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TUITION FEES AND ADMISSION STANDARDS: HOW DO PUBLIC AND PRIVATE UNIVERSITIES REALLY COMPETE FOR STUDENTS?

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Tuition fees and admission standards: how do public and private universities really compete for students?*

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Abstract

We study a market where two universities, a public and a private one, compete for students by setting admission standards. Students differ in ability and receive a wage premium for participating in higher education. This wage increases with the quality of the university attended. The private university maximizes profits, the public university maximizes welfare. We show that there is no "same-standard" equilibrium. In a specific example we show that multiple equilibria can exist. In one equilibrium the private university sets a higher admission standard, and in the other equilibrium the public university sets a higher admission standard.

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1 Introduction

The market for education displays several distinctive features. Often, large public and private sectors coexist. In addiction, higher education provision is an example of a customer-input technology (Rothschild and White (1995)): the production of a good or service depends on the characteristics of the customers. Allocation mechanisms are not limited to pricing: devices like exams are often used by higher education institutions to ration demand and to determine the offers of places made to future students. In fact, because of these features standard theoretical results may no longer apply. We observe that universities do compete for students and they do so by setting prices and admission policies. Bearing this in mind, we study the higher education market, concentrating on the competition between private and public universities for students.

Compared with other aspects of the economics of education, models of competition between universities are scarce in the literature.[1] Like Del Rey (2001) and De Fraja and Iossa (2002) we also use a duopoly model of university competition, but unlike these papers where the two universities considered are identical, we model specifically the effects of different objectives pursued by private and public universities. The interaction between these types of education institutions has received little attention. One exception is Epple and Romano (1998) where, however, the public sector does not behave strategically and offers lower quality education. The later is a good first approximation of the American reality, but in Europe, tuition fees are often very low in public universities, and many private universities are often considered to be of lower quality than their state run equivalents.[2]

In this paper we abstract from all differences between public and private universities except for their objectives. The private university maximizes profits, while the public maximizes social welfare. Both can make strategic choices to further their objectives, specifically: they both can choose their students by imposing a standard for admission; in addition the private institution can choose its price, while the state university is constrained to

[1] De Fraja (2001) provides a review of the literature on private and public schools.

[2] A good example: Portugal has a private sector with a similar size to the American one (measured by the number of students enrolled in private higher education): 31% in the USA and 36% in Portugal, but here public universities set high admission thresholds and charge minimal tuition fees.

charging tuition fees imposed by the state educational authority. Students decide to enroll in the university that maximizes their wage minus attendance costs. This wage depends on their individual ability and the quality of their education. Quality, in turn, is determined by the average ability of the student body.

We study a simultaneous move game between the universities and show that there is no "same-standard"[3] equilibrium: in no equilibria the standards set are such that the least able student in each university is of the same ability. If equilibria exist, they must be asymmetric. We therefore proceed to show, with a specific example, that multiple asymmetric equilibria can happen for a set of reasonable values of the parameters. This multiplicity may correspond to the different patterns observed in higher education markets in various countries. The intuition can be described as follows. Suppose the public university (which must set a low, or zero, tuition fee) imposes a high admission standard. Faced with this the private university has two options: it can either set a lower standard, thus meeting the unsatisfied demand from students not able to pass the public university's admission threshold; or it can set a stricter standard and admit few high ability students who can be charged a higher fee because they receive higher quality education than in the public university. For sufficiently high standard set by the state university, the former may be preferable. On the other hand, for the same parameters, if the public university sets a lower standard, the private university will be able to choose a stricter admission threshold and charge correspondingly high fees.

The model is presented in Section 2, with the specifics of students and universities detailed in separate subsections. In Section 3 we show that a "same-standard" equilibrium cannot exist and describe the two types of asymmetric equilibria. A computed example is provided and analyzed in Section 4. The final section concludes.

2 The model

A population of potential university students who differ in their ability is considered. A private and a state university supply higher education. They

[3]The term symmetric equilibrium cannot strictly be applied here as the universities are different. In Subsection 3.2 this is further discussed.

set standards of admission and prices, although the state one must charge a price decided by the educational authority, and is normalized to zero. Each university sets a minimum admission score required for a student to be accepted. We assume it is possible to represent the admission requirement in one dimension and call it the standard of admission.[4] Let this standard be denoted by $x_i, i = p, s$ and let it vary in the subset of the real line X: $x_i \in X \subseteq \mathbb{R}, i = p, s$, where the subscripts p and s refer to private and state university respectively. A student can enroll in university i if and only if her admission score is x_i or higher. The price or tuition charged by the private university is denoted by t.

2.1 The students

Ability is measured by a parameter $\theta \in [0, 1]$. The students' distribution by ability is given by the differentiable function $F(\theta)$, with $F(0) = 0$, $F(1) = 1$, with density $f(\theta) = F'(\theta)$.

Assumption 1 *There are no borrowing constraints in the market for higher education.*

If this were not the case, ability to pay the tuition fees would depend to some extent on parental income or some measure of family wealth, and a second dimension of differentiation of students would need considering.[5] This also tallies with empirical evidence: Carneiro and Heckman (2002) provide a discussion on the topic suggesting that short run income constraints play a minor role in explaining gaps in university enrollment by family income in

[4]The practice shows that admission can be determined by an exam or a composite measure of students' quality. Admission policies in many American universities require a comprehensive evaluation of each applicant based on all the information available, including letters of reference, essays, previous grade point averages, standardized test scores, extracurricular activities, and sports performance, just to mention a few. We can find examples of these policies described in detail in the American Supreme Court decisions on the affirmative action landmark case of Regents of University of California v Bakke (www.landmarkcases.org/bakke/home.html), and more recently the cases against the University of Michigan (Grutter v Bollinger *et al* and Gratz *et al* v Bollinger *et al*).

[5]Fernandez (1998) provides a theoretical approach of the properties of exams and tuition fees as alternative allocation devices under borrowing constraints, assuming no peer effects. Romero and Del Rey (2004) analyze competition between public and private universities under borrowing constraints. Their result that the public university is of higher quality than the private one rests heavily on students being credit constrained. By lifting this restriction we can analyze the robustness of their result.

the US; for the UK, Dearden *et al.* (2004) also find little evidence that the decision to participate in higher education is credit constrained.

Higher education determines a wage premium in expected lifetime earnings. Evidence on wage differentials between secondary school and university graduates strongly supports this statement.[6] We also assume that the wage is an increasing function of the student's own ability and the quality of the higher education received.

Firstly, we assume that wages are an increasing monotonic function of ability. The return to schooling has risen throughout the eighties in the US and several authors have argued that a substantial part of that increase is related to higher returns to ability.[7] Although early work on the size of ability bias (Chamberlain and Griliches (1975)) reported it to be small, more recently, Herrnstein and Murray (1994) sustain that, with an increasing application of meritocratic principle in education, the interaction between ability and schooling has become stronger and with it the importance of the ability bias. Furthermore, recent evidence on the returns to cognitive ability (Galindo-Rueda and Vignoles (2005)) also provides empirical support for the assumption.

Regarding the role of quality, Brewer *et al* (1999) find evidence of a significant, and increasing with time, economic return to attending an elite private university. For the UK, Chevalier and Conlon (2003) also find that there is a wage premium attached to degrees from more prestigious universities. The quality of higher education has largely been measured using indicators of the academic ability of the student body. It is common to assume that students' productivity is likely to improve in the presence of abler and more motivated peers. Epple and Romano (1998), de Bartolomé (1990), and Nechyba (2000) point out the importance of peer group effects in the context of education analyzing its role in theoretical models of school choice. The existence of such effects is now established in the literature, although there is still a wide debate about the form of such effects and the correct way

[6] Greenaway and Haynes (2003) report that university graduates earn around £400,000 more over their lifetime than non-graduates. In the UK, the empirical evidence on wage equations points to returns to higher education qualifications for individuals with at least one A-level that range from 15-26% for men and 26-43% for women (Blundell *et al.*(2000)).

[7] Taber (2001) finds results that suggest that the growing university wage premium in the last twenty years is mainly brought about by an increase in the demand for unobserved ability.

of testing them empirically.[8] The measure of educational quality considered is the average ability of the students in the university attended, $\overline{\theta}_i$, $i = p, s$. Formally:

Assumption 2 *The wage of a student of ability θ associated with higher education is given by $w(\theta, \overline{\theta})$, where $\overline{\theta}$ is the average ability of the students in the university attended; the function $w(\theta, \overline{\theta})$ satisfies $w_\theta, w_{\overline{\theta}} > 0, w_{\theta\theta} \leq 0, w_{\overline{\theta}\overline{\theta}} \leq 0$.*

We also assume, for convenience, that the wage function satisfies a weak single crossing property in ability: in the space of wage and ability, the wage curve for a higher quality university intersects that of a lower quality university from below. This assumption implies that the benefit of an increase in the peer group quality (or simply the university quality) is weakly higher for brighter students. It captures the idea that brighter students can better take advantage of a better peer group.

Assumption 3 $w_{\theta\overline{\theta}} \geq 0$.

The students who do not participate in higher education expect to receive a reservation future wage w_0. A student of ability θ_j chooses to attend university i if the future lifetime wage net of tuition is higher than in any of the other two alternatives: going to the other university or not attending university at all.

2.2 The universities

Universities incur a cost of educating their students which depends only on the number of students they enroll, $n \in [0, 1]$. Denoting it as $c(n)$ we assume that it satisfies: $c'(n) > 0$ for $n > 0$ and $c'(0) = 0$, $c''(n) \geq 0$, and that there are no fixed costs so $c(0) = 0$. We do not posit any difference in the cost technologies of the two universities. This allows us to isolate the effects of different objective functions in the results.

We consider the game where the universities choose simultaneously the standards of admission and the level of private tuition fee. Unlike other types of firms where adjustments in the quality of the product require more time to become effective, so there is a clear case for choosing prices first, in this setting the decisions on tuition and admission are likely to be taken contemporaneously.

[8] Empirical evidence on peer effects is provided by Epple *at al* (2003), Robertson and Symons (2003), and Sacerdote (2001).

2.2.1 Private university

The private university aims to maximize its profits by setting tuition fees and admission standards.[9] Universities can seek to maximize profits of their teaching activity, so that they can use the available funds for research or to raise the profile of the institution.

The private university problem can be written as:

$$\max \ \pi_p = tn_p - c_p(n_p) \tag{1}$$

The tuition fee charged to the students does not depend on individual characteristics. That is, unlike Epple and Romano (1998), private universities do not attract poorer but brighter students with lower or even negative fees (scholarships). In our model where income does not vary across households, this possibility would be redundant. Moreover, this reflects the systems in place in many countries, in several European countries tuition costs are fixed.[10]

2.2.2 Public university

The degree of autonomy of public universities varies across countries, but we believe that it is uncontroversial that somehow public universities have to follow governments' directives. The objective of the government is to maximize social welfare, measured by total income minus the costs of educating the students in the public university, including the marginal cost of public funds employed, $\lambda \geq 0$. The source of this additional cost is the distortionary and administrative costs of tax systems, thus raising the costs of public provision (this has been often estimated empirically, see Browning (1987)). The public university does not control the tuition charged in the public sector, but we consider the case where this fee is lower than the one

[9] Epple and Romano (1998) also use such a function to describe the universities' behavior, but point out that considering alternative objective functions can make sense given the existence of a significant proportion of non-profit schools. Of course, this does not mean that they are not institutions allowed make profits, but simply that profits cannot legally be distributed to outsiders. Alternatively, De Fraja and Iossa (2002) maximize a measure of the prestige of the institution, which is related to the number and quality of its students and expenditure in research activities.

[10] Data from the European Commission shows that in the majority of the countries students pay registration and tuition fees that can differ by degree course, but which do not usually depend on individual ability.

charged by the private university.[11] The tuition in the state university does not affect the welfare function as it enters both as a revenue for the state sector and a cost to the students, so to ensure that in every equilibrium it is lower than private tuition we normalize it to 0. The objective function of the state university is given by:

$$\pi_s = \int_{\theta\in\Theta_0} w_0 d\theta + \int_{\theta\in\Theta_p} (w_p - t) d\theta + \int_{\theta\in\Theta_s} w_s d\theta - (1+\lambda) c_s(n_s) \qquad (2)$$

The sets $\Theta_i, i = 0, p, s$, denote the range of possible ability respectively for the individuals that do not participate in higher education, the ones that attend the private university, and the ones that enroll in the public university. These sets are derived below in Subsection 3.3.

3 Results

3.1 Students' behavior

It is natural to assume that the probability of admission is an increasing function of ability: more able students are more likely to pass a certain test. Since our focus is on the competition between universities rather than the test technology, we take a very simple formulation for this probability. For any given standard x set by a university, there exists a function $\theta^t(x)$ such that the probability that student j passes the test is 0 if $\theta_j < \theta^t(x)$ and is 1 if $\theta_j \geq \theta^t(x)$. Our first result is that stratification by ability of students occurs under the assumptions of the model.

Proposition 1 *Let x_p and x_s be given, with $x_p \neq x_s$. Then there exist $\widehat{\theta}(x_p, x_s, t)$ and $\widetilde{\theta}(x_p, x_s, t)$, with $\widehat{\theta}(.) < \widetilde{\theta}(.)$ and $\widetilde{\theta}(.) \leq 1$, such that:*

a) all students with ability $\theta \geq \widetilde{\theta}$ attend the university that sets the higher standard,

b) all students with ability $\widehat{\theta} < \theta \leq \widetilde{\theta}$ attend the university that sets the lower standard,

c) all students with ability $\theta < \widehat{\theta}$ do not go to university.

[11] Traditionally, in Europe, there has been free university education. The trend now is to have increasing tuition fees in the public sector, but there is still a significant gap between public and private tuition.

Proof A student with ability $\theta \geq \theta^t(\max\{x_p, x_s\})$ has three options. She will choose the one that yields a higher wage:

$$\max\{w_0, w(\theta, \overline{\theta}_s), w(\theta, \overline{\theta}_p) - t\}$$

where $\overline{\theta}_p$ and $\overline{\theta}_s$ are the average abilities of students enrolled in the private and state university respectively. Now, note that, since $w_{\theta\overline{\theta}} \geq 0$, then if student with ability $\theta^t(\max\{x_p, x_s\})$ chooses one university, then all students with ability above her also choose the same university. This shows stratification above $\widetilde{\theta}(.)$.

Next consider students with ability $\theta \in [\theta^t(\min\{x_p, x_s\}), \theta^t(\max\{x_p, x_s\})[$. These students can choose to go to the university that sets the lower standard or not to go. They decide to attend university as long as the wage net of tuition (if the private university is setting the lower standard) is greater then w_0. As $w_\theta > 0$, and w_0 is constant, under Assumption 3 there exists $\widehat{\theta}$ correspondent to the point where $w(\theta, \overline{\theta}_p) - t$ or $w(\theta, \overline{\theta}_s)$ intersect $w = w_0$. If $\widehat{\theta} < \widetilde{\theta}$, students with ability above $\widehat{\theta}$ go to the lower standard university while the ones with ability below this threshold do not go. If $\widehat{\theta} > \widetilde{\theta}$, these students do not go to university.

Students with ability $\theta < \theta^t(\min\{x_p, x_s\})$ do not go to university.■

3.2 "Same-standard" Nash Equilibrium

Since the players have different strategy spaces, we cannot define a symmetric Nash equilibrium. In this subsection we investigate whether there are equilibria such that the lowest ability student accepted in both universities has the same ability. We call this "same-standard" equilibrium. This does not necessarily mean that the standard of admission is the same. The next proposition establishes the impossibility of such equilibrium.

Proposition 2 *There is no "same-standard" equilibrium in pure strategies where the universities set strictly positive standards.*

Before moving to the general case, we can prove a useful result under a strict single crossing condition of the wage function:

Lemma 1 *If $w_{\theta\overline{\theta}} > 0$, for every level of ability θ, there is a single level of ability that makes the students indifferent between going to private or state university. All the students with ability above that level prefer to attend the private university.*

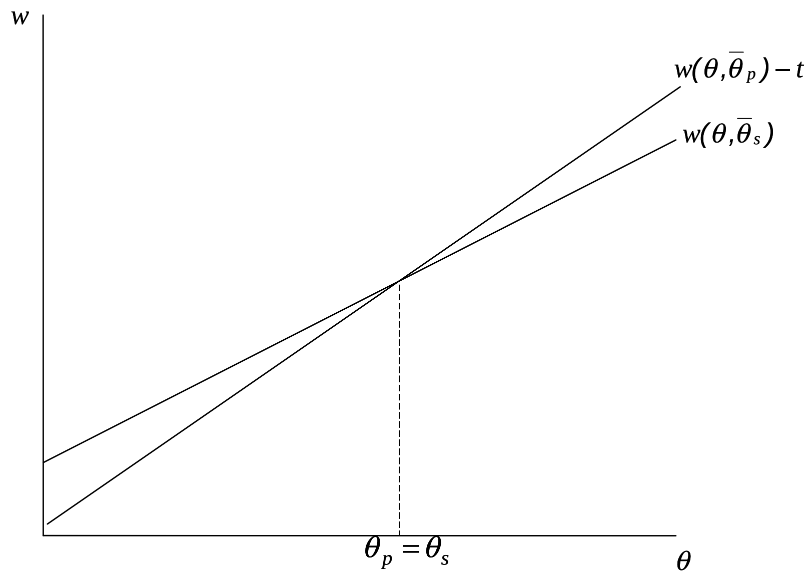

Figure 1: Illustration of Lemma 1

Proof Let the standards set by the universities, x_p and x_s, be such that the ability of the least able student still satisfying those standards is $\theta_p = \theta_s$. From Assumption 2 we know that if a student is indifferent between going to a private or a public university it must be the case that $\overline{\theta}_p > \overline{\theta}_s$. On the other hand, with $w_{\theta\overline{\theta}} > 0$ the wage functions cross only once, so all the students of ability higher than $\theta_p = \theta_s$ strictly prefer to enroll in the private university. Figure 1 illustrates that case.■

We can now proceed to prove Proposition 2.

Proof Suppose that there is a "same-standard" equilibrium. Let $\theta^* = \theta_p = \theta_s$ denote the equilibrium ability level of the least able student. It must be that students of this ability are indifferent between going to the two universities. Because the distribution of ability is differentiable, there exists $\delta > 0$ such that students of ability $\theta \in [\theta^*, \theta^* + \delta]$ are also indifferent. By Lemma 1 we know that if $w_{\theta\overline{\theta}} > 0$ this is not the case.

Let now $w_{\theta\overline{\theta}} = 0$ for some θ. Note that the argument given in Lemma 1 applies to any right neighborhood of θ^*, therefore lets consider that $w_{\theta\overline{\theta}} = 0$ in a right neighborhood of θ^*. Let $p(\theta)$ be the proportion of students of ability θ that attend the state university and $\psi < \delta$ such that:

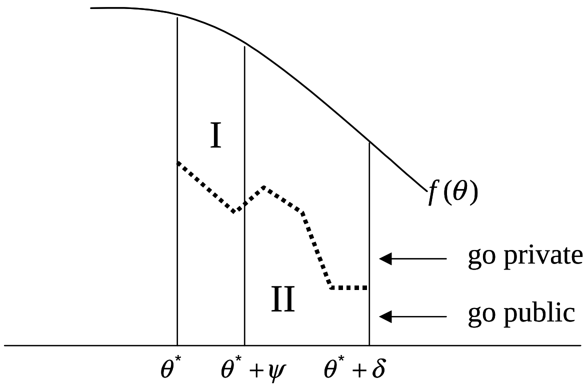

Figure 2: Illustration of condition (5)

$$\int_{\theta^*}^{\theta^*+\psi} (1 - p(\theta))f(\theta)d\theta = \int_{\theta^*+\psi}^{\theta^*+\delta} p(\theta)f(\theta)d\theta \tag{3}$$

To clarify condition (3) consider Figure 2. Above the dotted line we have the proportion of students of each ability level that go to the private university, while the proportion of students below, $p(\theta)$, go to the public university. It is then possible to find ψ such that sections I and II have the same area, therefore satisfying condition (3). If the private university increases the standard by ψ it will capture all students in the interval $[\theta^* + \psi, \theta^* + \delta]$. The cost of this move is null as the number of students it now attracts is the same as the one of students lost (given by areas II and I in the picture), as they can no longer meet the new standard. The average ability of the students attending the private university increases, so this university can charge higher tuition and increase profit. So there is an incentive to deviate from θ^*, hence it cannot be an equilibrium.■

Intuitively it seems reasonable to think that a "same-standard" equilibrium could not occur. A standard that leads to least able students accepted of the same ability gives a signal that the quality in both universities is the same. If so, there is no reason to pay higher tuition in a private university and get a degree of the same quality as one that could be achieved in the public university.

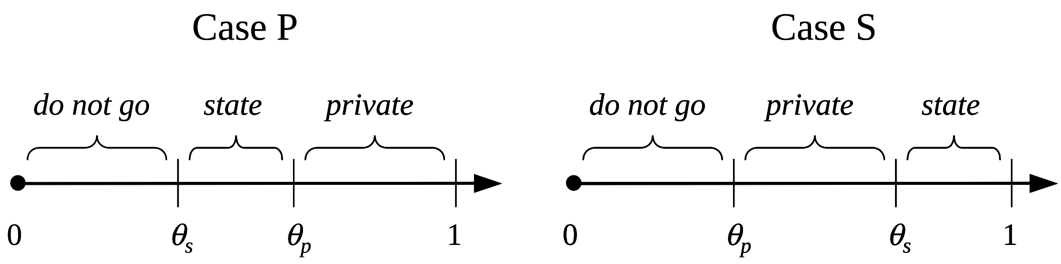

Figure 3: Stratification of students by ability in each case

3.3 Asymmetric Nash Equilibria

Having excluded the possibility of "same-standard" equilibrium, we turn to situations of asymmetry. Only two possible equilibrium configurations need to be considered given our result of stratification by ability (Proposition 1). Figure 3 depicts both these patterns in the space of ability.

- Case P: the stratification pattern is such that, the lowest ability students do not go to university, the students with intermediate ability go to the state university, and the higher ability students choose to attend the private university;
- Case S: the stratification pattern holds that the lowest ability students do not participate in university, the intermediate ability students go to the private university, while the higher ability ones enroll in the state university.

3.3.1 Case P: $x_p > x_s$

Let θ_i, $i = p, s$, be the ability level of the lowest ability students accepted in university i. In this stratification pattern the number of students in each university is:

$$n_p(\theta_p) = 1 - F(\theta_p) \qquad n_s(\theta_p, \theta_s) = F(\theta_p) - F(\theta_s) \tag{4}$$

The quality of each university, measured by the average ability of the student body, is:

$$\overline{\theta}_p(\theta_p) = \frac{\int_{\theta_p}^{1} \theta f(\theta) d\theta}{1 - F(\theta_p)} \qquad \overline{\theta}_s(\theta_p, \theta_s) = \frac{\int_{\theta_s}^{\theta_p} \theta f(\theta) d\theta}{F(\theta_p) - F(\theta_s)} \tag{5}$$

The private university charges a tuition fee to all students such that the least able student admitted is indifferent between attending the private or the public university:

$$t(\theta_p, \theta_s) = w(\theta_p, \overline{\theta}_p(\theta_p)) - w(\theta_p, \overline{\theta}_s(\theta_p, \theta_s))$$

Substituting $t(.)$ in the private university's objective function, we can write the problem as:

$$\max_{\theta_p} \pi_p = \{w(\theta_p, \overline{\theta}_p(\theta_p)) - w(\theta_p, \overline{\theta}_s(\theta_p, \theta_s)\}n_p(\theta_p) - c(n_p(\theta_p)) \quad (6)$$

The state university sets a lower standard, so it is able to supply higher education to students of ability in the interval $[\theta_s, \theta_p[$. The public university maximizes social welfare solving:

$$\begin{aligned} \max_{\theta_s} \ \pi_s = & \int_0^{\theta_s} w_0 d\theta + \int_{\theta_s}^{\theta_p} w\left(\theta, \overline{\theta}_s(\theta_p, \theta_s)\right) d\theta + \\ & + \int_{\theta_p}^{1} \left(w\left(\theta, \overline{\theta}_p(\theta_p)\right) - t(\theta_p, \theta_s)\right) d\theta - (1+\lambda)\, c\left(n_s(\theta_p, \theta_s)\right) \quad (7) \\ s.t \quad & w\left(\theta_s, \overline{\theta}_s(\theta_p, \theta_s)\right) \geq w_0 \end{aligned}$$

3.3.2 Case S: $x_p < x_s$

In the second equilibrium configuration public university has a higher standard of admission. Again, by stratification we have the students with ability between $[\theta_s, 1]$ attending the public university, the students of ability in the interval $[\theta_p, \theta_s]$ going to private higher education, and the remaining students not enrolled in this level of education. The number of students in each type of education is:

$$n_p(\theta_p, \theta_s) = F(\theta_s) - F(\theta_p) \qquad n_s(\theta_s) = 1 - F(\theta_s) \quad (8)$$

In this setting the average ability in each university is:

$$\overline{\theta}_p(\theta_p, \theta_s) = \frac{\int_{\theta_p}^{\theta_s} \theta f(\theta) d\theta}{F(\theta_s) - F(\theta_p)} \qquad \overline{\theta}_s(\theta_s) = \frac{\int_{\theta_s}^{1} \theta f(\theta) d\theta}{1 - F(\theta_s)} \quad (9)$$

It is possible to determine the tuition fee that the private university can charge by finding the maximum the least able accepted student is willing to pay to enroll:

$$t(\theta_p, \theta_s) = w(\theta_p, \overline{\theta}_p(\theta_p, \theta_s)) - w_0$$

Therefore the private university solves:

$$\max_{\theta_p} \pi_p = \{w(\theta_p, \overline{\theta}_p(\theta_p, \theta_s)) - w_0\} n_p(\theta_p, \theta_s) - c(n_p(\theta_p, \theta_s)) \tag{10}$$

The public university sets the higher standard, so the problem can be described by the following expression:

$$\begin{aligned} \max_{\theta_s} \ \pi_s = & \int_0^{\theta_p} w_0 d\theta + \int_{\theta_p}^{\theta_s} \left(w\left(\theta, \overline{\theta}_p(\theta_p, \theta_s)\right) - t(\theta_p, \theta_s)\right) d\theta + \\ & + \int_{\theta_s}^{1} w\left(\theta, \overline{\theta}_s(\theta_s)\right) d\theta - (1+\lambda)\, c\left(n_s(\theta_s)\right) \qquad (11) \\ s.t \quad & w\left(\theta_s, \overline{\theta}_s(\theta_s)\right) \geq w(\theta_p, \overline{\theta}_p(\theta_p, \theta_s)) - t(\theta_p, \theta_s) \end{aligned}$$

4 The example

To show that the asymmetric equilibrium can exist for a set of parameter values we will specify functional forms for the equations in the model. As the objective of the paper is to show that both configurations of equilibria can be found for the same parameter set this is sufficient. The following functions are considered:

$$w(\theta_i, \overline{\theta}_i) = \theta_i + a\overline{\theta}_i \ , \qquad i = p, s \tag{12}$$

$$c(n_i) = \frac{c}{2} n_i^2, \qquad i = p, s \tag{13}$$

$$F(\theta) = \theta, \qquad f(\theta) = 1 \tag{14}$$

The wage function considered is a linear function of own ability and university peer quality, where the parameter a reflects the strength of the second effect, with $0 < a < 1$. The reservation wage is normalized to zero, $w_0 = 0$, and no shadow cost of public funds is considered, $\lambda = 0$.

4.1 Case P

Using the specific functions $(12 - 14)$ we can write the objective functions of the two universities (6) and (7). With those we can solve the game and find sets of values for the parameters where the solutions exist.

$$\pi_p = \frac{1}{2}(\theta_p - 1)(c - a - c\theta_p + a\theta_s) \tag{15}$$

$$\pi_s = \frac{1}{2} \int_{\theta_s}^{1} (2\theta + a\theta_p + a\theta_s)\, d\theta - \frac{c}{2}(\theta_p - \theta_s)^2 \tag{16}$$

Proposition 3 *When a Nash equilibrium exists the universities will set standards that imply that the ability of the least able students offered a place in each university is:*

$$\theta_p^{NP} = \frac{4c^2 + c\,(2a+4) - a^2 - 2a}{4c^2 + c\,(2a+4) + a^2} \quad \textit{and} \quad \theta_s^{NP} = \frac{4c^2 + a^2 - 2ac}{4c^2 + c\,(2a+4) + a^2}$$

Proof Solving for the first order conditions of equations (15) and (16), the best reply functions are:

$$\begin{aligned} \theta_p^{BR} &= \left(1 - \frac{a}{2c}\right) + \frac{a}{2c}\theta_s \\ \theta_s^{BR} &= \frac{a}{2\,(a+c+1)} + \left(\frac{2c-a}{2\,(a+c+1)}\right)\theta_p \end{aligned}$$

The Nash equilibrium satisfies these two equations simultaneously.■

4.2 Case S

To find the specific expressions for the universities objective functions we substitute $(12 - 14)$ in the equations (10) and (11). The simplified objective functions are:

$$\pi_p = \frac{1}{2}\,(\theta_s - \theta_p)\,(2\theta_p + a\theta_p + c\theta_p + a\theta_s - c\theta_s) \tag{17}$$

$$\pi_s = \int_{\theta_p}^{\theta_s} (\theta - \theta_p)\, d\theta + \frac{1}{2}\int_{\theta_s}^{1} (a + 2\theta + a\theta_s)\, d\theta - \frac{c}{2}\,(1-\theta_s)^2 \tag{18}$$

Proposition 4 *If a Nash equilibrium of this type exists the standards set will be such that the ability of the least able student accepted is respectively:*

$$\theta_p^{NS} = \frac{c^2 + c}{2a + a^2 + c^2 + c\,(2a+3) + 1} \quad \textit{and} \quad \theta_s^{NS} = \frac{c^2 + c\,(a+2)}{2a + a^2 + c^2 + c\,(2a+3) + 1}$$

Proof As in proposition 4 we can solve the maximization problem of each university and find the best response functions:

$$\begin{aligned} \theta_p^{BR} &= \left(\frac{1+c}{a+c+2}\right)\theta_s \\ \theta_s^{BR} &= \frac{c}{a+c} - \left(\frac{1}{a+c}\right)\theta_p \end{aligned}$$

The above expressions for θ_p and θ_s solve this system of two simultaneous equations.■

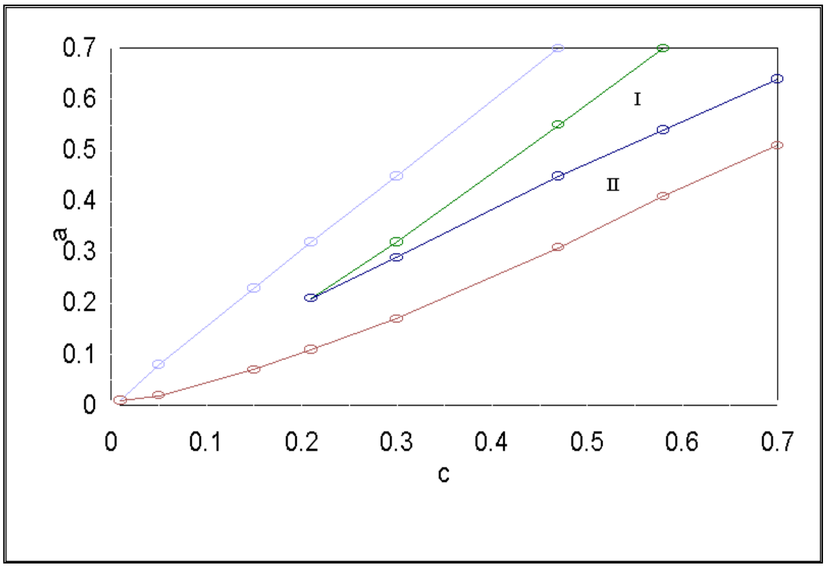

Figure 4: Characterization of equilibria

4.3 Nash Equilibria

We assume free tuition in public university and a welfare maximizer government and find that multiple equilibria can arise. We can then characterize the equilibria in a Cartesian diagram in the space (c, a), Figure 4 (Appendix A describes the procedure followed to draw this figure).

Area II comprises the parameter combinations that yield type P equilibrium, one where the universities set standards such that the ability of the least able student accepted in the private university is higher than the ability of the corresponding student in the public university. Area I includes the parameters' pairwise combinations that satisfy multiple equilibria solution. We can see that the equilibrium configuration where the private university sets a higher standard of admission (area II) happens in a bigger area in the parameter space, but multiplicity of equilibria is also a possibility (area I). This gives theoretical background for the coexistence of systems described in Section 1.

To describe some of the properties of the equilibria found we present in Table 1 the values of some of the critical variables. We take as reference the cost parameter and set it equal to one half. If the strength of the peer effect is high, for instance $a = 0.6$, or low, $a = 0.4$, the equilibrium is of type P.

a		Case P θ_i	π_i	t_i	$w(\theta_i,\bar{\theta}_i)$	Case S θ_i	π_i	t_i	$w(\theta_i,\bar{\theta}_i)$
0.6	p	0.515	0.059	0.242	0.970				
	s	0.192	0.627	0.000	0.404				
0.51	p	0.591	0.042	0.204	0.997	0.165	0.042	0.292	0.292
	s	0.199	0.603	0.000	0.401	0.331	0.574	0.000	0.671
0.49	p	0.609	0.038	0.196	1.003	0.168	0.042	0.291	0.291
	s	0.201	0.597	0.000	0.399	0.335	0.565	0.000	0.662
0.4	p	0.685	0.025	0.157	1.022				
	s	0.213	0.563	0.000	0.393				

Table 1: Equilibrium values when c=0.5

Table 1 gives the respective values for the ability of the least able student accepted in each university, the value of each objective function, the tuition charged, and the gross wage of the least able student. It is possible to see that the number of students going to university is higher if the peer effect is weaker, but the social welfare is also lower. The private university charges lower tuition, getting a lower number of students, although of higher average ability.

The observation of two points in area I, with $a = 0.51$ and $a = 0.49$ respectively, allows us to notice that the type S equilibria is socially inferior. Although the total number of students is higher under an S pattern, the private university is able to charge higher tuition for lower quality education. This occurs because the improvement in wage terms of a university degree makes students willing to participate. When multiple equilibria exist we can look at the Pareto ranking of the solutions. When the strength of the peer effect is higher than the cost parameter the equilibrium of type P Pareto dominates, but if a is lower than c there is no dominance. The private university reaps higher profits under an S equilibrium. In type P equilibria the level of the standard set by the private university is greater than the one the public university sets in type S, for the same values of all the other parameters. This reflects the concern of the public authority with the welfare of the students in society. By setting lower standards the total number of students with higher education is higher than in case P.

5 Conclusion

Universities can choose their students using prices and/or measures of students suitability. We assume that the universities impose an admission requirement and can charge tuition fees. Moreover the competition between private and public universities is described. Two equilibrium configurations are presented. The first one finds private universities setting higher standards and being of higher quality and is consistent with the findings of Epple and Romano (1998). In equilibrium the public university concerned with total welfare chooses to locate in an area of lower ability to allow a greater number of graduates. The second one has the public university with higher quality. This arises if the public university sets such a restrictive admission policy that the private university finds it better to locate at the lower end of the ability distribution, but capturing the unsatisfied demand for university places.

We find that in the case of a government concerned with the aggregate welfare of the potential higher education students and profit maximizing private university it is possible to find multiple equilibria. The configuration where private education is of higher quality is an equilibrium for a larger area in the parameter set, but the multiplicity result cannot be ignored. In the multiple equilibria area, configuration S yields a lower social welfare. The assumption that the private universities are better can be misleading, but the model presented here shows that this configuration of equilibrium is socially preferable.

The results found occur when the private university sets a tuition fee independent of the students' ability. This is a feature of most of European systems. A possible extension be explored is to consider different timing for the competition game, specifically studying the effects of having each of the sectors as a leader. As it is not clear to us who should have first mover advantage, we restricted ourselves to the simultaneous case.

A Characterization of equilibrium

Figure 4 is constructed following the procedure described below.

A.1 Case P

First, the ability levels candidate to local maxima that satisfy Proposition 3 are θ_p^{NP} and θ_s^{NP}. For these ability levels, the value of each university objective functions is π_p^{NP} and π_s^{NP}, respectively.

Second, the highest profit the private university can achieve if deviates and decides to set a lower standard is π_p^{DP}. Also, it must be that this ability level, θ_p^{DP}, is really lower than θ_s^{NP}.

Thirdly, if that is not the case we further need to check if the profit when both universities set θ_s^{NP}, π_p^{EP}, is lower than π_p^{NP}. The condition for the private university not to have an incentive to deviate can be summarized as:

$$\begin{aligned} \pi_p^{NP} &\geq \pi_p^{DP} \text{ if } \theta_p^{DP} \leq \theta_s^{NP} \\ \pi_p^{NP} &\geq \pi_p^{EP} \text{ if } \theta_p^{DP} > \theta_s^{NP} \end{aligned} \tag{19}$$

The respective condition for the state university is:

$$\begin{aligned} \pi_s^{NP} &\geq \pi_s^{DP} \text{ if } \theta_s^{DP} \geq \theta_p^{NP} \\ \pi_s^{NP} &\geq \pi_s^{EP} \text{ if } \theta_p^{DP} < \theta_p^{NP} \end{aligned} \tag{20}$$

In areas I and II these conditions are satisfied, so the private university setting a higher standard is an equilibrium.

A.2 Case S

Using the ability levels that satisfy Proposition 4, θ_p^{NS} and θ_s^{NS}, we find the optimal values of each universities' objective function, π_p^{NS} and π_s^{NS}.

If the private university deviates and sets a higher admission standard, θ_p^{DS}, the maximum profit possible is π_p^{DS}. If the state university sets a lower standard than the private, θ_s^{DS}, then the highest achievable welfare is π_s^{DS}. As before, we also need to ensure that the deviation ability levels are in the appropriate interval. When that doesn't happen π_i^{NS} must be greater or equal to π_i^{ES}.

Therefore in area I the following conditions are satisfied:

$$\begin{aligned} \pi_p^{NS} &\geq \pi_p^{DS} \text{ if } \theta_p^{DS} \geq \theta_s^{NS} \\ \pi_p^{NS} &\geq \pi_p^{ES} \text{ if } \theta_p^{DS} < \theta_s^{NS} \end{aligned} \tag{21}$$

$$\begin{aligned} \pi_s^{NS} &\geq \pi_s^{DS} \text{ if } \theta_s^{DS} \leq \theta_p^{NS} \\ \pi_s^{NS} &\geq \pi_s^{ES} \text{ if } \theta_s^{DS} > \theta_p^{NS} \end{aligned} \tag{22}$$

We then computed the values of each of the π_i^{ki} functions for ranges of a and c, and verify when conditions (19)-(22) hold. Figure 4 represents the results of this simulation procedure in the Cartesian space (c, a).

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