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A SIMPLE MODEL OF OPTIMAL TAX SYSTEMS: TAXATION, MEASUREMENT AND UNCERTAINTY

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Abstract

The neglect of administrative issues is a serious limitation of optimal tax theory, with implications for its practical applicability. Under uncertainty, the problems for optimal tax theory are compounded when the full set of tax instruments is neglected. These twin issues are addressed in this paper, by focussing on a fundamental implication of administrative problems, namely, that the tax bases are measured with some error. Consumption taxes can perform the 'social insurance role of taxation'; a role previously ascribed only to income taxes. A combination of income and consumption taxes can hedge income and measurement-error risks better, relative to the imposition of these taxes alone. The optimal taxes are decreasing in the imprecision with which the corresponding tax base is measured. The taxpayer engages in precautionary savings, in response to uncertainty arising on account of income and measurement problems. Differential commodity taxes, tailored to the measurability characteristics of the different tax bases, dominate uniform commodity taxes. Furthermore, the paper provides a simple, tractable framework for optimal tax theorists interested in diverse kinds of uncertain situations.

Keywords: Social Insurance, Measurability of tax bases, Yardstick Competition, Differentiated taxes.

JEL Classification: H21 (Efficiency; Optimal Taxation), D82 (Asymmetric and Private Information).

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"As economists have been aware, the omitted constraints on communication, calculation and administration of an economy...limit the direct applicability of the implications of this theory to policy problems...." Diamond and Mirrlees (1971).

1. Introduction

Two important considerations motivate this paper. First, despite their crucial importance in actual policymaking, administrative issues are typically ignored in tax theory. Second, the practice of ignoring the full set of tax instruments, especially under uncertainty, leads to misleading results. The model is simple, tractable, and provides a useful template for optimal tax theorists working in a range of problems where uncertainty of some sort is important and explicit modeling is difficult or impossible. Hence, the scope of the paper goes beyond the issues it directly addresses. It is perhaps instructive to look more closely at the two considerations that motivate the paper.

1.1. The importance of administrative issues

In normative tax theory, issues of taxation and proposals for tax reform are typically evaluated on the basis of efficiency and equity considerations[1]. Administrative issues are ignored in tax theory despite their importance in the actual implementation of tax policies. The meaning of the generic term 'administrative issues' obviously depends on the context. Nevertheless, if all tax bases were costlessly and publicly observed, then several important categories of administrative issues would disappear. Hence, probably the most interesting overarching implication of several kinds of 'administrative issues' must be that tax bases are measured with some measurement error. As the discussion below will indicate, several well-known discussions of administrative issues focus on the *measurement error aspect of administrative issues* and this is the (reduced form) implication of administrative issues that this paper concentrates on. In what follows, the terms 'measurement errors' and 'administrative problems', are used synonymously.

Despite the neglect of administrative problems in the theory of taxation, general commentaries on tax policy often pose the choice between critical taxes, such as that between consumption and income taxes, in terms of the relative difficulty of measuring the two tax bases. For instance, Devereux (1996: 14) writes, "it is on administrative grounds that the proponents of the expenditure tax have the strongest case. This has largely to do with the problems of implementing a truly comprehensive income tax." Bradford (1980) is very explicit: "From this perspective, the winner of the great debate over the relative merits of the consumption versus the income tax rests on an issue of measurability."

In the context of income taxation, Boadway and Wildasin (1996: 98) point to severe problems in the measurement of 'capital income'. They write: " In principle this should include all forms of returns to assets including interest, dividends, accrued capital gains, capital income from unincorporated business, imputed rent on consumer durables (especially housing) and the imputed return of assets such as transactions balances and

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[1]Textbook treatments can be found in Atkinson and Stiglitz (1980) and Myles (1995).

insurance. These should all be indexed for inflation and should include an appropriate risk premium. Unfortunately the measurement of these items is difficult or impractical." Mintz (1997: 467-68) lists several problems in the measurement of a consumption tax base, both, in its VAT version and in its registered versus non-registered asset treatment. These problems include identification of taxpayers, and issues of consumption versus business expenses, real versus financial transactions, wage versus self-employed income, treatment of losses and tracking of transactions etc.

The measurement of several tax bases can be especially difficult for developing countries. Burgess and Stern (1993: 798-99) identify some of the relevant factors: insufficient staff with the appropriate skills, equipment, motivation, or honesty; complex legal and tax structures; poor and inconsistent records that are often under the control of different tax authorities and lack of incentive based remuneration etc. Although rarely acknowledged, developed countries often face similar problems. Fortin (1995: 2) writes: "A substantial portion of Revenue Canada employees fails elementary tests of the knowledge of the tax system. Even our best experts admit that they find it very hard to keep up."

Surveys of optimal taxation generally point to the lack of real world applicability of tax theory on administrative grounds. Heady (1996: 33) writes that "One way in which many models are unrealistic has already been mentioned: their neglect of administrative costs..." Burgess and Stern (1993: 798) make similar remarks in the context of developing countries. Slemrod (1990: 157) provides a cogent overview of the issues and writes that "Differences in the ease of administrating various taxes have been and will continue to be a critical determinant of appropriate tax policy." Slemrod advocates incorporating administrative issues into 'optimal tax theory' to generate a unified '*Theory of Optimal Tax Systems*'. This paper can be viewed as one attempt in that direction.

1.2. The full set of tax instruments and uncertainty

A desirable modelling practice in optimal tax theory is to explicitly consider the full set of tax instruments. Considering only a subset of the available taxes often results in erroneous conclusions about the optimality of certain taxes. For instance, under certainty, Atkinson and Stiglitz (1976), Atkinson (1977) and Deaton and Stern (1986) have shown that under certain conditions, commodity taxes are redundant in the presence of income taxes.

However, the full set of tax instruments are generally omitted in optimal tax models involving uncertainty, as they were, in the seminal papers by Varian (1980) and Eaton and Rosen (1980). The essential contribution of these papers was to identify the 'social insurance role of taxation' in the presence of income uncertainty[2]. Cremer and Gavhari (1995, 1999) and Mirrlees (1990) provide further insights on optimal taxation in the presence of uncertainty[3]. Cremer and Gahvari uncover a novel role for consumption taxes by

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[2]For extensions, but mostly with a partial set of tax instruments, see the survey in Myles (1995).

[3]Cremer and Gahvari (1995, 1999) and Mirrlees (1990) consider consumption taxes but omit income taxes. The argument is that 'Given that the full set of commodity taxes is being used, the tax rate on the wage is superfluous and can be set equal to zero without imposing any restrictions'; see for instance Cremer and Gavhari (1995: 298). However, this does not hold true when both tax bases are observed with some measurement error. In that case (see below) the optimal income and consumption taxes will typically be strictly positive .

distinguishing between goods that are consumed prior and posterior to the resolution of income uncertainty, however, their focus is not on administrative issues. Mirrlees (1990) comes closest to providing a theory of optimal tax systems under uncertainty, but assumes that while the income tax base is observed with some error, there are no such problems with measuring the consumption tax base.

This paper extends the basic model in Varian (1980) to take account of measurement problems with the income as well as the consumption tax bases and the full set of taxes is considered. The results are not tied to any particular source of measurement problems; such problems are taken as given. Attention is focussed on 'pure' consumption and income taxes, rather than on the specific institutional detail of any particular tax system, however, the model seems reasonably amenable to such extensions.

The results are as follows. In the absence of income uncertainty and administrative problems, a poll tax is optimal. Under income uncertainty, strictly positive income and consumption taxes perform the social insurance role of taxation; that such a role is not the exclusive domain of income taxes is not always reflected in the existing literature. When tax administration issues are taken into account, strictly positive income and consumption taxes are often optimal; some combination of these taxes typically provides superior hedging of the income and the measurement error risk for the taxpayer. This result suggests a role that is similar to the idea of yardstick competition in the moral hazard literature. That the optimal consumption tax should be positive in the presence of an income tax is an important result given the originally pessimistic role for indirect taxes in Atkinson and Stiglitz (1976); in this respect the results in the paper also contribute to a growing literature that justifies a role for indirect taxes[4].

Measurement errors in a tax base reduce the optimal tax on that base and also have 'spillover effects' on taxes levied on other tax bases. The relative magnitudes of the two taxes are inversely proportional to the relative difficulty of measuring the two tax bases. The magnitude of any tax is directly proportional to its social insurance role relative to the measurement error risk that it imposes. The taxpayer engages in precautionary savings, in response to uncertainty arising on account of income and tax administration. Finally, differential commodity taxes, tailored to the measurability characteristics of the different tax bases, dominate uniform commodity taxes.

In addition to providing sharp, closed form, results that have pedagogical merit, the main attractiveness of the model is its simplicity and tractability in dealing with fairly vexed questions. The questions posed in this paper are hardly novel, as the discussion above shows, however, one suspects that the lack of theoretical progress in the area owes much to lack of formal models that could provide a useful template for research. This paper is an attempt to fill that lacuna and hopefully provide the basis of a simple off-the-shelf model that can be used to model a range of optimal tax situations under uncertainty.

Sections 2 through 4 adapt the model in Varian (1980) to the full set of tax instruments and measurement problems. Subsection 4.3 explores the implications for precautionary

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[4]For instance, Boadway and Pestieau (1994) deal with tax evasion features, Cremer and Gavhari (1999) distinguish between those goods that are consumed before and after the resolution of income uncertainty. Cremer, Pestieau and Rochet (2001) introduce multidimensional heterogeniety among individuals. Each of these amendments to the Atkinson-Stigiltz framework uncovers a role for consumption taxes.

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savings. Finally Section 5 examines the issue of uniform versus differentiated taxes. Section 6 concludes followed by the appendix. All proofs are collected in the appendix.

2. Model

Consider the following stylized two-period model as in Varian (1980). A representative taxpayer, when young, allocates first period income I_1 between first period consumption C_1 and savings S. In the second period, the taxpayer is old and second period income $I_2 = S + \eta$, where η is a normally distributed random shock with mean zero and variance σ_η^2; Varian (1980) interprets η as 'idiosyncratic uncertainty'.

2.1. Description of the tax system

Taxation is levied only in the second period and the tax system has full loss-offsets[5]. The government levies a linear, progressive income tax with constant marginal rate θ and a lumpsum payment α to the taxpayer; $\alpha > 0$ denotes transfer payments while $\alpha < 0$ signifies a poll tax. After deducting the income tax, denote the taxpayer's second period disposable income by I_2^D, which identically equals before-tax second period consumption. The government levies a linear consumption tax at the constant marginal tax rate τ on disposable income, hence, the tax system is *sequential*; the consumption tax follows the imposition of the income tax. Denote by $\Gamma = (\theta, \alpha, \tau)$, the tax vector chosen by the government.

Due to administrative problems, the government observes the following imperfect signals, I^O and C^O, of the income and the consumption tax bases respectively

$$I^O = I_2 + \epsilon_I \tag{2.1}$$

$$C^O = I_2^D + \epsilon_C \tag{2.2}$$

where ϵ_I and ϵ_C are respectively, the measurement errors associated with measuring the income and the consumption tax bases[6]. The measurement errors are independent of the idiosyncratic uncertainty term η and are jointly normally distributed with zero mean, respective variances σ_I^2 and σ_C^2 and with covariance given by σ_{IC}. Since the two tax bases are measured with different methods and techniques in practice[7], thus, the distributions

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[5]First period taxation introduces unnecessary complexity without affecting the qualitative results. Asides from imparting greater tractibility to the analysis, the full loss offset assumption is not entirely unrealistic. Most real world tax systems allow investors to set their losses from one source of income against other sources of income. If all sources of income are taxed at the same rate (as in this paper) then the tax system approximately behaves as if there were full loss offsets.

[6]Measurement problems could arise from a wide variety of sources discussed in the introduction; the model looks at a fairly generic problem however, that does not specify the precise source of the measurement problem.

[7]The issues are succinctly summarized in Boadway and Wildasin (1996: 98-9). In a consumption tax, relative to an income tax, "it is no longer imperative to measure capital income on an accrual basis or to index capital income for the effect of inflation on asset values. Thus all accounting can be done on a cash flow basis which is relatively easier to administer. Furthermore, unlike in a comprehensive income tax,

of ϵ_I and ϵ_C are likely to be different.

2.2. The Government Budget Constraint

The government has exogenous revenue requirements equal to R. Denoting by E, the expectation operator with respect to the joint distribution of ϵ_I, ϵ_C and η, the government budget constraint is given by[8]:

$$E\left[\theta I^O + \tau C^O\right] - \alpha = R \qquad (2.3)$$

Hence the government budget constraint holds in expected value terms[9].

2.3. The Taxpayer's Budget Constraint

Since no taxes are levied in the first period, thus, the first period budget constraint is

$$C_1 + S = I_1 \qquad (2.4)$$

Second period disposable income is $I_2^D = (S + \eta) - \theta I^O = (1 - \theta)(S + \eta) - \theta\epsilon_I + \alpha$, hence, after the imposition of the consumption tax, second period consumption is

$$C_2 = (1 - \theta)(1 - \tau)(S + \eta) - \theta(1 - \tau)\epsilon_I - \tau\epsilon_C + (1 - \tau)\alpha \qquad (2.5)$$

Substituting 2.1 and 2.2 in 2.3, the government budget constraint can be simplified and rewritten as

$$(1 - \tau)\alpha = S\{\theta + \tau(1 - \theta)\} - R \qquad (2.6)$$

Using 2.6 to eliminate α from 2.5, the second period budget constraint of the taxpayer is

$$C_2(\Gamma) = (S - R) + \eta(1 - \theta)(1 - \tau) - \theta(1 - \tau)\epsilon_I - \tau\epsilon_C \qquad (2.7)$$

Since the average tax proceeds are returned back to the taxpayer, net taxes on the non-stochastic component of income are zero. It is obvious from 2.7 that the two types of risks- the income risk (captured by the term η) and the measurement error risk (captured by the terms ϵ_I and ϵ_C) can be partially or fully offset by taxation.

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returns to capital which take an imputed form, such as rent on housing, need not be measured".

[8]Cremer and Gavhari (1995, 1999) and Mirrlees (1990) write the government budget constraint by setting $\theta = 0$, which is admissible in their model because one of the two taxes is redundant in the presence of the other. However, in the presence of administrative problems, where the two tax bases might be faced with different measurement problems, setting $\theta = 0$ is not admissible; indeed as shall be demonstrated below, both optimal taxes are generally positive.

[9]On such issues see the discussion in Myles (1995). One possible justification for the form of the government budget constraint in 2.3 is that given the 'information currently available' to the government and the 'relevant economic theory' i.e. the income generation process, the LHS of 2.3 is the rational expectations prediction of the tax revenues.

2.4. Sequence of Moves

The government acts as the Stackelberg leader and commits to a tax vector Γ. The taxpayer observes Γ and then makes the first period savings-consumption choice, following which, in the second period, the announced tax policy is implemented with measurement errors. The two stage game is solved by backward induction.

2.5. Preferences of the Taxpayer

The taxpayer's preferences are of the CARA form, and are additively separable in C_1 and C_2, thus, expected utility is given by

$$E\left[U\left(C_1, C_2\right)\right] = E\left[-\exp\left(-\rho C_1\right)\right] + E\left[-\exp\left(-\rho C_2\right)\right] \tag{2.8}$$

where $\rho > 0$ is the coefficient of absolute risk aversion. The strategy of using normally distributed error terms and CARA preferences is adopted from a related literature in agency theory[10], which substantially simplifies a fairly complex problem, to which, the general case might yield either no results or those of limited significance.

2.6. Preferences of the Government

After substituting for the taxpayer's optimal choices in 2.8, the indirect utility of the taxpayer is given by $V\left(\Gamma\right)$. The objective of the government is to maximize $V\left(\Gamma\right)$ by a suitable choice of Γ. The lumpsum transfer α is already substituted out using 2.6, hence, the government's problem is an unconstrained one.

3. Solution to the Optimal Tax Problem

Start with the second stage of the game, namely, the saving-consumption decision of the taxpayer. The taxpayer's problem, conditional on the tax vector Γ, is to choose C_1 and C_2 in order to maximize 2.8 subject to the two budget constraints, 2.4 and 2.7. First period consumption is non-random, so $E\left[-\exp\left(-\rho C_1\right)\right] = -\exp\left(-\rho C_1\right)$. From 2.7, C_2 is normally distributed, and using a standard result in statistics[11], one can evaluate expected utility explicitly and show that $E\left[-\exp\left(-\rho C_2\right)\right] = -\exp\left(-\rho\xi\left(S, \Gamma\right)\right)$, where $\xi\left(S, \Gamma\right)$, the certainty equivalent, is given by

$$\xi\left(S, \Gamma\right) = \{S - R\} - \frac{\rho}{2}\left\{(1-\tau)^2\left\{(1-\theta)^2\sigma_\eta^2 + \theta^2\sigma_I^2\right\} + \tau^2\sigma_C^2 + 2\theta\tau\left(1-\tau\right)\sigma_{IC}\right\} \tag{3.1}$$

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[10]In multidimensional moral hazard models, a principal observes several imperfect signals of an agent's effort; for example Holmstrom and Milgrom (1990, 1991). Holmstrom and Milgrom also show that for this case, the optimal (private) incentive scheme between the principal and an agent is linear in the observed signals. Hence, veiwing taxation as a (social) contract between the government and the taxpayer, Holmstrom and Milgrom's results suggest that the linear taxes used in this paper do not involve loss in generality.

[11]If Z is normally distributed with mean μ and variance σ^2 then $E\left[\exp\left(-\rho Z\right)\right] = \exp\left(-\rho\mu + \frac{1}{2}\rho^2\sigma^2\right)$.

The certainty equivalent is increasing in the expected value of consumption (first term in 3.1) and decreasing in its variance (second term in 3.1). Substituting the two budget constraints, 2.4 and 2.7 into the objective function 2.8, the taxpayer's optimal savings choice, conditional on Γ, $S^* = S^*(\Gamma)$, is found from the following unconstrained problem

$$S^* \in \arg\max E\left[U\left(S,\Gamma\right)\right] = -\exp\left\{-\rho\left(I-S\right)\right\} - \exp\left\{-\rho\xi\left(S,\Gamma\right)\right\}$$

The first order condition to the taxpayer's problem can be seen to imply that $\xi\left(S,\Gamma\right) = I - S$; solving out for S^*, one gets

$$S^*\left(\Gamma\right) = \frac{\{I+R\}}{2} - \frac{\rho}{4}\left\{\left(1-\tau\right)^2\left\{\left(1-\theta\right)^2\sigma_\eta^2 + \theta^2\sigma_I^2\right\} + \tau^2\sigma_C^2 + 2\theta\tau\left(1-\tau\right)\sigma_{IC}\right\} \quad (3.2)$$

The first term in 3.2 captures the *intertemporal consumption smoothing role of savings* while the second term, which is more fully explored in Section 4.3, illustrates the *precautionary savings*. Substituting $S^*\left(\Gamma\right)$ into 3.1 one gets $\xi\left(\Gamma\right) = \xi\left(S^*\left(\Gamma\right),\Gamma\right)$.

Since the government is the Stackelberg leader, its first stage problem is to choose Γ to maximize the indirect utility of the representative taxpayer, given by

$$V\left(\Gamma\right) = -\exp\left\{-\rho\left(I-S^*\left(\Gamma\right)\right)\right\} - \exp\left\{-\rho\xi\left(S^*\left(\Gamma\right),\Gamma\right)\right\}$$

Using the envelope theorem, the first order conditions are

$$\frac{\partial V\left(\Gamma\right)}{\partial\theta} = \left(1-\tau\right)^2\left(1-\theta\right)\sigma_\eta^2 - \theta\left(1-\tau\right)^2\sigma_I^2 - \tau\left(1-\tau\right)\sigma_{IC} \leq 0; \quad \theta \geq 0 \quad (3.3)$$

$$\frac{\partial V\left(\Gamma\right)}{\partial\tau} = \left(1-\tau\right)\left(1-\theta\right)^2\sigma_\eta^2 + \theta^2\left(1-\tau\right)\sigma_I^2 - \theta\left(1-2\tau\right)\sigma_{IC} - \tau\sigma_C^2 \leq 0; \quad \tau \geq 0 \quad (3.4)$$

The first term in 3.3 captures 'income-risk sharing' between the government and the taxpayer on account of the income tax; this is the 'social insurance effect' in Varian (1980) and Eaton and Rosen (1980). The second term is the increased risk to the taxpayer on account of measurement errors in income; this is roughly analogous to various forms of the 'measurement risk effects' in Stern (1982), Mirrlees (1990) and Dhami (2002). The third term is the 'covariance effect'; correlation in the two measurement errors affects the overall risk facing the taxpayer, and hence, has an affect on the optimal tax rates. The precise affect depends on whether risks increase or decrease; this is examined in more detail below. An interpretation similar to that of 3.3 applies to 3.4 except for the last term, which takes account of measurement errors in the consumption tax base.

Denote the solution to the optimal taxes, found by jointly solving 3.3 and 3.4, by $\left(\theta^*,\tau^*\right)$; α^* can be found residually from the government budget constraint 2.3.

3.1. Some benchmark and limiting results

This section derives some useful limiting results that help to build subsequent intuition about the results.

Proposition 1 : *If there are no tax administration problems (i.e. $\sigma_I^2 = \sigma_C^2 = \sigma_{IC} = 0$) and there is no income uncertainty (i.e. $\sigma_\eta^2 = 0$), then, a poll tax is optimal i.e. $\theta^* = \tau^* = 0$ and $\alpha^* = -R$. However, when income is uncertain but administration problems are absent, then either $(\theta^*, \tau^*) = (1, 0)$ or $(\theta^*, \tau^*) = (0, 1)$.*

The first part of Proposition 1 establishes the optimality of a poll tax under certainty; this result is also derived in Atkinson and Stiglitz (1976), Atkinson (1977), Deaton and Stern (1986)[12].

The second part of Proposition 1 illustrates the 'social insurance role of taxation' as in Varian (1980) and Eaton and Rosen (1980), namely, that income taxes allow the government and the taxpayer to share risks, however, its implications go slightly further. First, in the absence of tax administration issues, as $(\theta^*, \tau^*) = (1, 0)$ and $(\theta^*, \tau^*) = (0, 1)$ are equally admissible, the consumption and the income taxes are equally suited to the social insurance role. Hence, the presumed superiority of income taxes in sharing risks arises because the full set of tax instruments is generally not considered. Second, Cremer and Gahvari (1995: 53) ask why, given that labor supply is exogenous in Varian (1980), is not the optimal tax a 100% income tax "; Proposition 1 shows that a 100% income tax is indeed optimal, but so is a 100% consumption tax.

Define $r = \frac{\sigma_{IC}}{\sigma_I \sigma_C}$ as the correlation coefficient between ϵ_I and ϵ_C.

Corollary 1 : *Under income certainty (i.e. $\sigma_\eta^2 = 0$), but in the presence of tax administration problems (i.e. $\sigma_I^2, \sigma_C^2, \sigma_{IC} \neq 0$) if $r \neq \pm 1$ then a poll tax is optimal. But, if $r = \pm 1$ then there is a multiplicity of non-zero tax solutions.*

The intuition behind Corollary 1 is that, in the absence of the social insurance role of taxation, if income and consumption taxes are associated with measurement error risks then it is best to impose a poll tax that imposes no such risks on the representative taxpayer. In the special case where $r = \pm 1$ i.e. the two measurement errors are perfectly correlated then the measurement error risk imposed on the taxpayer through one tax is completely offset through the other tax. If $r = -1$ then risks are completely offset by strictly positive taxes while if $r = +1$ then a combination of a consumption tax and an income subsidy is optimal. In general, this special case ($r = \pm 1$) allows for a multiplicity of optimal tax solutions.

Lemma 1 : *If σ_I^2, σ_C^2, and σ_{IC} are bounded but $\sigma_\eta^2 \to \infty$ then $\theta^* = \tau^* = 1$.*

The intuition can be seen from the term $(1 - \tau)^2 (1 - \theta)^2 \sigma_\eta^2$ in the expression for the taxpayer's certainty equivalent (equation 3.1). Thus, when σ_η^2 is high, the effect of income

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[12]Although these results were derived using non-linear taxes in this literature, linear taxes are optimal in this paper; see footnote *9* above

uncertainty can be countered by increasing the magnitude of both taxes. High taxes also increase the measurement error risk (see the other terms in 3.1), however, when $\sigma_\eta^2 \to \infty$, considerations of 'social insurance' overweigh measurement risk effects.

Lemma 2 : *If σ_η^2, σ_C^2, and σ_{IC} are bounded and $\sigma_I^2 \to \infty$ then $\theta^* = \tau^* = 0$.*

Lemma 2 reflects the sequential nature of the two taxes; the consumption tax is levied after the imposition of the income tax, so it inherits the measurement problems associated with the income tax base. Hence, when $\sigma_I^2 \to \infty$, both tax rates are optimally zero, thus, a particular tax might provide an excellent social insurance role, but measurability issues might limit its use

Lemma 3 : *If σ_η^2, σ_I^2, and σ_{IC} are bounded and $\sigma_C^2 \to \infty$ then $\theta^* \neq 0$, $\tau^* = 0$.*

Thus, if the consumption tax base is impossible to measure, it should not be taxed. However, the income tax base is unaffected by measurement problems in the consumption tax base, hence, the optimal income tax is non-zero.

4. Comparative Static Results and Optimal Tax Formulae

The comparative static results are substantially simplified when $\sigma_{IC} = 0$ and one can derive explicit formulae for the optimal tax rates that have substantial pedagogical merit. Hence, the discussion below is separated into the two cases, namely, $\sigma_{IC} = 0$ and $\sigma_{IC} \neq 0$.

4.1. Uncorrelated measurement errors $(\sigma_{IC} = 0)$

When $\sigma_{IC} = 0$, the first order conditions 3.3 and 3.4 can be jointly solved, explicitly, for the optimal tax rates, θ^* and τ^*; the solution is given below.

$$\theta^* = \left\{ 1 + \frac{\sigma_I^2}{\sigma_\eta^2} \right\}^{-1} \tag{4.1}$$

$$\tau^* = \left\{ 1 + \frac{\sigma_C^2}{\sigma_I^2} + \frac{\sigma_C^2}{\sigma_\eta^2} \right\}^{-1} \tag{4.2}$$

Proposition 2 : *The optimal tax rates θ^* and τ^* are increasing in the extent of income uncertainty, σ_η^2, and decreasing in the imprecision associated with measuring their respective tax bases, σ_I^2 and σ_C^2. Furthermore, measurement problems create "spillover effects"- τ^* is increasing in σ_I^2 while θ^* is unaffected by σ_C^2.*

As income uncertainty (captured by σ_η^2) increases, both tax rates optimally perform a 'social insurance role'. As the informativeness of an observed tax base decreases (i.e. σ_I^2 or σ_C^2 increases) that tax is optimally reduced to mitigate the measurement error risk facing the taxpayer. In different contexts, Stern (1982), Mirrlees (1990), and Dhami (2002) also

find that measurement problems reduce the optimal tax rate. Finally, $\frac{\partial \tau^*}{\partial \sigma_I^2} > 0$ demonstrates 'spillover effects' from one tax base to the other; despite $\sigma_{IC} = 0$, administrative problems with one tax base affect the optimal tax rate on the other tax base. The intuition can be seen by an examination of the expression for the taxpayer's net consumption in 2.7; an increase in τ reduces $\theta (1 - \tau) \epsilon_I$, which is the exposure to measurement error risk on account of measurement problems in the income tax base. Finally, given the sequential nature of the two taxes, with the income tax levied before the consumption tax, the income tax is unaffected by measurement errors with the consumption tax base.

Divide 4.1 by 4.2 and simplify to get the relative taxes:

$$\frac{\theta^*}{\tau^*} = \frac{\sigma_C^2}{\sigma_I^2} + \left\{ 1 + \frac{\sigma_I^2}{\sigma_\eta^2} \right\}^{-1} \tag{4.3}$$

Proposition 3 : *The relative optimal tax rates, $\frac{\theta^*}{\tau^*}$, depend inversely on the following. (a) The relative imprecision, $\frac{\sigma_I^2}{\sigma_C^2}$, in measuring the two tax bases. (b) The imprecision in measuring the income tax base relative to income risk, $\frac{\sigma_I^2}{\sigma_\eta^2}$. (c) Income-risk relative to imprecision in the measurement of the consumption tax base, $\frac{\sigma_\eta^2}{\sigma_C^2}$.*

Proposition 3 formalizes at least two intuitive ideas. First, ceteris paribus, the debate on the relative magnitude of the income and the consumption taxes rests, at least partially, as Bradford (1980) argues, on an issue of measurability; optimal taxes on relatively easier to measure tax bases are relatively higher. Second, the optimal taxes are relatively higher when, roughly, their social insurance role is more important than the measurement error risk that they impose. The results in Propositions 2 and 3 can have interesting implications for the following issues in tax theory.

1. *Direct versus indirect taxes*: Why do developing countries, unlike developed countries, raise the bulk of their tax revenues through indirect taxes relative to direct taxes? It is often argued that the explanation lies in the relative difficulty of measuring income in developing countries for reasons such as the paucity of recorded transactions, corrupt tax administration etc.; for example Burgess and Stern (1993). This conforms to the result in Proposition 3.

2. *Taxation of Fixed Factors*: Economic theory demonstrates that taxes on fixed factors (for example land and capital) are efficient; indeed in the presence of such taxes there is no need for other taxes. However, why do such taxes account for a relatively small proportion of actual governmental tax revenues? One possible explanation, consistent with the predictions of Proposition 3, lies in the relative difficulty of measuring fixed factors. Two such taxes are considered below.

2(a) *Land taxes*: Bird (1974: 223) contends that "...the administrative constraint on effective land tax administration is so severe in most developing countries today that virtually all the more refined fiscal devices beloved of theorists can and should be discarded for this reason alone." Similar problems are raised in Newbery (1987) and

Skinner (1996). Land quality, which is one of the crucial elements in the definition of the land tax base, is hard to measure and requires ascertaining the soil type and quality, rainfall, irrigation facilities etc. Proxies for the land tax base such as capital value assessment, value of the produce on land, site value etc. are riddled with similar measurement problems; for example Bird (1974). Hence usage of the land tax is extremely limited and has historically declined.

2(b) *Capital stock taxes*: A capital tax is levied directly on the capital stock by state or federal authorities in many countries such as United States, Canada, and Germany, at fairly low tax rates ranging from 0.25 to 0.50 percent, with generous exemptions. Although, a tax on the stock of capital that a firm owns is non distortionary, there exist well known difficulties in the measurement of the capital stock justifying the low taxation (or even exemption) of the tax base.

3. *Time Inconsistency Issues*: In an often cited example in the time inconsistency literature, the government announces that new capital is tax exempt, but once the new capital is in place, the government can renege and impose a 100 percent capital tax which is ex-post non-distortionary. Propositions 2 and 3 suggest that if measurement problems associated with the fixed tax base are acute, it is not efficient to impose a high, confiscatory tax, even if the government has the discretion to do so. This argument provides a possible 'optimal tax' supplement to reputation based explanations for the absence of 100 percent capital taxes[13].

4.2. Correlated measurement errors $(\sigma_{IC} \neq 0)$

Correlation in the measurement errors (i.e. $\sigma_{IC} \neq 0$) can have an important affect on the overall risk facing the taxpayer. This can be seen easily from the expression for the certainty equivalent in 3.1 or from the first order conditions in 3.3 and 3.4. To focus on the affects of σ_{IC} assume that all other exogenous variables are fixed. In general, the comparative static affects of σ_{IC} are not easy to sign. Proposition 4 provides some results in a subset of the cases.

Proposition 4 : If $\frac{\tau}{(1-\tau)} \geq \frac{\sigma_\eta^2}{\sigma_C^2}$ then $\frac{d\tau}{d\sigma_{IC}} < 0$. If $\tau < \frac{1}{2}$, then $\frac{d\theta}{d\sigma_{IC}} < 0$ if $\frac{1+(\sigma_C^2/\sigma_\eta^2)}{1+(\sigma_I^2/\sigma_\eta^2)} < \theta^2$.

The covariance term, σ_{IC}, performs a role similar to that of 'yardstick competition' in the moral hazard literature, whereby the observation of two correlated signals of an agent's effort allows the principal to filter some of the risk facing a risk-averse agent and allows for improved incentives; for example Holmstrom (1982) and Holmstrom and Milgrom (1990, 1991). Proposition 4 shows when a variant of these results applies to a social contract between a government and a taxpayer. One is looking at circumstances when an increase in σ_{IC} increases the taxpayer's share of the cake (in the analogous agency situation this is the agent's share of the surplus). The first condition in the Proposition shows that when

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[13]This is not meant to trivialize the time inconsistency literature which is clearly important, but to suggest a range of factors or assets to which the problem might not apply in a particularly serious manner.

$\frac{\sigma_\eta^2}{\sigma_C^2}$ is low then τ decreases with σ_{IC} for in this case, the social insurance role of taxation foregone relative to the measurement risks it imposes is small. Hence better risk sharing implied by a higher σ_{IC} filters out some of the risk facing the taxpayer allowing for a reduction in taxes. A similar interpretation can be given to the second half of Proposition 4 which requires that σ_I^2/σ_η^2 be large relative to σ_C^2/σ_η^2 for θ to respond negatively to σ_{IC}.

4.3. Precautionary behavior

The model brings out some simple but important implications for precautionary behavior. When income is ex-ante uncertain and taxpayers are risk averse, one would expect them to engage in precautionary savings. Varian (1980) uses quadratic preferences, thus, the zero third derivative precludes precautionary behavior[14]. Strawczynski (1998) identifies precautionary savings by performing simulation techniques on a log utility version of the Varian (1980) model.

Since the third derivative is strictly positive for CARA preferences, the taxpayer engages in precautionary savings. A drawback of CARA preferences is that precautionary savings are independent of wealth, however, this does not affect other qualitative results. Since the taxpayer treats the tax vector Γ as given when making the consumption-saving decision, successive differentiation of 3.2 with respect to σ_η^2, σ_I^2 and σ_C^2 gives:

$$\frac{\partial S}{\partial \sigma_\eta^2} = \frac{\rho}{4}\left(1-\tau\right)^2\left(1-\theta\right)^2 > 0$$

$$\frac{\partial S}{\partial \sigma_I^2} = \frac{\rho}{4}\left(1-\tau\right)^2\theta^2 > 0$$

$$\frac{\partial S}{\partial \sigma_C^2} = \frac{\rho}{4}\left(1-\tau\right)^2\tau^2 > 0$$

All three partial derivatives are positive, therefore, the taxpayer engages in precautionary savings with respect to future uncertainty arising from (1) income and (2) tax administration problems. Within models of precautionary savings, the second effect is a relatively novel result. These results complement the results in Strawczynski (1998) and provide the theoretical counterpart to his simulation results.

5. Optimal Commodity Taxation: Differentiated or Uniform?

The Ramsey model derives optimal consumption taxes in a representative taxpayer setting when efficiency is the sole objective of taxation. If the two consumption goods are identical in all respects (for instance, identical compensated elasticities) then uniform commodity taxation is optimal. The stylized model of this section shows that for two identical commodities, if the respective tax bases are measured with different degrees of imprecision,

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[14]Precautionary savings require a positive third derivative, see for example Leland (1968).

then uniform commodity taxation is not optimal. Indeed, if one of the two commodities is measured relatively more imprecisely it will be taxed at a lower rate[15].

To fix ideas in a simple manner, modify the model in Section 2 as follows. Interpret the two dated consumption goods, C_1 and C_2, as two different consumption goods in a static timeless model[16]. The two goods are identical for all purposes, except for their measurability characteristics (for purposes of taxation). The before-tax price of each good is identically equal to unity. Preferences take the CARA form and are additively separable in the two goods:

$$E\left[U\left(C_1, C_2\right)\right] = E\left[-\exp\left(-\rho C_1\right)\right] + E\left[-\exp\left(-\rho C_2\right)\right] \tag{5.1}$$

The government levies the income tax (θ, α), and a consumption tax on each the two goods at respective rates τ_1 and τ_2. For tax purposes, the observed signal on the income tax base continues to be given by 2.1, while 2.2 is modified to reflect the two observed signals on the two consumption tax bases, C_1^O and C_2^O respectively, as follows:

$$C_1^O = I_D + \gamma_1$$

$$C_2^O = I_D + \gamma_2$$

where γ_1 and γ_2 are respectively zero mean, normally distributed, measurement errors with respective variances σ_1^2 and σ_2^2; both errors are assumed to be uncorrelated with the income uncertainty term η and, for analytical simplicity, also with each other.

The sequence of moves is as follows. The government is the Stackelberg leader and announces the tax vector $\Gamma = (\alpha, \theta, \tau_1, \tau_2)$ followed by the (static) allocation of income, I, by the taxpayer among the two consumption goods, C_1 and C_2. Since the solution to the model is similar to that in Section 3, the details are confined to the appendix.

Proposition 5 : *The relative optimal tax rates on the two consumption goods, $\frac{\tau_1^*}{\tau_2^*}$, are equal to the relative imprecision in measuring the two tax bases, $\frac{\sigma_2^2}{\sigma_1^2}$. Unless $\sigma_1^2 = \sigma_2^2$, differential commodity taxes dominate uniform commodity taxation.*

The main message of Proposition 5 is that optimal consumption taxes ought to be tailored to the measurability characteristics of various commodities. Consumption goods that can be measured less precisely ought to be taxed at lower rates. An extreme example is the tax exemption of the returns from certain assets such as a house, car, equity of certain types, bequests, and inheritances, under the 'registered asset version' of consumption taxes, on account of the extreme difficulty of measuring such returns.

It is, however, possible to derive an even stronger and unexpected result when the measurement errors in the two consumption tax bases are correlated with the income

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[15]The results in this section can be easily modified to address the issue of uniform versus differentiated taxation of different sources of income. Since the treatment of these issues is analogous, but it is issues of uniform versus differentiated commodity taxes that typically receive more attention, it is omitted.

[16]For a treatment of alternative consumption tax systems in a dynamic setting with consumption-savings choice under uncertainty and measurement problems, see Dhami (2002).

uncertainty term. The presence of such correlation depends on the plausible assumption that the size of the measurement errors depends on the size of the disposable income. In this case, uniform commodity taxation is not optimal *even when the relative imprecision in measuring the two consumption goods is identical.*

To demonstrate this result, assume that the imprecision in measuring each of the two consumption tax bases is $\sigma_1^2 = \sigma_2^2 = \sigma^2$. Denote the covariance between the measurement errors ϵ_I and γ_j, $j = 1, 2$, as σ_{Ij}. Proposition 6 below shows that if $\sigma_{I1} \neq \sigma_{I2}$ then commodities that provide a better risk-hedging role are taxed at a relatively higher rate; uniform commodity taxation is not optimal.

Proposition 6 : *If $\sigma_{I1} \neq \sigma_{I2}$ then $\tau_1^* \neq \tau_2^*$ and $\tau_1^* - \tau_2^*$ is increasing in $\sigma_{I1} - \sigma_{I2}$.*

Greater covariance between two sources of uncertainty allows the risks facing the tax-payer to be filtered out to a greater extent, hence, it is optimal to differentiate the tax treatment of the two consumption goods. Propositions 5 and 6 show, in two stylized contexts, that administrative issues have important, but hitherto ignored, implications for the choice between uniform and differential commodity taxes. Future research can use the template provided above in a fairly abstract and general context, to look at specific sources of measurement errors and institutional detail.

6. Conclusions

This paper incorporates administrative issues into optimal tax models under uncertainty; an endeavour termed by Slemrod (1990) as a theory of 'Optimal Tax Systems'. It focuses on one important implication of administrative problems, namely, that tax bases must be difficult or costly to measure. In spirit, the model is the one originally used in Varian (1980). However, it is extended to allow for tax administration considerations and the full set of (linear) tax instruments. One of the strengths of the model is its simplicity, and pedagogical merit in providing several closed form solutions. The model also provides a useful template for optimal tax theorists working in the general area of taxation under uncertainty- an area which has not progressed a great deal since the seminal works of Varian, Eaton and Rosen more than two decades ago.

Some of the results are as follows. Measurement problems reduce the optimal tax rates; in the limit as such problems become too severe, they might even override the 'social insurance role' of taxation. The social insurance role of taxation can be provided by consumption taxes − a role that often seems in the literature to be ascribed to income taxes alone. The relative magnitudes of the income and consumption taxes are proportional to the relative ease of measuring the income tax base relative to the consumption tax base − a conjecture made by Bradford (1980) as the most important distinction in the choice between direct and indirect taxes. Errors in the measurement of a tax base can have 'spillover effects' by affecting the optimal tax rate on another base. In a stylized application of the basic model to consumption taxes, it is shown that even in circumstances where the Ramsey taxes are predicted to be uniform, differences in the measurability characteristics

of different commodities imply differentiated optimal commodity taxes. The model also derives implications for precautionary savings in the presence of income and administrative uncertainty.

Although measurement problems could arise due to a wide variety of reasons, these are considered exogenous in the paper. Endogenous treatment of measurement problems has the potential to produce a rich range of differentiated models of 'Optimal Tax Systems'. This remains an important challenge and will reduce the gap between the theory and practice of taxation − an endeavour that seemed vital for the pioneers of optimal tax theory.

7. Appendix

Proof of Proposition 1: Evaluating 3.3 and 3.4 at $\sigma_\eta^2 = \sigma_I^2 = \sigma_C^2 = \sigma_{IC} = 0$, it follows that $\theta^* = \tau^* = 0$. Hence, a poll tax optimally raises all revenues i.e. $\alpha^* = -R$. Evaluating 3.3 and 3.4 at $\sigma_I^2 = \sigma_C^2 = \sigma_{IC} = 0$ one gets

$$\frac{\partial V(\Gamma)}{\partial \theta} = (1 - \tau)^2 (1 - \theta) \sigma_\eta^2 \le 0; \quad \theta \ge 0$$

$$\frac{\partial V(\Gamma)}{\partial \tau} = (1 - \tau)(1 - \theta)^2 \sigma_\eta^2 \le 0; \quad \tau \ge 0$$

$(\theta^*, \tau^*) = (0, 0)$ cannot be a solution, for in that case, $\frac{\partial V(\Gamma)}{\partial \theta} > 0$ and $\frac{\partial V(\Gamma)}{\partial \tau} > 0$, contradicting the optimality of $(\theta^*, \tau^*) = (0, 0)$. It is easy to check that $(\theta^*, \tau^*) = (1, 0)$ and $(\theta^*, \tau^*) = (0, 1)$ satisfy the complementary slackness conditions of the Kuhn-Tucker theorem, namely, $\theta \frac{\partial V(\Gamma)}{\partial \theta} = 0$ and $\tau \frac{\partial V(\Gamma)}{\partial \tau} = 0$. ■

Proof of Corollary 1: Rewriting 3.3 and 3.4 when $\sigma_\eta^2 = 0$:

$$\frac{\partial V(\Gamma)}{\partial \theta} = -\theta (1 - \tau)^2 \sigma_I^2 - \tau (1 - \tau) \sigma_{IC} \le 0; \quad \theta \ge 0 \tag{7.1}$$

$$\frac{\partial V(\Gamma)}{\partial \tau} = \theta^2 (1 - \tau) \sigma_I^2 - \theta (1 - 2\tau) \sigma_{IC} - \tau \sigma_C^2 \le 0; \quad \tau \ge 0 \tag{7.2}$$

Two corner solutions are readily eliminated. First, $\theta^* \ne 0, \tau^* = 0$, is not a solution because $\theta \frac{\partial V(\Gamma)}{\partial \theta} = -\theta \sigma_I^2 \ne 0$, which violates the complementary slackness condition in 7.1. Second, $\theta^* = 0, \tau^* \ne 0$, cannot be solution because $\tau \frac{\partial V(\Gamma)}{\partial \tau} = -\tau \sigma_C^2 \ne 0$, which violates the complementary slackness condition in 7.2. Using analogous reasoning it can be checked that $\theta^* = \tau^* = 0$ is a valid solution which does not violate the complementary slackness conditions. Hence, in this case, a poll tax, $\alpha^* = -R$, is optimal. It remains to check the final possibility i.e. $\theta^* \ne 0$ and $\tau^* \ne 0$. In this case, using 7.1, θ can be solved in terms of τ as $\theta = \frac{\tau \sigma_C}{(1 - \tau) \sigma_I} r$. Substituting θ in 7.2 the latter can be rewritten as $\frac{\partial V(\Gamma)}{\partial \tau} = \sigma_C^2 (r^2 - 1)$. The complementary slackness conditions imply that the solution $\theta^* \ne 0$ and $\tau^* \ne 0$ can only be supported if $\frac{\partial V(\Gamma)}{\partial \tau} = 0$, which requires that $r = \pm 1$. ■

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Proof of Lemma 1: Evaluating 3.3 and 3.4 at $\theta^* = \tau^* = 0$, in the limit as $\sigma_\eta^2 \to \infty$, it follows that $\frac{\partial V(\Gamma)}{\partial \theta} > 0$ and $\frac{\partial V(\Gamma)}{\partial \tau} > 0$, violating the supposed optimality of $\theta^* = \tau^* = 0$. It can be checked that the only solution which does not violate the complementary slackness condition is $\theta^* = \tau^* = 1$. ∎

Proof of Lemma 2: Evaluating 3.3 and 3.4 in the limit as $\sigma_I^2 \to \infty$, the complementary slackness conditions $\theta \frac{\partial V(\Gamma)}{\partial \theta} = 0$ and $\tau \frac{\partial V(\Gamma)}{\partial \tau} = 0$ require that $\theta^* = \tau^* = 0$. ∎

Proof of Lemma 3: The first order condition 3.3 is independent of σ_C^2. Suppose that $\tau^* \neq 0$, then as $\sigma_C^2 \to \infty$, it follows that $\tau \frac{\partial V(\Gamma)}{\partial \tau} = -\infty$, violating the complementary slackness condition in 3.4, hence, $\tau^* = 0$. ∎

Proof of Proposition 2: Straightforwardly check by differentiating 4.1 and 4.2 with respect to σ_η^2, σ_I^2 and σ_C^2, that $\frac{\partial \theta^*}{\partial \sigma_\eta^2} > 0$, $\frac{\partial \tau^*}{\partial \sigma_\eta^2} > 0$, $\frac{\partial \theta^*}{\partial \sigma_I^2} < 0$, $\frac{\partial \tau^*}{\partial \sigma_C^2} < 0$, $\frac{\partial \theta^*}{\partial \sigma_C^2} = 0$ and $\frac{\partial \tau^*}{\partial \sigma_I^2} > 0$. ∎

Proof of Proposition 3: The proof follows by differentiating 4.3 with respect to $\frac{\sigma_I^2}{\sigma_C^2}$, $\frac{\sigma_I^2}{\sigma_\eta^2}$ and $\frac{\sigma_\eta^2}{\sigma_C^2}$. ∎

Proof of Proposition 4: Totally differentiate 3.3 and 3.4. holding fixed, all exogenous variables, other than σ_{IC}, to get the following matrix equation system

$$AX = Y \tag{7.3}$$

where the matrices are defined as follows
$$A = \begin{bmatrix} -(1-\tau)\left\{\sigma_\eta^2 + \sigma_I^2\right\} & \theta\sigma_I^2 - (1-\theta)\sigma_\eta^2 - \sigma_{IC} \\ 2(1-\tau)\left\{\theta\sigma_I^2 - (1-\theta)\sigma_\eta^2\right\} - (1-2\tau)\sigma_{IC} & -(1-\theta)^2\sigma_\eta^2 - \theta^2\sigma_I^2 - \sigma_C^2 + 2\theta\sigma_{IC} \end{bmatrix}$$
$$X = \begin{bmatrix} d\theta \\ d\tau \end{bmatrix} \text{ and } Y = \begin{bmatrix} \tau d\sigma_{IC} \\ \theta(1-2\tau)d\sigma_{IC} \end{bmatrix}$$

The matrix A is simply the Hessian matrix of second order partial derivatives i.e. $D^2V(\Gamma)$ which is negative semi-definite, hence, the determinant of A, denoted as $\det A$ is positive. Use Cramer's rule to find the following partial derivatives.

$$\frac{d\tau}{d\sigma_{IC}} = \frac{1}{\det A}\left\{-(1-\tau)^2\theta\left\{\sigma_\eta^2 + \sigma_I^2\right\} - \frac{\tau}{2}\left\{\tau\sigma_C^2 - (1-\tau)\sigma_\eta^2\right\}\right\} \tag{7.4}$$

Using 7.4, $\frac{d\tau}{d\sigma_{IC}} < 0$ if $\tau\sigma_C^2 - (1-\tau)\sigma_\eta^2 \geq 0$.

$$\frac{d\theta}{d\sigma_{IC}} = \frac{1}{\det A}\left\{\theta\left\{\sigma_\eta^2 + \sigma_{IC}\right\} - \theta^2(1-\tau)\left\{\sigma_\eta^2 + \sigma_I^2\right\} - \tau\left\{\sigma_C^2 + \sigma_\eta^2\right\}\right\} \tag{7.5}$$

Using the $(2,2)^{th}$ element in the matrix A, to substitute for $\theta\left\{\sigma_\eta^2 + \sigma_{IC}\right\}$ in 7.5 it follows that

$$\frac{d\theta}{d\sigma_{IC}} < \left(\frac{1}{2} - \tau\right)\left\{\left\{\sigma_\eta^2 + \sigma_C^2\right\} - \theta^2\left\{\sigma_\eta^2 + \sigma_I^2\right\}\right\} \tag{7.6}$$

If $\tau < \frac{1}{2}$, then using 7.6, $\frac{d\theta}{d\sigma_{IC}} < 0$ if $\frac{1+(\sigma_C^2/\sigma_\eta^2)}{1+(\sigma_I^2/\sigma_\eta^2)} < \theta^2$. ∎

Proof of Proposition 5: The government's income tax revenues equal $\theta I^O - \alpha$, while consumption tax revenues equal $\tau_1 C_1^O + \tau_2 C_2^O$, hence, its budget constraint is given by

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$E\left[\theta I^O + \tau_1 C_1^O + \tau_2 C_2^O\right] - \alpha = R$. The budget constraint of the taxpayer is given by $C_1 + C_2 = I + \alpha - \left[\theta I^O + \tau_1 C_1^O + \tau_2 C_2^O\right]$. Using the government budget constraint to eliminate α from the taxpayer's budget constraint, the latter can be written as a function of C_2, conditional on the tax vector Γ announced by the government:

$$C_1 = C_1\left(C_2 \mid \Gamma\right) = (I - R) + (1 - \tau_1 - \tau_2)\left\{(1 - \theta)\eta - \theta\epsilon_I\right\} - \tau_1\gamma_1 - \tau_2\gamma_2 - C_2 \quad (7.7)$$

Since the government is the Stackelberg leader, conditional on the tax vector Γ, the taxpayer chooses C_1 and C_2 in order to maximize 5.1 subject to the constraint $C_1 = C_1\left(C_2 \mid \Gamma\right)$ in 7.7. Substitute the constraint into 5.1 to get an unconstrained problem. Note that $E\left[-\exp\left(-\rho C_2\right)\right] = -\exp\left(-\rho C_2\right)$, since C_2 is parametrically fixed while $E\left[-\exp\left(-\rho C_1\right)\right] = -\exp\left(-\rho\xi_1\right)$, where ξ_1, the certainty equivalent, is given by

$$\xi_1\left(\Gamma, C_2\right) = \left\{I - R - C_2\right\} - \frac{\rho}{2}\left\{(1 - \tau_1 - \tau_2)^2\left\{(1 - \theta)^2\sigma_\eta^2 + \theta^2\sigma_I^2\right\} + \tau_1^2\sigma_1^2 + \tau_2^2\sigma_2^2\right\}$$

Hence, the taxpayer chooses C_1 and C_2 to solve the following unconstrained problem

$$Maximize\ E\left[U\left(C_1, C_2\right)\right] = -\exp\left\{-\rho\xi_1\left(\Gamma, C_2\right)\right\} - \exp\left\{-\rho C_2\right\}$$

The first order condition to the taxpayer's problem can be seen to imply that $\xi_1\left(\Gamma, C_2\right) = C_2$; solving out for C_2, one gets:

$$C_2\left(\Gamma\right) = \frac{\left\{I - R\right\}}{2} - \frac{\rho}{4}\left\{(1 - \tau_1 - \tau_2)^2\left\{(1 - \theta)^2\sigma_\eta^2 + \theta^2\sigma_I^2\right\} + \tau_1^2\sigma_1^2 + \tau_2^2\sigma_2^2\right\}$$

Substituting $C_2\left(\Gamma\right)$ into 7.7 one gets the optimal choice of the taxpayer, $C_1 = C_1\left(\Gamma\right)$. Hence, the optimal solution to the taxpayer's problem is summarized as $\left(C_1\left(\Gamma\right), C_2\left(\Gamma\right)\right)$.

Since the government is the Stackelberg leader, the first stage problem is to choose $\Gamma = \left(\alpha, \theta, \tau_1, \tau_2\right)$ to maximize the indirect utility of the representative taxpayer given by:

$$E\left[U\left(C_1\left(\Gamma\right), C_2\left(\Gamma\right)\right)\right] = -\exp\left(-\rho\xi_1\left(\Gamma, C_2\left(\Gamma\right)\right)\right) - \exp\left(-\rho C_2\left(\Gamma\right)\right)$$

The first order conditions are:

$$\frac{\partial E\left[U\left(\Gamma\right)\right]}{\partial\theta} = (1 - \theta)\sigma_\eta^2 - \theta\sigma_I^2 \leq 0; \quad \theta \geq 0 \quad (7.8)$$

$$\frac{\partial E\left[U\left(\Gamma\right)\right]}{\partial\tau_1} = (1 - \tau_1 - \tau_2)\left\{(1 - \theta)^2\sigma_\eta^2 - \theta^2\sigma_I^2\right\} - \tau_1\sigma_1^2 \leq 0; \quad \tau_1 \geq 0 \quad (7.9)$$

$$\frac{\partial E\left[U\left(\Gamma\right)\right]}{\partial\tau_2} = (1 - \tau_1 - \tau_2)\left\{(1 - \theta)^2\sigma_\eta^2 - \theta^2\sigma_I^2\right\} - \tau_2\sigma_2^2 \leq 0; \quad \tau_2 \geq 0 \quad (7.10)$$

Dividing 7.9 by 7.10, it follows that $\frac{\tau_1^*}{\tau_2^*} = \frac{\sigma_2^2}{\sigma_1^2}$, as claimed in the proposition.∎

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Proof of Proposition 6: The proof follows that of Proposition 5 closely; it can be checked that the certainty equivalent in this case is:

$$\xi_1\left(\Gamma, C_2\right) = \{I - R - C_2\} - \frac{\rho}{2} \left\{ \begin{array}{c} (1 - \tau_1 - \tau_2)^2 \left\{(1-\theta)^2 \sigma_\eta^2 + \theta^2 \sigma_I^2\right\} \\ + \sigma^2 \left\{\tau_1^2 + \tau_2^2\right\} + 2\left(1 - \tau_1 - \tau_2\right)\theta \left\{\tau_1 \sigma_{I1} + \tau_2 \sigma_{I2}\right\} \end{array} \right\}$$

The first order conditions with respect to τ_1 and τ_2 for the government's problem are:

$$\frac{\partial E\left[U\left(\Gamma\right)\right]}{\partial \tau_1} = (1 - \tau_1 - \tau_2)\left\{(1-\theta)^2 \sigma_\eta^2 - \theta^2 \sigma_I^2\right\} - \tau_1 \sigma^2 - \theta \sigma_{I1}\left\{1 - 2\tau_1 - \tau_2\right\} + \theta \tau_2 \sigma_{I2} = 0 \tag{7.11}$$

$$\frac{\partial E\left[U\left(\Gamma\right)\right]}{\partial \tau_2} = (1 - \tau_1 - \tau_2)\left\{(1-\theta)^2 \sigma_\eta^2 - \theta^2 \sigma_I^2\right\} - \tau_1 \sigma^2 - \theta \sigma_{I2}\left\{1 - 2\tau_2 - \tau_1\right\} + \theta \tau_1 \sigma_{I1} = 0 \tag{7.12}$$

Dividing 7.11 by 7.12 and simplifying, check that the following condition holds:

$$\tau_1 - \tau_2 = \left\{1 + \frac{\sigma^2}{\sigma_{I1} - \sigma_{I2}}\right\}^{-1} \tag{7.13}$$

When $\sigma_{I1} \neq \sigma_{I2}$ it follows that $\tau_1 \neq \tau_2$. To check the second part of the proposition differentiate 7.13 with respect to $\sigma_{I1} - \sigma_{I2}$ to get $\frac{\partial(\tau_1 - \tau_2)}{\partial(\sigma_{I1} - \sigma_{I2})} = \frac{(\tau_1 - \tau_2)^2}{(\sigma_{I1} - \sigma_{I2})^2} > 0.\blacksquare$

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