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REVISITING MONEY-OUTPUT CAUSALITY FROM A BAYESIAN LOGISTIC SMOOTH TRANSITION VECM PERSPECTIVE

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Working Paper No. 08/5
January 2008

Revisiting money-output causality from a Bayesian logistic smooth transition VECM perspective

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January 29, 2008

Abstract

This paper proposes a Bayesian approach to explore money-output causality within a logistic smooth transition VECM framework. Our empirical results provide substantial evidence that the postwar US money-output relationship is nonlinear, with regime changes mainly governed by the lagged inflation rates. More importantly, we obtain strong support for long-run non-causality and nonlinear Granger-causality from money to output. Furthermore, our impulse response analysis reveals that a shock to money appears to have negative accumulative impact on real output over the next fifty years, which calls for more caution when using money as a policy instrument.

*This paper has benefited from the helpful discussions with Stephen Hall, Gary Koop, Roberto Leon-Gonzalez, Roderick McCrorie, Emi Mise and Rodney Strachan. Any remaining errors are the author's responsibility. Financial support from the Department of Economics, University of Leicester is gratefully acknowledged.

1 Introduction

From the late 1980s through the early 2000s, with the prevalence of interest rate based Taylor rule (Taylor, 1993), the role of money (monetary base or monetary aggregates) had been deemphasized in much research on monetary policy and macroeconomic modeling (see, e.g., Barro (1989), Taylor (1999), Clarida, Galí and Gertler (2000)). However, there has been a renewed interest in the effect of money in recent years. Meltzer (2001), Nelson (2002, 2003), Duca and VanHoose (2004), among others, raise the issue that money constitutes a crucial channel for the transmission mechanism of monetary policy, and the role of money cannot be simply replaced by any other policy instruments. Moreover, we find money reemerges as an important variable of concern in a number of most recent empirical monetary analysis (for instance, Wang and Wen (2005), Sims and Zha (2006), Hill (2007), to mention a few).[1]

This paper contributes to the discussion on whether money matters by revisiting an old topic: examining the causal effects from money to output in the postwar US data.[2] However, the current research departs from the literature in two main aspects. First, to capture the possible regime changes in US monetary policy, we adopt a smooth transition vector error correction

[1]As of the time of writing, Federal Reserve, the European Central Bank, Bank of England and central banks from Canada and Switzerland jointly announced cash injection plans to lessen the credit squeeze triggered by the sub-prime mortgages losses. Although the consequence of the intervention is yet to know, this unprecedent operation clearly implies that money remains a vital instrument for monetary policy.

[2]The money-output relationship has been intensively investigated in the literature. However, there is much less consensus about how money affects output (see, e.g. Sims (1972, 1980), Stock and Watson (1989), King and Watson (1997), Coe and Nason (2004)).

model (STVECM) incorporating cointegration of an unknown form. Second, we develop a simple Bayesian approach to investigating the causal effects from money to output.

Single-equation smooth transition error correction models have been widely used in the literature to capture the possible nonlinear money-output relationship (Lütekepohl, Teräsvirta and Wolters (1999), Teräsvirta and Eliasson (2001), Escribano (2004), Haug and Tam (2007), to mention a few). However, considering the interplay between endogenously determined money, interest rates and the ultimate policy targets output and inflation, we believe STVECM can be more effective in capturing both the long run and short run dynamics in the linkages among all the variables. Perhaps the reason why researchers have not followed this route is due to the lack of a fully developed statistics tool that can directly test the cointegration (or no cointegration) null in a nonlinear VECM against its both linear and nonlinear alternatives (see Seo (2004), Seo (2006), Kapetanios, Shin and Snell (2006) for details). In the literature, only Rothman, van Dijk and Frances (2001) apply a multivariate STVECM framework which is closest to us to study the money-output relationship.[3] Yet, Rothman, van Dijk and Frances (2001) pre-impose a theory based long-run cointegrating relationship in their estimation. While recognizing that the actual money-output interrelation is rather complex, unlike Rothman, van Dijk and Frances (2001), we let both the cointegration rank and cointegrating vectors to be determined by the data.

[3]Rothman, van Dijk and Frances (2001) test Granger causality from money to output in a classical context involving rolling window forecasting.

Our estimation technique is Bayesian. Specifically, we extend the Bayesian cointegration space approach introduced in Strachan and Inder (2004) and collapsed Gibbs sampler developed in Koop, Leon-Gonzalez and Strachan (2005) into the nonlinear framework. Our method jointly captures the equilibrium and presence of nonlinearity in the STVECM in a single step. Compared with the available classical estimation techniques which often require multiple steps and Taylor expansions, our approach is less susceptible to the sequential testing and inaccurate approximation problems. Furthermore, the commonly used maximum likelihood estimation in classical works is subject to the multi-mode problem caused by the nuisance parameters in the transition function of the STVECM. Yet, jagged likelihood functions do not create any particular problems in our Gibbs sampling scheme.

Considering the large model we employed is subject to the criticism of being too parameter rich, we use Bayes Factors for model comparison in order to reward more parsimonious models.[4] Alternative models are specified by placing zero restrictions on certain parameters of the unrestricted STVECM. Our approach to examining whether money long-run causes output is in spirit to that in Hall and Wickens (1993), Hall and Milne (1994) and Granger and Lin (1995). With respects to the Granger causality test from money to output, aside from considering if money directly enters the output equation as described in Rothman, van Dijk and Frances (2001), we look into whether money indirectly affects output through the channels of price and interest rate.

[4]Bayes Factors include an automatic penalty for more complex models (see Koop and Potter, (1999a, 1999b) for details).

An important finding of our study is that the postwar US money-output relationship is nonlinear, with the regime shifting mainly driven by the lagged inflation rates. In terms of triggering regime changes, compared with the key role played by inflation rates, the role of lagged annual growth rates of output is less important, while the roles played by changes in oil prices, money and interest rates are nearly negligible. However, it is worth stressing that, in our study, nonlinear models consistently outperform linear models.

We find substantial evidence that money does not long-run cause output in the postwar US data. Additionally, consistent with the in-sample testing results in Rothman, van Dijk and Frances (2001), our studies show that money is nonlinearly Granger-causal for output. The impulse response analysis shows that the dynamic paths of output given a shock to money is rather complex. Most strikingly, we find that the accumulated effect of a shock to money is negative on real output in the next 50 years, regardless of the size and sign of the initial shock. This result calls for a word of caution when using money as a policy instrument.

The outline of this paper is as follows. Section 2 describes the model and the Bayesian estimation technique. Section 3 reports the empirical results. Section 4 concludes.

2 STVECM Model and Bayesian Inference

Following a majority of empirical work (for example, Lütekepohl, Teräsvirta and Wolters (1999), Rothman, van Dijk and Frances (2001)), we investigate the money-output relationship in a system of output, money, prices and interest rates.

We use the monthly US data spanning from 1959:1 to 2006:12. The data are obtained from the database of Federal Reserve Bank of St. Louis. Various measures of output, money, prices and interest rates are used in the literature. In this paper, we adopt the seasonally adjusted industrial production index (i_t), the seasonally adjusted M2 money stock (m_t), the producer price index for all commodities (p_t), and the secondary market rate on 3-month Treasury bills (r_t) for the measures of output, money, prices and interest rates, respectively. All variables are in logarithms except for interest rates which are in percent.

To catch the possible regime changes in US monetary policy, we model the interrelationship among output, money, prices and interest rates in a STVECM.[5] Let $y_t = [i_t \quad m_t \quad p_t \quad r_t]$, the STVECM of the 1×4 vector time series process y_t, t=1,...,T, conditioning on the p observations t= -p+1,...,0, can be specified as

$$\begin{aligned} \triangle y_t = & y_{t-1}\beta\alpha + \xi + \Sigma_{h=1}^{p} \triangle y_{t-h}\Gamma_h \\ & + F(z_t)(y_{t-1}\beta^z\alpha^z + \xi^z + \Sigma_{h=1}^{p} \triangle y_{t-h}\Gamma_h^z) + \varepsilon_t \end{aligned} \tag{1}$$

ε_t is a Gaussian white noise process where $E(\varepsilon_t) = 0$, $E(\varepsilon_s'\varepsilon_t) = \Sigma$ for $s = t$, and $E(\varepsilon_s'\varepsilon_t) = 0$ for $s \neq t$. Note that $\triangle y_t = y_t - y_{t-1}$. The dimensions of Γ_h and Γ_h^z are $n \times n$, and the dimensions of β, α', β^z, and $\alpha^{z'}$ are $n \times r$. Since

[5] The possible regime changes in US monetary policy have been well documented in the literature (see, e.g., Weise (1999), Clarida, Galí and Gertler (2000), Leeper and Zha (2003)).

we are using monthly data, without loss of generality, we set $p = 6$.

In model (1), the dynamics of the regime changes are assumed to be captured by the first order logistic smooth transition function introduced in Granger and Teräsvirta (1993) and Teräsvirta (1994):

$$F(z_t) = \{1 + exp[-\gamma(z_t - c)/\sigma]\}^{-1} \tag{2}$$

where z_t is the transition variable determining the regimes. Note that z_t can be any exogenous or endogenous variables of interest. In this paper, following Rothman, van Dijk and Frances (2001), we set z_t to be the lagged annual growth rates of output, the lagged annual growth rates of money, the lagged annual inflation rates, the lagged annual changes in interest rates and the lagged annual growth rates in oil prices, respectively.[6] In particular, we allow the lag length of the transition variables to vary from 1 to 6.

The transition function $F(z_t)$ is bounded by 0 and 1. As convention, we define $F(z_t) = 0$ and $F(z_t) = 1$ corresponding to the lower and upper regimes, respectively. In function (2), the smoothing parameter γ (which is non-negative) determines the speed of the smooth transition. Observe that when $\gamma \to \infty$, the transition function becomes a Dirac function, then model (1) becomes a two-regime threshold VECM model along the lines of Tong (1983). When $\gamma = 0$, the logistic function becomes a constant (equal to 0.5), and the nonlinear model (1) collapses into a linear VECM. The transition parameter c is the threshold around which the dynamics of the

[6]Rothman, van Dijk and Frances (2001) point out that using annual growth rates instead of monthly changes as plausible transition variables is in accord with the commonly accepted perception that the regimes in the money-output relationship are quite persistent.

model change. The value for the parameter σ is chosen by the researcher and could reasonably be set to one. In this study, we set σ equal to the standard deviation of the process z_t. This effectively normalizes γ such that we can give γ an interpretation in terms of the inverse of the number of standard deviations of z_t. The transition from one extreme regime to the other is smooth for reasonable values of γ.

Observe that model (1) encompasses a set of models distinguished by the number of the long-run equilibrium relationships, the cointegrating vectors, the order of the autoregressive process, the existence of the nonlinear effects, the choice of the transition variable, and whether Granger non-causality or long-run non-causality from money to output is imposed.

2.1 Likelihood Function

Koop, Leon-Gonzalez and Strachan (2005) develop an efficient collapsed Gibbs sampler for the VECM estimation in linear contexts, which provides great computation advantages over conventional methods. To incorporate the collapsed Gibbs sampler into our posterior simulation algorithm, following Koop, Leon-Gonzalez and Strachan (2005), we obtain two representations of the likelihood .

To start with, restricting β and β^z to be semi-orthogonal, we write (1) as

$$\triangle y_t = x_{1,t-1}\beta\alpha + x_{2,t}\Phi + F(z_t)(x_{1,t-1}\beta^z\alpha^z + x_{2,t}\Phi^z) + \varepsilon_t \tag{3}$$

where $x_{1,t-1} = y_{t-1}$, $x_{2,t} = (1, \triangle y_{t-1}, ..., \triangle y_{t-p})$, $\Phi = (\xi', \Gamma_1', ..., \Gamma_p')'$, $\Phi^z = (\xi^{z'}, \Gamma_1^{z'}, ..., \Gamma_p^{z'})'$. To simplify the notation, we then define the $T \times n$ matrix $X_0 = (\triangle y_1', \triangle y_2', ..., \triangle y_T')'$ and the $T \times 2(r+1+np)$ matrix $X = (X_1\beta \quad X_2 \quad F^z X_1 \beta^z \quad F^z X_2)$, where $X_1 = (x_{1,1}', \quad x_{1,2}', \quad ..., \quad x_{1,T}')'$, $X_2 = (x_{2,1}', \quad x_{2,2}', \quad ..., \quad x_{2,T}')'$, and $F^z = diag(F(z_1), F(z_2), ..., F(z_T))$. Next, we set $B = (\alpha' \quad \Phi' \quad \alpha^{z'} \quad \Phi^{z'})'$, and stack the error terms ε_t in the $T \times n$ matrix E, where $E = (\varepsilon_1', \varepsilon_2', ..., \varepsilon_T')'$. Finally, we rewrite model (1) as

$$X_0 = X_1\beta\alpha + X_2\Phi + F^z X_1 \beta^z \alpha^z + F^z X_2 \Phi^z + E = XB + E \tag{4}$$

It is seen that the likelihood function of (4) is

$$L(y|\Sigma, B, \beta, \beta^z, \gamma, c) \propto |\Sigma|^{-\frac{T}{2}} exp\{-\frac{1}{2} tr\Sigma^{-1}E'E\} \tag{5}$$

Vectorizing (4), model (1) can be transformed into

$$x_0 = xb + e \tag{6}$$

where $x_0 = vec(X_0)$, $x = I_n \otimes X$, $b = vec(B)$, and $e = vec(E)$. Note that $E(ee') = V_e = \Sigma \otimes I_T$.

Given that

$$\begin{aligned} tr\Sigma^{-1}E'E &= e'(\Sigma^{-1} \otimes I_T)e \\ &= s^2 + (b - \widehat{b})'V^{-1}(b - \widehat{b}) \end{aligned} \tag{7}$$

where $s^2 = x_0' M_v x_0$, $M_v = \Sigma^{-1} \otimes [I_T - X(X'X)^{-1}X']$, $\widehat{b} = [I_n \otimes (X'X)^{-1}X']x_0$ and $V = \Sigma \otimes (X'X)^{-1}$. The likelihood (5) can be written as

$$L(y|\Sigma, B, \beta, \beta^z, \gamma, c) \propto |\Sigma|^{-\frac{T}{2}} exp\{-\frac{1}{2}[s^2 + (b - \widehat{b})'V^{-1}(b - \widehat{b})]\} \tag{8}$$

Observe that the likelihood of b is Normal conditional on all other parameters.

With a Normal form for the likelihood of b, we next obtain a Normal form for the likelihood of the cointegration vectors.

For any positive definite matrix κ and κ^z of rank r, we have $\beta\alpha = \beta\kappa\kappa^{-1}\alpha = \beta^*\alpha^*$ and $\beta^z\alpha^z = \beta^z\kappa^z\kappa^{z(-1)}\alpha^z = \beta^{z*}\alpha^{z*}$, where$\beta^* = \beta\kappa$ and $\alpha^* = \kappa^{-1}\alpha$, $\beta^{*z} = \beta^z\kappa^z$ and $\alpha^{*z} = \kappa^{z(-1)}\alpha^z$. Moreover, restricting $\kappa = (\alpha\alpha')^{\frac{1}{2}} = (\beta^{*'}\beta^*)^{\frac{1}{2}}$, and $\kappa^z = (\alpha^z\alpha^{z'})^{\frac{1}{2}} = (\beta^{z*'}\beta^{z*})^{\frac{1}{2}}$, we find $\alpha^{*'}$ and $\alpha^{z*'}$ are semi-orthogonal if β and β^z are semi-orthogonal. Therefore, we can reexpress equation (4) as

$$\begin{aligned} X_0 - X_2\Phi - F^zX_2\Phi^z &= X_1\beta\alpha + F^zX_1\beta^z\alpha^z + E \\ &= X_1\beta^*\alpha^* + F^zX_1\beta^{*z}\alpha^{*z} + E \end{aligned} \tag{9}$$

Setting $\widetilde{x_0} = vec(X_0 - X_2\Phi - F^zX_2\Phi^z)$, $\widetilde{x} = [\alpha^{*'} \otimes X_1 \quad \alpha^{*z'} \otimes F^zX_1]$, $\widetilde{b} = [vec(\beta^*)' \quad vec(\beta^{*z})']'$, we find equation (9) can be written as

$$\widetilde{x_0} = \widetilde{x}\widetilde{b} + e \tag{10}$$

where the dimension of $\widetilde{x_0}$ is $Tn \times 1$, the dimension of $\widetilde{x}$ is $Tn \times 2nr$, and the dimension of $\widetilde{b}$ is $2nr \times 1$.

Thus, we find the second likelihood representation from (10) is

$$L(y|\Sigma, B, \beta, \beta^z, \gamma, c) \propto |\Sigma|^{-\frac{T}{2}} exp\{-\frac{1}{2}[s^2_{\beta^*} + (b_{\beta^*} - \widehat{b_{\beta^*}})'V^{-1}_{\beta^*}(b_{\beta^*} - \widehat{b_{\beta^*}})]\} \tag{11}$$

where $s^2_{\beta^*} = (\widetilde{x_0} - \tilde{x}\widehat{b_{\beta^*}})'(\Sigma^{-1} \otimes I_T)(\widetilde{x_0} - \tilde{x}\widehat{b_{\beta^*}})$, $\widehat{b_{\beta^*}} = (\tilde{x}'\widetilde{x})^{-1}\tilde{x}'\widetilde{x_0}$, $V^{-1}_{\beta^*} =$

$\tilde{x}'(\Sigma^{-1} \otimes I_T)\tilde{x}$.

2.2 Priors

Although the most commonly elicited quantity money demand equation indicates that the velocity of money is stationary (see, e.g., Rothman, van Dijk and Frances (2001), Teräsvirta and Eliasson (2001)), empirical work does not rule out the possibility that the number of the long run cointegration relationships and the cointegration vectors are in fact data-based (see, e.g., Ambler (1989), Friedman and Kuttner (1992), Swanson (1998)). Furthermore, it is impossible to impose meaningful informative priors for the coefficients of the long run/short run adjustment in the VECM nor for parameters that indicates the speed of regime changes in the transition function. Hence, we use uninformative or weakly informative priors to allow the data information to dominate any prior information. To start with, we assume that all possible models are to be independent and, *a priori*, equally likely.

Before setting our priors for the parameters, it is worthwhile to stress the identification problems in our model setting. Note that both the linear VECM and smooth transition VAR model (STVAR) suffer from identification problems.

As well documented in the literature, a linear VECM suffers from both the global and local nonidentifications of the cointegration vectors and parameters corresponding to the long-run adjustments. In Bayesian literature, a great effort has been made to surmount this problem. In earlier research, to set uninformative prior for the cointegration vector β, researchers first normalize β into $\beta = [I_r \quad V']'$, then impose uninformative prior on the sub-vector V. However, as argued by Strachan and van Dijk (2004), this approach has an undesirable side-effect that it favors the regions of cointegration space where the imposed linear normalization is actually invalid. In most recent work, researchers have worked on putting uninformative priors on the cointegration space (see, e.g., Strachan (2003), Strachan and Inder (2004), Villani (2005)). As noted in Koop, Strachan, van Dijk and Villani (2006), since only the space of the cointegration vector can be derived from the data, it is better to elicit priors in terms of the cointegration space than in terms of cointegration vectors.

With regards to the smooth transition part of the model, as explained in Lubrano (1999a), since Bayesians have to integrate over the whole domain of the smooth parameter, the identification problem that arises from $\gamma = 0$ (the so called Davies' problem (Davies, 1977), see Koop and Potter (1999a) for further explanation) becomes more serious in the Bayesian context than in classical framework. Bauwens, Lubrano and Richard (1999) and Lubrano (1999a, 1999b) introduce a number of prior settings to solve the problem. Following Gefang and Strachan (2007), we tackle this problem by simply setting the prior distribution of γ as Gamma.

The nonidentification problem faced by the STVECM is slightly different. Although the Davies' problem remains relatively the same as in the STVAR, the problem in identifying the cointegration vector and its adjustment parameters is subject to the additional influence from the transition parameters. Here the cointegration vectors come forth in two combinations, namely $\beta\alpha$ and $\beta^z\alpha^z$. However, this difference does not render the identification problem more complicated than what we have to deal with in a linear VECM or a STVAR. As long as we can rule out the possibility that $\gamma = 0$, we can identify β, β^z, α and α^z sequentially once we choose a way to normalize β and β^z.

In the rest of the section, we construct prior distributions for all the parameters. With regards to the variance covariance matrix of the error terms, following Zellner (1971), we set standard diffuse prior for Σ.

$$p(\Sigma) \propto |\Sigma|^{-\frac{n+1}{2}}$$

For the purpose of our research, we need to calculate posterior model probabilities to compare across different possible models. As the dimension of b changes across different model specifications, to have the Bayes Factors well defined, we are not allowed to set flat prior for b (see Bartlett (1957) and O'Hagan (1995) for details). Therefore, following Strachan and van Dijk (2006), we set weakly informative conditional proper prior for b as:

$$P(b|\Sigma, \beta, \gamma, c, M_\omega) \propto N(0, \eta^{-1} I_k)$$

where $b = vec(B)$, $k = 2(r + 1 + np)$. η is the shrinkage prior as proposed by Ni and Sun (2003). As practiced in Koop, Leon-Gonzalez and Strachan (2006), we draw η from the Gibbs sampler. In our case, we set the relatively uninformative prior distribution of η as Gamma with mean $\underline{\mu_\eta}$, and degrees of freedom $\underline{\nu_\eta}$, where $\underline{\mu_\eta}$=10, $\underline{\nu_\eta}$=0.001.

Following the arguments of Koop, Strachan, van Dijk and Villani (2006), we elicit the uninformative prior of β and β^z indirectly from the prior expressed upon the cointegration space. In particular, following Strachan and Inder (2004), for $r \in (0,4)$, we specify $\beta'\beta = I_r$ and $\beta^{z\prime}\beta^z = I_r$ to express our ignorance about the cointegration space.[7] Moreover, in lines with Koop, Leon-Gonzalez and Strachan (2005), we set the prior for b_{β^*} as $p(b_{\beta^*}|\eta) \sim N(0, \eta^{-1} I_{2nr})$ in order to obtain a Normal form for the posterior.

To avoid the Davies' problem in the nuisance parameter space, following Lubrano (1999a, 1999b) and Gefang and Strachan (2007), we set the prior distribution for γ as Gamma, which exclude *a priori* the point $\gamma = 0$ from the integration range. Since the nonlinear part of b can still be a vector of zeros as $\gamma > 0$, the prior specification of γ does not render model (1) in favor of the nonlinear effect. In empirical work, we use Gamma(1,0.001) to allow the data information to dominate the prior of γ.

As to the prior of c, to make more sense in the context of economic interpretation, we elicit the conditional prior of c as uniformly distributed between the middle 80% ranges of the transition variables.

2.3 Posterior Computation

Using the priors just identified and the likelihood functions in (8) and (11), we obtain the full conditional posteriors as follows.

Conditional on β, β^z, γ, c, and b, the posterior of Σ is Inverted Wishart (IW) with scale matrix $E'E$, and degree of freedom T; Conditional on Σ, β, β^z, γ, and c, the posterior of b is Normal with mean $\overline{b} = \overline{V}_b V^{-1} \hat{b}$ and

[7] Note that the priors over the cointegration spaces of β and β^z are proper. See James (1954), Strachan and Inder (2004) for further explanation on the uniform distribution of the cointegration space.

covariance matrix $\overline{V}_b = \Sigma \otimes (X'X + \eta I_k)^{-1}$. Conditional on Σ, b, γ, and c, the posterior of b_{β^*} is Normal with mean $\bar{b}_{\beta^*} = \overline{V}_{\beta^*} V_{\beta^*}^{-1} \widehat{b_{\beta^*}}$ and covariance matrix $\overline{V}_{\beta^*} = [V_{\beta^*}^{-1} + \eta I_{nr}]^{-1}$.

To obtain the conditional posterior for η, we combine the prior and likelihood to obtain the expression

$$p(\eta|b, \Sigma, \gamma, c, y, x) \propto \eta^{\frac{\underline{\nu_\eta}+nk-2}{2}} exp(-\frac{\eta \underline{\nu_\eta}}{2\underline{\mu_\eta}} - \frac{1}{2}b'b\eta) \tag{12}$$

Thus with a Gamma prior, the conditional posterior distribution of η is Gamma with degrees of freedom $\overline{\nu_\eta} = nk + \underline{\nu_\eta}$, and mean $\overline{\mu_\eta} = \frac{\overline{\nu_\eta}\underline{\mu_\eta}}{\underline{\nu_\eta}+\underline{\mu_\eta}b'b}$.

The posterior distributions for the remaining parameters, γ and c, have nonstandard forms. However, we can use Metropolis-Hastings algorithm (Chib and Greenberg, 1995) within Gibbs to estimate γ, and the Griddy Gibbs sampler (Ritter and Tanner, 1992) to estimate c.

Following Koop, Leon-Gonzalez and Strachan (2005), we construct the collapsed Gibbs sampler as following.

1. Initialize $(b, \Sigma, b_\beta, \gamma, c)$;

2. Draw $\Sigma|b, b_\beta, \gamma, c$ from $IW(E'E, T)$;

3. Draw $b|\Sigma, b_\beta, \gamma, c$ from $N(\bar{b}, \overline{V}_b)$;

4. Calculate $\alpha^* = (\alpha\alpha')^{-\frac{1}{2}}\alpha$, $\alpha^{z*} = (\alpha^z\alpha^{z'})^{-\frac{1}{2}}\alpha^z$;

5. Create $\widetilde{x_0}$;

6. Draw $b_{\beta^*}|\Sigma, b, \gamma, c, \widetilde{x_0}$ from $N(\bar{b}_{\beta^*}, \overline{V}_{\beta^*})$;

7. construct $\kappa = (\beta^{*'}\beta^*)^{\frac{1}{2}}$, calculate $\beta = \beta^*\kappa^{-1}$. Construct $\alpha = \kappa\alpha^*$. Use the same procedure to derive β^z and α^z;

8. Draw $\gamma|\Sigma, b, b_\beta, c$ using M-H algorithm;

9. Draw $c|\Sigma, b, b_\beta, \gamma$ using Griddy-Gibbs sampler;

10. Repeat steps 2 to 9 for a suitable number of replications.

We consider a wide range of models to investigate the causal effects from money to output. Alternative models are distinguished by the number of the long run cointegration relationship, the lag length of the autoregressive process, the existence of the nonlinear effects, and the transition variable triggering regime changes.

Similar to Rothman, van Dijk and Frances (2001), we specify that if money does not Granger-cause output, the lagged money variables do not enter the equation for output, and money can not be identified as the transition variable triggering regime changes. Moreover, enlightened by Hill (2007), we define that if money does not Granger-cause output, the lagged money does not enter the equations for price and interest rate.[8] In terms of long-run causality, following Hall and Wickens (1993), Hall and Milne (1994) and Granger and Lin (1995), we specify that if money does not appear in any cointegration relationships which enter the output equation, money is not long-run causal (or weakly causal) for output.[9]

[8]As explained in Hill (2007), the situation that A causes B and B causes C implies A eventually causes C.

[9]See Hall and Wickens (1993), Hall and Milne (1994) and Granger and Lin (1995) for details.

Bayesian methods provide us a formal approach to evaluating the support for alternative models by comparing posterior model probabilities. These posterior probabilities can be used to select the best model for further inference, or to use the information in all or an important subset of the models to obtain an average of the economic object of inference by Bayesian Model Averaging. The posterior odds ratio - the ratio of the posterior model probabilities - is proportional to the Bayes factor. Once we know the Bayes factors and prior probabilities, we can compute the posterior model probabilities.

The Bayes Factor for comparing one model to a second model where each model is parameterized by $\zeta = (\zeta_1, \zeta_2)$ and ψ respectively, is

$$B_{12} = \frac{\int \ell(\zeta)p(\zeta)d(\zeta)}{\int \ell(\psi)p(\psi)d(\psi)},$$

where $\ell(.)$ is the likelihood function and $p(.)$ is the prior density of the parameters for each model.

If the second model nests within the first at the point $\zeta_2 = \zeta^*$, then, subject to further conditions, we can compute the Bayes factor B_{12} via the Savage-Dickey density ratio (see, for example, Koop and Potter (1999a), Koop, Leon-Gonzalez and Strachan (2006) for further discussion in this class of models). For the simple example discussed here, the Savage-Dickey density ratio is:

$$B_{12} = \frac{p(\zeta_2 = \zeta^* | Y)}{p(\zeta_2 = \zeta^*)},$$

where the numerator is the marginal posterior density of ζ_2 for the unrestricted model evaluated at the point $\zeta_2 = \zeta^*$, and the denominator is the prior density of ζ_2 also evaluated at the point $\zeta_2 = \zeta^*$.

Since the conditional posterior of b is Normal, it is easy to incorporate the estimation of the numerator of the Savage-Dickey density ratio in the Gibbs sampler. As to the denominator of the Savage-Dickey density ratio, using the properties of the Gamma and Normal distributions, we derive the marginal prior for a sub-vector of b evaluated at zeros as

$$\{(\frac{\underline{\mu_\eta}}{\pi\underline{\nu_\eta}})^{\omega/2}\Gamma(\frac{\omega+\underline{\nu_\eta}}{2})\}/[\Gamma(\frac{\underline{\nu_\eta}}{2})]$$

where $\Gamma(.)$ is the Gamma function, and ω is the number of elements in b restricted to be zeros.

Note that Bayes factors enable us to derive the posterior probabilities for restricted models nested in different unrestricted models. A simple restriction in our application to choose is the point where all lag coefficients are zero, i.e., $\Gamma_h = \Gamma_h^z = 0$, at which point we have the model with $p = 0$. This restricted model is useful as it nests within all models. Once we have the Bayes factor for each model to the zero lag model, via simple algebra we can back out the posterior probabilities for all models.

Taking a Bayesian approach we have a number of options for obtaining inference. If a single model has dominant support, we can model the data generating process via this most preferred model. However, if there is considerable model uncertainty then it would make sense to use Bayesian Model Averaging and weight features of interest across different models using posterior model probabilities (as suggested by Leamer (1978)).

3 Empirical Results

In empirical work, we allow the cointegration rank of the unrestricted model (1) to vary from 1 to 3.[10] For unrestricted models with a specific cointegration rank, we allow for 5 types of possible transition variables to trigger the regime changes, namely the lagged annual output growth, the lagged annual money growth, the lagged annual inflation rates, the lagged annual changes in interest rates and lagged annual growth rates in oil prices, respectively. Among these models, both the maximal order of the autoregressive process and longest lag length of the transition indicator are allowed to be 6. In total, we investigate the causal effects from money to output in the postwar US data by estimating 90 unrestricted STVECM models.

Altogether, we run 90 Gibbs sampling schemes to derive our interests of concern. Each Gibbs sampler is run for 12,000 passes with the first 2,000 discarded. The convergence of the sequence draws is checked by the Convergence Diagnostic measure introduced by Geweke (1992). We use the MATLAB program from LeSage's Econometrics Toolbox (LeSage, 1999) for the diagnostic.

3.1 Model Comparison Results

In this section, we report the results relating to the posterior model probabilities associated with a set of 2766 possible models nested in the original

[10]We don't consider unrestricted models with rank 0 since they can be derived by imposing zero restrictions on the long-run adjustment parameters of the unrestricted models with rank 1, 2 or 3. In addition, we rule out the possibility that the cointegration rank is equal to 4 for that can only happen when the time series i_t, m_t, p_t and r_t are stationary.

90 unrestricted models.[11] Assuming the 2766 models are mutually independent, in calculating the Bayes Factors, we have each of the 2766 models receive an *a priori* equal weight.

We find compelling evidence that money does not long-run cause output in the postwar US data. First, assuming all the 2766 models are mutually independent and exhaustive, we find money long-run non-causality models jointly account for 95.16% of the posterior mass.[12] Second, assuming all models nested in the unrestricted models with the same number of cointegration ranks are independent and exhaustive, we observe that money long-run non-causality models are predominant in each of the three cases. Specifically, for models nested in the unrestricted STVECM models with only one cointegration relationship, money long-run non-causality models jointly account for 96.06% of the posterior probabilities; for models nested in the unrestricted STVECM models with two cointegration relationships, overall, money long-run non-causality models receive 97.68% of the posterior mass; for models nested in the unrestricted STVECM models with three stationary cointegration relationships, money long-run non-causality models altogether get 95.16% of the posterior probability. Finally, if we assume models nested in each of the 90 unrestricted STVECM models are independent and exhaustive, we find that in each cases, money long-run non-causality models

[11] Altogether, we examine 66 linear models and 2700 nonlinear models. Namely 6 linear VARs, 6 linear VARs with money Granger non-causality restriction, 18 linear VECMs, 18 linear VECMs with money Granger non-causality restriction, 18 linear VECMs with money long-run non-causality restriction, 540 nonlinear VARs, 540 nonlinear VARs with money Granger non-causality restriction, 540 nonlinear VECMs, 540 nonlinear VECMs with money Granger non-causality restriction, and 540 nonlinear VECMs with money long-run non-causality restriction.

[12] In the remainder of the paper, we use money long-run non-causality model to indicate the restricted model where money does not long-run cause output.

are constantly overwhelmingly supported over other types of models.[13]

Assuming models nested within STVECMs with the same number of cointegration ranks (from 1 to 3) to be exhaustive, we reports the top 10 models with the highest posterior model probabilities in table 1. Note that the top 10 models of all the 2766 models are exactly the same as the top 10 models nested in the STVECM models with three cointegration relationships, for nonlinear models of rank 3 get nearly 100% of the posterior mass among all the 2766 models. Table 1 reinforces the substantial support for money long-run non-causality models. It is worth noting that the most preferred models for all cases are nonlinear money long-run non-causality models. Another interesting finding is that there is no pronounced model uncertainty if we focus on all the 2766 possible models or a subset of models nested within the unrestricted STVECM models with 2 or 3 stochastic trends. Yet, more evidence of model uncertainty emerges if we pre-impose the cointegration rank of the unrestricted models to be 1. Finally, note that the most preferred model among all the possible 2766 models is the restricted money long-run non-causality STVECM of rank 3, order 6, and with lagged 2 inflation rates as the transition indicator.

Overall, we find little support for models indicating money is not Granger-causal for output. The posterior mass for all models (linear types of VAR, VECM models and nonlinear types of STVAR, STVECM models) with zero restrictions on the lagged money in the equations for output, price and interest rates is nearly negligible. Furthermore, observing that the total

[13]The model comparison results for models nested in each of the 90 unrestricted STVECM models are available upon request.

posterior model probability associated with the unrestricted STVECMs and the restricted money long-run non-causality STVECMs is almost 100%, we proclaim that money nonlinearly Granger-causes output, which is in accord with the in-sample evidence in Rothman, van Dijk and Frances (2001).

Given the substantial support for nonlinear models, it is interesting to examine which transition variable plays a more important role in triggering regime changes. Examining all the possible nonlinear models, we find that lagged annual inflation rates consistently predominate over other candidate transition variables in driving regime changes. All together, nonlinear models with lagged inflation rates as transition variables receive 89.17% of the posterior mass. The next important triggers for regime changes are lagged annual output growth rates. Note that nonlinear models with regime shifting governed by the lagged annual output growth rates account for 10.83% of the posterior mass. Last, compared with lagged inflation and output growth, lagged changes in money, interest rates and oil prices play trivia roles in triggering regime changes.

To highlight the nonlinear feature of the interrelationship among money, output, prices and interest rates, in figure 1 we plot the values of the smooth transition function over time for the most preferred model chosen from all the 2766 candidate models.[14] Observe that although the plot is quite volatile, the values of the transition function are almost always bounded by 0.4 and 0.6 throughout the time. This result implies that the regime changes in the postwar US money-output relationship are quite modest, which is in line with the findings of Primiceri (2005) and Sargent, Williams and Zha

[14] The whole set of the time profiles of the transition functions are available upon request.

(2006). However, given the compelling support for nonlinear models over linear models, it is worth stressing that we find it improper to model the post-war US money-output relationship in linear models.

Table 2 contains the estimates of the cointegration vectors and transition parameters for the most preferred models nested in the unrestricted STVECM models of rank 1, 2, 3, respectively. Recall that the most preferred model among the whole set of 2766 candidate models is exactly the same most preferred model selected from all the possible models nested in the unrestricted STVECM models of rank 3.

To aid in interpretation, in table 2, we normalize the cointegration vectors on output and money, respectively. Assuming that the cointegration rank is 1, we find the parameters for output, money and price levels appear to have reasonable economic interpretations. For example, inflation brings about (nominal) higher output level. Yet, it is not so straightforward to explain why effects of interest rates are quite different between the lower and upper regimes. Focusing on the model with 2 cointegration relationships, we find that in each regimes, the first cointegrating vectors can be explained as the (log) quasi-velocity of money as defined in Rothman, van Dijk and Frances (2001), while it is hard to find an economic theory to explain the second long-run equilibrium relationship. For the most preferred model among all the possible 2766 models (or the most preferred model among all the models nested in STVECMs of rank 3), we find it even more difficult to find a theory-based explanation for the long-run stationery interrelationships. Yet, it is clear that there are enormous differences in the cointegration vectors between the upper and lower regimes.

The estimated values of the smoothing parameter γ presented in table 2 are relatively small. With the speed of the transition determined by γ, small value of γ indicates that the transition between regimes is rather smooth. As to the estimated value of c, recall that for all cases, the transition variable is the lagged inflation rates. In our sample, the mean of inflation rates is 0.0352. Given the threshold c is greater than 0.05 for each cases, it is seen that the upper regimes only become active when the transition variable is very large.

Finally, it is illuminating to look into the model comparison results in the linear framework. Assuming the 66 linear models are exhaustive, we find that the unrestricted linear VECM of rank 3 and order 6 receives nearly 100% of the posterior mass. Thus, models denoting long-run money non-causality are no longer supported in the linear frameworks. Furthermore, we find unrestricted VECM of order 6 dominates money long-run non-causality models when we pre-specify the rank of the cointegration space to be 1 or 2. Nevethless, these results prove that ignoring nonlinear effects can lead to quite misleading conclusions, such as money is long-run causal for output.

3.2 Impulse Response Analysis

To shed further light on the causal effects from money to output, we analyze the impulse responses of output given a shock to money. The nonlinear STVECM allows for asymmetries in the behaviour of the money-output linkages. In this study, we are interested in two types of asymmetric effects. First, whether positive and negative shocks to money have unbalanced effects on real output. Second, whether big and small money shocks have disproportionate effects.

It is acknowledged that the impulse response functions of the nonlinear models are history- and shock- dependent (e.g. Potter (1994), Koop, Pesaran and Potter (1996)). We use the generalized impulse response function proposed in Koop, Pesaran and Potter (1996) to examine the response of output to a money shock. In particular, we examine the generalized impulse response functions of GI_P for a shock, υ_t, and a history, ω_{t-1} as follows

$$GI_P(n, \upsilon_t, \omega_{t-1}) = E[P_{t+n|\upsilon_t, \omega_{t-1}}] - E[P_{t+n}|\omega_{t-1}] \tag{13}$$

where n is the time horizon. By averaging out the future shocks, in (13), we treat the impulse responses as an average of what might happen given what has happened. Using Bayesian approach, we calculate the generalized impulse responses by averaging out the history uncertainties, future uncertainties and parameter uncertainties.

In each replication of the Gibbs Sampler after the initial burning runs, we calculate the generalized impulse response functions for all the alternative models as follows.

1. Randomly draw an ω_{t-1} in the observed sample as the history.

2. For a pre-specified shock hits money, randomly draw the corresponding shocks hit the other three variables at time t from Σ.

3. Set the maximum horizon as n and randomly sample $n+1$ four by one vectors of innovations from Σ.

4. Calculate the expected realizations of output using the shocks calculated in step 2 and the last n innovations in step 3.

5. Calculate the shock-independent expected realizations of output using all the $n + 1$ innovations in step 3.

6. Take the difference of the results from step 4 and step 5 to generate the impulse responses of output for the current draw.

At the end of the Gibbs sampling scheme, we derive the generalized impulse response functions for each possible model by integrating out all the parameter uncertainties. Note that if there is a great deal of model uncertainty, we can also average across models to derive the impacts of money on output weighting by the posterior model probabilities.

We set the magnitudes of the initial shocks amounting to ± 1 and ± 2 times the standard deviation of monthly money growth rates, namely ± 1 unit and ± 2 units of shocks. The time horizon of the impulse responses is set as 600 months (50 years). Given the large number of models and four different shocks, we only present the impulse response functions for the most preferred model among all the 2766 models in figures 2-3. For comparison, both the impulse response functions of output (nominal output) and real output are provided. The following observations are noteworthy in figures 2-3.

1. Positive and negative money shocks of the same magnitude appear to have asymmetric affects on both nominal output and real output. Observe that the time path of the impulse responses to positive shocks

never mirror that of the impulse responses to negative shocks.

2. Impacts on both nominal output and real output appear to vary disproportionately with the size of the shock to money.

3. Impacts on nominal output appear to steadily increase in the same direction of the initial shocks in the first 10 years. After that, the impact responses become more volatile.

4. Compared with the responses of nominal output, the impact responses of real output to a money shock are rather volatile. More strikingly, the accumulated effect on real output appear to be negative in the next 50 years after a shock to money, regardless of the size and sign of the shock.

4 Conclusion

This paper investigates the causal effects from money to output using postwar US data. We develop a Bayesian approach to catch the interrelationship among money, output, prices and interest rates in a STVECM model. Different from similar nonlinear modeling method in the literature, we jointly estimate the cointegration relationship and nonlinear effects in a single step without pre-imposing any theory based restrictions.

Our model comparison results indicate that the postwar US money-output relationship is nonlinear. Yet, we find that the transition between regimes is rather smooth, and it is improper to use any abrupt transition framework to model the money-output linkage. Through model comparison, we find substantial evidence in favor of money long-run non-causality for output. In addition, we find little evidence against Granger causality from money to output. More precisely, our result strongly support that money nonlinearly Granger-causes output during the postwar period in the US.

Our impulse response analysis sheds further light on the nonlinear causal effects from money to output. An important finding that leaps out is that although a positive money shock can increase nominal output, we have to be cautious in using money as a policy instrument, for it appears that a shock to money will have negative accumulative effects on real output over the next fifty years, regardless of the size and sign of the shock.

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Table 1: Most preferred models

	Rank1		Rank2		Rank3/All 2766 models	
No.	Model	Posterior Mass	Model	Posterior Mass	Model	Posterior Mass
1	$STVECMb(1,3,5,6)$	0.4409	$STVECMb(2,3,5,6)$	0.9606	$STVECMb(3,3,2,6)$	0.8771
2	$STVECMb(1,1,4,6)$	0.2098	$STVECM(2,3,5,6)$	0.0217	$STVECM(3,1,1,6)$	0.0737
3	$STVECMb(1,3,1,6)$	0.1689	$STVECMb(2,1,5,6)$	0.0150	$STVECM(3,1,1,6)$	0.0342
4	$STVECMb(1,3,6,6)$	0.1028	$STVECM(2,1,5,6)$	0.0015	$STVECM(3,3,2,6)$	0.0141
5	$STVECM(1,1,4,6)$	0.0301	$STVECMb(2,3,2,6)$	0.0010	$STVECMb(3,3,1,6)$	0.0006
6	$STVECMb(1,5,4,6)$	0.0266	$STVECMb(2,1,3,6)$	0.0002	$STVECMb(3,1,3,6)$	0.0003
7	$STVECMb(1,2,4,6)$	0.0037	$STVECMb(2,3,6,6)$	0.0000	$STVECM(3,1,3,6)$	0.0001
8	$STVECM(1,3,5,6)$	0.0030	$STVECMb(2,3,2,6)$	0.0000	$STVECM(3,1,3,6)$	0.0000
9	$STVECMb(1,5,5,6)$	0.0026	$STVECM(2,1,3,6)$	0.0000	$STVECM(3,1,4,6)$	0.0000
10	$STVECM(1,5,4,6)$	0.0021	$STVECMb(2,4,2,6)$	0.0000	$STVECM(3,3,1,6)$	0.0000

Notes:

i. The columns labeled Rank 1 report the top 10 most preferred models and their corresponding posterior probabilities when we assume all possible models nested within the STVECM models with only one cointegration relationship are exhaustive. Same procedure applies to columns labeled Rank 2 and Rank 3.

ii. STVECMb indicates the STVECM model is restricted so money is long-run non-causal for output.

iii. In parenthesis, the first subscript indicates the rank of the unrestricted model; The second subscript indicates the transition variable causing regime changes, where 1,2,3,4 and 5 denote annual output growth, annual money growth, annual inflation, annual growth in interest rates, and annual growth in oil prices; The third subscript indicates the lag length of the transition variable; The fourth subscript indicates the order of the model.

Table 2: Cointegration vectors and smooth transition parameters

	Rank1	Rank2	Rank3
Lower Regime	*i-0.2031m-0.2855p+0.0950r*	*i-1.1168m-0.2555p* *m+0.2149p+0.2829r*	*i+0.1923m* *m+1.8885r* *m-10.5341p*
Upper Regime	*i-0.4150m-0.6940p-0.0986r*	*i-0.4831m-0.7252p* *m+1.0513p+0.7710r*	*i-0.1417m* *m+0.8494r* *m-0.8133p*
γ	0.4068 (*0.0001 2.2946*)*	0.1957 (*0.0002 1.2811*)	0.1826 (*0.0000 1.2678*)
c	0.0518 (*0.0301*)**	0.0519 (*0.0301*)	0.0513 (*0.0302*)

Notes:

* The 95% highest posterior density intervals (HPDIs) for γs are reported in parenthesis.

** Standard deviations of c are reported in parenthesis.

Figure 1
Smooth transition function

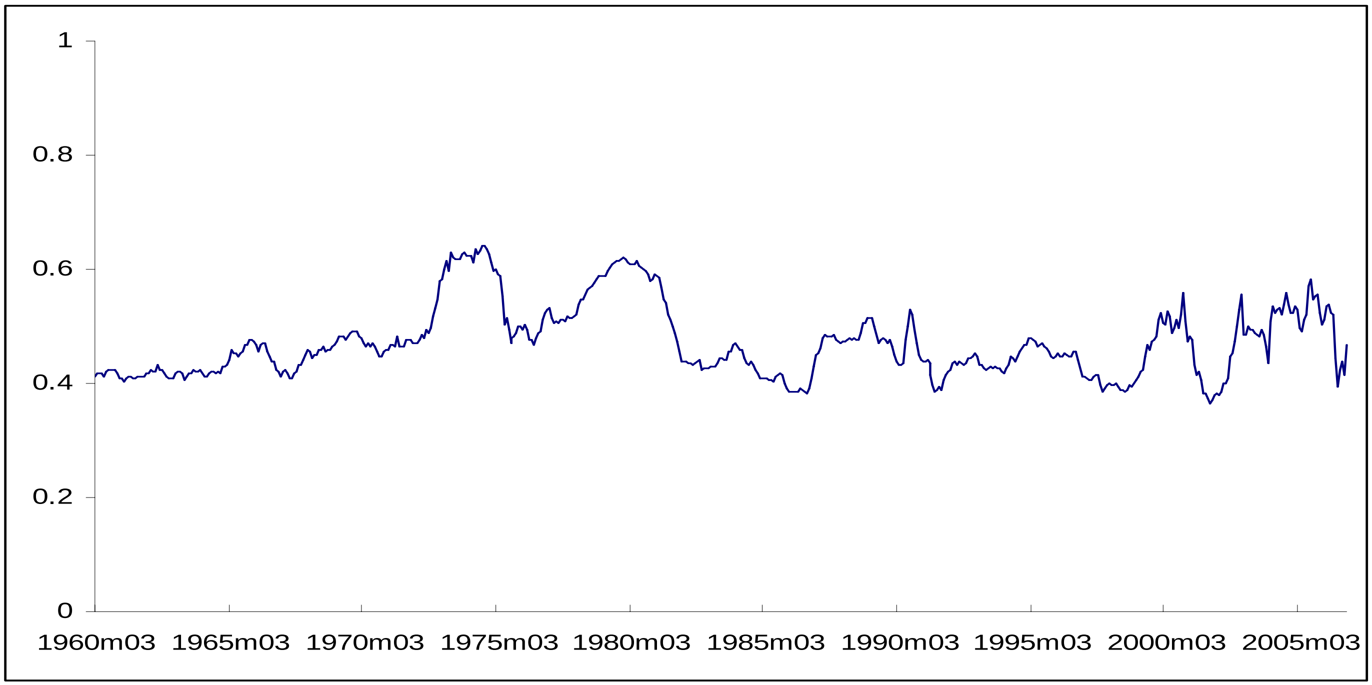
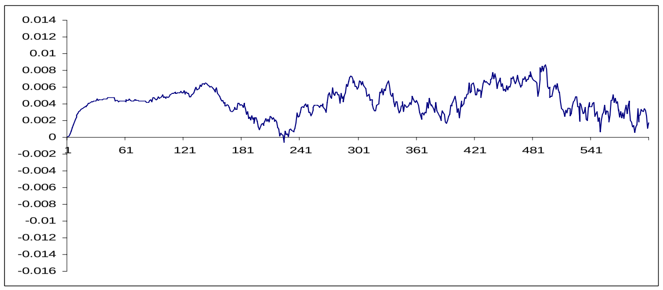
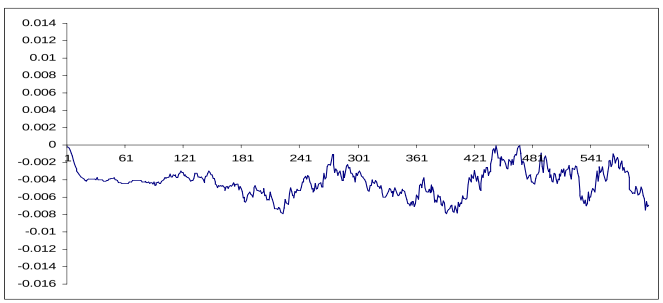


Figure 2
Impulse response functions of (nominal) output

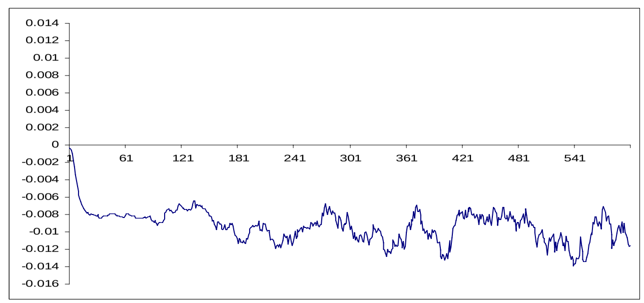


Impacts of one unit of positive shock to money

Impacts of two units of positive shock to money

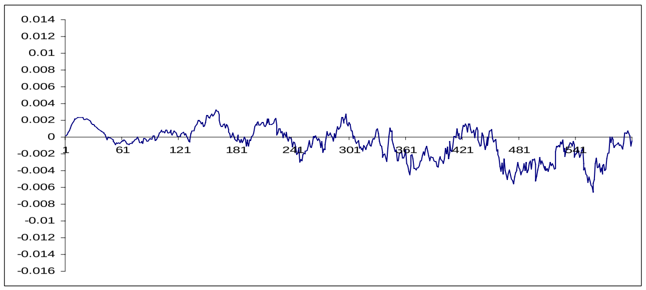


Impacts of one unit of negative shock to money

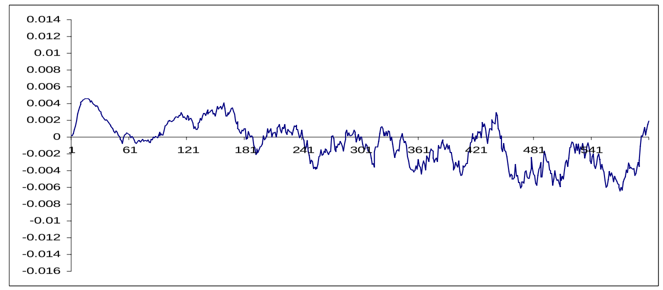


Impacts of two units of negative shock to money

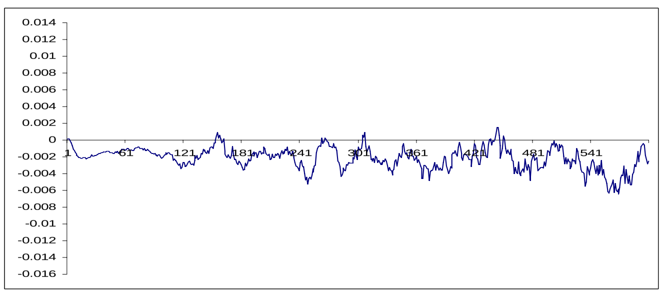
Figure 3
Impulse response functions of real output



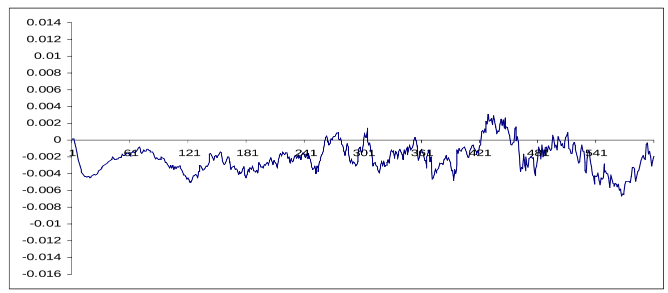
Impacts of one unit of positive shock to money



Impacts of two units of positive shock to money



Impacts of one unit of negative shock to money



Impacts of two units of negative shock to money