-1}(e) \)). In particular, an individual tries to manipulate the education signal by acquiring more education than others of similar ability, in order to obtain, in the future, a partner with higher ability than her. But in equilibrium, every individual takes into account assortative matching and tries to do precisely this, hence with probability \( \beta \) everyone is matched with a partner of same ability.

The first order condition for the maximisation of equation (2) is:

\[
\theta + \alpha \beta \frac{d}{de} E^{-1}(e) - ce = 0.
\]

(3)

The following lemma shows the solution of equation (3).

**Lemma 1** The level of education chosen by type \( \theta \) in equilibrium is \( e_{av} = \frac{\frac{\theta}{\theta + (\theta^2 + 4c\alpha \beta)^{\frac{1}{2}}}}{2c} \).

**Proof.** Equation (3) is a differential equation which has solution:

\[
\frac{d}{de} E^{-1}(e) = \frac{ce - \theta}{\alpha \beta}.
\]

Since an individual with ability \( \theta \) acquires a level of education \( e \) and with assortative matching a partner with ability \( \theta_p = E^{-1}(e) \) acquires an amount of
education \( e \) too, then necessarily \( \theta_p = \theta \). Hence we can substitute \(^{13}E^{-1}(e) = \theta \).

By integrating:

\[
\theta = \frac{(ce - \theta) e}{\alpha \beta}.
\]

We obtain a second grade equation with solutions \( e = \frac{\theta + (\theta^2 + 4c\theta \alpha \beta)}{2c}, \frac{\theta - (\theta^2 + 4c\theta \alpha \beta)}{2c} \).

We can easily see that \( \frac{\theta - (\theta^2 + 4c\theta \alpha \beta)}{2c} < 0 \), and hence this is not a feasible solution as \( e > 0 \).

In order \( e_{ov} \) to be invertible, it needs to be a strictly increasing function. Differentiating \( e_{ov} \) with respect to \( \theta \) yields \( \frac{\partial}{\partial \theta} \frac{\theta + (\theta^2 + 4c\theta \alpha \beta)}{2c} = \frac{(2\theta + 4c\theta \alpha \beta + 1)}{2c} > 0 \).

### 3 Results

In the equilibrium presented in the previous section, a part of the education acquired by individuals is to improve the quality of the potential partner. But since everyone does this, the expected quality of partners does not improve. Thus although individuals choose their optimal amount of education, the overall education is not socially efficient. Indeed the bit acquired for increasing the chance of a better potential partner is not helpful in it, and hence is wasted.

In this section we investigate the equilibrium where individuals exploit the socially optimal educational resources. We assume that education is determined by a planner aiming to maximise social welfare. This is given by the unweighted sum of the individual utilities when \( \beta = 0 \):

\[
W = \int_{\theta}^{\bar{\theta}} \left( \theta + \alpha \int_{\theta}^{\bar{\theta}} \theta_p f(\theta_p) d\theta_p - \frac{c}{2} e^2 \right) d\theta.
\]

In other words, the social welfare function considered does not take into account assortative matching, in order to rule out the cause of inefficiency from the problem. For every \( \theta \), the social planner problem is the maximisation of equation (2) when \( \beta = 0 \).

\(^{13}\)Note that we can substitute \( E^{-1}(e) = \theta \) only once that \( e \) has been maximised. If we do it before the maximisation is like to keep as fixed the partner’s education. But this is a simultaneous game where every individual is also a partner, so the result would not be a Nash equilibrium.
The solution of Lemma 1 becomes $e^* = \frac{\theta}{c}$. In order to have over-education, it is necessary that $e_{ov} > e^*$, $\frac{\theta + (\theta^2 + 4c\theta\alpha\beta)^{\frac{1}{2}}}{2c} > \frac{\theta}{c}$, which is always verified since $4c\theta\alpha\beta > 0$. This is intuitive. In the presence of assortative matching, individuals observe the potential partners’ education and try to look more able. This extra amount of education is not considered by the social planner. Individuals obtain the same result in terms of optimal choice (i.e., same income and partner), but employing less educational resources than in the presence of assortative matching and thus optimising social welfare (Figure 1). Hence we refer to $e^*$ as the first best equilibrium. Over-education is defined as the difference between $e_{ov}$ and $e^*$.

**Definition 1** $\Delta e = \frac{\theta + (\theta^2 + 4c\theta\alpha\beta)^{\frac{1}{2}}}{2c} - \frac{\theta}{c} = \frac{(\theta^2 + 4c\theta\alpha\beta)^{\frac{3}{2}} - \theta}{2c}$ is the level of over-education.

The following proposition summarises the comparative statics properties of $\Delta e$. 

$\theta$ $ce$ $\theta + a\frac{d}{dt}E^*(e)$

$e^*$ First best equilibrium

$e$ Level of over-education

FIGURE 1

$ce \theta = \theta$, $e_{ov} \theta^2 \alpha \beta + 4c \theta\alpha\beta \theta^{\frac{1}{2}} $ $\frac{1}{2c}$

$\Delta e = \frac{(\theta^2 + 4c\theta\alpha\beta)^{\frac{3}{2}} - \theta}{2c}$
Proposition 1  An increase either in assortative matching or in the relative importance of the partner’s quality leads to an increase in over-education. Also, the higher the ability, the more an individual is over-educated. Finally, as the cost of education increases, the level of over-education diminishes.

Proof. Differentiation of $\Delta e$ with respect to $\beta, \alpha, \theta$ and $c$ yields $\theta \alpha (\theta^2 + 4c \alpha \beta \theta)^{-\frac{1}{2}} > 0$, $\beta (\theta^2 + 4c \alpha \beta \theta)^{-\frac{1}{2}} > 0$, $\frac{\beta - (\theta^2 + 4c \alpha \beta \theta)^{-\frac{1}{2}}}{2c^2} + \frac{\alpha \beta (\theta^2 + 4c \alpha \beta \theta)^{-\frac{1}{2}}}{c} > 0$, respectively.

By looking at $\Delta e$, we can observe that an increase either in $\beta$ or in the relative importance of the partner’s quality $\alpha$ leads to an increase in over-education. Clearly, individuals acquire more education the more likely they meet their partner among their school friends ($\beta$ high). Also, they invest more in education if $\alpha$ is high, since having a high-quality partner is more valuable. This leads to more over-education. Moreover, over-education proportionally increases the higher the individuals ability. Finally, as the cost of education increases, the level of over-education increases.

4 Government intervention

In this section, we assume that there is a government whose objective is to reach the first best education level. To accomplish this, the government considers to levy a tax. We focus on a first best solution through a progressive taxation on income. To do that we need the strong assumption that the government is able to perfectly discriminate taxation according to individual type. This indeed implies that the government can observe individuals’ ability, which is clearly not possible in the reality.

With progressive taxation, the tax rate increases the higher the income. We denote it as $\tau = \gamma^2 (1 - \frac{e \theta}{e \theta L}) \in [0, 1]$, where $\gamma$ represents the tax progression and $e \theta L$ is the lowest income in the population considered (the income of the least able individuals). For every $\theta$, equation (2) becomes:

$$e \theta \left(1 - \gamma^2 \left(1 - \frac{e \theta L}{e \theta}\right)\right) + \alpha \left(1 - \beta \frac{\bar{f} \theta_p f(\theta_p) d\theta_p}{e} + \beta \left(\alpha \frac{E^{-1}(e)}{2e^2}\right)\right) - \frac{c}{2} e^2. \quad (4)$$
The first order condition for the maximisation of (4) is:

$$\theta + \beta \alpha \frac{d}{dc} E^{-1}(e) = ce + \gamma^2 \theta,$$

and the level of education is determined by the following lemma.

**Lemma 2** With a progressive tax on income, the education in equilibrium is

$$e^* = \frac{\theta(1-\gamma^2) + (\theta^2(1-\gamma^2)^2 + 4e\theta_0 \beta)^{\frac{1}{2}} +}{2c \theta(1-\gamma^2) - (\theta^2(1-\gamma^2)^2 + 4e\theta_0 \beta)^{\frac{1}{2}}} +.$$  

**Proof.** Equation (5) is a differential equation which has solution:

$$\frac{d}{de} E^{-1}(e) = \frac{ce + \gamma^2 \theta - \theta}{\alpha \beta}.$$  

By integrating and substituting $E^{-1}(e) = \theta$:

$$\theta = \frac{(ce + \gamma^2 \theta - \theta) e}{\alpha \beta}.$$  

We obtain a second grade equation with solutions $e = \frac{\theta(1-\gamma^2) + (\theta^2(1-\gamma^2)^2 + 4e\theta_0 \beta)^{\frac{1}{2}} +}{2c \theta(1-\gamma^2) - (\theta^2(1-\gamma^2)^2 + 4e\theta_0 \beta)^{\frac{1}{2}}} +$.

In order to reach the first best level of education, $e^*$ needs to be equal to $e^*$, thus: $\frac{\theta(1-\gamma^2) + (\theta^2(1-\gamma^2)^2 + 4e\theta_0 \beta)^{\frac{1}{2}} +}{2c \theta(1-\gamma^2) - (\theta^2(1-\gamma^2)^2 + 4e\theta_0 \beta)^{\frac{1}{2}}} = \frac{\theta \gamma}{e^*}$. By explicating $\gamma$ we find two solutions, $\gamma = \frac{(e\theta_0 \beta)^{\frac{1}{2}}}{\theta}$ and $\gamma = -\frac{(e\theta_0 \beta)^{\frac{1}{2}}}{\theta}$. Since $-\frac{(e\theta_0 \beta)^{\frac{1}{2}}}{\theta} < 0$, we have a unique feasible solution.

**Proposition 2** The optimal progressive income tax is $\tau^* = (\gamma^*)^2 \left(1 - \frac{e\theta_0}{e\theta}\right)$, where $\gamma^* = \frac{(e\theta_0 \beta)^{\frac{1}{2}}}{\theta}$.  

Figure 2 shows the equilibrium where the progressive income tax is levied. These results may justify the introduction of income progressive taxation on efficiency grounds, with no appeal to equity or redistributive reasons.

The following corollary illustrates the relationship between the education in equilibrium and the tax progression $\gamma$.


Corollary 1. The more progressive the taxation on income, the less the incentive to acquire education.

Proof. Differentiation of $e^\gamma$ with respect to $\gamma$ yields $\frac{\partial e^\gamma}{\partial \gamma} = -\frac{\partial e}{e} \left( 1 + (1 - \gamma^2) \left( \theta (\gamma^2 - 1)^2 + 4\theta \alpha \beta \right)^{-\frac{3}{2}} \right) < 0.$

As the tax progression increases, the incentives of acquiring education diminish. This result is in line with Lommerud (1989), where progressive income taxation corrects over-education but blunts the incentive to undertake education.

5 Concluding remarks

In the presence of assortative matching, individuals increase their education to improve the quality of colleagues or spouses. But as everyone is more educated, the extra education acquired does not improve the chance of a good match. Hence over-education emerges, since individuals can obtain the same result in terms of optimal choice but exploiting less educational resources. Public
An interesting extension of the paper may be to consider assortative matching in terms of social class. Although educational and social class assortative matching are positively correlated, individuals with different social background may acquire the same level of education. Introducing assortative matching by social class may have different effects according to the social group we regard. On the one hand, the opportunity cost to acquire more education is generally higher for advantaged individuals since, for instance, they may have better job opportunities through the parental network. On the other hand, this can strengthen the effect on over-education for disadvantaged people, as assortative matching by class is a further barrier in the attempt to improve the matching through education. The introduction of assortative matching by social class is left for future work.

References


