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FAIRNESS AND DIRECT DEMOCRACY

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Fairness and Direct Democracy*

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Abstract

The median voter model (direct democracy) has wide applicability, but it is based on *selfish voters* i.e. voters who derive utility solely from 'own' payoff. The recent literature has pointed to fairness and concern for others as basic human motives that explain a range of economic phenomena. We examine the implications of introducing *fair voters* who have a preference for *fairness* as in Fehr and Schmidt (1999). Within a simple general equilibrium model, we demonstrate the existence of a Condorcet winner for fair voters using the *single crossing property* of voters' preferences. In a fair voter model, unlike a selfish voter model, poverty can lead to increased redistribution. Mean preserving spreads of income increase equilibrium redistribution. Greater fairness leads to greater redistribution. The introduction of selfish voters in an economy where the median voter is fair can have a large impact on the redistributive outcome. An empirical exercise using OECD data illustrates the potential importance of fairness in explaining redistribution.

Keywords: Redistribution, other regarding preferences, single crossing property, income inequality, American Exceptionalism.

JEL Classification: D64 (Altruism); D72 (Economic Models of Political Processes: Rent-Seeking, Elections, Legislatures, and Voting Behavior); D78 (Positive Analysis of Policy-Making and Implementation).

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1. Introduction

All societies face the issue of aggregating individual preferences into social outcomes. In actual practice, such policies are chosen by the elected representatives of the citizens (i.e. *representative democracy*). Furthermore, the political process is complicated by issues of political agency, information asymmetries, and legislative logrolling etc.[1].

However, in a range of applications in political economy, one often needs to abstract away from some of these issues. Indeed, there is a rich literature in public economics that relies on the simpler notion of a median voter who directly chooses the social outcome; for instance, Romer (1975), Roberts (1977), Meltzer and Richard (1981).

Recent experience in Western democracies suggests that direct democracy is more than a useful benchmark. Figures given in Matsusaka (2005a) (who terms the increasing trend in direct democracy as the "storm in ballot box lawmaking") are instructive. In the US, 70 percent of the population lives in a state or city where the apparatus of direct democracy is available. There have been at least 360 citizen initiated measures in the last 10 years in the US and at least 29 referenda on monetary and market integration in Europe have already been held. Matsusaka (2005b) argues that there is a fundamental shift in how policy decisions are made. The implications of *direct democracy* for *representative democracy* are profound[2].

1.1. The RRMR model

The pioneering work on redistribution within a direct democracy framework was done by Romer (1975), Roberts (1977), Meltzer and Richard (1981).[3] The, commonly used, collective name for this class of models is the 'RRMR' model. Since voters in the RRMR model care only about their narrow self-interest, we call it the *selfish voter model*.

In the basic model, the problem is to choose a linear progressive income tax rate that accomplishes redistribution in the sense that the post tax distribution of income reflects

[1] See, for instance, Persson and Tabellini (2000) for a comprehensive exposition of such issues.

[2] Direct citizen initiatives and referenda can override broken campaign promises and reduce the likelihood that elected representatives choose policies that the majority will not desire. This mitigates agency problems to an extent. When information is dispersed among the electorate, a referendum can elicit such information better, hence, direct democracy improves the quality of information. If legislatures bundle contentious issues together, the possibility of referenda can force them to be unbundled. The same applies to the attempts of politicians to bundle multiple issues. Thus, direct democracy induces greater transparency of issues. Limited information on the part of voters might not be a serious hindrance. Voters might use *information cues* that permit a fairly informed decision. For instance, voters voting on the environment might just mimic the stance taken by Greenpeace or by Ralph Nader. For these and other issues see Matsusaka (2005a).

[3] The pioneering work on the existence of a Condorcet winner in a unidimensional policy space is Black (1948).

relatively greater equality. Romer (1975) laid out the conditions for single-peaked preferences when labor supply is endogenous. Roberts (1977) weakened the single-peakedness condition to *hierarchical adherence*. Gans and Smart (1996) proposed the *single crossing property* as an alternative method of determining a Condorcet winner and demonstrated that hierarchical adherence and single crossing are equivalent in a redistributive context.

Meltzer and Richard (1981) derived the testable prediction that the extent of redistribution directly depends on the ratio between mean and median income. The intuition is that as inequality increases, the median voter is relatively poorer and, hence, chooses greater redistribution.[4] The evidence on the relation between inequality and redistribution is mixed, however. Positive support is found by Meltzer and Richard (1981), Easterly and Rebelo (1993), Alesina and Rodrick (1994), Persson and Tabellini (1994), and Milanovic (2000). However, Lindert (1996), Perotti (1996) do not find any support.

The RRMR framework treats only a specific kind of transfer, namely, the intra-generational transfers of income. In actual practice, the growth of government transfers in recent decades has been driven by a range of other considerations. For instance, the increased (inter-generational) transfers to the old could possibly reflect their growing political clout in Western democracies. Regional transfers could arise from special interest group considerations. Unemployment and health insurance can only be understood within the model insofar as these entail intra-generational transfers. These issues are possibly better analyzed within a dynamic model[5].

1.2. Why fairness?

Traditional economic theory relies on the twin assumptions of rationality and self-interested behavior. The latter is generally taken to imply that individuals are interested mainly in their own pecuniary payoffs (*selfish preferences*). This view is not always in conformity with the evidence. The purely selfish individual model is unable to explain a range of phenomena from many diverse areas such as collective action, contract theory, the structure of incentives, political economy and the results of several experimental games. Individuals are also often motivated by the pecuniary and non-pecuniary payoffs of others. A substantial fraction of individuals exhibit *social preferences*, i.e., care about the consumption and well being of others. Evidence from a range of experimental games, such as the ultimatum game, the gift exchange game, the public good game with punishment etc. can easily be

[4] It is not commonly realized that this follows from the special case of quasi-linear preferences with quadratic disutility from labour.

[5] Alesina and Rodrik (1994) discuss issues of growth and redistribution. Persson and Tabellini (1994) deal with a model of reputation in which the device of inter-generational punishment can sustain redistribution to the old. Tabellini (1991) shows how altruism among the young can sustain transfers to the old. Galasso and Conde Ruiz (2005) consider two policy tools: intra-generational and inter-generational transfers. Several models of unemployment insurance are surveyed in Persson and Tabellini (2000).

reconciled if we assume individuals to have social preferences.[6]

It may seem obvious to many that issues of fairness and regard for others (social preferences) motivative the human desire to redistribute. The experimental results of Ackert et al. (2007), Tyran and Sausgruber (2006) and Bolton and Ockenfels (2002) are strongly supportive of the importance of social preferences in the domain of voting models.

Tyran and Sausgruber (2006) examine pure transfers of income from the rich to the poor that do not affect the middle income voter. Some rich voters, on account of their fairness, vote for the transfers to the poor in circumstances where a rich, but selfish, voter would have voted otherwise. Hence, a majority of the fair voters might vote for redistribution while, under similar circumstances, a selfish voter model might predict no redistribution; this is the important contribution of that paper.

Bolton and Ockenfels (2002) examine the preference for equity versus efficiency in a voting game. Groups of three subjects are formed and are presented with two alternative policies: one that promotes equity while the other promotes efficiency. The final outcome is chosen by a majority vote. About twice as many experimental subjects preferred equity as compared to efficiency. Furthermore, even those willing to change the status-quo for efficiency are willing to pay, on average, less than half relative to those who wish to alter the status-quo for equity.

Our innovation is to replace the selfish voters in the RRMR *selfish voter model* by fair voters: this we will call the *fair voter model*.

1.3. Which model of fairness?

There are several models of fairness. We choose to use the Fehr-Schmidt (1999) (henceforth, FS) approach to fairness[7]. In this approach, voters care not only about their own payoffs but their payoffs relative to those of others. If their payoff is greater than other voters then they suffer from advantageous inequity (arising from, say, altruism) and if their payoff is lower than other voters they suffer from disadvantageous inequity (arising from, say, envy).

Several reasons motivate our choice of the FS model. The FS model is tractable and explains the experimental results arising from several games where the prediction of the standard game theory model with selfish agents yields results that are not consistent with the experimental evidence. These games include the ultimatum game, the gift exchange

[6] The references are too numerous to list. A good place to start is the book by Camerer (2003) and the survey article by Fehr and Fischbacher (2002). The neuroeconomic foundations of reciprocity are surveyed in Fehr et al. (2005).

[7] Bolton and Ockenfels (2000) provide yet another approach of inequity averse economic agents, but it cannot explain the outcome of the public good game with punishment, which is a fairly robust experimental finding (see below).

game, the dictator game as well as the public good game with punishment[8].

The FS model focusses on the role of inequity aversion. However, a possible objection is that it ignores the role played by intentions that have been shown to be important in experimental results (Falk et al. (2002)) and treated explicitly in theoretical work (Rabin (1993), Falk and Fischbacher (2006)). However, experimental results on the importance of intentions come largely from bilateral interactions. Economy-wide voting, on the other hand, is impersonal and anonymous, thereby ruling out any important role for intentions.

Tyran and Sausgruber (2006) explicitly test for the importance of the FS framework in the context of direct voting. They conclude that the FS model predicts much better than the standard selfish voter model. In addition, the FS model provides, in their words, "strikingly accurate predictions for individual voting in all three income classes." The econometric results of Ackert et al. (2007), based on their experimental data, lend further support to the FS model in the context of redistributive taxation. The estimated coefficients of altruism and envy in the FS model are statistically significant and, as expected, negative in sign. Social preferences are found to influence participant's vote over alternative taxes. They find evidence that some participants are willing to reduce their own payoffs in order to support taxes that reduce advantageous or disadvantageous inequity.

1.4. A critique of the literature on voting and fairness

There is a relatively small theoretical literature that considers fair voters. We concentrate below on the papers that are directly relevant to our work. Tyran and Sausgruber (2006), reviewed above, do not analyze the relation between inequality and redistributive taxes which is important in the RRMR framework (and ours'). Their's is not a general equilibrium model, does not analyze the efficiency costs of redistribution, does not look at a mixture of fair and selfish voters and, probably most importantly, does not provide existence results for there to be a decisive median voter. Furthermore, they consider a more restricted tax policy choice than us. While we consider changes in a linear progressive income tax that affect all taxpayers they focus attention only on redistributions from the rich to the poor that leave the middle income voters unaffected[9].

Galasso (2003) modifies the RRMR model to allow for fairness concerns. However, his notion of fairness is not only one-sided but it is of a very specific form; it is not fully

[8] In the first three of these games, experimental subjects offer more to the other party relative to the predictions of the Nash outcome. In the public good game with punishment, the possibility of ex-post punishment dramatically reduces the extent of free riding in voluntary giving towards a public good. In the standard theory with selfish agents, bygones are bygones, so there is no ex-post incentive for the contributors to punish the free-riders. Foreseeing this outcome, free riders are not deterred, which is in disagreement with the evidence. Such behavior can be easily explained within the FS framework.

[9] They do introduce a cost of such redistribution to the middle income voters, but it is not an integral part of the redistributive fiscal package considered.

consistent with any of the accepted models of fairness. In particular, fair voters care about their own payoffs but suffer disutility through a term that is linear in their payoffs relative to the worse off voter in society.[10] Since this concern for fairness arises from a linear term, preferences continue to be strictly concave and a median voter equilibrium exits. Within this framework there is greater redistribution when there is a mean preserving spread in inequality. However, this leaves open the question of whether a median voter equilibrium will exist in a standard model of fairness, such as the FS model, and what the properties of the resulting equilibrium will be?

1.5. An example: European versus American redistribution

Europe, where (disposable income) inequality is lower, undertakes greater redistribution than America, where such inequality is higher. This would seem to contradict the RRMR model. Alesina and Angeletos (2003) have a novel explanation. The key to understanding their model is the individual beliefs on the source of poverty (or affluence). More Americans than Europeans believe that poverty (or affluence) is caused by individual effort than luck. A crucial assumption of the model is that voters expect there to be greater public redistribution if income outcomes are governed by luck rather than effort. Hence, in the European equilibrium, more people believe that income is caused by luck, so put in less effort and, hence, actual outcomes are indeed governed more by luck rather than effort. Given the assumption on public redistribution, there is greater redistribution in equilibrium. The American, high effort - low redistribution equilibrium can be understood analogously.

Benabou (2000) develops a stochastic growth model with incomplete asset markets and heterogeneous agents who vote over redistributive policies. He shows that multiple equilibria can exist, some featuring low inequality and high redistribution, while others exhibit high inequality and low redistribution. Thus countries with similar preference, technologies and political systems can feature very different levels of inequality and redistribution.

Theory is very explicit about the inequality variable required: it should be pre-tax/transfer distribution of income i.e. the factor income distribution. Theory does not necessarily predict any relation between disposable income and redistribution. However, most work on the relation between inequality and redistribution uses disposable income data. Factor income data has recently been made available in Milanovic (2000). Are inequality fig-

[10] The latter term captures some notion of social justice. Others have included such a term to incorporate social justice e.g. Charness and Rabin (2002). However, they posit preferences, different from Galasso (2003), that are a convex combination of the total payoff of the group (this subsumes selfishness, in so far as one's own payoff is part of the total, and altruism) and a Rawlsian social welfare function. These sorts of models are able to explain positive levels of giving in dictator games, and reciprocity in trust and gift exchange games. However, they are not able to explain situations where an individual tries to punish others in the group at some personal cost, for instance, punishment in public good games.

	DII	SS/GDP	FII	Multi-Aid/GDP
Swe/US	0.6	2	1	9

Figure 1.1: **An illustrative comparison beween Sweden and the US**

ures based on factor incomes and disposable income significantly different? The results in Milanovic (2000) show that almost a third of factor income inequality is removed by government tax and transfer programs. As an illustrative example, consider a relative comparison of Sweden and the US in Figure 1.1. DII denotes disposable income inequality, SS/GDP denotes the ratio of social spending to gross domestic product, FII denotes factor income inequality and Multi-Aid/GDP is multilateral aid to the GDP ratio.

Disposable income inequality in Sweden is about 60 percent that of the US. However factor income inequality is almost identical. Sweden social spending to GDP ratio is about twice that of the US. Since FII is the relevant income inequality variable that theory uses, it would be hard to explain to the Swedish versus US comparison based on income inequality alone.

We provide an alternative explanation for greater European redistribution relative to America by using the argument that Europeans are relatively more inequity averse on average and that there is basis for '*American Exceptionalism*'[11]. Our first explanation requires us to construct some empirical measure of fairness. Measuring fairness presents a challenging, and understandably contentious, set of issues. We now turn to these.

One could possibly use charitable giving per capita as an indicator of fairness. However, charitable contributions are endogenous in a model with fair voters. So, for instance, if government redistribution is perceived to be inadequate, citizens might attempt to compensate by donating more to charity. For that reason we do not believe that the relatively greater per capita giving of Americans necessarily indicates that they are more inequity averse than Europeans.

A better measure of fairness/ inequity aversion seems to be aid given to other countries, particularly, developing countries. A criticism, similar to our objection against using charitable contributions to proxy inequity aversion, also applies for aid because aid given by a country to any particular developing country might reflect the low volume of aggregate

[11] A good summary (and some historical background) of American Exceptionalism is provided in Glaeser (2005); the reasons include proportional versus majoritarian representation, greater ethnic heterogeneity in America, and the US tradition of federalism that gives redistributive powers to the individual States, among others.

giving to that country in the first place. However, crucially, this applies equally to all giving countries. Hence, relative giving of countries potentially reflects relative fairness/ inequity aversion. This is the measure that we will use.

While the US is the single largest contributor to development aid, in per capita terms its contribution is lower than most European countries. According to OECD figures, the US contributed only 0.15 percent of its GDP to development assistance, placing it last in a list of 21 western (mostly European) countries. The Center for Global Development estimated that US development assistance per capita is one eighth that of Norway, one sixth that of Denmark and close to half of the average contributions of Belgium, France, Finland and Britain.

Coming back to the Swedish versus US comparison in Figure 1.1, our proxy for inequity aversion, multilateral aid to GDP ratio, for Sweden is about 9 times that of the US. We argue that this reveals greater inequity aversion of the Swedes relative to the US and potentially helps to explain the differences in the social spending to GDP ratios between the two countries; a difference that factor income inequality is unable to explain.

1.6. Results and plan of the paper

Our main theoretical results are as follows. First, we demonstrate the existence of a Condorcet winner for voters who have the FS preferences for fairness. Insofar as one believes that issue of fairness and concern for others underpin the human tendency to redistribute, this result opens the way for modelling such concerns in the context of direct democracy. Second, if voters are fair, then increased poverty can lead to increased redistribution and the ratio of social spending to GDP would move countercyclically, which is at variance with the selfish voter model, but in agreement with the evidence. Third, the introduction of selfish poor voters in an economy where the median voter is fair can have a large impact on the redistributive outcome and may actually reduce redistribution. In other settings, even in the presence of a majority of fair voters the redistributive outcome is identical to that of an economy comprising solely of selfish voters.

Due to the paucity of data on factor incomes, we claim to provide no more than an illustrative empirical exercise that provides empirical support to the idea that fairness is an important determinant of redistribution. Conditional on the limitations of our exercise, our regression results, based on data from 20 OECD economies, can be summarized as follows. First, factor income inequality leads to a better specification than disposable income inequality[12]. Second, the results support the idea of 'American Exceptionalism' relative to Europe. Third, our fairness variable is a significant determinant of redistribution, while

[12]Theory only predicts a relation between pre-tax or factor income distribution and redistribution (not between post-tax income distribution and redistribution). However, the existing empirical work largely uses disposable (or post-tax/transfer) income instead of factor income.

income inequality is not significant.

The plan of the paper is as follows. Section 2 describes the theoretical model and derives some intermediate results. Section 3 derives the conditions needed for the existence of a Condorcet winner for fair voters. Comparative static results along with some calibration exercises are derived and discussed in Section 4. Section 5 considers the relationship between income distribution and the tax rate using a discrete analogue of second order stochastic dominance. Section 6 considers an economy where there is a mixture of fair and selfish voters. Section 7 presents our illustrative empirical exercise. Finally, section 8 concludes. The results of regression analysis are presented in Appendix 1. All proofs are relegated to Appendix 2.

2. Model

We consider a general equilibrium model as in Meltzer and Richard (1981). Let there be $n = 2m - 1 \geq 3$ voters, where m is the median voter. Let the skill level of voter i be s_i, where

$$0 < s_i < s_j < 1, \text{ for } i < j, \tag{2.1}$$

and $\mathbf{s} = (s_1, s_2, ..., s_n)$. Each voter has a fixed time endowment of one unit and supplies l_i units of labor and so enjoys $L_i = 1 - l_i$ units of leisure, where

$$0 \leq l_i \leq 1. \tag{2.2}$$

Labour markets are competitive and each firm has access to a linear production technology such that production equals $s_i l_i$. Hence, the wage rate offered to each worker-voter coincides with the marginal product, i.e., the skill level, s_i. Thus, the before-tax income of a voter is given by

$$y_i = s_i l_i. \tag{2.3}$$

Note that 'skill' here need not represent any intrinsic talent, just ability to translate labour effort into income[13]. Let the average before-tax income be $\overline{y} = \frac{1}{n}\Sigma_{i=1}^{n} y_i$. We make the empirically plausible assumption that the income of the median voter, y_m, is less than the average income,

$$y_m < \frac{1}{n}\Sigma_{i=1}^{n} y_i. \tag{2.4}$$

Since typical income distributions are skewed to the left, (2.4) is empirically plausible.

[13] For example, a highly talented classical musician may be able to earn only a modest income, while a merely competent 'pop' musician may earn millions. In our model, the former would be classified as having a low s while the latter would be classified as having a high s. Similarly, in recent years, there has been a record level of skilled (in the ordinary sense of the word) migration into Britain from Eastern Europe. However, since they are predominantly accepting low pay work, they would be classified in our model as having low s.

The government operates a linear progressive income tax that is characterized by a constant marginal tax rate, $t \in [0,1]$, and a uniform transfer, b, to each voter that equals the average tax proceeds,

$$b = t\overline{y} = \frac{t}{n}\Sigma_{i=1}^{n} y_i \geq 0. \tag{2.5}$$

Thus, the tax rate is also the ratio of social spending to aggregate income,

$$t = \frac{nb}{\Sigma_{i=1}^{n} y_i}. \tag{2.6}$$

Remark 1 *From (2.6), changes in the tax rate can equivalently be viewed as changes in the ratio of social spending to aggregate income.*

The budget constraint of voter i is given by

$$0 \leq c_i \leq (1-t)\, y_i + b. \tag{2.7}$$

In view of (2.3), the budget constraint (2.7) can be written as

$$0 \leq c_i \leq (1-t)\, s_i l_i + b. \tag{2.8}$$

2.1. Preferences, labour supply and indirect utility of a selfish voter

Voter i (whether selfish or fair) has a utility function, $u(c_i, 1-l_i)$, over own consumption, c_i, and own leisure, $1-l_i$. In common with the literature, we assume that all voters have the same utility function. Hence, voters differ only in that they are endowed with different skill levels, s_i. We assume that the utility function has the following, plausible, properties. It is thrice continuously differentiable and

$$\text{(a) } u_1 > 0, \text{ (b) } l > 0 \Rightarrow u_2(c, 1-l) > 0, \text{ (c) } u_2(c,1) = 0, \text{ (d) } u_1(c,0) \leq u_2(c,0), \tag{2.9}$$

$$\text{(a) } u_{11} \leq 0, \text{ (b) } u_{12} \geq 0, \text{ (c) } l > 0 \Rightarrow u_{22}(c, 1-l) < 0, \tag{2.10}$$

From (2.9a), the marginal utility of consumption is positive, while (2.9b) implies that marginal utility of leisure is positive, unless $l = 0$ in which case (2.9c) says that the consumer is satiated with leisure. From (2.9d), when a consumer has no leisure she always (weakly) prefers one extra unit of leisure to one extra unit of consumption. (2.10a) says that marginal utility of consumption is non-increasing. Consumption and leisure are complements (see, (2.10b)) while (2.10c) implies that the marginal utility of leisure is strictly declining unless, possibly, the consumer is satiated with leisure (in which case $u_{22}(c,1) = 0$). The conditions (2.9) and (2.10) will guarantee that a maximum exists, that it is unique and that it is an interior point ($0 < l_i < 1$), unless $t = 1$ in which case the maximum will lie at $l_i = 0$. Alternative conditions can also guarantee a unique interior maximum but

(2.9) and (2.10) are chosen because they can be satisfied by a quasi-linear utility function, which is the most commonly used functional form in various applications of the median voter theorems.

Since $\frac{\partial u}{\partial c_i} > 0$, the budget constraint (2.8) holds with equality. Substituting $c_i = (1-t)\, s_i l_i + b$ in the utility function of voter i, we get

$$U(l_i; t, b, s_i) = u((1-t)\, s_i l_i + b, 1 - l_i). \tag{2.11}$$

Given t and b, voter i chooses labour supply, l_i, in order to maximize the objective function in (2.11). We list, in lemmas 1, 2, below, some useful results.

Lemma 1 *(Properties of labour supply): (a) Given t, b and s_i, there is a unique labour supply for voter i, $l_i = l(t, b, s_i)$, that maximizes utility (2.11),*
(b) $t \in [0,1) \Rightarrow 0 < l_i < 1$,
(c) $l_i = 0$ at $t = 1$,
(d) $t \in [0,1] \Rightarrow \left[\frac{\partial U}{\partial l_i}(l_i; t, b, s_i)\right]_{l_i = l(t,b,s_i)} = 0$,
(e) $l(t, b, s_i)$ is twice continuously differentiable,
(f) $\frac{\partial l(t,b,s)}{\partial b} \leq 0$,
(g) for each $t \in [0,1]$, the equation $b = \frac{1}{n} t \Sigma_{i=1}^{n} s_i l(t, b, s_i)$ has a unique solution $b(t, \mathbf{s}) \geq 0$; and $b(t, \mathbf{s})$ is twice continuously differentiable.

Substituting labour supply, given by Lemma 1 (a), in (2.11) we get the indirect utility function of voter i:

$$v_i = v(t, b, s_i) = U(l(t, b, s_i); t, b, s_i). \tag{2.12}$$

Lemma 2 *(Properties of the indirect utility function): (a) $\frac{\partial v(t,b,s)}{\partial b} > 0$,*
(bi) $\frac{\partial v(1,b,s)}{\partial s} = 0$, (bii) $t \in [0,1) \Rightarrow \frac{\partial v(t,b,s)}{\partial s} > 0$,
(ci) $\left[\frac{\partial v(t,b,s)}{\partial t}\right]_{t=1} = 0$, (cii) $t \in [0,1) \Rightarrow \frac{\partial v(t,b,s)}{\partial t} < 0$.

Substituting labour supply, $l(t, b, s_i)$, into (2.3) gives before-tax income:

$$y_i(t, b, s_i) = s_i l(t, b, s_i). \tag{2.13}$$

Substitute $b(t, \mathbf{s})$, given by Lemma 1(g), into the indirect utility (2.12), to get

$$w_i(t, \mathbf{s}) = v(t, b(t, \mathbf{s}), s_i). \tag{2.14}$$

2.2. Preferences of fair voters

Fair voters have *other regarding preferences* as in Fehr-Schmidt (1999). These preferences are as follows

$$V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)=v\left(t,b,s_j\right)-\frac{\alpha}{n-1}\sum_{k\neq j}\max\left\{0,v\left(t,b,s_k\right)-v\left(t,b,s_j\right)\right\} \qquad (2.15)$$
$$-\frac{\beta}{n-1}\sum_{i\neq j}\max\left\{0,v\left(t,b,s_j\right)-v\left(t,b,s_i\right)\right\},$$

where

$$\text{for } \textit{selfish} \text{ voters } \alpha=\beta=0, \qquad (2.16)$$

$$\text{for } \textit{fair} \text{ voters } 0<\beta<1,\ \beta<\alpha. \qquad (2.17)$$

From (2.15), the fair voter cares about own payoff (first term), payoff relative to those where inequality is disadvantageous (second term) and payoff relative to those where inequality is advantageous (third term). The second and third terms which capture respectively, *envy* and *altruism*, are normalized by the term $n-1$ where n is the number of voters. Notice that in FS preferences, inequality is *self-centered*, i.e., the individual uses her own payoff as a reference point with which everyone else is compared to. Also, while the Fehr-Schmidt specification is directly in terms of monetary payoffs, it is also consistent with comparison of payoffs in utility terms. These and related issues are more fully discussed in Fehr and Schmidt (1999). From (2.17), β is bounded below by 0 and above by 1 and α. On the other hand, there is no upper bound on α.[14]

In the light of Lemmas 2(bi) and 2(bii), (2.15) becomes

$$V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)=v\left(t,b,s_j\right)-\frac{\alpha}{n-1}\sum_{k>j}\left[v\left(t,b,s_k\right)-v\left(t,b,s_j\right)\right]-\frac{\beta}{n-1}\sum_{i<j}\left[v\left(t,b,s_j\right)-v\left(t,b,s_i\right)\right], \qquad (2.18)$$

or, equivalently,

$$V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)=\frac{\beta}{n-1}\sum_{i<j}v\left(t,b,s_i\right)+\left(1-\frac{\left(j-1\right)\beta}{n-1}+\frac{\left(n-j\right)\alpha}{n-1}\right)v\left(t,b,s_j\right)-\frac{\alpha}{n-1}\sum_{k>j}v\left(t,b,s_k\right). \qquad (2.19)$$

Remark 2 : *(Weighted utilitarian preferences) Note that (2.19) can be written in the (weighted utilitarian) form*

$$V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)=\sum_{i=1}^{n}\omega_{ij}v\left(t,b,s_i\right), \qquad (2.20)$$

[14] $\beta\geq 1$ would imply that individuals could increase utility by simply giving away all their wealth; this is counterfactual. The restriction $\beta<\alpha$ is based on experimental evidence. Finally the lack of an upper limit on α implies that 'envy' is unbounded.

where

$$i<j \Rightarrow \omega_{ij}=\frac{\beta}{n-1}>0,\ \omega_{jj}=1-\frac{(j-1)\beta}{n-1}+\frac{(n-j)\alpha}{n-1}>0,\ i>j \Rightarrow \omega_{ij}=-\frac{\alpha}{n-1}<0,\ \sum_{i=1}^{n}\omega_{ij}=1. \tag{2.21}$$

Let

$$W_j(t,\alpha,\beta,\mathbf{s})=V_j(t,b(t,\mathbf{s}),\alpha,\beta,\mathbf{s}), \tag{2.22}$$

where $b(t,\mathbf{s})$ is given by Lemma 1(g). Then (2.14), (2.18) and (2.22) give

$$W_j(t,\alpha,\beta,\mathbf{s})=w_j(t,\mathbf{s})-\frac{\alpha}{n-1}\sum_{k>j}[w_k(t,\mathbf{s})-w_j(t,\mathbf{s})]-\frac{\beta}{n-1}\sum_{i<j}[w_j(t,\mathbf{s})-w_i(t,\mathbf{s})]. \tag{2.23}$$

Remark 3 : *(Existence of a maximum) Since $u(c_i,1-l_i)$ and $l(t,b,s_i)$ are continuous, it follows from (2.11), (2.12), (2.14) and (2.23) that $W_j(t,\alpha,\beta,\mathbf{s})$ attains a maximum at some $t_j\in[0,1]$.*

2.3. Sequence of moves

We consider a two-stage game. In the first stage, voters choose a tax rate, t, and a lumpsum benefit, b, anticipating the outcome of the second stage. Consumer j exhibits fairness by voting for the tax rate, t, and the lumpsum benefit, b, that would maximize social welfare, $V_j(t,b,\alpha,\beta,\mathbf{s})=\sum_{i=1}^{n}\omega_{ij}v(t,b,s_i)$, as seen from her own perspective (see Remark 2). In the second stage, consumer j chooses own labour supply, l_j, so as to selfishly maximize own utility, $U(l_j;t,b,s_j)$, given t,b,s_j. This determines labour supplies, $l_i=l(t,b,s_i)$, and indirect utilities, $v_i=v(t,b,s_i)=U(l(t,b,s_i);t,b,s_i)$, $i=1,2,...,n$.

Remark 4 : *One might wonder, why should the consumer not exhibit fairness in the second stage (when choosing own labour supply) as well as the first stage (when choosing the tax rate)? However, it would make no difference. To see this, suppose that in the second stage consumer j chooses l_j so as to maximize $\sum_{i=1}^{n}\omega_{ij}U(l_i;t,b,s_i)=\omega_{jj}U(l_j;t,b,s_j)+\sum_{i\neq j}^{n}\omega_{ij}U(l_i;t,b,s_i)$ given t,b,s_j and given s_i, l_i for all $i\neq j$. Then, since in the Fehr-Schmidt theory, $U(l_i;t,b,s_i)$, $i\neq j$, enter additively, and $\omega_{jj}>0$, it follows that maximizing $\sum_{i=1}^{n}\omega_{ij}U(l_i;t,b,s_i)$ is equivalent to maximizing $U(l_j;t,b,s_j)$. One can also give general arguments: 1. Bounded rationality: it is difficult to calculate the effect of all of ones consumption and labour supply decisions on the rest of society. But, come elections, we are more inclined to think about these issues or seek advice. 2. Insignificance of an individual: by giving to charity I (say) suffer a large loss but make very little difference. However, by voting for a redistributive tax, I force all others to make contributions, hence, have a much greater effect.*

3. Existence of a Condorcet winner

We now ask if a median voter equilibrium exists in a model with fair voters? As expected, single peakedness of preferences turns out to be a very strong restriction. We instead use the single crossing property of Gans and Smart (1996).

Definition 1 *: (Gans and Smart, 1996) The 'single crossing' property holds if for tax rates t, T and voters j, J,*

$t < T,\ j < J,\ W_j(t, \alpha, \beta, \mathbf{s}) > W_j(T, \alpha, \beta, \mathbf{s}) \Rightarrow W_J(t, \alpha, \beta, \mathbf{s}) > W_J(T, \alpha, \beta, \mathbf{s})$.[15]

Lemma 3 *: (Gans and Smart, 1996) The 'single crossing' property holds if* $-\frac{\partial V_j}{\partial t} / \frac{\partial V_j}{\partial b}$ *is an increasing function of* j.

Lemma 4 *: (Gans and Smart, 1996) If the 'single crossing' property holds, then the median voter is decisive, i.e., a majority chooses the tax rate that is optimal for the median voter.*

The proofs of lemmas 3, 4 can be found in Gans and Smart (1996). The intuition behind these lemmas is straighforward to illustrate in the following diagram in (b, t) space. In Figure 3.1, the aggregate budget constraint of the economy, given in (2.5), is shown by the straight upward sloping line, BB', that has slope $\overline{y}$. We show two indifference curves belonging to a poor $(I_p I_p)$ and a rich $(I_r I_r)$ voter respectively. Lemma 3 requires that $-\frac{\partial V_j}{\partial t} / \frac{\partial V_j}{\partial b}$ be an increasing function of j i.e. $I_p I_p$ is relatively flatter (we show this in section 3.1 below). The most preferred tax rate of the poor, t_p, is greater than the most preferred tax rate of the rich, t_r. Hence, the preferred tax rates can be uniquely ordered from the rich to the poor. This monotonicity property gives rise to the result in Lemma 4.

3.1. Main results

We now introduce two further assumptions, A1 and A2, followed by the main result of the paper.

A1: $\frac{\partial}{\partial s}\left(\frac{\partial v}{\partial b}\right) \leq 0$ and $t \in [0, 1) \Rightarrow \frac{\partial}{\partial s}\left(\frac{\partial v}{\partial t}\right) < 0$.

[15] Here we use "$<$" to denote the usual ordering of real numbers. In the more general setting of Gans and Smart (1996), "$<$" is used to denote several (possibly different) abstract orderings. In particular, a literal translation of Gants and Smart (1996) would give: $T < t$, $j < J$, $W_j(t, \alpha, \beta, \mathbf{s}) > W_j(T, \alpha, \beta, \mathbf{s}) \Rightarrow W_J(t, \alpha, \beta, \mathbf{s}) > W_J(T, \alpha, \beta, \mathbf{s})$, where "$j < J$" has the usual meaning "j is less than J" but "$T < t$" means "t is less than T".

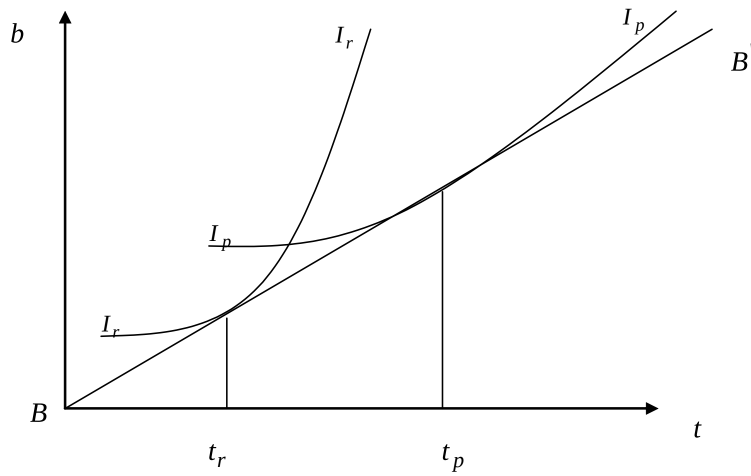


Figure 3.1: **Illustration of the Gans-Smart single crossing property.**

Recall, from Lemma 2(a), that $\frac{\partial v}{\partial b} > 0$. Thus, we can interpret $\frac{\partial}{\partial s}\left(\frac{\partial v}{\partial b}\right) \leq 0$ as saying that the marginal utility of an extra pound of benefit for a rich person is no more than that for a poor person. Also recall, from Lemma 2(cii), that $t \in [0,1) \Rightarrow \frac{\partial v}{\partial t} < 0$. Hence, $\frac{\partial}{\partial s}\left(\frac{\partial v}{\partial t}\right) < 0$ can be interpreted as saying that an extra 1% on the tax rate hurts a rich person more than a poor person. Thus Assumption A1 roughly says that benefits help the poor more than the rich while taxes hurt the rich more than the poor. This is the basic foundation of the modern welfare state.

A2: $\frac{\partial V_j(t,b,\alpha,\beta,\mathbf{s})}{\partial t} \leq 0$.

Recall, from Lemma 2(cii), that $\frac{\partial v}{\partial t} < 0$, for $t < 1$. Thus, an increase in tax (benefits remaining fixed), is undesirable for a selfish-voter which is, of course, entirely reasonable. Assumption A2 extends this to fair-voters as well. This puts an upper bound on envy. Envy is not so great as to make a fair-voter like an increase in tax, even if it has no gain for any one at all (in terms of an increase in benefit).[16]

Proposition 1 : *Under assumptions A1 and A2 a majority prefers the tax rate that is optimal for the median-voter.*

[16] From (2.19) we can compute the derivative

$$\frac{\partial V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)}{\partial t} = \frac{\beta}{n-1}\sum_{i<j}\frac{\partial v\left(t,b,s_i\right)}{\partial t} + \left(1-\frac{(j-1)\beta}{n-1}+\frac{(n-j)\alpha}{n-1}\right)\frac{\partial v\left(t,b,s_i\right)}{\partial t} - \frac{\alpha}{n-1}\sum_{k>j}\frac{\partial v\left(t,b,s_i\right)}{\partial t}.$$

From Lemma 2, we know that $\frac{\partial v(t,b,s)}{\partial t} \leq 0$. The first two terms are, therefore, negative while the last term (whose magnitude depends on the envy parameter, α) is positive. It is straightforward to show that if α is suitably bounded above, then assumption A2 holds.

Corollary 1 : *Under assumption A1, if utility is quasi-linear or if voters are selfish ($\alpha = \beta = 0$), then a majority prefers the tax rate that is optimal for the median-voter.*

The result in Proposition 1 is potentially of fundamental importance for political economy. If one takes the view that issues of fairness and concern for others are important human motivations that play a significant part in the actual design of redistributive tax policies, then Proposition 1 provides conditions that allow one to model these concerns within a direct democracy framework. Insofar as actual applications of a direct democracy framework largely use quasi-linear preferences, Corollary 1 shows that fairly weak conditions will allow the existence of a Condorcet winner.

4. Comparative static results

From now on, we concentrate on the quasi-linear case with constant elasticity of labour supply i.e.

$$u(c, 1-l) = c - \frac{\epsilon}{1+\epsilon} l^{\frac{1+\epsilon}{\epsilon}}, \tag{4.1}$$

where ϵ is a constant satisfying

$$0 < \epsilon \leq 1. \tag{4.2}$$

The case $\epsilon = 1$ has special significance in the literature. In this case,

$$u(c, 1-l) = c - \frac{1}{2} l^2 \tag{4.3}$$

Meltzer and Richard (1981) use (4.3) to derive the celebrated result that the extent of redistribution varies directly with the ratio of the mean to median income. Piketty (1995) restricts preferences to the quasi-linear case with disutility of labour given by the quadratic form, (4.3). Benabou and OK (2001) do not actually consider a production side and their model has exogenously given endowments which evolve stochastically. Benabou (2000) considers the additively separable case with log consumption and disutility of labor given by the constant elasticity case, (4.1).

Lemma 5 : *(a) Conditions (2.9) and (2.10) hold,*
(b) labour supply of consumer j is $l(t, b, s_j) = (1-t)^{\epsilon} s_j^{\epsilon}$ and is independent of b,
(c) for the median skill level, s_m, $s_m^{1+\epsilon} < \frac{1}{n}\Sigma_{i=1}^{n} s_i^{1+\epsilon}$,
(d) assumption A1 holds,
(e) the tax rate, t_m, chosen by the median voter, is given by

$$t_m = \frac{\frac{1}{n}\Sigma_{i=1}^{n} s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1} \sum\limits_{k>m} \left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1} \sum\limits_{i<m} \left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)}{\frac{1+\epsilon}{n}\Sigma_{i=1}^{n} s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1} \sum\limits_{k>m} \left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1} \sum\limits_{i<m} \left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)}. \tag{4.4}$$

From Proposition 1 and Lemma 5 (a) and (d) we get:

Corollary 2 : *For the quasi-linear utility function (4.1), (4.2) with constant elasticity of labour supply, ϵ, a majority prefers the tax rate that is optimal for the median voter.*

Proposition 2, below, gives the change in the tax rate chosen by the median voter, t_m, as various parameters in the model are changed.

Proposition 2 : *(a) For fair voters,* $\frac{\partial t_m}{\partial \alpha} > 0$, $\frac{\partial t_m}{\partial \beta} > 0$.
(b) A fair median voter chooses a higher tax rate than a selfish median voter.
(c) For selfish and fair voters, $j > m \Rightarrow \frac{\partial t_m}{\partial s_j} > 0$.
(d) For selfish and fair voters, $\frac{\partial t_m}{\partial s_m} < 0$.
(e) For selfish voters, $j < m \Rightarrow \frac{\partial t_m}{\partial s_j} > 0$.
(f) For fair voters, for $j < m$,

$$\frac{\partial t_m}{\partial s_j} \gtrless 0 \Leftrightarrow \alpha \sum_{k>m} \left(s_k^{1+\epsilon} - s_m^{1+\epsilon} \right) + \beta \left(\Sigma_{i=1}^n s_i^{1+\epsilon} + \sum_{i<m} \left(s_m^{1+\epsilon} - s_i^{1+\epsilon} \right) \right) \lesseqgtr (n-1)\, s_m^{1+\epsilon}$$

(g) For fair voters, for $j < m$, $\alpha \geq \frac{(n-1) s_m^{1+\epsilon}}{\sum\limits_{k>m} \left(s_k^{1+\epsilon} - s_m^{1+\epsilon} \right)} \Rightarrow \frac{\partial t_m}{\partial s_j} < 0$.
(h) For fair voters, for $j < m$, $\beta \geq \frac{(n-1) s_m^{1+\epsilon}}{s_m^{1+\epsilon} + 2 \sum\limits_{k>m} s_k^{1+\epsilon}} \Rightarrow \frac{\partial t_m}{\partial s_j} < 0$.

From part (a), the tax rate (equivalently, the ratio of social spending to GDP, see (2.6)) is increasing in α, β. The intuition is that an increase in α increases disutility arising from disadvantageous inequity. By increasing the redistributive tax rate, the median voter reduces, relatively, the utility of anyone who is richer, hence, reducing disadvantageous inequity. On the other hand, an increase in β increases disutility arising from advantageous inequity. An increase in the redistributive tax benefits everyone poorer than the median voter relatively more, reducing advantageous inequity.

Selfish median voters would like to redistribute because they are poorer than the average voter. Part (b) follows by simply noting that fair median voters have an additional tendency to redistribute on account of their fairness.

From Remark 1 and Proposition 2, selfish and fair voters alike, respond to increased affluence of the very rich by redistributing more (part (c)) and so also raising the ratio of social spending to GDP. Selfish voters would like to redistribute more when the rich get richer because average incomes increase and so the lumpsum available for redistribution is higher. Fair voters have an additional motive to redistribute more, namely, that it reduces disadvantageous inequity.

Parts (e)-(h) point out to an important difference in the predictions of the fair and selfish voter models. From part (e), for selfish voters, an increase in poverty reduces the tax rate and the ratio of social spending to GDP. The intuition is that poverty reduces average income available for redistribution, hence, reducing the marginal benefits of increasing the tax rate. For fair voters, however, the results can go either way; part (f) gives the appropriate condition. The reason is that on the one hand, the fair voter is influenced by very similar considerations as the selfish voter (because the fair voter also cares about 'own' payoff). However, on the other hand, the empathy/concern for poorer voters, on account of the disutility arising from advantageous inequity induces the fair voter in the opposite direction i.e. greater redistribution. The interplay between these two opposing factors determines if the fair voter will respond, unlike the selfish voter, by redistributing more in response to poverty. From parts (g), (h), for fair voters, if α or β is sufficiently high, then empathy for the poorer voters (as well as envy of richer voters) becomes stronger, which increases the tax rate and the ratio of social spending to GDP in response to increased poverty.

4.1. Social spending in recessions

The selfish and fair voter models alike, predict that a reduction in the skill of voters *above* the median will reduce the ratio of social spending to GDP (Remark 1 and Proposition 2(c)). However, the two models differ on what would happen to this ratio in response to a decline in the skill of voters *below* the median (equivalently, an increase in poverty). The selfish voter model predicts that the ratio of social spending to GDP would *decline* (Proposition 2(e)). On the other hand, the fair voter model predicts that this ratio will *increase*, if α and/or β is sufficiently high (Proposition 2(g) and 2(h)). Recall that, in our model, 'skill' is just a measure of the ability of a voter to translate labour time into income. We may, therefore, identify periods of high unemployment with episodes where the 'skills' of below median voters receive strong negative shocks. The selfish voter model would then predict a *decline* in the ratio of social spending to GDP, while the fair voter model would predict an *increase* in this ratio. Thus, the selfish voter model predicts *pro*cyclic movement of the social spending to GDP ratio, while the fair voter model predicts a *counter*cyclical movement. For the US data, the prediction of the selfish voter model is inconsistent with the evidence, while the prediction of the fair voter model is consistent with the evidence; see, for instance, Auerbach (2003).

4.2. Some illustrative calibration results

Consider a calibration exercise in a three voter model ($m = 2$, $n = 3$) with skill levels $s_1 < s_2 < s_3$. For simplicity of illustration, we take $\epsilon = 1$. This accords with partitioning society into three broad groups, namely, *poor*, *middle class* and *rich*. Proposition 2(f) can be illustrated diagrammatically. Figure 4.1 plots the surface $\left(\alpha, \beta, \frac{\partial t_2}{\partial s_1}\right)$ as well as the horizontal plane passing through $(0, 0, 0)$. In the plot we use $\beta \in [0, 1]$, $\alpha \in [0.5, 5]$ and $s_3^2/s_2^2 = 2.5$ (hence, the income of the rich is 2.5 times the income of the middle income voters, which is not unrealistic). Since, $\frac{\partial t_2}{\partial s_1}$ is plotted along the vertical axis, the selfish voter model predicts that the graph will lie above the plane passing through $(0, 0, 0)$, while the fair voter model predicts that the graph will lie below the plane, especially when α, β are high. Keeping in mind that $\alpha > \beta$, the calibrations are consistent with the fair voter model but not the selfish voter model.[17]

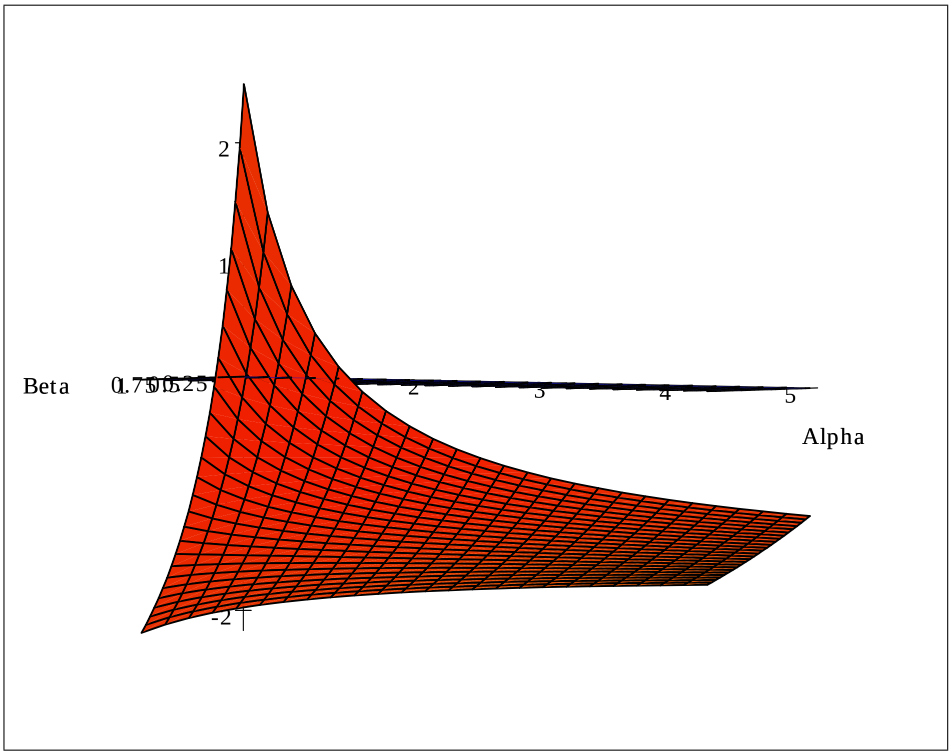


Figure 4.1: **Plot of** $\partial t_2^* / \partial s_1^2$.

4.3. Fairness and inequality

We plot in Figure 4.1 the redistributive tax chosen by a fair median voter, as given in (4.4), when inequality and fairness vary simultaneously. We consider a three voter model ($m = 2$, $n = 3$). The increase in inequality is caused by the rich getting richer (an increase in s_3), with s_1 and s_2 held fixed. This leads to the same comparative static effect in the selfish and the fair voter models i.e. an increase in redistributive tax (see Proposition

[17] The graph also plots the surface for $\alpha < \beta$ while the restriction in the FS model is that $\alpha > \beta$. This rules out about half the area of the surface that we observe above the plane passing through $(0, 0, 0)$. Hence, the predictions of the selfish voter model are even more at variance with the calibrations than is suggested by a look at the plot.

2(c)). In Figure 4.1 we use the following simulations values, $s_1 = 1$, $s_2 = 2.5$ and s_3, which captures *inequality*, varies between 8 and 12. We fix $\beta = 0.5$ and allow α, which captures *fairness* to vary between 0.6 and 5.

5. Income distribution and the tax rate

In section 4 we have already seen the redistributive effect of changes in skill levels of voters above and below the median voter. We are now interested in the redistributive outcome of a change in the entire discrete distribution of skills (the analogue of second order stochastic dominance when skills are continuous). There are a large number of measures of income inequality. For discussions of these see, for example, Atkinson (1970), Marshall and Olkin (1979), Preston (1990, 2006) and Zheng (2006). Here we shall consider two such measures.

Definition 2 : *Consider the set of income vectors*

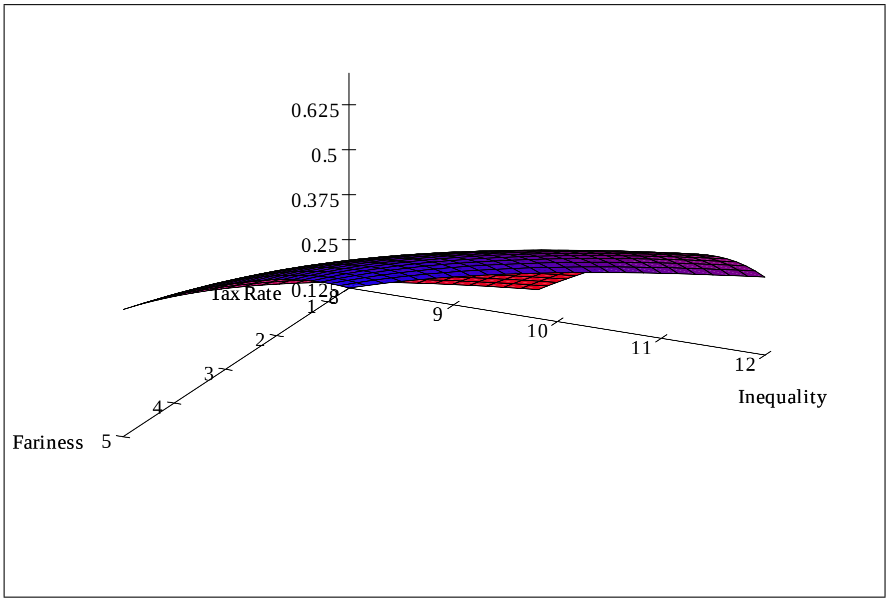
$$\mathbf{I} = \left\{ \mathbf{x} : 0 < x_1 \leq x_2 \leq \ \dots \ \leq x_n \text{ and } \frac{1}{n}\Sigma_{i=1}^{n} x_i = \mu \right\}.$$

(a) If $\Sigma_{i=1}^{j} x_i \geq \Sigma_{i=1}^{j} y_i$, $j = 1, 2, \dots, n-1$, *we say that* $\mathbf{x}$ *Lorenz dominates* $\mathbf{y}$. *If one of these inequalities is strict, we say* $\mathbf{x}$ *strictly Lorenz dominates* $\mathbf{y}$. *(Atkinson, 1970).*
(b) If $i < j \Rightarrow x_j - x_i \leq y_j - y_i$, $i, j = 1, 2, ..., n$, *we say that* $\mathbf{x}$ *difference dominates* $\mathbf{y}$. *If one of these inequalities is strict, we say* $\mathbf{x}$ *strictly difference* dominates $\mathbf{y}$. *(Marshall and Olkin, 1979).*

Remark 5 : *It is easy to see that the condition of (b) is satisfied if, and only if,* $x_i - y_i$ *is non-increasing in* i. *Marshall and Olkin (1979) showed that difference dominance implies Lorenz dominance, but not the reverse.*

Remark 6 : *Under (4.1), the pre-tax income of voter* i *is* $y_i = s_i l_i = (1-t)^{\epsilon} s_i^{1+\epsilon}$, *after tax income is* $(1-t)\, y_i + b$ *and indirect utility is* $\frac{1-t}{1+\epsilon} y_i + b$ *(from (2.3), (2.12), (4.1) and (10.24)). Hence, for our model, the two inequality measures, difference dominance (Marshall and Olkin, 1979) and utility gap dominance (Zheng, 2006), coincide.*

Proposition 3 : *For selfish and fair voters, if the vector of incomes* $\mathbf{x} \in \mathbf{I}$ *(strictly) difference dominates the vector of incomes* $\mathbf{y} \in \mathbf{I}$, *then the tax rate associated with* $\mathbf{y}$ *is (strictly) higher than the tax rate associated with* $\mathbf{x}$.



The relation between redistribution, fairness and inequality

For the selfish and fair voter models alike, an increase in inequality, as measured by an increase in (before-tax) income differences, will increase the tax rate (Definition 2, Remark 5 and Proposition 3). For the fair voter model, there is another possible cause for an increase in the tax rate, namely, an increase in 'fairness' as measured by an increase in α and/or β (see Proposition 2(a), 2(g), 2(h)).

The existing literature ignores issues of fairness. In actual practice, empirical researchers could be picking up any sequence of points along the surface in Figure 5. This practice is likely to lead to mixed and possible contradictory results. As the figure clarifies, low inequality-high fairness countries have the same redistribution as high inequality-low fairness countries. However, controlling for fairness, a prediction of the model is that one should have greater redistribution where inequality is higher[18]. To the best of our knowledge this test has not been carried out. This issue, we believe, could have seriously contaminated the existing literature's attempt at finding an empirical relation between inequality and the extent of redistribution.

6. An economy with selfish and fair voters

Experimental evidence indicates that there is a very large fraction (roughly 40-60 percent depending on the experiment) of purely self-interested individuals. The behavior of these individuals accords well with the predictions of the selfish preferences model, even in

[18]Because the results are sensitive with respect to the definition of inequality, we remind the reader that in this exercise an increase in inequality arises from the rich getting richer which gives the same comparative static results in the fair and the selfish voter models.

bilateral interactions. An important and interesting issue for theoretical and empirical research is to examine the implications of heterogeneity of preferences in the population. A range of theoretical and experimental work indicates that even a minority of individuals with social preferences can significantly alter the standard predictions.

What is the predicted redistributive outcome when there are both selfish and fair voters? To keep the analysis simple, we concentrate on a 3 voter economy ($m = 2$, $n = 3$). Here, there are $2^3 = 8$ possible combinations of voters. Denoting by S and F respectively, a selfish and a fair voter, the 8 possible combinations of the voters (each combination arranged in order of increasing skill level from left to right) are: SSS, FFF, SFF, SFS, FSF, FFS, SSF, FSS.[19] We have already looked at the first two cases, SSS and FFF, of purely selfish and purely fair voters above, hence, we concentrate below on the remaining six cases that consider mixtures of the selfish and the fair types.

If the j^{th} voter, $j = 1, 2, 3$, is fair, then, his preferences and indirect utility are given in (2.15), (2.16), (2.18) and (2.23). But if the j^{th} voter, $j = 1, 2, 3$, is selfish then his preferences are given by the analogous expressions but with $\alpha = \beta = 0$. Hence, there is intra-group homogeneity of preferences within the groups of fair and selfish voters but inter-group heterogeneity across the two groups. A second source of heterogeneity across all voters is the level of skill.

We consider the case $\epsilon = 1$, (see (4.3) and the discussion following it). Denote by t_j^k, the most preferred tax rate of a voter with skill level s_j, $j = 1, 2, 3$. The superscript $k = F, S$ denotes respectively a fair and a selfish voter.

Proposition 4 *: For the three voter model, the most preferred tax rates of a fair voter with the three levels of skills are given by:*

$$t_1^F = \frac{\frac{1}{3}\Sigma_{i=1}^n s_i^2 - s_1^2 + \frac{\alpha}{2}\left(s_2^2 - s_1^2\right) + \frac{\alpha}{2}\left(s_3^2 - s_1^2\right)}{\frac{2}{3}\Sigma_{i=1}^n s_i^2 - s_1^2 + \frac{\alpha}{2}\left(s_2^2 - s_1^2\right) + \left(s_3^2 - s_1^2\right)} > 0,$$

$$t_2^F = \frac{\frac{1}{3}\Sigma_{i=1}^n s_i^2 - s_2^2 + \frac{\alpha}{2}\left(s_3^2 - s_2^2\right) + \frac{\beta}{2}\left(s_2^2 - s_1^2\right)}{\frac{2}{3}\Sigma_{i=1}^n s_i^2 - s_2^2 + \frac{\alpha}{2}\left(s_3^2 - s_2^2\right) + \frac{\beta}{2}\left(s_2^2 - s_1^2\right)} > 0,$$

$$t_3^F = \frac{\frac{1}{3}\Sigma_{i=1}^n s_i^2 - s_3^2 + \frac{\beta}{2}\left(s_3^2 - s_1^2\right) + \frac{\beta}{2}\left(s_3^2 - s_2^2\right)}{\frac{2}{3}\Sigma_{i=1}^n s_i^2 - s_3^2 + \frac{\beta}{2}\left(s_3^2 - s_1^2\right) + \frac{\beta}{2}\left(s_3^2 - s_2^2\right)} > 0, \text{ if } \beta > \frac{2}{3}. \text{ Otherwise, } t_3 = 0.$$

In any mixture of the two types of voters, the redistributive outcome is altered if and only if, relative to the case of purely selfish or purely fair voters, the identity of the median voter alters. Proposition 5 checks the various cases.

[19] For instance, SFF denotes an economy in which the lowest skill voter is selfish and the middle and high skill voters are both fair.

Proposition 5 : *For the three voter model:*
(a) When all voters are fair then $t_1^F > t_2^F > t_3^F$*. When all voters are selfish then* $t_1^S > t_2^S > t_3^S$*. Furthermore,* $t_j^S > t_j^F$ $j = 1, 2, 3$*.*
(b) In cases FSF*,* FFS*,* SSF*,* FSS*, the restriction* $\beta < 1$ *ensures that the identity of the median voter is the same as the median skill voter i.e. voter 2.*
(c) In cases SFS *and* SFF*, it is possible that the identity of the median voter and the median skill individual might diverge. In particular, if*

$$\frac{2-\beta}{\alpha} < \frac{s_3^2 - s_2^2}{s_2^2 - s_1^2} \tag{6.1}$$

then the Condorcet winner is the lowest skill voter, voter 1. In the complementary case, the Condorcet winner is the median skill voter, voter 2.

In cases FSF, FFS, SSF, FSS the median skill voter is decisive in the redistributive tax choice and, hence, the redistributive outcome in the case of a mixture of voter types is identical to an economy in which all voters are of the same type as the median skill voter. In cases FSF, FFS two thirds of the voters are fair while a third are selfish. However the redistribute outcome in the former is controlled by a selfish voter while in the latter it is controlled by a fair voter. In cases SSF, FSS where a majority of the voters are selfish, the redistributive outcome is always controlled by a selfish voter.

The interesting case arises when the inequality in (6.1) is satisfied. We illustrate the implications for the case of a three voter SSF economy in (t, s) space in Figure 6.1 .

The poorest voter has selfish preferences, but the middle class and the rich have fair preferences. We depict two possible situations. The first is depicted by the (lighter) curve labelled I. The height of each dot represents the most preferred tax rate of a voter with the corresponding skill level. In this case, the median voter is the middle class voter who is a Condorcet winner, condition (6.1) is not satisfied, and the tax rate t_2 is implemented.

Now suppose that condition (6.1) is satisfied. Three factors are conducive to the inequality in (6.1) being satisfied. First inequity aversion, as captured by the magnitudes of α, β, should be high. Second, inequality at the upper end of the skill distribution, as measured by the range, $s_3 - s_2$, should be high. Third, inequality at the lower end of the income distribution, as measured by the range, $s_2 - s_1$, should be low. The new set of most preferred tax rates are shown by graph II. The most preferred tax rate of the selfish voter is unaffected. However, now the rich-fair voter finds the most preferred tax rate of the poor-selfish voter, t_1, to be closer to her most preferred tax rate. A coalition of these voters can now ensure that the Condorcet winner is t_1.

Higher inequality at the upper end of the skill distribution, as measured by the range, $s_3 - s_2$, ensures that the tax rates of the middle class-fair voter and the rich-fair voter are sufficiently distant that the later find the tax policy of a poor-selfish voter more attractive.

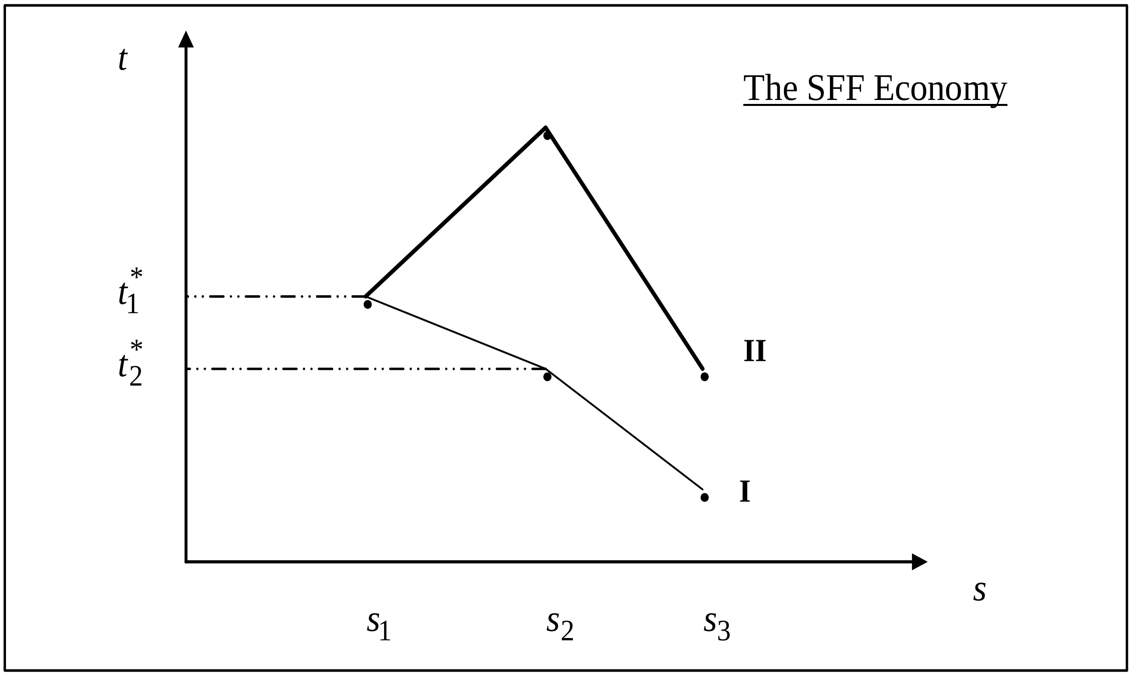


Figure 6.1: **Big policy jumps when small changes take place.**

Also note that in graph II, the poor-selfish voter is more 'conservative' than the middle class-fair voter, in the sense that he/she prefers a relatively *lower* tax rate.

There are two interesting implications of Proposition 5(b):

1. In several cases, i.e., FSF, SSF, FSS the selfish median skill voter is decisive . The presence of fair voters does not alter the policy choice.

2. The introduction of low skill, selfish voters can have *large* effects on policy when the median voter is fair. This is most striking in the case SFF when condition (6.1) is met. In this case, even when 2/3 of the voters are fair, redistribution is controlled by a 1/3 of the poorest, but selfish, voters. Thus, immigrants, insofar they are low skilled can sometimes have large redistributive consequence. Over a longer horizon this raises interesting issues of the dynamics of economic, cultural and social change.

7. An illustrative empirical exercise

In this section we test if redistribution is affected by (1) fairness, and (2) inequality. While earlier empirical studies have also looked at the effect of income inequality on redistribution, the novelty of our empirical exercise is twofold. First, we test for the effect of fairness on redistribution. Second, we use factor incomes rather than disposable incomes to generate the inequity measure.

The empirical exercise is only of an illustrative nature because data on factor incomes, which is crucial to testing the theory, is available only over a very short duration. This prevents one from conducting a more satisfactory econometric exercise that would, for instance, control extensively for country-specific effects. Nevertheless, our results are in line with other empirical results based on larger data sets and use disposable incomes (rather than factor incomes) but ignore issues of fairness.

7.1. Description of the data

We use cross-country data for OECD economies for 2003[20]. The list of variables and their explanation is as follows.

Dependent Variables: The dependent variable, denoted by SS/GDP, is social spending as a percentage of GDP at current prices taken from Lindert (2003)[21]. We have also tried as our dependent variable, tax revenues as a percentage of GDP, TR/GDP.

Fairness Variables: We use three possible measures in this regard. The first, denoted by ODA/GDP, is the ratio of 'Official development assistance' to GDP for each country. However, such assistance might also reflect motives other than fairness such as 'strategic giving', motivated by political considerations. For that reason we use a second measure of aid, namely, 'multilateral aid as a percentage of GDP; this is denoted by MA/GDP. Data on these is available through OECD statistics[22]. Our third measure, denoted by QAA/GDP is 'quality adjusted aid' to GDP ratio drawn from an index compiled by Roodman (2005).[23]

Inequality Variables: We have noted above the argument against using disposable or post-tax incomes to construct inequality measures. However, factor income data is very difficult to obtain and till recently has been unavailable. Hence, most existing empirical work has used data from disposable income to measure inequality. There have been two main sources of data for measures of inequality, based on disposable income. The main source is the 'Luxembourg Income Study' (LIS), which collates micro-data from various OECD countries based on survey information. The information is not contemporaneous. So, for instance, while the data for the Scandinavian countries, United Kingdom and Italy dates from 1995, that for Germany and France dates from 1994. See for instance, Figure 1 in Smeeding (2002). However, this is not a particularly serious problem because income

[20] The list of 20 countries that we use is as follows. Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and the United States.

[21] The underlying source is OECD, *Social Expenditure Database* 1980-1996.

[22] Our source is 'OECD in Figures: Statistics on the Member Countries', 2004 edition.

[23] Aid is adjusted for quality in that it takes account, among other things, of (1) the recipients of aid (relatively prosperous eastern European countries or abysmally poor sub-saharan African countries) (2) tied versus untied aid (3) whether cancelled interest payments are counted as aid (4) quality of governance.

inequality moves relatively slowly. The second main source of data is from the World Bank (e.g. the 2005 World Development Indicators, Table 2.7). We indicate the Gini coefficients obtained from these two data sources, respectively, by $Gini(LIS)$ and $Gini(World\ Bank)$. Disaggregated data is also available. LIS provides data on income by percentiles and the World Bank breaks down the income distribution into 5 parts. Using each of these more disaggregated data has its problems[24].

Our point of departure from the existing literature in the use of inequality variables is to rely instead on the newly created dataset on factor incomes that has been made available in Milanovic (2000). Denote the Gini calculated on the basis of factor incomes as $Gini(Factor\ Income)$. We shall compare alternative regression specifications based on the various Gini coefficients.

Control Variables: The final two variables are control variables that are not necessarily related to inequality or to fairness. These are as follows. In line with several other empirical studies, our first variable is the proportion of population aged 65 or over, denoted by Pop_{65}. This takes account of transfers to the old. The second variable, denoted by D_{US}, is a dummy variable that takes a value 1 for United States and zero for all other countries in the sample. The reasons for including a US dummy (relative to Europe) can be found in several places in the literature and is referred to as 'American Exceptionalism'; for instance, Glaeser (2005).

7.2. Results

Tables I, II in Appendix 1 report the regression results. There are 20 observations and we report the results of robust OLS estimation in Stata[25]. The Akaike information criteria, AIC, and the Schwarz Bayesian information criteria, BIC, are used as specification tests (lower values of these two indicators reflect a better specification).

Table-I reports the relation between inequality and redistribution in the absence of fairness concerns. The three columns for results in the table correspond respectively to

[24] LIS provides data on the ratio of 90th percentile to the median percentile and the 10th percentile to the median percentile. However, each of these variables are highly correlated (the correlation is about -0.85) hence they cannot be used simultaneously. The World Bank provides disaggregated information on the bottom 10% , top 10%, bottom 20%, next 20% and so on. However, where does the median voter lie among these? What would be an objective agglomeration of these figures when one imagines society as comprised of three broad groups: poor, middle class and rich? To us, the answer to these questions is not clear. Hence, we focus only on the Gini coefficients. This might not be a bad approximation because the Gini is very highly correlated with the ratio of 90th percentile to the median percentile and with the 10th percentile to the median percentile.

[25] The Stata regress command includes a robust option for estimating the standard errors using the Huber-White sandwich estimators. This allows one to deal with problems about normality, heteroscedasticity, or some observations that exhibit large residuals. With the robust option, the point estimates of the coefficients are exactly the same as in ordinary OLS, but the standard errors take into account the issues mentioned above.

the use of *Gini* (*LIS*), *Gini* (*World Bank*) and *Gini* (*Factor income*) as alternative proxies for income inequality. It is clear that the AIC and the BIC unambiguously pick the regression with *Gini* (*Factor income*) as the best specified. In this regression (the last column), inequality has an insignificant effect on redistribution, while both controls are significant and have the correct signs.

Table-II uses the *Gini* (*Factor income*) as our inequality measure but introduces three alternative notions of fairness, respectively, MA/GDP, ODA/GDP and QAA/GDP. The regression corresponding to each is reported in columns 2, 3 and 4. The specification tests unambiguously pick out the first regression, using MA/GDP, as the best specified. Relative to Table-I, the AIC and BIC are substantially lower for all regressions in Table-II. The fairness variables are all highly significant and have the correct signs. The intercept is no longer significant and the two controls have the correct sign and are generally significant. However, Pop_{65} is more significant across the regressions. The inequality variable is not significant in any of these regressions[26].

On the basis of the illustrative regression results we have three main conclusions. First, inequality measured on the basis of factor incomes (rather than disposable incomes) leads to a better specification. Second, the negative (and mostly significant) coefficient on the dummy variable for the US supports the idea of 'American Exceptionalism'. Third, the fairness variable is a very important determinant of redistribution. Certainly, based on the evidence, it is more important than the inequality variable, which the literature has focussed on so far.

8. Conclusions

We replace the self interested voters in the Romer-Roberts-Meltzer-Richard (RRMR) framework with voters who have a preference for fairness (as in Fehr-Schmidt (1999)) and ask the following questions. First, does a median voter equilibrium exist? Second, if yes, then what are its properties? Third, what are the features of the equilibrium redistributive policy when there is a mixture of fair and selfish voters? Fourth, conditional on data limitations, is fairness an important factor in determining redistribution?

Our findings are as follows. The single crossing property of Gans-Smart (1996) can be used to demonstrate the existence of a Condorcet winner when voters are fair. Increased fairness leads to a more redistributive outcome. Fair voters, if they are very fair, will

[26] This contrasts our results with Milanovic (2000) who finds a positive and significant effect of factor income inequality on redistribution to the poor. However the spirit of the theoretical predictions is that everyone (rich, middle class and poor) receive a lumpsum transfer. For instance, expenditure on health that everyone benefits from but the poor possibly benefit more than the rich because they might not have access to private medical care. Hence, the measure of redistribution often used in the empirical literature i.e. social spending to GDP is the one we prefer to use.

respond to poverty by redistributing more (and not less as the selfish voter model predicts). The ratio of social spending to GDP moves countercyclically in the fair voter model but pro-cyclically in the selfish voter model. The latter is not consistent with the evidence.

Introducing selfish poor voters in an economy populated by fair voters can have large effects on the redistributive equilibrium outcome. In particular, selfish poor voters may be more conservative than middle income fair voters, in the sense that the former may prefer a *lower* tax rate than the latter. Thus, introducing selfish poor voters in an economy populated by fair voters can result in a *reduction* in redistribution. On the other hand, introducing rich selfish voters in an economy populated by fair voters will have no effect.

Our illustrative empirical exercise show that factor income inequality, which is the appropriate variable suggested by theory, outperforms disposable income inequality, which is largely used in existing empirical work. Fairness is a very significant variable in explaining redistribution in OECD economies.

9. Appendix 1: Fairness, Inequality and Redistribution

Table-I: Inequality and Redistribution

	1	2	3
Constant	35.96^{***} 2.87	24.79 1.29	13.22 0.74
Gini (LIS)	-110.21^{***} -4.43	-	-
Gini (World Bank)	-	-0.63^{*} -1.80	-
Gini (Factor Income)	-	-	-0.34 -1.13
D_{US}	3.03 1.28	-0.14 -0.06	-5.47^{***} -3.87
Pop_{65}	1.23^{**} 2.37	1.19^{*} 1.96	1.91^{***} 3.78
R^2	0.72	0.48	0.59
F	95.33	31.85	37.32
AIC	110.19	122.39	92.76
BIC	113.18	125.37	95.08
n	20	20	16

Note: $t-$values in parentheses. Superscript $*$, $**$, $***$ denotes significance at the 10%, 5% and 1% level respectively.

Table-II: Fairness, Inequality and Redistribution

	1 (MA/GDP)	2 (ODA/GDP)	3 (QAA/GDP)
Constant	7.54 0.40	4.70 0.21	6.00 0.30
Fairness	37.58^{***} 3.34	9.4^{*} 2.10	20.59^{***} 2.85
Gini (Factor Income)	-0.15 -0.42	-0.18 -0.45	-0.19 -0.51
D_{US}	-2.90 -1.75	-3.61^{**} -2.73	-3.85^{***} -3.33
Pop_{65}	1.23^{***} 4.82	1.63^{***} 5.79	1.58^{***} 6.03
R^2	0.80	0.72	0.71
F	92.41	65.20	0.55
AIC	85.20	88.93	89.10
BIC	89.06	92.01	92.19
n	16	16	16

Note: $t-$values in parentheses. Superscript $*$, $**$, $***$ denotes significance at the 10%, 5% and 1% level respectively.

10. Appendix 2: Proofs

10.1. Proof of Lemma 1

Given t, b and s_i, $U(l_i; t, b, s_i)$ is a continuous function of l_i on the non-empty compact set $[0,1]$. Hence, a maximum exists. Since u is thrice differentiable, so is U and, from (2.11), we get

$$\frac{\partial U}{\partial l_i} = (1-t) s_i u_1 ((1-t) s_i l_i + b, 1 - l_i) - u_2 ((1-t) s_i l_i + b, 1 - l_i), \quad (10.1)$$

$$\frac{\partial^2 U}{\partial b \partial l_i} = (1-t) s_i u_{11} - u_{12}, \quad (10.2)$$

$$\frac{\partial^2 U}{\partial l_i^2} = (1-t)^2 s_i^2 u_{11} - 2(1-t) s_i u_{12} + u_{22}. \quad (10.3)$$

From (2.10a), (2.10b) and (10.2) we get

$$\frac{\partial^2 U}{\partial b \partial l_i} \leq 0, \quad (10.4)$$

and from (2.10a), (2.10b), (2.10c) and (10.3) we get

$$l_i > 0 \Rightarrow \frac{\partial^2 U}{\partial l_i^2} < 0. \quad (10.5)$$

First, consider the case $t = 1$. From (2.11), $U(l_i; 1, b, s_i) = u(b, 1 - l_i)$. From (2.9b), $u(b, 1 - l_i)$ is a strictly decreasing function of l_i on $(0, 1]$. By continuity, $u(b, 1 - l_i)$ must be a strictly decreasing function of l_i on $[0, 1]$. Hence, the optimum must be

$$l_i = 0 \text{ at } t = 1. \quad (10.6)$$

Now suppose $t \in [0, 1)$. From (2.1), (2.9a), (2.9c) and (10.1) we get:

$$\frac{\partial U}{\partial l_i}(0; t, b, s_i) = (1-t) s_i u_1 (b, 1) - u_2 (b, 1) = (1-t) s_i u_1 (b, 1) > 0,$$

and, using (2.9d),

$$\frac{\partial U}{\partial l_i}(1; t, b, s_i) = (1-t) s_i u_1 ((1-t) s_i + b, 0) - u_2 ((1-t) s_i + b, 0) < u_1 (0, 0) - u_2 (0, 0) \leq 0.$$

Hence, a maximum is an interior point, i.e.,

$$0 < l_i < 1. \quad (10.7)$$

From (10.7) and (10.5) it follows that $\frac{\partial^2 U}{\partial l_i^2} < 0$. Hence, the maximum is unique and is given by

$$\frac{\partial U}{\partial l_i}(l_i; t, b, s_i) = 0. \quad (10.8)$$

Since, from (2.9c), $u_2(b,0)=0$, it follows, from (10.1) and (10.6), that (10.8) also holds for $t=1$. Hence, for any voter i, the labor supply,

$$l_i = l(t,b,s_i), t \in [0,1], \tag{10.9}$$

can be found by solving (10.8).

Since u is thrice continuously differentiable it follows, from (2.11) and (10.8) that $l(t,b,s_i)$ is twice continuously differentiable. If $t=1$ then, from (10.6), $\frac{\partial l_i}{\partial b}=0$. Now suppose $t<1$. From (10.5), $\frac{\partial^2 U}{\partial l_i^2}<0$. Hence, from (10.4) and the implicit function theorem (or differentiating the identity (10.8)), we get $\frac{\partial l_i}{\partial b} = -\left[\frac{\partial^2 U}{\partial b \partial l_i} / \frac{\partial^2 U}{\partial l_i^2}\right]_{l_i=l(t,b,s_i)} \leq 0$. Hence, for all $t \in [0,1]$,

$$\frac{\partial l_i}{\partial b} \leq 0. \tag{10.10}$$

Let $f(b) = \frac{1}{n} t \Sigma_{i=1}^{n} s_i l(t,b,s_i)$. Since $f(b) \geq 0$, $f(b)$ is twice differentiable (hence continuous) and $f'(b) \leq 0$ (from (10.10)), it follows that $f(b)=b$ has a unique solution, $b(t,\mathbf{s}) \geq 0$; and $b(t,\mathbf{s})$ is twice continuously differentiable. ■

10.2. Proof of Lemma 2

From (2.11), (2.12) and the envelope theorem (or Lemma 1 (d)), we get

$$\frac{\partial v(t,b,s)}{\partial b} = \left[\frac{\partial U(l;t,b,s)}{\partial b}\right]_{l=l(t,b,s)} = \left[u_1((1-t)sl+b,1-l)\right]_{l=l(t,b,s)}, \tag{10.11}$$

$$\frac{\partial v(t,b,s)}{\partial s} = \left[\frac{\partial U(l;t,b,s)}{\partial s}\right]_{l=l(t,b,s)} = \left[(1-t)lu_1((1-t)sl+b,1-l)\right]_{l=l(t,b,s)}, \tag{10.12}$$

$$\frac{\partial v(t,b,s)}{\partial t} = \left[\frac{\partial U(l;t,b,s)}{\partial t}\right]_{l=l(t,b,s)} = -\left[slu_1((1-t)sl+b,1-l)\right]_{l=l(t,b,s)}. \tag{10.13}$$

Part (a) follows from (2.9a) and (10.11). Part (b) follows from (2.9a) and (10.12). Part (c) follows from (2.9a), Lemma 1 (b) and (c), and (10.13). ■

10.3. Proof of Proposition 1

From (2.19)

$$\frac{\partial V_{j+1}(t,b,\alpha,\beta,\mathbf{s})}{\partial t} - \frac{\partial V_j(t,b,\alpha,\beta,\mathbf{s})}{\partial t} = \left(1-\frac{j\beta}{n-1}+\frac{(n-j)\alpha}{n-1}\right)\left(\frac{\partial v(t,b,s_{j+1})}{\partial t}-\frac{\partial v(t,b,s_j)}{\partial t}\right). \tag{10.14}$$

From (2.17), Assumption A1 and (10.14), it follows that, for $t \in [0,1)$,

$$\frac{\partial V_{j+1}(t,b,\alpha,\beta,\mathbf{s})}{\partial t} - \frac{\partial V_j(t,b,\alpha,\beta,\mathbf{s})}{\partial t} < 0. \tag{10.15}$$

From (2.18)

$$\frac{\partial V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)}{\partial b} = \frac{\partial v\left(t,b,s_j\right)}{\partial b} - \frac{\alpha}{n-1}\sum_{k>j}\left[\frac{\partial v\left(t,b,s_k\right)}{\partial b} - \frac{\partial v\left(t,b,s_j\right)}{\partial b}\right] \quad (10.16)$$
$$-\frac{\beta}{n-1}\sum_{i<j}\left[\frac{\partial v\left(t,b,s_j\right)}{\partial b} - \frac{\partial v\left(t,b,s_i\right)}{\partial b}\right]$$

From Lemma 2 (a), Assumption A1 and (10.16)

$$\frac{\partial V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)}{\partial b} > 0. \quad (10.17)$$

From (2.19)

$$\frac{\partial V_{j+1}\left(t,b,\alpha,\beta,\mathbf{s}\right)}{\partial b} - \frac{\partial V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)}{\partial b} = \left(1-\frac{j\beta}{n-1}+\frac{(n-j)\,\alpha}{n-1}\right)\left(\frac{\partial v\left(t,b,s_{j+1}\right)}{\partial b} - \frac{\partial v\left(t,b,s_j\right)}{\partial b}\right). \quad (10.18)$$

From (2.17), Assumption A1 and (10.18)

$$\frac{\partial V_{j+1}\left(t,b,\alpha,\beta,\mathbf{s}\right)}{\partial b} - \frac{\partial V_j\left(t,b,\alpha,\beta,\mathbf{s}\right)}{\partial b} \leq 0. \quad (10.19)$$

Now,

$$\left(\frac{-\frac{\partial V_{j+1}}{\partial t}}{\frac{\partial V_{j+1}}{\partial b}}\right) - \left(\frac{-\frac{\partial V_j}{\partial t}}{\frac{\partial V_j}{\partial b}}\right) = \frac{\left(\frac{\partial V_{j+1}}{\partial b}-\frac{\partial V_j}{\partial b}\right)\frac{\partial V_j}{\partial t}+\frac{\partial V_j}{\partial b}\left(\frac{\partial V_j}{\partial t}-\frac{\partial V_{j+1}}{\partial t}\right)}{\frac{\partial V_j}{\partial b}\frac{\partial V_{j+1}}{\partial b}}. \quad (10.20)$$

From (10.15), (10.17), (10.19), (10.20) and Assumption A2, we get that, for $t \in [0,1)$,

$$\left(\frac{-\frac{\partial V_{j+1}}{\partial t}}{\frac{\partial V_{j+1}}{\partial b}}\right) - \left(\frac{-\frac{\partial V_j}{\partial t}}{\frac{\partial V_j}{\partial b}}\right) > 0, \quad (10.21)$$

hence,

$$-\frac{\partial V_j}{\partial t}/\frac{\partial V_j}{\partial b} \text{ is strictly increasing in } j. \quad (10.22)$$

From Lemma 4 and (10.22) we get that the median-voter is decisive, i.e., a majority chooses the tax rate that is optimal for the median-voter. This establishes Proposition 1. ■

10.4. Proof of Corollary 1

Note that if u is quasi-linear, then $u(c,1-l) = c - f\left(l\right)$. Hence, $U\left(l;t,b,s\right) = \left(1-t\right)sl + b - f\left(l\right)$. By the envelope theorem (or direct calculation), $\frac{\partial v(t,b,s)}{\partial b} = \frac{\partial U(l;t,b,s)}{\partial b} = 1$. From this, and (10.16), we get that $\frac{\partial V_j}{\partial b} = 1$. Hence, (10.20) reduces to

$$\left(\frac{-\frac{\partial V_{j+1}}{\partial t}}{\frac{\partial V_{j+1}}{\partial b}}\right) - \left(\frac{-\frac{\partial V_j}{\partial t}}{\frac{\partial V_j}{\partial b}}\right) = \frac{\partial V_j}{\partial t} - \frac{\partial V_{j+1}}{\partial t} > 0, \quad (10.23)$$

where the inequality in (10.23) comes from (10.15). Hence, (10.22) again holds, but we have not used Assumption A2.

If voters are selfish, so that $\alpha = \beta = 0$, then Assumption A2 reduces to $\frac{\partial v}{\partial t} \leq 0$, which we know holds from Lemma 2(c). This establishes Corollary 1. ■

10.5. Proof of Lemma 5

Part (a) follows from (4.1) and (4.2) by straightforward calculation. For $t \in [0, 1)$, part (b) follows from (2.11), Lemma 1 (a), (d) and (4.1). From Lemma 1 (c), it follows that part (b) also holds for $t = 1$. Thus ϵ is elasticity of labour supply with respect to disposable income. From (2.13) and part (b), we get $y_i = (1-t)^{\epsilon} s_i^{1+\epsilon}$ and hence, from (2.4), we get part (c). Using (2.11), (2.12), (4.1), (4.2) and part (b), we can verify that part (d) holds. We now turn to part (e). From (2.1), (2.3), (2.5), (2.11), (2.12), (2.14), (2.16), (2.17), (2.23), (4.1), (4.2) and parts (b), (c), we get:

$$v(t, b, s) = \frac{1}{1+\epsilon}(1-t)^{1+\epsilon} s^{1+\epsilon} + b, \tag{10.24}$$

$$b(t, \mathbf{s}) = \frac{t(1-t)^{\epsilon}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon}, \tag{10.25}$$

$$w_j(t, \mathbf{s}) = \frac{1}{1+\epsilon}(1-t)^{1+\epsilon} s_j^{1+\epsilon} + \frac{t(1-t)^{\epsilon}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon}, \tag{10.26}$$

$$\frac{\partial w_j(t, \mathbf{s})}{\partial t} = -(1-t)^{\epsilon} s_j^{1+\epsilon} + \frac{(1-t)^{\epsilon}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon} - \frac{\epsilon t (1-t)^{\epsilon-1}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon}, \text{ for } t < 1, \tag{10.27}$$

$$\frac{\partial w_m(t, \mathbf{s})}{\partial t} = -(1-t)^{\epsilon} s_m^{1+\epsilon} + \frac{(1-t)^{\epsilon}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon} - \frac{\epsilon t (1-t)^{\epsilon-1}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon}, \text{ for } t < 1, \tag{10.28}$$

$$\frac{\partial w_m(t, \mathbf{s})}{\partial t} - \frac{\partial w_i(t, \mathbf{s})}{\partial t} = -(1-t)^{\epsilon}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right), \text{ for } t < 1, \tag{10.29}$$

$$\frac{\partial w_k(t, \mathbf{s})}{\partial t} - \frac{\partial w_m(t, \mathbf{s})}{\partial t} = -(1-t)^{\epsilon}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right), \text{ for } t < 1, \tag{10.30}$$

$$\begin{aligned}\frac{\partial^2 w_j(t, \mathbf{s})}{\partial t^2} &= \epsilon(1-t)^{\epsilon-1} s_j^{1+\epsilon} - \frac{\epsilon(1-t)^{\epsilon-1}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon} - \frac{\epsilon(1-t)^{\epsilon-1}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon} \\ &\quad - \frac{\epsilon(1-\epsilon) t (1-t)^{\epsilon-2}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon}, \text{ for } t < 1,\end{aligned} \tag{10.31}$$

$$\begin{aligned}\frac{\partial^2 w_m(t, \mathbf{s})}{\partial t^2} &= \epsilon(1-t)^{\epsilon-1} s_m^{1+\epsilon} - \frac{\epsilon(1-t)^{\epsilon-1}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon} - \frac{\epsilon(1-t)^{\epsilon-1}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon} \\ &\quad - \frac{\epsilon(1-\epsilon) t (1-t)^{\epsilon-2}}{n} \Sigma_{i=1}^{n} s_i^{1+\epsilon}, \text{ for } t < 1,\end{aligned} \tag{10.32}$$

$$\frac{\partial^2 w_m(t,\mathbf{s})}{\partial t^2} = -\epsilon(1-t)^{\epsilon-1}\left(\frac{1}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon}\right) - \frac{\epsilon(1-t)^{\epsilon-1}}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} \tag{10.33}$$
$$-\frac{\epsilon(1-\epsilon)t(1-t)^{\epsilon-2}}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} < 0, \text{ for } t<1,$$

$$\frac{\partial^2 w_m(t,\mathbf{s})}{\partial t^2} - \frac{\partial^2 w_i(t,\mathbf{s})}{\partial t^2} = \epsilon(1-t)^{\epsilon-1}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right) > 0, \text{ for } i<m, \text{ for } t<1, \tag{10.34}$$

$$\frac{\partial^2 w_k(t,\mathbf{s})}{\partial t^2} - \frac{\partial^2 w_m(t,\mathbf{s})}{\partial t^2} = \epsilon(1-t)^{\epsilon-1}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) > 0, \text{ for } k>m, \text{ for } t<1, \tag{10.35}$$

$$W_m(t,\alpha,\beta,\mathbf{s}) = t(1-t)^{\epsilon}\frac{1}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} + (1-t)^{1+\epsilon}\left(s_m^{1+\epsilon} - \frac{\epsilon}{1+\epsilon}s_m^{1+\epsilon}\right) \tag{10.36}$$
$$-(1-t)^{1+\epsilon}\left(\frac{\alpha}{n-1}\frac{1}{1+\epsilon}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\frac{1}{1+\epsilon}\sum_{j<m}\left(s_m^{1+\epsilon} - s_j^{1+\epsilon}\right)\right).$$

$$\frac{\partial W_m(t,\alpha,\beta,\mathbf{s})}{\partial t} = (1-t)^{\epsilon}\left[\frac{1}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon} - \frac{t\epsilon}{1-t}\frac{1}{n}\Sigma_{i=1}^n s_i^{1+\epsilon}\right] + \tag{10.37}$$
$$(1-t)^{\epsilon}\left[\frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right], \text{ for } t<1,$$

$$\frac{\partial^2 W_m(t,\alpha,\beta,\mathbf{s})}{\partial t^2} = \frac{\partial^2 w_m(t,\mathbf{s})}{\partial t^2} - \frac{\alpha}{n-1}\sum_{k>j}\left(\frac{\partial^2 w_k(t,\mathbf{s})}{\partial t^2} - \frac{\partial^2 w_m(t,\mathbf{s})}{\partial t^2}\right) \tag{10.38}$$
$$-\frac{\beta}{n-1}\sum_{i<j}\left(\frac{\partial^2 w_m(t,\mathbf{s})}{\partial t^2} - \frac{\partial^2 w_i(t,\mathbf{s})}{\partial t^2}\right) < 0, \text{ for } t<1.$$

Set $\frac{\partial W_m(t,\alpha,\beta,\mathbf{s})}{\partial t} = 0$ in (10.37) and solve to get

$$t_m = \frac{\frac{1}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)}{\frac{1+\epsilon}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)}. \tag{10.39}$$

Hence, clearly,

$$0 < t_m < 1. \tag{10.40}$$

In the light of (10.38), $W_m(t,\alpha,\beta,\mathbf{s})$ attains a unique global maximum on $(0,1)$ at $t = t_m$. We shall show that this is also the unique global maximum on $[0,1]$. From (10.37) we get

$$\left[\frac{\partial W_m(t,\alpha,\beta,\mathbf{s})}{\partial t}\right]_{t=0} = \frac{1}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right) > 0 \tag{10.41}$$

Hence, $t = t_m$ is the unique global maximum of $W_m(t, \alpha, \beta, \mathbf{s})$ on $[0, 1)$. Next, from (10.36), we get

$$W_m(1, \alpha, \beta, \mathbf{s}) = 0, \tag{10.42}$$

and, from (10.36) and (10.39),

$$W_m(t_m, \alpha, \beta, \mathbf{s}) = \frac{1}{1+\epsilon}\left[\frac{1}{\epsilon}\left(\frac{1}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum_{j<m}\left(s_m^{1+\epsilon} - s_j^{1+\epsilon}\right)\right]$$

$$\times\left[\frac{\frac{\epsilon}{n}\Sigma_{i=1}^n s_i^{1+\epsilon}}{\frac{\epsilon}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} + \frac{1}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum\limits_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum\limits_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)}\right]^{1+\epsilon} > 0. \tag{10.43}$$

From (10.42) and (10.43), we see that $t = t_m$ is the unique global maximum of $W_m(t, \alpha, \beta, \mathbf{s})$ on $[0, 1]$. ■

10.6. Proof of Proposition 2

Let

$$x = \frac{1}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right). \tag{10.44}$$

From Lemma 5 (e) and (10.44), we get

$$t_m = \frac{1}{1 + \frac{\epsilon}{nx}\Sigma_{i=1}^n s_i^{1+\epsilon}}, \text{ where } x > 0 \text{ and } \frac{\epsilon}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} > 0. \tag{10.45}$$

It follows that an increase in x will increase t_m. In particular, an increase in α or an increase in β (or both) will increase t_m. This establishes part (a). It also follows that a fair median voter ($\alpha > 0, \beta > 0$) will vote for a higher tax rate than a selfish median voter ($\alpha = \beta = 0$). This establishes part (b).

For $j > m$,
$$\frac{\partial t_m}{\partial s_j} = \frac{\epsilon(1+\epsilon)s_j^\epsilon\left[\left(1 + \frac{\alpha}{n-1} + \frac{\alpha-\beta}{2}\right)s_m^{1+\epsilon} + \frac{\alpha+\beta}{n-1}\sum\limits_{i<m} s_i^{1+\epsilon}\right]}{n\left[\frac{1+\epsilon}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum\limits_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum\limits_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right]^2} > 0. \tag{10.46}$$

Hence, an increase in the skill of voter-workers above the median will increase the tax rate, whether the median voter is selfish or fair. This establishes part (c).

$$\frac{\partial t_m}{\partial s_m} = \tag{10.47}$$

$$-\frac{\epsilon(1+\epsilon)s_m^\epsilon\left[s_m^{1+\epsilon} + \left(1 + \frac{\alpha-\beta}{2}\right)\Sigma_{i=1}^n s_i^{1+\epsilon} + \frac{\alpha}{n-1}\sum\limits_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum\limits_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right]}{n\left[\frac{1+\epsilon}{n}\Sigma_{i=1}^n s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum\limits_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum\limits_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right]^2} < 0,$$

Hence, for selfish and fair voters alike, a reduction of the skill of the median voter with increase the tax rate. Conversely, an increase in the skill of the median voter will reduce the tax rate. This establishes part (d).

$$\text{For } j < m,\ \frac{\partial t_m}{\partial t_j} = \frac{\epsilon\left(1+\epsilon\right) s_j^{\epsilon}\left[s_m^{1+\epsilon} - \frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) - \frac{\beta}{n-1}\left(\Sigma_{i=1}^{n} s_i^{1+\epsilon} + \sum_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right)\right]}{n\left[\frac{1+\epsilon}{n}\Sigma_{i=1}^{n} s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right]^2}, \tag{10.48}$$

Part (f) follows from (90) since the coefficients of α and β in the numerator are positive. Set $\alpha = \beta = 0$ in (10.48) to get, for a selfish median voter,

$$\alpha = \beta = 0 \Rightarrow \text{for } j < m,\ \frac{\partial t_m}{\partial t_j} = \frac{\epsilon\left(1+\epsilon\right) s_j^{\epsilon} s_m^{1+\epsilon}}{n\left[\frac{1+\epsilon}{n}\Sigma_{i=1}^{n} s_i^{1+\epsilon} - s_m^{1+\epsilon}\right]^2} > 0, \tag{10.49}$$

Thus, a selfish median voter responds to a reduction in the skill of consumers below the median by *reducing* the tax rate. This establishes part (e). Recall, from Remark 1, that the tax rate in our model equals the ratio of social spending to aggregate income. Hence, to the extent that low skill workers are more adversely affected in a recession, the selfish voter model predicts pro-cyclic movement of the ratio of social spending to aggregate income.

Also, from (10.48), it follows that

$$\text{for } j < m,\ \frac{\partial t_m}{\partial t_j} < 0, \text{ if } \alpha \text{ or } \beta \text{ is sufficiently large.} \tag{10.50}$$

By contrast, a fair median voter responds to a reduction in the skill of consumers below the median by *increasing* the tax rate, provided that voter cares sufficiently about inequality. Hence, to the extent that low skill workers are more adversely affected in a recession, the fair voter model predicts counter cyclic movement of the ratio of social spending to aggregate income, in agreement with the evidence.

Since $\beta > 0$ and $\Sigma_{i=1}^{n} s_i^{1+\epsilon} + \sum_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right) > 0$ it follows, from (10.48), that

$$\text{for } j < m,\ \frac{\partial t_m}{\partial t_j} < \frac{\epsilon\left(1+\epsilon\right) s_j^{\epsilon}\left[s_m^{1+\epsilon} - \frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right)\right]}{n\left[\frac{1+\epsilon}{n}\Sigma_{i=1}^{n} s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right]^2}. \tag{10.51}$$

Hence, from (10.51),

$$\text{for } j < m,\ \alpha \geq \frac{\left(n-1\right) s_m^{1+\epsilon}}{\sum_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right)} \Rightarrow \frac{\partial t_m}{\partial t_j} < 0. \tag{10.52}$$

Hence, if the median voter sufficiently dislikes disparity with higher skill consumers, then he/she will vote for a higher tax rate in response to a reduction in the skill of low skill workers. This establishes part (g).

Since $0 < \beta < \alpha$ and $\sum\limits_{k>m} \left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) > 0$ we get, form (10.48),

$$\text{for } j < m,\ \frac{\partial t_m}{\partial t_j} < \frac{\epsilon\left(1+\epsilon\right)s_j^{\epsilon}\left[s_m^{1+\epsilon} - \frac{\beta}{n-1}\sum\limits_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) - \frac{\beta}{n-1}\left(\Sigma_{i=1}^{n}s_i^{1+\epsilon} + \sum\limits_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right)\right]}{n\left[\frac{1+\epsilon}{n}\Sigma_{i=1}^{n}s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum\limits_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum\limits_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right]^2}. \tag{10.53}$$

Simplifying, we get

$$\text{for } j < m,\ \frac{\partial t_m}{\partial t_j} < \frac{\epsilon\left(1+\epsilon\right)s_j^{\epsilon}\left[s_m^{1+\epsilon} - \frac{\beta}{n-1}\left(s_m^{1+\epsilon} + 2\sum\limits_{k>m}s_k^{1+\epsilon}\right)\right]}{n\left[\frac{1+\epsilon}{n}\Sigma_{i=1}^{n}s_i^{1+\epsilon} - s_m^{1+\epsilon} + \frac{\alpha}{n-1}\sum\limits_{k>m}\left(s_k^{1+\epsilon} - s_m^{1+\epsilon}\right) + \frac{\beta}{n-1}\sum\limits_{i<m}\left(s_m^{1+\epsilon} - s_i^{1+\epsilon}\right)\right]^2}. \tag{10.54}$$

Thus,

$$\text{for } j < m,\ \beta \geq \frac{\left(n-1\right)s_m^{1+\epsilon}}{s_m^{1+\epsilon} + 2\sum\limits_{k>m}s_k^{1+\epsilon}} \Rightarrow \frac{\partial t_m}{\partial t_j} < 0. \tag{10.55}$$

Hence, provided the median voter cares sufficiently about consumers with lower skill, he/she will vote for a higher tax rate in response to a reduction in the skill of low skill workers. However, we need to check that the lower bound on β in (10.55) is feasible, i.e., is consistent with $\beta < 1$. That this is the case, is established by (10.56), below, using (2.1) and the fact that $n = 2m - 1$.

$$\frac{\left(n-1\right)s_m^{1+\epsilon}}{s_m^{1+\epsilon} + 2\sum\limits_{k>m}s_k^{1+\epsilon}} < \frac{2\left(m-1\right)}{1 + 2\left(m-1\right)} < 1 \tag{10.56}$$

This completes the proof of part (h). ■

10.7. Proof of Proposition 3

Proof: Let t_m and T_m be the tax rates corresponding to $\mathbf{x} \in \mathbf{I}$ and $\mathbf{y} \in \mathbf{I}$, respectively. Then, from (2.13) and Lemma 5 (b), (e):

$$t_m = \frac{\frac{1}{n}\Sigma_{i=1}^{n}x_i - x_m + \frac{\alpha}{n-1}\sum\limits_{k>m}\left(x_k - x_m\right) + \frac{\beta}{n-1}\sum\limits_{i<m}\left(x_m - x_i\right)}{\frac{1+\epsilon}{n}\Sigma_{i=1}^{n}x_i - x_m + \frac{\alpha}{n-1}\sum\limits_{k>m}\left(x_k - x_m\right) + \frac{\beta}{n-1}\sum\limits_{i<m}\left(x_m - x_i\right)}, \tag{10.57}$$

$$T_m = \frac{\frac{1}{n}\Sigma_{i=1}^{n} y_i - y_m + \frac{\alpha}{n-1}\sum_{k>m}(y_k - y_m) + \frac{\beta}{n-1}\sum_{i<m}(y_m - y_i)}{\frac{1+\epsilon}{n}\Sigma_{i=1}^{n} y_i - y_m + \frac{\alpha}{n-1}\sum_{k>m}(y_k - y_m) + \frac{\beta}{n-1}\sum_{i<m}(y_m - y_i)}. \tag{10.58}$$

Let x and y be defined by:

$$x = \frac{1}{n}\Sigma_{i=1}^{n} x_i - x_m + \frac{\alpha}{n-1}\sum_{k>m}(x_k - x_m) + \frac{\beta}{n-1}\sum_{i<m}(x_m - x_i), \tag{10.59}$$

$$y = \frac{1}{n}\Sigma_{i=1}^{n} y_i - y_m + \frac{\alpha}{n-1}\sum_{k>m}(y_k - y_m) + \frac{\beta}{n-1}\sum_{i<m}(y_m - y_i), \tag{10.60}$$

then

$$t_m = \frac{x}{x + \frac{\epsilon}{n}\Sigma_{i=1}^{n} x_i} = \frac{1}{1 + \frac{\epsilon}{nx}\Sigma_{i=1}^{n} x_i}, \tag{10.61}$$

$$T_m = \frac{y}{y + \frac{\epsilon}{n}\Sigma_{i=1}^{n} y_i} = \frac{1}{1 + \frac{\epsilon}{ny}\Sigma_{i=1}^{n} y_i}. \tag{10.62}$$

If $\mathbf{x}$ difference dominates $\mathbf{y}$ (Definition 2), then $0 < x \leq y$. Hence, $t_m \leq T_m$. If $\mathbf{x}$ strictly difference dominates $\mathbf{y}$, then $0 < x < y$. Hence, $t_m < T_m$. ■

10.8. Proof of Proposition 4

Proof: For the case $\epsilon = 1$ we get

$$\begin{aligned} \frac{\partial W_j(t)}{\partial t} &= \frac{1}{n}\Sigma_{i=1}^{n} s_i^2 - s_j^2 + \frac{\alpha}{n-1}\sum_{k>j}\left(s_k^2 - s_j^2\right) + \frac{\beta}{n-1}\sum_{i<j}\left(s_j^2 - s_i^2\right) \\ &\quad - t\left[\frac{2}{n}\Sigma_{i=1}^{n} s_i^2 - s_j^2 + \frac{\alpha}{n-1}\sum_{k>j}\left(s_k^2 - s_j^2\right) + \frac{\beta}{n-1}\sum_{i<j}\left(s_j^2 - s_i^2\right)\right], \end{aligned} \tag{10.63}$$

$$\frac{\partial^2 W_j(t)}{\partial t^2} = -\left[\frac{2}{n}\Sigma_{i=1}^{n} s_i^2 - s_j^2 + \frac{\alpha}{n-1}\sum_{k>j}\left(s_k^2 - s_j^2\right) + \frac{\beta}{n-1}\sum_{i<j}\left(s_j^2 - s_i^2\right)\right]. \tag{10.64}$$

In particular, for the three voter model, we get

$$\begin{aligned} \frac{\partial W_1(t)}{\partial t} &= \frac{1}{3}\Sigma_{i=1}^{3} s_i^2 - s_1^2 + \frac{\alpha}{2}\left(s_2^2 + s_3^2 - 2s_1^2\right) \\ &\quad - t\left[\frac{2}{3}\Sigma_{i=1}^{3} s_i^2 - s_1^2 + \frac{\alpha}{2}\left(s_2^2 + s_3^2 - 2s_1^2\right)\right], \end{aligned} \tag{10.65}$$

$$\left[\frac{\partial W_1}{\partial t}\right]_{t=0} = \frac{1}{3}\Sigma_{i=1}^{3} s_i^2 - s_1^2 + \frac{\alpha}{2}\left(s_2^2 + s_3^2 - 2s_1^2\right) > 0, \tag{10.66}$$

$$\left[\frac{\partial W_1}{\partial t}\right]_{t=1} = -\frac{1}{3}\Sigma_{i=1}^{3} s_i^2 < 0, \tag{10.67}$$

$$\frac{\partial^2 W_1(t)}{\partial t^2} = -\left[\frac{2}{3}\Sigma_{i=1}^{3} s_i^2 - s_1^2 + \frac{\alpha}{2}\left(s_2^2 + s_3^2 - 2s_1^2\right)\right] < 0, \tag{10.68}$$

$$\frac{\partial W_2(t)}{\partial t} = \frac{1}{3}\Sigma_{i=1}^3 s_i^2 - s_2^2 + \frac{\alpha}{2}\left(s_3^2 - s_2^2\right) + \frac{\beta}{2}\sum_{i<j}\left(s_2^2 - s_1^2\right)$$
$$-t\left[\frac{2}{3}\Sigma_{i=1}^3 s_i^2 - s_2^2 + \frac{\alpha}{2}\left(s_3^2 - s_2^2\right) + \frac{\beta}{2}\sum_{i<j}\left(s_2^2 - s_1^2\right)\right], \tag{10.69}$$

$$\left[\frac{\partial W_2}{\partial t}\right]_{t=0} = \frac{1}{3}\Sigma_{i=1}^3 s_i^2 - s_2^2 + \frac{\alpha}{2}\left(s_3^2 - s_2^2\right) + \frac{\beta}{2}\sum_{i<j}\left(s_2^2 - s_1^2\right) > 0, \tag{10.70}$$

$$\left[\frac{\partial W_2}{\partial t}\right]_{t=1} = -\frac{1}{3}\Sigma_{i=1}^3 s_i^2 < 0, \tag{10.71}$$

$$\frac{\partial^2 W_2(t)}{\partial t^2} = -\left[\frac{2}{3}\Sigma_{i=1}^3 s_i^2 - s_2^2 + \frac{\alpha}{2}\left(s_3^2 - s_2^2\right) + \frac{\beta}{2}\left(s_2^2 - s_1^2\right)\right] < 0, \tag{10.72}$$

$$\frac{\partial W_3(t)}{\partial t} = \frac{1}{3}\Sigma_{i=1}^3 s_i^2 - s_3^2 + \frac{\beta}{2}\left(s_3^2 - s_1^2\right) + \frac{\beta}{2}\left(s_3^2 - s_2^2\right)$$
$$-t\left[\frac{2}{3}\Sigma_{i=1}^3 s_i^2 - s_3^2 + \frac{\beta}{2}\left(s_3^2 - s_1^2\right) + \frac{\beta}{2}\left(s_3^2 - s_2^2\right)\right], \tag{10.73}$$

$$\left[\frac{\partial W_3}{\partial t}\right]_{t=1} = -\frac{1}{3}\Sigma_{i=1}^3 s_i^2 < 0, \tag{10.74}$$

$$\left[\frac{\partial W_3}{\partial t}\right]_{t=0} > 0 \Leftrightarrow \beta > \frac{2}{3}, \tag{10.75}$$

$$\frac{\partial^2 W_3(t)}{\partial t^2} = -\left[\frac{2}{3}\Sigma_{i=1}^3 s_i^2 - s_3^2 + \frac{\beta}{2}\left(2s_3^2 - s_1^2 - s_2^2\right)\right]. \tag{10.76}$$

From (10.66), (10.67) and (10.68), W_1 attains a unique interior maximum, t_1, which is found by solving $\frac{\partial W_1(t)}{\partial t} = 0$. Similarly, from (10.70), (10.71) and (10.72), W_2 attains a unique interior maximum, t_2, which is found by solving $\frac{\partial W_2(t)}{\partial t} = 0$. If $\beta > \frac{2}{3}$ then, from (10.76), it can be checked that $\frac{\partial^2 W_3(t)}{\partial t^2} < 0$. It then follows from (10.74) and (10.75) that W_3 attains a unique interior maximum, t_3^*, which is found by solving $\frac{\partial W_3(t)}{\partial t} = 0$. Suppose now that $\beta \leq \frac{2}{3}$. From (10.75), $\left[\frac{\partial W_3}{\partial t}\right]_{t=0} \leq 0$. If $\frac{\partial^2 W_3(t)}{\partial t^2} < 0$, then $t_3 = 0$ is the unique optimum. If $\frac{\partial^2 W_3(t)}{\partial t^2} \geq 0$ then, from (10.74), $t_3 = 0$ is, again, the unique optimum. ■

10.9. Proof of Proposition 5:

The proof of part (a) simply follows from the general case of n voters considered earlier (see Proposition 1) and Proposition 2(b). To prove (b) we now consider the various cases below.

1. In the case FSF, $t_2^F > t_2^S$ (from part a), so two cases arise. (1) $t_1^F > t_2^S > t_3^F$, in which case the median voter and median skill individuals coincide. From Proposition

4, $\beta_3 \leq \frac{2}{3}$ is sufficient for this case. (2) $t_1^F > t_3^F > t_2^S$. in which case, voter 3 becomes the median voter. However, when $\beta_3 > \frac{2}{3}$ then, using Proposition 4, $t_3^F > t_2^S$ requires that

$$\frac{s_3^2 - s_2^2}{s_3^2 - s_1^2} < \frac{\beta}{2-\beta} \tag{10.77}$$

The LHS of (10.77) is greater than 1. The RHS is monotonically increasing in β. Since $0 \leq \beta < 1$, the RHS is bounded within $[0,1)$. Thus we get a contradiction and the inequality in (10.77) cannot hold. So we cannot have $t_3^F > t_2^S$. This rules out the second case. If the first case holds then the median voter and median skill individuals coincide.

2. In the case FFS, since $t_3^F > t_3^S$ we get that $t_1^F > t_2^F > t_3^F > t_3^S$. Hence, voter 2 is also the median voter.

3. In the case SSF we know that $t_1^S > t_2^S$ so the only two outcomes that will alter the identity of the median voter are as follows (1) $t_1^S > t_3^F > t_2^S$ in which case the fair-rich voter wins. (2) $t_3^F > t_1^S > t_2^S$ in which case the selfish-poor voter wins. We can simply rule out both cases because the inequality in (10.77) does not hold when $0 \leq \beta < 1$.

4. Finally, consider the case FSS. Using (a) we know that $t_1^F > t_1^S > t_2^S > t_3^S$, hence, the median voter and the median skill individual coincide.

To prove (c) we need to consider the following two cases:

1. SFF: From (a) we know that $t_1^F > t_2^F > t_3^F$, so if $t_1^S > t_2^F > t_3^F$ we are done and the median voter coincides with the median skill voter. However, if $t_1^S < t_2^F$ then have a potential reversal of the median voter. Using Proposition 4, $t_1^S < t_2^F$ if

$$\frac{2-\beta}{\alpha} < \frac{s_3^2 - s_2^2}{s_2^2 - s_1^2} \tag{10.78}$$

If (10.78) holds then we get that $t_1^S < t_2^F > t_3^F$, so we need to compare t_1^S, t_3^F to determine the identity of the Condorcet winner. Using Proposition 4, $t_1^S < t_3^F$ if

$$(2-\beta)\left(s_3^2 - s_1^2\right) - \beta\left(s_3^2 - s_2^2\right) < 0 \tag{10.79}$$

Since $\beta < 1$ so $2 - \beta > \beta$ and $s_3^2 - s_1^2 > s_3^2 - s_2^2$ which contradicts the supposed inequality in (10.79), hence, $t_1^S \not< t_3^F$. Hence, if (10.78) holds then the only possible case is that $t_3^F < t_1^S < t_2^F$ and so the poorest skill voter becomes the decisive median voter.

2. SFS : From (a) we know that $t_1^S > t_3^S$. From the previous case we know that if inequality (10.78) holds, then $t_1^S < t_2^F$. In which case, we get $t_3^S < t_1^S < t_2^F$ and so the poorest skill voter becomes the decisive median voter in choosing the redistributive tax rate. ■

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