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VERTICAL INTEGRATION AND
PRODUCT INNOVATION

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Vertical integration and product innovation

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We study vertical integration and product innovation (in the form of horizontal product
differentiation) as interdependent strategic choices of vertically related firms. We consider
product innovation in the downstream market as a strategic decision of innovative firms
facing a threat of vertical integration and market foreclosure by an upstream monopolist.
Our main finding is that, although product differentiation allows to soften product market
competition and to avoid market foreclosure, the downstream market may prefer less
product differentiation to deter vertical integration. Therefore, less product innovation
can be a possible social cost of a lenient antitrust policy.

Key Words: Vertical Integration; product innovation; market foreclosure; duopoly.
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1. INTRODUCTION

One of the most controversial issues in industrial organization is market foreclosure through vertical integration. Although one major advantage of vertical integration is to eliminate the problem of double marginalization, the major criticism against vertical integration is market foreclosure, which has generated a significant amount of literature to examine the competitive structure of the upstream and downstream industries and welfare.\(^3\) However, surprisingly enough the previous literature on vertical integration has mainly concentrated on the production activities of the firms, paying less attention to the non-production activities of the firms and particularly, of the downstream firms.\(^4\)

In this paper, we consider product innovation, in the form of horizontal product differentiation in the downstream market, as a strategic choice of innovative firms facing a prospective threat of vertical integration and market foreclosure by an upstream monopolist. We examine how product innovation in the downstream market affects the upstream monopolist’s incentive for vertical integration and market foreclosure, and how the possibility of vertical integration affects the downstream firms’ incentive to differentiate products.

We use a simple model in which, without vertical integration, an upstream monopolist charges a linear price for the sole input required by two downstream firms in order to produce the final product. In the downstream market, firms compete in quantities. In this setting, vertical integration of the upstream monopolist with one of the two downstream firms eliminates double marginalisation in one segment of the final product market, and gives the integrated firm a cost advantage over the downstream rival. Moreover, by setting the input price, the integrated firm affects its downstream competitor’s cost, and it may choose to foreclose the downstream

\(^3\)Vertical foreclosure refers to restrictions in supply (resp. demand) a vertically integrated firm would apply to its downstream (resp. upstream) competitors, extending in this way its market power in the industry. Contrary to the benign view of the so-called Chicago school (e.g., Bork (1978)), denying vertical foreclosure as an equilibrium consequence of vertical mergers, subsequent works have proved vertical foreclosure in different models of vertical integration. Among many others: Salinger (1988), Ordover, Saloner and Salop (1990), Hart and Tirole (1990), Riordan (1998), Chen (2001), Riordan and Chen (forthcoming). Rey and Tirole (2007) and Riordan (forthcoming) provide excellent surveys of this literature.

\(^4\)One exception is Baake, Kamecke and Norman (2004), who extend Hart and Tirole (1990) model by considering physical capital investment as a non-production strategic decision of an upstream monopolist. In their model, banning vertical integration has the social cost of a sub-optimal level of capital investment, leading to productive inefficiency in the market, while vertical integration guarantees the efficient investment level but output is monopolistically restricted.
market (i.e., to price the essential input sufficiently high to drive the competitor out of the industry).

The vertical structure of the industry (i.e., vertical integration vs. vertical separation) is endogenously determined by an integration game modeled as a sale auction between the downstream firms. If the gain from vertical integration exceeds a fixed integration cost, the upstream monopolist calls the downstream firms for offers in order to integrate one of them. Then, competition between the downstream firms in the integration game allows the upstream monopolist to appropriate more than the full surplus from integration, and reap most of the profit created in the final product market. Therefore, vertical integration is a threat to the downstream firms at the initial stage of the game, when they can invest in R&D to differentiate products.

Besides the usual effect of softening competition in the downstream market, product differentiation exerts two more effects in our model: it eliminates market foreclosure under vertical integration, and it affects the possibility of vertical integration. The elimination of market foreclosure encourages innovation in the downstream market. However, the trade-off between the benefits from eliminating market foreclosure and softening product market competition, on one hand, and the loss from vertical integration, on the other hand, makes the impact of vertical integration on innovation ambiguous.

In fact, we show that whether vertical integration is more likely for higher or lower degrees of product differentiation is ambiguous and depends on the cost of integration. If the cost of integration is low, vertical integration always occurs. If the cost of integration is moderate, vertical integration occurs for small and for large, but not for intermediate, degrees of product differentiation. If the cost of integration is sufficiently high, but not large enough to prevent vertical integration, then vertical integration only occurs for large degrees of product differentiation. Therefore, while higher product differentiation softens competition in the final goods market, it may also create the threat of vertical integration, which helps the upstream monopolist to extract more rent from the downstream firms. As a consequence, there are situations where the downstream market prefers relatively lower degrees of product differentiation to prevent vertical integration, so that, instead of market foreclosure, we highlight less product innovation as a possible social cost of vertical integration.
integration.

Our paper is related to two literatures: the literature on vertical integration and foreclosure, and the literature on product innovation. In the vertical integration literature, previous works have analysed the incentives to vertically integrate and foreclose the downstream market when the final products are differentiated (e.g. Ordover, Saloner and Salop (1990), Economides (1994), Colangelo (1995), Hackner (2001), Chen (2001)). However, the degree of product differentiation is exogenous in all these studies, so that they do not consider the effect exerted by the possibility of vertical integration on product innovation.

The literature on product innovation has mainly focused on the effects of product market competition and R&D competition on the incentive to innovate. For instance, Lambertini and Rossini (1998) and Lin and Saggi (2002) study R&D and product market competition in a setting similar to ours, where product innovation takes the form of horizontal product differentiation. A related literature analyzes product differentiation in upstream and downstream markets under alternative market structures (e.g. Pepall and Norman (2001), Belleflamme and Toulemonde (2003), Matsushima (2004)). Also this literature essentially concentrates on the relationship between product differentiation and the intensity of product market competition. On the contrary, we focus on a different source of strategy, viz. vertical integration, that may affect both product market competition and the incentive for product innovation by innovative firms vertically related to an upstream monopolist.

A handful of papers explicitly consider the role played by vertical relations in the innovative firms’ incentive to invest in process or product innovations. Banerjee and Lin (2003) focus on process innovations by oligopolistic downstream firms vertically related to an upstream monopolist. They highlight a demand effect associated with a downstream process innovation, leading to an increase in the input price, and hence, in the unit cost of the innovator’s competitors. This indirect effect may foster the downstream firms’ incentive to innovate. Economides (1999) considers the choice of the quality levels of the upstream and the downstream components of a final product in a successive monopoly, and shows that vertical integration leads to higher quality than vertical separation. In these papers, however, the vertical structure of the industry is exogenous, and hence they do not address the strategic
interaction between the incentive to innovate and the incentive to vertically merge.

Brocas (2003) and Buehler and Schmutzler (2007) consider the interplay between process innovation and endogenous vertical structure of the industry. Brocas (2003) focuses on process innovations discovered by oligopolistic upstream firms and licensed to the oligopolistic downstream firms. She shows that vertical integration can effectively shelter an innovator from the competitive pressure of the other potential innovators when the switching costs between different technologies are low. In this case, vertical integration and process innovation mutually reinforce. Buehler and Schmutzler (2007) consider downstream process innovations in a successive oligopoly with endogenous vertical integration. They highlight an intimidation effect which increases the incentive to innovate of a vertically integrated firm and decreases the incentive to innovate of the integrated firm’s competitors. They further show that downstream process innovations decreases the likelihood of a complete vertically separated structure of the industry. Besides our focus on product instead of process innovation, we differ from the two above mentioned papers for the nature of the effects exerted by vertical integration on innovation. We stress that vertical integration can be a competitive threat from an upstream monopolist to innovative downstream firms which may stifle socially valuable innovations.

The rest of the paper is organised as follows. In Section 2 we present the model, which consists of a three-stage game with the following timing: innovation stage (first stage), integration stage (second stage), market stage (final stage). In Section 3 we solve the market stage under the two alternative vertical structures of the industry (vertical integration vs. vertical separation) and we discuss the effect of product differentiation on market foreclosure. Section 4 analyzes the vertical integration game and shows how the vertical structure of the industry depends on product differentiation and integration costs. In Section 5 we study the effects of vertical integration on product innovation. In Section 6 we point out that the possibility of vertical integration can cause the social cost of less product innovation. Finally, Section 7 provides some concluding remarks.

2. THE MODEL

We consider an industry with upstream and downstream markets. In the upstream market, a monopolist (firm \(U\)) produces the sole input needed by two down-
stream firms (firms $D_1$ and $D_2$) in order to produce their final products. For simplicity, we assume that the upstream monopolist produces the essential input at zero-cost. Both downstream firms require one unit of input to produce one unit of the final product, and they can differentiate the final product at the outset by investing in R&D.

More precisely, on the demand side of the downstream market, the degree of product substitutability perceived by consumers, $\gamma$, leads to the inverse demand system:

$$p_i = a - q_i - \gamma q_j \quad (i,j = 1, 2; \; i \neq j),$$

where $\gamma \in [0, 1]$.\(^5\) With $\gamma = 0$, consumers perceive products 1 and 2 as independent goods, while $\gamma = 1$ corresponds to the consumers’ perception of perfect substitutes. According to the demand system (1), a lower degree of product substitutability rises consumers’ willingness to pay for the two differentiated products: for any given quantities, $p_1$ and $p_2$ increase as $\gamma$ falls. Thus, product differentiation generates social value through consumers’ preference for variety.

The degree of product substitutability is set by the downstream firms’ R&D investment. Since our main focus is on the effect exerted by a prospective threat of vertical integration on the incentive to innovate (i.e., horizontally differentiate) products, we abstract from any strategic consideration related to R&D competition by assuming that the two firms cooperate at the innovation stage.\(^6\) By paying a fixed R&D cost $k$, they can reduce the degree of product substitutability from $\gamma = 1$ (perfect substitutes goods) to $\gamma = \tilde{\gamma} \in [0, 1]$. With no R&D investment, the final

\(^5\)The demand side of the downstream market is a simplified version of Singh and Vives (1984). The inverse demand sistem (1) arises from the representative consumer’s maximization of the utility function: $U = a(q_1 + q_2) - \frac{1}{2} (q_1^2 + q_2^2 + 2\gamma q_1 q_2) + m$ (where $m$ is a numeraire good), subjected to a standard budget constraint.

\(^6\)Lambertini and Rossini (1998) study R&D competition for product differentiation in the same differentiated duopoly model we adopt here for the downstream market. Due to the strong positive externality exerted by the R&D investment of one firm on the rival’s profit, a simultaneous R&D game can lead to a prisoner dilemma equilibrium, where the final products remain homogeneous even if both downstream firms would gain from product differentiation. One one hand, R&D cooperation may be seen as a natural solution of such a prisoner dilemma problem, possibly agreed by the downstream firms in a preliminary (implicit) stage of the model. On the other hand, allowing for R&D competition complicates our model without changing the nature of the effects we focus on, and without qualitatively affecting our main results. Furthermore, since the downstream firms turn out to be symmetric in any respect at the innovation stage of our model, all our results go through if we assume that only one firm has the capability of innovating (our measure of the incentive to invest would simply scale down by one half). This suggests an alternative interpretation of our model as an entry model, where one innovative firm can enter the downstream market with a differentiated or with an homogeneous version of a final product originally produced by a downstream monopolist vertically separated from the upstream monopolist. The effects of the threat of vertical integration on the entrant’s incentive to differentiate the final product would be qualitatively similar to those presented in Section 5.
products are perceived as perfect substitutes by consumers.

After the R&D decision is taken, the vertical integration stage takes place. We model the vertical integration game as a first-price sale auction. First, the upstream monopolist decides whether to ask the downstream firms for simultaneous and independent price offers in order to integrate one of them. If the auction is not organised, vertical integration does not occur. If the auction is organised, each downstream firm decides whether to submit an offer. On the basis of the offers received, the upstream monopolist then decides whether to integrate the downstream firm asking for the lower price, paying the lowest bid (in the case of tie, we assume that both downstream firms have fifty percent probability of merging with the upstream firm).\(^7\) We further assume that vertical integration involves a fixed integration cost, denoted by \(E.\)\(^8\)

The outcome of the integration game sets vertical structure of the industry. If vertical integration does not occur, the upstream monopolist supplies the essential input to the downstream firms charging a linear price \(w_u.\)\(^9\) The input price acts as the marginal cost of production for both downstream firms, which finally compete à la Cournot in the downstream market. If vertical integration occurs, the downstream market is populated by a vertically integrated firm (firm \(V\)), and an independent firm (firm \(I\)). The integrated firm can use the essential input at zero-cost, and optimally sets the price of the input supplied to the rival, \(w_v.\)\(^{10}\) Finally,  

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\(^7\) As many other works on vertical integration (e.g., Hart and Tirole (1990)), we assume away the possibility of horizontal mergers (in our case, between the two downstream firms), likewise the complete integration of the industry in a single monopoly. Hence, if vertical integration occurs, the upstream monopolist merges with one downstream firm only. While both assumptions may derive from the antitrust authorities banning horizontal or vertical mergers resulting in the monopolization of the downstream market or the whole industry, the second assumption may simply be justified by the cost of vertically integrating both downstream firms being prohibitive. As to the first assumption, we recognize that the horizontal merger of the two downstream firms may well represent an alternative strategy for them to face the threat of vertical integration other than their strategic decision on product differentiation. We leave this extension of our model to future research.

\(^8\) See Hart and Tirole (1990) for the interpretation of the cost of vertical integration.

\(^9\) Linear pricing of the essential input is a common assumption in the vertical integration literature (e.g., Salinger (1988), Ordover, et al. (1990), Colangelo (1995), Economides (1998), Hackner (2001), Arya et al. (2007)). It is worth noting that although allowing for non linear pricing in our model may solve the upstream monopolist problem of reaping profit in the downstream market, making vertical integration unnecessary, it would exacerbate the downstream firms’ incentive problem of (not) investing in socially valuable innovations.

\(^10\) We consider the integrated firm as behaving as a single firm. However, all our results still hold if we model the integrated firm as a multi-division firm that can optimally and credibly (from the viewpoint of the independent firm) set the internal price at which the upstream division provides the essential input to the downstream division. Interestingly, unless the final products are perfect substitutes (\(\gamma = 1\)) or independent (\(\gamma = 0\)), the integrated firm would internally price the essential input above the marginal cost, using the internal price as an effective instrument to decrease the price and profit erosion due to competition in the downstream market (while still
the two firms compete á la Cournot in the downstream market.

Summarising, the model consists of three stages. In stage 1, the R&D decision is taken by the downstream firms, and the degree of product differentiation is determined. In stage 2, the vertical integration game takes place, and the vertical structure of the industry is determined. In stage 3, the price of the essential input is set by firm $U$ (or firm $V$, under vertical integration), and Cournot competition takes place in the downstream market. Production and profits are finally determined.

The timing of the model reflects the idea that product innovation may require longer time span investments than vertical integration. More specifically, the design of new products may involve higher sunk costs and irreversible investments than the process of vertical merger. Furthermore, product innovation often requires specific skills and innovative knowledge difficult to codify and transmit, which may impede an external assessment of the innovation market value before the innovation is fully developed, and raise standard agency issues in integrating innovative firms before the innovation is fully developed. These considerations explain both our assumptions that the innovation stage precedes the integration stage, and that the characteristics of the final products cannot be further altered after the innovation stage.

Our solution concept is perfect subgame equilibrium. We therefore solve the model by backward induction starting from the market stage.

3. THE MARKET STAGE

At the market stage, the degree of product differentiation, $\gamma$, and the vertical structure of the industry (i.e., vertical integration or vertical separation) are already determined.

Consider first the market equilibrium under vertical separation. Given the input price, $w$, the downstream firms ($D_1$ and $D_2$) face the same marginal cost. Hence, Cournot competition leads to a symmetric equilibrium in the downstream market, where the downstream firms produce

$$q_1^D(w) = q_2^D(w) = \frac{a - w}{2 + \gamma},$$

setting the external price to optimally fix the cost differential between the downstream division and the independent firm. The proof of this extension is available on request.
and earn profits

$$\pi^D_1(w_u) = \pi^D_2(w_u) = \left[ \frac{a - w_u}{2 + \gamma} \right]^2.$$ 

The upstream monopolist faces the demand function for the essential input

$$\pi^D_1(w_u) = \pi^D_2(w_u) = \frac{2(a - w_u)}{2 + \gamma}.$$ 

and sets the input price, $w_u$, to maximize

$$\pi^U(w_u) = w_u \frac{2(a - w_u)}{2 + \gamma}.$$ 

This leads to the input equilibrium price:

$$w_u^* = \frac{a}{2}.$$ (2)

Equilibrium profits under vertical separation are finally given by:

$$\pi^D_1 = \pi^D = \left( \frac{a}{2} \right)^2 \left[ \frac{1}{2 + \gamma} \right]^2,$$ (3)

$$\pi^U = \left( \frac{a}{2} \right)^2 \frac{2}{2 + \gamma}.$$ (4)

We turn now to the market equilibrium under vertical integration. The integrated firm produces its final product at zero-cost, and charges the linear price $w_v$ on the input sold to the independent firm. For a given input price $w_v > 0$, Cournot competition yields an asymmetric equilibrium in the downstream market, where the independent firm (I) and the integrated firm (V) produce, respectively:

$$q^I(w_v) = \frac{a(2 - \gamma) - 2w_v}{4 - \gamma^2}, \quad q^V(w_v) = \frac{a(2 - \gamma) + \gamma w_v}{4 - \gamma^2}.$$  

The corresponding profits are:

$$\pi^I(w_v) = \left[ \frac{a(2 - \gamma) - 2w_v}{4 - \gamma^2} \right]^2$$ for the independent firm, and

$$\pi^V(w_v) = \left[ \frac{a(2 - \gamma) + \gamma w_v}{4 - \gamma^2} \right]^2 + w_v \frac{a(2 - \gamma) - 2w_v}{4 - \gamma^2}$$ for the integrated firm, where the second term of the integrated firm’s profit arises from its sales of the essential input to the rival. The input price, $w_v$, is set by the integrated firm to maximise $\pi^V(w_v)$, leading to:

$$w_v^* = \frac{a(2 - \gamma)(2\gamma + 4 - \gamma^2)}{2(8 - 3\gamma^2)}.$$ (5)
Firms’ equilibrium profits under vertical integration are finally given by:

\[
\pi^I = \left(\frac{a}{2}\right)^2 \frac{4(1-\gamma)}{(8-3\gamma^2)}^2, \tag{6}
\]

\[
\pi^V = \left(\frac{a}{2}\right)^2 \frac{(2-\gamma)(6-\gamma)}{(8-3\gamma^2)}. \tag{7}
\]

The following Lemma establishes some useful comparative results between the market equilibria under vertical separation and under vertical integration.

**Lemma 1.** i) Unless products are independent or perfect substitutes, the input price charged to the independent firm under vertical integration is lower than the input price charged to the downstream firms under vertical separation (i.e., \(w^*_u < w^*_v\) for \(\gamma \in (0,1)\), while \(w^*_u = w^*_v\) for \(\gamma = 0\) and \(\gamma = 1\)). ii) Unless products are independent, the independent firm earns lower profits under vertical integration than under vertical separation (i.e., \(\pi^I < \pi^D\) for \(\gamma \in (0,1)\)). iii) Both the equilibrium profit of the independent firm under vertical integration, \(\pi^I\), and the equilibrium profit of a downstream firm under vertical separation, \(\pi^D\), increase with product differentiation (i.e., decrease with \(\gamma\)).

**Proof.** i) Using equations (2) and (5), we calculate:

\[
w^*_u - w^*_v = a \frac{\gamma(1-\gamma)}{(8-3\gamma^2)}. \tag{8}
\]

From the expression above it follows immediately that \(w^*_u - w^*_v > 0\) for any \(\gamma \in (0,1)\), while \(w^*_u - w^*_v = 0\) for \(\gamma = 0\) and \(\gamma = 1\). \(^{11}\)

ii) From equations (3) and (6), we find that \(\pi^D \geq \pi^I\) is equivalent to:

\[
\frac{8 - 4\gamma - 4\gamma^2}{8 - 3\gamma^2} \leq 1,
\]

which is strictly satisfied for \(\gamma \in (0,1)\). Equality clearly holds for \(\gamma = 0\).

iii) Differentiating the profit function (6), we get:

\[
\frac{\partial \pi^I}{\partial \gamma} = -\frac{32(1-\gamma)(8+3\gamma^2-6\gamma)}{(8-3\gamma^2)^3} \left(\frac{a}{2}\right)^2,
\]

which is strictly negative for \(\gamma \in [0,1)\) (it equals zero for \(\gamma = 1\)). Similarly, from the profit function (3) we obtain:

\[
\frac{\partial \pi^D}{\partial \gamma} = -\frac{2}{(2+\gamma)^3} \left(\frac{a}{2}\right)^2
\]

which is strictly negative for \(\gamma \in [0,1]\). \(\blacksquare\)

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\(^{11}\)More precisely, while the input price under vertical separation is independent of \(\gamma\) (see eq. (2)), it is easy to show that the input price under vertical integration is a U-shaped function of \(\gamma\) in the interval \([0,1]\).
3.1. Vertical integration and market foreclosure

Before proceeding to the previous stages of the game, we pause here to discuss the effect of product differentiation on the possibility of market foreclosure under vertical integration. Market foreclosure occurs if only the vertically integrated firm is active in the downstream market, i.e., if \( q^I(w^*_v) = 0 \).

**Proposition 1.** Vertical integration leads to market foreclosure only when products are perfect substitutes (i.e., only for \( \gamma = 1 \)).

**Proof.** From the expression \( q^I(w_v) = \frac{a(2-\gamma)-2w_v}{\gamma} \), it follows immediately that \( q^I(w_v) = 0 \) iff \( w^*_v \geq \frac{a(2-\gamma)}{2} \). Using equation (5), we find that \( w^*_v < \frac{a(2-\gamma)}{2} \) for any \( \gamma \in [0,1) \), and \( w^*_v = \frac{a(2-\gamma)}{2} \) for \( \gamma = 1 \). That is, market foreclosure occurs only for \( \gamma = 1 \). In contrast, the independent firm remains active in the market for any \( \gamma \in [0,1) \). \( \blacksquare \)

The interpretation of Proposition 1 is as follows. The integrated firm has a strategic incentive to raise the input price charged to the independent firm, since its price, production, and profit in the downstream market increase with the rival’s marginal cost. On the other hand, its sales of the essential input decrease. Intuitively, the strategic incentive to rise the rival’s cost strengthens with the degree of product substitutability (it actually vanishes if products are independent, i.e., for \( \gamma = 0 \)). According to Proposition 1, only when products are perfect substitutes the strategic incentive is strong enough to induce the integrated firm to foreclose the market and stop supplying the essential input to the rival.

Proposition 1 has an interesting implication for the previous stages of the game. Since product differentiation allows to avoid market foreclosure, it also guarantees both downstream firms positive profits under vertical integration. On one hand, the independent firm can assure a positive profit only if products are differentiated. On the other hand, by allowing the independent firm to gain a positive profit, product differentiation helps the downstream firm that vertically integrates to extract a positive profit from vertical integration even if the upstream firm has full bargaining power.
4. THE VERTICAL INTEGRATION GAME

Having solved the market stage under the two alternative vertical structures of the industry, we are now in the position to examine the incentive for vertical integration. Recall that, at the vertical integration stage, the degree of product differentiation is already determined. We start by noting that there is a positive surplus to gain from vertical integration when the integrated firm’s profits, net of the fixed cost of integration, exceed the joint profits of the two firms involved in the merger (i.e., the upstream monopolist and one downstream firm) under vertical separation. Let us denote with \( S = \pi^V - (\pi^U + \pi^D) \) the surplus from vertical integration before the integration cost, so that the profitability condition for vertical integration is:

\[
S - E > 0. \tag{8}
\]
The upstream monopolist’s gain from integrating with a downstream firm asking for an integration price \( P \) is given by:

\[
\pi^V - P - E - \pi^U = (S - E) + (\pi^D - P).
\]

When vertical integration is profitable (i.e., when condition (8) holds), each downstream firm always has an incentive to make a price-offer to be vertically integrated. If firm \( D_2 \) does not make any offer, it is convenient for firm \( D_1 \) to make an offer between \( \pi^D \) and \( \pi^D + (S - E) \). Since a positive surplus is left to the upstream monopolist\(^{12}\), the offer will be accepted, and the bidder will gain a higher profit than under the alternative of not making any offer (without any offer, vertical integration does not occur, so that both downstream firms earn \( \pi^D \)). Alternatively, if firm \( D_2 \) makes the above offer, then it is convenient for firm \( D_1 \) to undercut the rival’s offer, since \( \pi^D > \pi^I \) (see Lemma 1). Furthermore, each downstream firm has always an incentive to undercut any rival’s offer \( P_j \) greater than \( \pi^I \). By bidding above the rival, a firm ends up being the independent firm under vertical integration, earning \( \pi^I \). By matching the rival’s offer, it has equal chances of being independent or integrated, with expected profit \( \frac{1}{2}(P_j + \pi^I) \). It is then optimal to bid just below the rival, say \( P_j - \epsilon \), which assures to be integrated with a profit \( P_j - \epsilon > \frac{1}{2}(P_j + \pi^I) > \pi^I \). Thus, the unique Nash equilibrium pair of

\(^{12}\)By integrating firm \( D_1 \) at a price \( P = \pi^D + \epsilon \), with \( 0 < \epsilon < S - E \), the upstream monopolist would gain \( (S - E) - \epsilon > 0 \).
offers from the downstream firms is \((\pi^I, \pi^I)\). The upstream monopolist is left with more than the full surplus from integration\(^{13}\), so that it will certainly call for offers at the outset, and vertical integration occurs. Notice that, due to competition in price-offers to be integrated, the downstream firm that is finally integrated only reaps its outside option under vertical integration (i.e. the equilibrium profit of the independent firm).

Assume now that vertical integration is not profitable (i.e., condition (8) does not hold). In this case, since the net surplus from integration is negative, the upstream firm would reject any price-offer equal to (or greater than) \(\pi^D\).\(^{14}\) Then, if the upstream firm calls for offers, the relevant Nash equilibrium of the auction is that both downstream firms make an offer which leaves the upstream firm with negative surplus (any offer above \(\pi^D - (E - S) < \pi^D\) will do), and the upstream firm rejects.\(^{15}\) Clearly, neither downstream firm has an incentive to deviate by making a price-offer low enough to be acceptable by the upstream firm, since the deviant firm would be integrated at a price below \(\pi^D\). Hence, vertical integration does not occur, and both downstream firms earn profit \(\pi^D\). Anticipating this equilibrium outcome, the upstream firm will not ask for offers at the outset.

We have proved:

**Lemma 2.** If the net surplus from integration is positive (i.e., \(S - E > 0\)), vertical integration occurs, and the downstream firm involved in the merger earns the same profit as the independent firm, \(\pi^I\). If the net surplus from integration is negative (i.e., \(S - E < 0\)), vertical integration does not occur, so that both downstream firms earn profit \(\pi^D\).

We next characterize the vertical structure that will arise in the industry after the integration stage as a function of the degree of product differentiation and

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\(^{13}\) By integrating one of the two downstream firms at the price \(P = \pi^f\), the upstream monopolist gains: \((S - E) + (\pi^D - \pi^f) > (S - E) > 0\).

\(^{14}\) By integrating a downstream firm at a price \(P = \pi^D + \epsilon\), with \(\epsilon \geq 0\), the upstream monopolist would get \((S - E) - \epsilon < 0\) (since \((S - E) < 0\)).

\(^{15}\) A qualification of this result is in order. When condition (8) does not hold, the pair of offers \((\pi^I, \pi^I)\) still identifies a Nash equilibrium. However, from the viewpoint of the downstream firms (who are the only active players at the bidding stage of the vertical integration game), such an equilibrium is strictly payoff (pareto) dominated by the equilibrium adopted in the text. Pareto dominance is therefore a first criterion to select away the "bad equilibrium" \((\pi^I, \pi^I)\). Furthermore, the "good equilibrium" indicated in the text would also be chosen against the "bad equilibrium" \((\pi^f, \pi^f)\) by other equilibrium selection criteria. For instance, since, for any player, the "good equilibrium" strategy weakly dominates the "bad equilibrium" strategy, risk dominance would also select the "good equilibrium".
the integration cost level. By Lemma 2, this amounts to evaluate the sign of the net surplus from integration, $S - E$, along the range of product substitutability $\gamma \in [0,1]$.

**Proposition 2.**  

a) When the integration cost is low, vertical integration occurs for any degree of product differentiation.  

b) When the integration cost is moderate, vertical integration occurs only for large or for small (but not for intermediate) degrees of product differentiation.  

c) When the integration cost is high (but not prohibitive), vertical integration occurs only for large degrees of product differentiation.

**Proof.** Using equations (3), (4) and (7), the surplus from integration before the integration cost, $S$, can be written as:

$$S(\gamma) = \left(\frac{a}{2}\right)^2 \frac{8 - \gamma^2 + 2\gamma^3 + \gamma^4}{(8 - 3\gamma^2)(2 + \gamma)^2}$$  

(9)

Inspection of equation (9) suffices to show that $S(\gamma) > 0$ for any $\gamma \in [0,1]$, taking values $S(1) < S(0)$. Furthermore, we prove in Appendix 1 that $S(\gamma)$ is a U-shaped function of $\gamma$ over the interval $[0,1]$, with a minimum value for $\gamma \approx 0.61037$ (see Figure 1 below). From the shape of $S(\gamma)$, the proof of Proposition 2 is straightforward. Let us denote with $S_m$ the minimum value of $S(\gamma)$.

a) If $E < S_m$, the net surplus from integration is positive, and vertical integration occurs, for any $\gamma \in [0,1]$;

b) if $S_m < E < S(1)$, there must be two critical degrees of product substitutability, $\gamma_{b_1}$ and $\gamma_{b_2}$ (with $\gamma_{b_1} < \gamma_{b_2}$), such that the net surplus from integration is positive, and vertical integration occurs, for $\gamma < \gamma_{b_1}$ and $\gamma > \gamma_{b_2}$, while the net surplus from integration is negative, and vertical integration does not occur, for $\gamma \in [\gamma_{b_1}, \gamma_{b_2}]$;

c) if $S(1) < E < S(0)$, there must be one critical degree of product substitutability, $\gamma_c$ ($< \gamma_{b_1}$), such that the net surplus from integration is positive, and vertical integration occurs, only for $\gamma < \gamma_c$.

Finally, if $E \geq S(0)$ the net surplus from integration is negative for any $\gamma \in [0,1]$, so that vertical integration never occurs (i.e., $S(0)$ identifies a threshold level above which the integration cost becomes prohibitive).
The interpretation of Proposition 2 relies on the U-shaped behaviour of the gross surplus from integration as the degree of product differentiation decreases (see Figure 1). Notice first that a positive surplus from integration may come from two sources in our model: 1) the avoidance of double marginalisation in one segment of the final product market; 2) the cost advantage (i.e., the lower marginal cost in producing the final product) of the integrated firm over the independent firm in the downstream market competition.16

Suppose now that products are independent (i.e., \( \gamma = 0 \)). In this case, only the first source of surplus is active, since the two segments of the downstream market are isolated. As the degree of product differentiation starts decreasing (i.e., \( \gamma \) starts increasing from 0), the total demand in the downstream market starts decreasing as well, since consumers value less any bundle of the two products relative to the numeraire good. The fall in the gross surplus from integration is then explained by the lower gain from avoiding double marginalisation in a smaller market, while the second source of surplus (i.e. the cost advantage) is still irrelevant since products are almost independent. Only when the degree of product differentiation is sufficiently low, the second source of surplus plays a significant role. Then, the cost advantage of the integrated firm allows it to soften the negative effect exerted by a further increase in \( \gamma \) on the demand for its final product, since consumers tend to substitute the high priced product of the independent firm for the low priced prod-

\[16\] The cost advantage is optimally set by the integrated firm, and allows it to expand its equilibrium production and profit in the downstream market at the expense of the independent firm. Clearly, it negatively affects the component of the integrated firm’s profit arising from the sales of the essential input to the rival.
uct of the integrated firm. Moreover, the integrated firm benefits from a higher reduction of the rival’s demand while playing the Cournot game in the product market. Hence, the integrated firm has an incentive to increase the rival’s cost (by rising the input price) as the degree of product differentiation further decreases. When products are sufficiently close substitutes, the second source plays a dominant role, reversing the sign of the relationship between product differentiation and surplus from integration.

5. VERTICAL INTEGRATION AND PRODUCT INNOVATION

In this section, we analyse the effects exerted by vertical integration on the incentive to differentiate products. Recall that the two downstream firms cooperate at the innovation stage: by paying a fixed R&D cost \( k \), they can reduce the perceived degree of product substitutability, \( \gamma \), from 1 to \( \hat{\gamma} \in [0, 1) \). Products are perceived as perfect substitutes \( (\gamma = 1) \) if firms do not invest.

For a given effectiveness of the R&D investment (i.e., for a given degree of product differentiation achievable by investing in R&D, \( 1 - \hat{\gamma} \)), we measure the downstream firms’ incentive to invest in R&D by the joint-gain they would obtain by decreasing the degree of product substitutability from 1 to \( \hat{\gamma} \). This clearly corresponds to the highest R&D cost the innovative firms would be willing to pay in order to differentiate their products to the degree \( 1 - \hat{\gamma} \), which we denote with \( \hat{k}(\hat{\gamma}) \).

To proceed, notice first that, at the innovation stage, the two downstream firms share identical profit expectations under any subsequent evolution of the game (i.e., the independent firm’s profit \( \pi^I(\gamma) \) under vertical integration, the downstream firm’s profit \( \pi^D(\gamma) \) under vertical separation). Let \( \pi(\gamma) \) denote their profit expectation as a function of the degree of product substitutability. We clearly have:

\[
\hat{k}(\hat{\gamma}) = 2[\pi(\hat{\gamma}) - \pi(1)]. \tag{10}
\]

Building upon Proposition 2, we next distinguish four cases according to the level of the integration cost.

- **Low integration cost (case (a) of Proposition 2).** Vertical integration will occur at the second stage of the game for any degree of product substitutability. Therefore, independently of both the effectiveness of the R&D investment and the investment decision, both downstream firms will end up with the independent firm’s
profit under vertical integration (Lemma 2), and the expected profit function at the innovation stage coincides with the independent firm’s profit function:

\[ \pi(\gamma) = \pi^I(\gamma), \forall \gamma \in [0, 1], \]

where \( \pi^I(\gamma) \) is given by equation (6). Notice that, if the downstream firms do not invest in R&D, products are perceived as perfect substitutes by consumers and vertical integration leads to market foreclosure (Proposition 1). Hence \( \pi(1) = \pi^I(1) = 0 \), so that our measure of the incentive to invest in R&D becomes:

\[ \tilde{k}_a(\tilde{\gamma}) = 2\pi^I(\tilde{\gamma}), \forall \tilde{\gamma} \in [0, 1]. \quad (10a) \]

Since the innovative firms’ profit expectation always coincides with the profit of the independent firm under vertical integration, the incentive to invest in R&D reflects the following three motives: 1) avoiding market foreclosure; 2) softening the competitive pressure of a more efficient firm (i.e., the integrated firm); 3) forcing the integrated firm to charge a lower input price.

**Moderate integration cost (case (b) of Proposition 2).** Vertical integration occurs at the second stage of the game only for large and for small, but not for intermediate, degrees of product substitutability. Consequently, the innovative firms’ profit expectation will jump between the profit of the independent firm under vertical integration and the profit of a downstream firm under vertical separation at the extremes of the interval of product substitutability where vertical integration does not occur. Denoting such an interval with \( [\gamma_{b_1}, \gamma_{b_2}] \) (see Figure 1), we have:

\[ \pi(\gamma) = \begin{cases} 
\pi^I(\gamma) & \text{for } \gamma \in [0, \gamma_{b_1}) \\
\pi^D(\gamma) & \text{for } \gamma \in [\gamma_{b_1}, \gamma_{b_2}] \\
\pi^I(\gamma) & \text{for } \gamma \in (\gamma_{b_2}, 1]
\end{cases} \]

where \( \pi^D(\gamma) \) and \( \pi^I(\gamma) \) are given by equations (3) and (6), respectively. Like in the case of low integration cost, if the downstream firms do not invest in R&D, then vertical integration and market foreclosure occur at the final stage of the game, so that \( \pi(1) = \pi^I(1) = 0 \). Hence, our measure of the incentive to invest in R&D becomes:

\[ \tilde{k}_b(\tilde{\gamma}) = \begin{cases} 
2\pi^I(\tilde{\gamma}) & \text{for } \tilde{\gamma} \in [0, \gamma_{b_1}) \\
2\pi^D(\tilde{\gamma}) & \text{for } \tilde{\gamma} \in [\gamma_{b_1}, \gamma_{b_2}] \\
2\pi^I(\tilde{\gamma}) & \text{for } \tilde{\gamma} \in (\gamma_{b_2}, 1)
\end{cases} \quad (10b) \]

\[ \text{17 For future reference, notice that } \tilde{k}_a(\tilde{\gamma}) \text{ always increases with the effectiveness of the R&D investment, } 1 - \tilde{\gamma}, \text{ as } \pi^I(\gamma) \text{ is monotonically decreasing in } \gamma \text{ (Lemma 1).} \]
When the R&D technology allows the innovative firms to target the interval \([\gamma_{b1}, \gamma_{b2}]\), the outcome of the vertical integration game depends on their investment decision. In such a case, the incentive to invest incorporates the additional motive of preventing vertical integration (recall that, by Lemma 1, \(\pi^D > \pi^I\) for any \(\gamma \in (0, 1]\)).

**High integration cost (case (c) of Proposition 2).** Vertical integration will occur only for small degrees of product substitutability (that is, for large degrees of product differentiation). Therefore, the innovative firms’ profit expectation jumps from the profit of the independent firm under vertical integration to the profit of a downstream firm under vertical separation at the critical degree of product substitutability below which vertical integration occurs. Denoting such a critical degree by \(\gamma_c\) (see Figure 1), we have:

\[
\pi(\gamma) = \begin{cases} 
\pi^I(\gamma) & \text{for } \gamma \in [0, \gamma_c) \\
\pi^D(\gamma) & \text{for } \gamma \in [\gamma_c, 1]. 
\end{cases}
\]

Contrary to the previous cases, vertical integration and market foreclosure will not occur in the subsequent stages of the game if the innovative firms do not invest in R&D, that is, \(\pi(1) = \pi^D(1) > 0\). On the other hand, vertical integration would follow the decision to invest in R&D when the resulting degree of product differentiation is high. In other words, in the case under examination, not investing in R&D may be the only way to prevent vertical integration at the following stage of the game. Our measure of the incentive to invest in R&D becomes:

\[
\hat{k}_c(\gamma) = \begin{cases} 
2[\pi^I(\gamma) - \pi^D(1)] & \text{for } \gamma \in [0, \gamma_c) \\
2[\pi^D(\gamma) - \pi^D(1)] & \text{for } \gamma \in [\gamma_c, 1]. 
\end{cases}
\]  

\[\text{(10c)}\]

**Prohibitive integration cost (benchmark case).** If the integration cost exceeds \(S(0)\) (i.e., the gross surplus from integration when \(\gamma = 0\)), then vertical integration never occurs at the second stage of the game. Therefore, the innovative firms’ profit expectation always coincides with the profit of a downstream firm under vertical separation,

\[
\pi(\gamma) = \pi^D(\gamma), \ \forall \ \gamma \in [0, 1],
\]

and our measure of the incentive to invest in R&D becomes:

\[
\hat{k}_* (\gamma) = 2[\pi^D(\gamma) - \pi^D(1)], \ \forall \ \gamma \in [0, 1].
\]  

\[\text{(10*)}\]

The vertical integration stage of the model is, in this case, irrelevant for the downstream firms’ incentive to differentiate products, which mainly reflects the usual
motive of softening the competitive pressure of a symmetric competitor in the product market. Henceforth, we use the case of prohibitive integration costs as a benchmark to contrast the effects on product innovation arising from the threat of vertical integration which characterises the previous cases.  

We start by comparing the case of low integration costs with the benchmark case of prohibitive integration costs.

**Proposition 3.** Unless the effectiveness of the R&D investment is very low (i.e. \( \tilde{\gamma} \) is very high), the innovative firms’ incentive to invest in R&D is stronger when the integration cost is low (so that vertical integration always occurs) than when the integration cost is prohibitive (so that vertical integration never occurs).

**Proof.** From equations (10a) and (10*), \( \hat{k}_a(\tilde{\gamma}) \geq \hat{k}_*(\tilde{\gamma}) \) iff

\[
\pi^I(\tilde{\gamma}) \geq \pi^D(\tilde{\gamma}) - \pi^D(1).
\]

Using equations (3) and (6), the last inequality reduces to:

\[
16(1 - \tilde{\gamma})(6 + 3\tilde{\gamma})^2 - (5 + \tilde{\gamma})(8 - 3\tilde{\gamma}^2)^2 \geq 0.
\]

Calculations with Mathematica show that the polynomial on the LHS has only one real root within the admissible range \( \tilde{\gamma} \in [0,1) \), that is \( \tilde{\gamma}_a \approx 0.81682 \). Since \( \pi^I(0) \geq \pi^D(0) - \pi^D(1) \) (recall that \( \pi^I(0) = \pi^D(0) \), by Lemma 1, and \( \pi^D(1) > 0 \)), it must be:

\[
\hat{k}_a(\tilde{\gamma}) > \hat{k}_*(\tilde{\gamma}) \text{ for } \tilde{\gamma} \in [0, \tilde{\gamma}_a),
\]

\[
\hat{k}_a(\tilde{\gamma}) < \hat{k}_*(\tilde{\gamma}) \text{ for } \tilde{\gamma} \in (\tilde{\gamma}_a, 1].
\]

\[18\] As in the case of low integration costs, also with prohibitive integration costs the innovative firms’ incentive to innovate products, \( \hat{k}_*(\tilde{\gamma}) \), always increases with the effectiveness of the R&D investment, \( 1 - \tilde{\gamma} \), as \( \pi^D(\gamma) \) is monotonically decreasing in \( \gamma \) (Lemma 1).
Figure 2a illustrates Proposition 3. The intuition is that, when the effectiveness of the R&D investment is very low, the gain from softening the competitive pressure of a more efficient competitor (i.e., the integrated firm under vertical integration) is smaller than the gain from softening the competitive pressure of a symmetric competitor (i.e., the other downstream firm under vertical separation). Although even mild degrees of product differentiation allow to avoid market foreclosure under vertical integration, the resulting profit of the independent firm is negligible because both the cost disadvantage relative to the integrated firm and its negative effect on the independent firm’s profit remain very strong when products are poorly differentiated.\textsuperscript{19} On the contrary, when the effectiveness of the R&D investment is sufficiently high, the gain from softening the competitive pressure of the integrated firm dominates the gain from softening the competitive pressure of a symmetric competitor. The independent firm’s profit is no more negligible when products are sufficiently differentiated, since both the cost disadvantage (up to a certain degree of differentiation) and its negative impact on the independent firm’s profit sharply decrease with product differentiation. This allows the incentive to avoid market foreclosure to play the dominant role: with low integration costs (and hence vertical integration), the downstream firms can guarantee a positive profit.

\textsuperscript{19}In fact, it is easy to check that the independent firm’s profit function, $\pi^I(\gamma)$, is flat at $\gamma = 1$. 
only by investing in R&D and differentiating products, while, with prohibitive integration cost (and hence, vertical separation), they gain a positive profit also without investing in R&D.

Consider now the case of moderate integration costs. Clearly, if the innovative firms cannot target the intermediate degrees of product differentiation where vertical integration does not occur, a comparison with the benchmark case exactly replicates Proposition 3. However, when the resulting degree of product substitutability lies in the crucial interval \([\gamma_{b_1}, \gamma_{b_2}]\), the possibility of preventing vertical integration strengthens the innovative firms’ incentive to invest relative to both the benchmark case and the case of low integration costs.

**Proposition 4.** Assume that the integration cost is moderate and the R&D investment leads to the intermediate degrees of product differentiation where vertical integration is prevented. Then the innovative firms’ incentive to invest in R&D is stronger than under the alternative cases of prohibitive and low integration costs.

**Proof.** Assume that \(\tilde{\gamma} \in [\gamma_{b_1}, \gamma_{b_2}]\). From equations (10b) and (10a) we get:

\[
\hat{k}_b (\tilde{\gamma}) - \hat{k}_a (\tilde{\gamma}) = \pi^D (\tilde{\gamma}) - [\pi^D (\tilde{\gamma}) - \pi^D (1)] = \pi^D (1) > 0.
\]

This proves that the incentive to invest in R&D is stronger in the case of moderate integration costs (where vertical integration and market foreclosure can be prevented only by investing in R&D) than in the benchmark case of prohibitive costs of integration (where vertical integration never occurs).

Similarly, using equations (10b) and (10a), we get:

\[
\hat{k}_b (\tilde{\gamma}) - \hat{k}_a (\tilde{\gamma}) = \pi^D (\tilde{\gamma}) - \pi^I (\tilde{\gamma}) > 0 \text{ (by Lemma 1)},
\]

This proves that the incentive to invest in R&D is stronger in the case of moderate integration costs (where the R&D investment allows to prevent vertical integration) than in the case of low integration costs (where vertical integration always occurs).

\[ \]

Figure 2b illustrates Proposition 4.
We turn now to the case of high integration costs, where vertical integration occurs only for high degrees of product differentiation. If the effectiveness of the R&D investment is not high (i.e., if $\hat{\gamma} > \gamma_c$), the incentive to invest in R&D identically coincides with that of the benchmark case. On the contrary, if the R&D effectiveness is high (i.e., if $\hat{\gamma} < \gamma_c$), the possibility of preventing vertical integration by not-differentiating products weakens the innovative firms’ incentive to invest relative to both the benchmark case and the case of low integration costs.

**Proposition 5.** Assume that the integration cost is high and the R&D investment leads to the high degrees of product differentiation where vertical integration will occur. Then the innovative firms’ incentive to invest in R&D is weaker than under the alternative cases of prohibitive and low integration costs.

**Proof.** Assume that $\hat{\gamma} < \gamma_c$. From equations (10c) and (10*), we get:

$$\hat{k}_c(\hat{\gamma}) - \hat{k}_*(\hat{\gamma}) = [\pi^I(\hat{\gamma}) - \pi^D(1)] - [\pi^D(\hat{\gamma}) - \pi^D(1)]$$

$$= \pi^I(\hat{\gamma}) - \pi^D(\hat{\gamma}) < 0 \quad \text{(by Lemma 1)}.$$  

This proves that the incentive to invest is lower in the case of high integration costs (where the R&D investment leads to vertical integration) than in the benchmark case of prohibitive integration cost (where vertical integration never occurs).
Similarly, from equations (10c) and (10a), we have:

$$\hat{k}_c(\hat{\gamma}) - \hat{k}_a(\hat{\gamma}) = [\pi^I(\hat{\gamma}) - \pi^D(1)] - \pi^I(\hat{\gamma}) = -\pi^D(1) < 0,$$

This proves that the incentive to invest is lower in the case of high integration costs (where vertical integration can be avoided only by not-investing in R&D) than in the case of low integration costs (where vertical integration always occurs).

Figure 2c illustrates Proposition 5.

To sum up, a prospective threat of vertical integration may have either positive or negative effects on the downstream firms’ incentive to innovate products. The nature of the effects crucially depends on how product innovation affects the upstream monopolist’s incentive to vertically integrate a downstream firm. When vertical integration is an unavoidable outcome because of low integration costs, the innovative firms’ incentive for product differentiation is stronger than under the benchmark case of prohibitive integration costs only if products can be sufficiently differentiated. With moderate integration costs, the incentive to differentiate products incorporates the strategic motive of preventing vertical integration. Both strong and weak degrees of product differentiation foster the upstream monopolist’s
incentive to vertically integrate. Then, the downstream firms have a strategic incentive to target intermediate degrees of product differentiation in order to prevent the upstream monopolist from reaping downstream profits through vertical integration. Finally, high integration costs impede vertical integration unless products are strongly differentiated. This gives the downstream firms a strategic incentive to avoid high degrees of
differentiation.

The following two examples may help to further illustrate our results.

**Example 1.** Suppose that the R&D investment allows the innovative firm to obtain (exactly) the degree of product substitutability \( \hat{\gamma} = \gamma_b \) (see Figure 2b). Given the "point-to-point" nature of the R&D technology, in equilibrium we will observe either no-differentiation (i.e., \( \gamma = 1 \)) if the innovative firms do not invest in R&D, or the degree of product differentiation \( 1 - \gamma_b \) (i.e., \( \gamma = \gamma_b \)) if the innovative firms do invest. Let the R&D cost, \( k_1 \), be sufficiently high such that \( k_a(\gamma_b) < k_1 < k_b(\gamma_b) \). Then, inspection of Figure 2b immediately reveals that we will observe product differentiation in the downstream market only when moderate integration costs give the innovative firms a strategic incentive to invest in R&D in order to deter vertical integration.

**Example 2.** Suppose that the R&D investment allows the innovative firms to reduce the degree of product substitutability up to a minimum level \( \hat{\gamma}_2 \), with \( \hat{\gamma}_2 \) slightly lower than \( \gamma_c \) (see Figure 3). Hence, if the innovative firms decide to invest in R&D, they can also select the optimal degree of product differentiation in the range \( (0, 1 - \hat{\gamma}_2] \). Let the fixed R&D cost, \( k_2 \), be sufficiently low such that \( k_2 < k_c(\gamma_c) \). Clearly, the innovative firms will choose the degree of differentiation that maximize \( \hat{k}(\hat{\gamma}) - k_2 = \pi(\hat{\gamma}) - \pi(1) - k_2 \). Then, inspection of Figure 3 immediately reveals that, whilst products will be differentiated in all cases, the innovative firms will select the maximum degree of differentiation (i.e., \( 1 - \hat{\gamma}_2 \)), only in the cases of low and prohibitive integration costs. On the contrary, the incentive to prevent vertical integration will lead them to select lower degrees of differentiation in both cases of high and moderate integration costs (i.e., \( 1 - \gamma_c \) and \( 1 - \gamma_b \), respectively).

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20 In this case, the surplus from integration exceeds the integration cost only when the market size is wide because products are strongly differentiated.
5.1. A new social cost of vertical integration

The above analysis suggests that a threat of vertical integration faced by innovative firms vertically related to a monopolistic supplier may decrease welfare by discouraging socially valuable innovations. The simplest way to show this is to reconsider Example 2, where the innovative firm can select the optimal degree of product differentiation up to a maximum level $1 - \gamma_2$. Suppose that the integration cost is high enough to make the innovative firms’ incentive to prevent vertical integration active (that is, cases b) or c) in Figure 3). As we have seen before, the innovative firm will deter vertical integration by choosing a lower degree of product differentiation relative to the benchmark case where the threat of vertical integration is absent. If we re-interpret the benchmark as the case of a severe antitrust policy which bans vertical mergers, we can say that a lenient antitrust policy would cause a lower degree of product differentiation, while the vertical structure of the market, that is, vertical separation, would be identical under the two policy regimes. On the other hand, it is easy to prove that, given vertical separation, social welfare, as measured by the total surplus in the market, increases with product differenti-
Consider first industry profits. The equilibrium profits in the downstream market increase with product differentiation (see Lemma 1 (point iii)). Similarly, equation (4) clearly shows that the equilibrium profit of the upstream monopolist also increases with product differentiation. Hence, industry profits are higher with more differentiation. Consider now the consumer surplus. As shown in Appendix 2, the consumer surplus can be expressed in terms of the equilibrium quantities as:

\[ CS = \frac{1}{2} \left[ (q_1^D)^2 + (q_2^D)^2 + 2\gamma q_1^D q_2^D \right]. \]

Since \( q_1^D = q_2^D = \frac{a_2}{2 + \gamma} \) in the symmetric equilibrium arising under vertical separation, we have:

\[ CS = (1 + \gamma) \left( \frac{a_2}{2} \right)^2 \left( \frac{1}{2 + \gamma} \right)^2. \]

Then, we evaluate:

\[ \frac{\partial CS}{\partial \gamma} = -\left( \frac{a_2}{2} \right)^2 \frac{\gamma}{(2 + \gamma)^3} < 0, \]

that is, the consumer surplus increases with product differentiation. Intuitively, consumers’ preference for variety and the increase in equilibrium quantities compound to increase consumers’ welfare even if equilibrium prices increase.

6. CONCLUSIONS

In this paper we have studied vertical integration and product innovation as interdependent strategic choices of vertically related firms. Our main innovation with respect to the previous literature on vertical integration is that we have considered product differentiation as a non-production strategic decision of the downstream firms, showing its impact on the incentive for vertical integration and market foreclosure. Our main innovation relative to the literature on product innovation, is that, besides product market competition, we have accounted for another source of competition capable of affecting product innovation by innovative firms vertically related to a monopolistic supplier, i.e., the threat of vertical integration. Due to the downstream firms’ inability to commit to a cooperative behaviour if asked for integration offers, the monopolistic supplier can use vertical integration as a means to reap profits in the downstream market. As a consequence, the incentive to differentiate products in the downstream market incorporates the strategic motive

\[ 21 \text{ More precisely, the expression above gives the consumer surplus as a function of the consumer’s optimal demands of goods } q_1 \text{ and } q_2 \text{ at given prices.} \]

26
of preventing vertical integration, which may lead to less innovation in the downstream market. Therefore, instead of market foreclosure, less product innovation may be the social cost of a lenient antitrust policy which allows vertical integration in innovative markets.

REFERENCES


Appendix 1

We prove that the surplus form integration before the integration cost, $S$, is a U-shaped function of the degree of product differentiation over the range $\gamma \in [0, 1]$. From equation (9), we calculate:

$$\frac{\partial S(\gamma)}{\partial \gamma} = \left(\frac{a}{2}\right)^2 2 \frac{(-64 + 32\gamma + 96\gamma^2 + 40\gamma^3 - \gamma^4 - 3\gamma^5)}{(8 - 3\gamma^2)^2 (2 + \gamma)^4}.$$

Since $(8 - 3\gamma^2)^2 (2 + \gamma)^3 > 0$ for $\gamma \in [0, 1]$, we find that the polynomial on the RHS has a unique real root within the admissible range $[0, 1]$, that is $\gamma_m \approx 0.61037$. Since $\frac{\partial S(\gamma)}{\partial \gamma}$ is continuous over $[0, 1]$, and takes values $\frac{\partial S(\gamma)}{\partial \gamma}|_{\gamma=0} = -0.25 \left(\frac{a}{2}\right)^2 < 0$ and $\frac{\partial S(\gamma)}{\partial \gamma}|_{\gamma=1} = 0.2963 \left(\frac{a}{2}\right)^2 > 0$, then it must be negative for $\gamma < \gamma_m$ and positive for $\gamma > \gamma_m$. Finally, with simple calculations we get: $S(0) = \frac{1}{4} \left(\frac{a}{2}\right)^2$ and $S(1) = \frac{10}{36} \left(\frac{a}{2}\right)^2$, so that $S(0) > S(1)$.

Appendix 2

The representative consumer’s optimisation problem is:

$$\text{Max } U = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + m$$

s.t. $p_1 q_1 + p_2 q_2 + m = I$

where $I$ is the consumer’s income in units of the numeraire good ($m$).

From the first order conditions $p_i = a - q_i - \gamma q_j$ ($i, j = 1, 2; i \neq j$) and the budget constraint, we get:

$$m = I - a(\hat{q}_1 + \hat{q}_2) + (\hat{q}_1^2 + \hat{q}_2^2 + 2\gamma \hat{q}_1 \hat{q}_2),$$

where $\hat{q}_i$ denotes the consumer’s optimal demand of good $i$ at given prices.

Substituting for $m$ into the utility function, we get:

$$\hat{U} = I + \frac{1}{2}(\hat{q}_1^2 + \hat{q}_2^2 + 2\gamma \hat{q}_1 \hat{q}_2).$$

Finally, the consumer surplus is:

$$CS = \hat{U} - I = \frac{1}{2}(\hat{q}_1^2 + \hat{q}_2^2 + 2\gamma \hat{q}_1 \hat{q}_2).$$