DEPARTMENT OF ECONOMICS

CORRUPTION AND THE PROVISION OF PUBLIC OUTPUT IN A HIERARCHICAL ASYMMETRIC INFORMATION RELATIONSHIP

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Abstract

This paper develops a hierarchical principal-agent model to explore the influence of corruption, bribery, and politically provided oversight of production on the efficiency and level of output of some publicly provided good. Under full information, an honest politician achieves the first best while a dishonest politician creates shortages and bribes. Under asymmetric information, however, an honest politician might create more shortages relative to a dishonest one, although, the latter creates greater bribes. Furthermore, the contracted output can be greater or smaller relative to that produced by an unregulated private monopolist. The model identifies a tradeoff between bribery and allocative efficiency. This helps to reconcile some conflicting results on the implications of corruption for the size of the public sector and provides new results on the circumstances under which an improvement in the auditing technology is beneficial. Relative to the static case, in the dynamic renegotiation-proof equilibrium, shortages fall but bribes can increase or decrease, raising important issues of the choice between long-term and short-term contracts.

Keywords: Corruption, Regulation, Asymmetric Information, Renegotiation-Proofness.

JEL Classification: D82 (Asymmetric and Private Information), D78 (Positive Analysis of Policy-Making and Implementation), L51 (Economics of Regulation)
1. Introduction

This paper develops a hierarchical principal-agent model to explore the influence of corruption, bribery, and politically provided oversight of production on the efficiency and level of output of some publicly provided good.

Consider the following generic situation. A possibly corrupt politician regulates a corrupt, monopolist, intermediary who provides some output or service to final consumers; the regulatory contract has the following essential features.

1. The politician enforces an official price which can be charged by the intermediary.
2. The contract specifies the volume of output to be sold by the intermediary.
3. The politician can freely audit the intermediary.

This regulatory framework characterizes at least two generic situations.

Example 1: The monopolist intermediary is an arm of the government, a public-agent, who supplies a ‘public output’ on behalf of the government. The public-agent could, for instance, be a civil servant or an executive branch of the government. There is no presumption that the output supplied by the public-agent has the nature of a public good.

Example 2: The intermediary is a monopolist private firm that supplies some ‘private output’. In particular, the private firm is not an arm of the government.

While the interpretations in Examples 1 and 2 are both plausible, we feel that the interpretation in Example 1 is more natural for the following reason. Whilst regulatory conditions (1) and (2) above are often observed separately in regulation of private firms, their simultaneous occurrence is less frequent. Furthermore, the government is constrained in several respects when it audits private firms, for instance, on account of various confidentiality clauses while it, as the notional owner on behalf of the citizens, has much greater powers in auditing public-agents. For these reasons, we will conduct our analysis within the context of Example 1 and interpret the intermediary, a public-agent, as an arm of the government. However, it is worth bearing in mind the interpretation given in Example 2.

1Possible normative explanations for the conferment of such monopoly rights might include market failures, merit goods, national security, national goals, homogeneity of standards or feasibility, issues. For many public outputs such as passports, industrial licenses etc. there are strong grounds for giving control to a single provider; see Bardhan (1997). The positive explanations view the conferment of such legal rights as a device to generate political rents; for example Shleifer and Vishny (1992, 1993) and Coolidge and Rose-Ackerman (1997). The effect of competition on corruption is not considered here but see Rose-Ackerman (1999) and Laffont and Guessan (1999). There is a sense in which the competition results of Laffont and Guessan (1999) can be applied to this paper; see Section 4 below.
1.1. Scarcity rents and shortages

The combination of a monopolist, and corrupt, public-agent is often cited as the reason for the existence of scarcity rents and shortages of public output in the literature; for instance Aidt (2003) and Bardhan (1997). Private individuals often require the consent of a monopolist public-agent to engage in some intermediate or final economic activity, the actual demand for which often exceeds its supply. The public-agent then charges a price in excess of the official price (scarcity rent per unit), to clear the market. Scarcity rents are extensively documented for a wide range of activities such as industrial licenses, export-import licenses, public housing, irrigation water, passports, driving licenses, public credit, exchange rates and old age pensions, in developed and developing countries2.

There are two main competing explanations of shortages and scarcity rents. In queuing models, for example Lui (1985), waiting in a queue for a public output is costly. The objective is to find the Nash equilibrium in bribing strategies for individuals who can pay bribes, in order to jump the queue. However, the results are very sensitive to the different methods of organizing the queue and are not robust to plausible extensions; see for example Bardhan (1997).

In the other explanation, due to Shleifer and Vishny (1993), the government has full information on the cost/demand conditions facing a monopolist public-agent who provides a non-contractible public output. Hence, the public-agent sells the monopoly output and collects a scarcity rent equal to the monopoly profit. However, under full information, the monopoly profits are public information, so charging the public-agent a transfer/franchise fee equal to the monopoly profit, at all output levels, ensures the first best, removal of corruption and an improvement in welfare. Corruption would then be non-distortionary, a prediction rejected by the empirical evidence; for example Mauro (1995).

One of the aims of this paper is to provide an extension of the basic Shleifer-Vishny model that enables an equilibrium with shortages and scarcity rents to be supported.

1.2. Basic building blocks of the model

1.2.1. The public-agent is better informed about costs

The notion that the government has access to information on all relevant aspects of the operation of a public-agent is quite strong; see for example, Acemoglu and Verdier (2000). Public-agents are likely to have superior information on, for instance, the physical and

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managerial technology used for producing the public output or indeed on their competence in using it. Insofar as these factors impinge mainly on costs, we assume that the public-agent has private information on costs. There are two types of public-agents, a low-cost type, \( c_L \), and a high-cost type, \( c_H \).

The costs in the model can be interpreted either as *production* or *provision* costs. In one possible interpretation, the public-agent engages directly in the production of the public output. However, in several examples of scarcity rents cited above, the public-agent engages in provision of the public output. Provision costs can of course be high, for instance, in the provision of scarce housing when expensive ‘means testing’ needs to be done. Or in the provision of industrial licenses where detailed feasibility studies and compliance criteria need to be checked.

Provision costs can also be small (relative to the costs of production which are sunk), nevertheless it is on the basis of the provision costs that the public-agent takes his decision. Essentially, the marginal costs, not the fixed costs, condition the corruption decision of the public-agent. Hence, inefficiencies or distortions might arise on account of these ‘small’ costs of provision. Furthermore, our results do not crucially hinge on the magnitude of the costs \( c_L \) and \( c_H \); the important condition is \( c_H > c_L \).

Costs of provision among public-agents can differ for several reasons. For instance, the public-agent can be particularly inefficient in processing the available information, or might lack in experience and insist on undertaking detailed means testing, feasibility studies and checking in minute detail all compliance criteria so that the costs in terms of resources or time foregone are very high. Also, a particularly conscientious public-agent could have high costs for similar reasons. Since ‘competence’ and ‘conscience’ are deep personal characteristics, cost becomes private information for the public-agent. We use the efficient/inefficient terminology to refer to types \( c_L \) and \( c_H \) respectively.

### 1.2.2. Public output is often observable and verifiable

Shleifer and Vishny (1993) assume that the government cannot contract on the quantity sold by the public-agent. However, in the current context, the converse assumption is often more realistic for the following reasons. First, for many types of outputs supplied by public-agents, the transaction must be officially recorded to be of any use to the consumer. Thus, for instance, public housing is of limited use if it is not officially issued; the same also applies to a passport, and several forms of industrial and export-import licenses. Once officially recorded, the output sold by the public-agent is fully observed by the government and can be contracted upon; indeed, it is common practice for governments to set quantity
targets for public-agents in both developed and developing countries\(^3\). Second, although the government often mandates the price at which the public-agent is required to sell its output\(^4\), it typically does not observe the actual price charged by the public-agent when the latter is dishonest. Indeed, the evidence suggests that when public output is scarce, public-agents often resort to scarcity rents in order to clear the market.

### 1.2.3. Dynamic considerations

In mechanism design games of the sort considered in this paper, where the government tries to elicit the public-agent’s hidden information by a choice of contracted output, a dynamic setting raises issues of renegotiation. Once the type of the public-agent is revealed, then the government can implement the first best contracts in subsequent periods. Anticipating such action in the future, the public-agent might not be willing to reveal hidden information, or might require additional information rents to do so. However, this distorts the contracts predicted by the static game.

### 1.3. Other features of the model

The politician reimburses the public-agent’s cost using non-distortionary taxation, instructs the latter to sell at some official price and contracts on its output. The public-agent engages in bribery by selling at a price above the official price\(^5\). An exogenously given auditing technology allows the politician to discover hard evidence of such bribery with some probability \(\rho > 0\). However, in return for a share in the bribe, certain kinds of politicians, the *venal* ones, are willing to hide evidence of the bribe. *Decent* politicians, on the other hand, eschew such corrupt side transactions. The ‘degree of venality’ of the politician is a parameter of her preferences. We characterize and analyze the comparative static properties of contracted output and bribes under these conditions.

### 1.4. Results

Under full information, shortages and corruption occur only if the politician is venal. Decent politicians, by virtue of their ability to contract on output, produce the first best

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\(^3\)Quantity targets can be explicit or implicit; in the latter case, fixing the budgetary allocation to a executive department implicitly defines the quantity that can be supplied.

\(^4\)Governments routinely announce an official price (or require the public-agent to announce one) at which the public-agent’s output will be sold. Examples include an official price for passports/permits/licenses or an official interest rate for borrowing from public financial institutions etc.

\(^5\)The politician is able to contract on the official price, but not the unofficial price. The latter might be considered a form of contractual incompleteness in the model. However, “incomplete contracts” are usually discussed in the context of legal activity. Of course, illegal or illicit activity could also be considered a form of contractual incompleteness. But we think it is useful, and in line with standard practice, to distinguish between legal but incomplete contracting on the one hand and illicit activity on the other.
outcome. This is in contrast to Shleifer and Vishny (1993) where bribery can occur even when the politician is decent because the latter cannot contract on output. Furthermore, under full information, the contracted output always exceeds that produced by a private unregulated monopolist.

Under asymmetric information, each type of politician creates shortages in order to limit information rents. Whilst limiting information rents is the primary focus of a decent politician, however, a venal politician, in addition, also cares about the bribes that he gets. For this reason, a venal politician creates further distortions in contracted output (in addition to those that arise from the desire to limit information rents). The direction of these distortions depends on the relation of the contracted output to that produced by a private unregulated monopolist. Because the direction of these distortions depends on the parameters, the asymmetric information case gives new insights relative to the full information case. Furthermore, it helps to reconcile apparently conflicting results on the effects of corruption on the size of the public sector.

An improvement in the auditing technology lowers the private marginal cost of a unit of bribes to the dishonest politician and increases her bargaining power. The dishonest politician then distorts output in the direction of increasing bribes. However, the distortion of output can, depending on the parameter values, be efficiency enhancing or efficiency reducing. In the former case, it creates tradeoffs, in welfare terms, between the bribe increasing (cost) and the efficiency enhancing (benefit) aspect of changes in the monitoring technology.

The analysis of the dynamic game follows Hart and Tirole (1988) and Laffont and Tirole (1993). Relative to the static case, in the dynamic renegotiation-proof equilibrium, shortages fall but bribes can either increase or decrease. This suggests important determinants of the choice between offering short-term and long-term contracts to public-agents. The type of equilibrium expected to prevail in the dynamic game, namely, hybrid, fully separating or fully pooling, depends on the time discount factor of the public-agent.

1.5. Schematic outline

The schematic outline of the paper is as follows. Section 2 describes the static model and Section 3 solves it in the benchmark case of full information. Section 4 derives the solution to the static model under asymmetric information. Section 5 analyzes the problem in its dynamic version, in the presence of renegotiation. Finally, some conclusions are presented in Section 6. All proofs are collected in the appendix.
2. The Model

An upper-tier of the government, referred to by the generic term, politician, contracts a monopolist lower-tier of the government, referred to by the generic name, public-agent, to supply some good or service, on its behalf, to final consumers. The publicly known inverse demand curve facing the public-agent is given by \( p(q) \) where \( p \) is the price, and \( q \) is the demand. The demand curve is downward sloping i.e. \( p' < 0 \). The cost curve of the public-agent is given by \( C(q) = cq \), where \( c > 0 \) is the constant marginal cost.

The marginal cost \( c \) is privately known to the public-agent and is referred to as her ‘type’. The type space is given by the discrete set \( \Theta = \{c_H, c_L\} \) where \( c_H > c_L \) and the subscripts ‘H’ and ‘L’ have the connotation of ‘high’ and ‘low’ cost respectively. We shall denote by \( \Delta c \), the cost difference \( c_H - c_L \). The prior belief that the type is efficient, i.e. \( c = c_L \), is given by \( \nu \in [0, 1] \).

All players, the consumers, politician and the public agent, are risk neutral.

The politician levies non-distortionary taxes on the consumers to finance the payment of a lumpsum transfer ‘\( t \)’ and the cost to provision \( C(q) \) to the public-agent. The politician announces the type contingent contracts \( (t_L, q_L, c_L) \), \( (t_H, q_H, c_H) \) respectively for the efficient and the inefficient types of the public-agent. Each of these contracts specifies a triple: a transfer \( t_i \), a quantity \( q_i \) and an official per unit price \( c_i \); \( i = H, L \).

2.1. Bribes

If a type-\( c_j \) public-agent \( (j = i \text{ or } j \neq i) \) accepts the contract \( (t_i, q_i, c_i) \), the bribe to the public-agent \( j \) is

\[
B_i = B_i(t_i, q_i, c_i) = q_i [p(q_i) - c_i] \quad ; \quad i = H, L. \tag{2.1}
\]

Note that the bribe received by type-\( c_j \) depends on the contracted output, \( q_i \), the consumer’s willingness to pay, \( p(q_i) \), and the official price, \( c_i \). In particular, the bribe does not depend on the unit cost, \( c_j \), of agent \( j \). Sometimes we will simply use the abbreviated notation \( B_i(q_i) \) for \( B_i(t_i, q_i, c_i) \). Figure 2.1 shows a situation in which the efficient type, type \( c_L \), accepts the contract designed for type \( c_H \).

The efficient type is then faced with an official price per unit, \( c_H \), but the consumers’ willingness to pay per unit is \( p_H \) and so type \( c_L \) receives a bribe \( B_H = q_H(p_H - c_H) \) (which is independent of \( c_L \)). Furthermore, by mis-stating costs type \( c_L \) derives an extra payoff equal to \( \Delta c q_H \) because her per unit costs are \( c_L \) but she is reimbursed at the rate of \( c_H \) per unit by the politician.

\(^6\)Shleifer and Vishny (1993) also set the official price equal to \( c_i \). This is not restrictive in the current context as the level of the official price has no bearing on the qualitative results provided that the official price is set no lower than the cost and no higher than the consumer’s maximum willingness to pay.
Also note that bribes, under contract \((t_i, q_i, c_i)\), are positive if, and only if, contracted output, \(q_i\), is below the first best, \(q_i^{FB}\), given by \(p(q_i^{FB}) = c_i\), so that \(p(q_i) > c_i\). Hence, a dishonest politician has an incentive to generate shortages to create the scope for bribes.

Bribes, defined in (2.1), are equivalent to the monopoly profits \(\Pi^M(q_i)\) of a private unregulated monopolist who has marginal cost \(c_i\). It is well known that if \(\Pi^M(q_i)\) is concave then it has a unique maximum at \(q_i = q_i^M\). Furthermore, \(\Pi^M(q_i)\) is increasing in \(q_i\) upto \(q_i = q_i^M\) and decreasing thereafter. This analogy can be used to infer the properties of the bribe function \(B_i(q_i)\).

**Remark 1**: If \(B_i(q)\) is concave in \(q\) then bribes are increasing in contracted output for all \(q < q_i^M\) and decreasing in output for all \(q > q_i^M\). At \(q = q_i^M\), \(B'_i(q_i^M) = 0\).

### 2.2. Sequence of moves in the static game

The sequence of moves in the static game (the dynamic game is considered in Section 5) is as follows.

The politician announces the type contingent contracts \((t_L, q_L, c_L), (t_H, q_H, c_H)\). The public-agent accepts or rejects the contracts. If the contracts are accepted, the public-agent decides whether to receive bribes from consumers. Then the politician discovers hard evidence of bribes with probability \(\rho > 0\). With probability \(1 - \rho\), the public-agent gets to keep the bribe. There are no penalties over and above the confiscation of the bribe. Such penalties do not qualitatively alter the results as long as they are not prohibitive in the sense that they completely eliminate the incentive for bribery. This conforms to
the experience in many countries, notably several developing countries; for instance Rose-Ackerman (1999).

If hard evidence is discovered, then the politician might (depending on the degree of venality) offer to suppress the evidence if the public-agent agrees to share the bribe; such sharing uses the Nash Bargaining solution. If the public-agent refuses to share the bribe, then the bribe is confiscated and returned back to the consumers. If the public-agent agrees to share the bribe then the game ends with the division of the bribe and no bribes are returned back to consumers. The solution is derived by backward induction.

2.3. Audits and information revelation

In our model, a successful audit merely reveals that a bribe has been paid (and its magnitude). In particular, even a successful audit does not reveal any new information about the cost parameter $c_i$. From (2.1) it is immediately apparent that bribes only depend on the type of the contract accepted by the public agent and not on the public-agent’s type. We explain this more fully below.

In a fully separating equilibrium, type $i$ (with marginal cost $c_i$) chooses contract
(t_i, q_i, c_i) and hence, obviously, reveals the type through her choice of contract^{7}.

At the other extreme, in a fully pooling equilibrium, where, say, both types c_L and c_H choose contract (t_H, q_H, c_H), auditing does not reveal any information about costs. The audit merely reveals, with probability \( \rho \), that a bribe \( q_H [p(q_H) - c_H] \) has been paid. This gives no new information about the true value of \( c_i \).

In a hybrid equilibrium where, say, type c_H chooses contract (t_H, q_H, c_H) with certainty and type c_L chooses \((t_L, q_L, c_L)\) with probability \( \pi > 0 \), the politician updates her belief, \( \text{Prob}(c = c_L) \), from \( \nu_1 \) to \( \nu_2 \) (see section 5). If contract \((t_L, q_L, c_L)\) is chosen, then \( \nu_2 = 1 \). A successful audit will reveal the bribe \( q_L [p(q_L) - c_L] \) indicating that \( c = c_L \). But this is already known from the fact that the agent has accepted the contract \((t_L, q_L, c_L)\). On the other hand, if contract \((t_H, q_H, c_H)\) is accepted, then \( \nu_2 = \frac{(1-\pi)\nu_1}{(1-\pi)\nu_1 + 1 - \nu_1} \). A successful audit would reveal a bribe of \( q_H [p(q_H) - c_H] \) confirming that \( \nu_2 = \frac{(1-\pi)\nu_1}{(1-\pi)\nu_1 + 1 - \nu_1} \), but adding no new information.

In an extended model we could allow ‘cost auditing’ as well as ‘honesty auditing’. But this lies beyond the scope of this paper.

2.4. Preferences of the Public-Agent

The expected utility of the public-agent of type \( c_j \) who accepts the contract \((t_i, q_i, c_i)\), \( E[V_j(t_i, q_i, c_i)] \), \( j, i = H, L \) is defined as

\[
E[V_j(t_i, q_i, c_i)] = t_i + E[B_i^A] + (c_i - c_j) q_i; \quad j = i \text{ or } j \neq i
\]  

where \( t_i \) is the transfer received from the politician \( E[B_i^A] \) is the expected bribe received and the term \((c_i - c_j) q_i \) arises because a type \( j \) has unit cost \( c_j \) but is reimbursed \( c_i \) by accepting the contract \((t_i, q_i, c_i)\) (see Figure 2.1). The expectation operator \( E \) runs over the ‘state of the world’- absence or presence of hard evidence of bribes and the type of the politician- venal or decent (these terms are formally defined below). The public-agent’s reservation utility is normalized to zero.

2.5. Preferences of Consumers

The expected utility of a representative consumer is defined as

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^{7} Once the choice of contracts reveals the type of the public-agent, why does not the politician tear up the original contract and offer the full information contracts? There are two reasons why this does not happen. First, the ability of the government to commit not to renegotiate its contracts underpins a large literature that uses mechanism design in the presence of asymmetric information. We find this to be a fairly plausible restriction given issues of reputation etc. Second, renegotiation might actually not be possible in several kinds of static games when previous events are irreversible; see for instance Laffont and Tirole (1993). Renegotiation is explicitly considered below in a dynamic version of the game.

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\[ E[U] = S(q_i) - (t_i + c_i q_i) - B_i + E[B_i^C] \ ; i = H, L \tag{2.3} \]

where \( S(q_i) = \int_0^{q_i} p(q) dq \) is the ‘gross consumer surplus’. Consumers pay taxes of an amount \( t_i + c_i q_i \) to finance the operation of the public-agent and bribes equal to \( B_i \) to gain access to the (possibly scarce) public output. In the event that hard evidence of bribes is discovered by the politician and if the bribe is confiscated (which is an endogenous decision), it is returned back to consumers as a lumpsum transfer; \( E[B_i^C] \) is the expected receipt of such bribes by the consumers from the politician when a public agent chooses contract \((t_i, q_i, c_i)\).

### 2.6. Preferences of the Politician

The objective function of the politician is given by

\[ E[W] = E[U] + \mu E[B_i^P] \tag{2.4} \]

where \( U \) is the utility of consumers (a measure of social welfare) and \( E[B_i^P] \) is the expected bribe received by the politician from the public-agent when the latter chooses contract \((t_i, q_i, c_i)\). The parameter \( \mu \in [0, \infty) \) is the weight placed by the politician on personal gratification relative to social welfare; it reflects the “degree of the politician’s venality”.

**Definition 1**: A “venal” politician cares relatively more for personal benefits i.e. \( \mu > 1 \) while a “decent” politician cares relatively more for social welfare i.e. \( \mu \leq 1 \). The “degree of venality” is given by the size of \( \mu \).

### 2.7. The Nash Bargaining Solution

Suppose that the politician discovers hard evidence of bribes after the public agent chooses contract \((t_i, q_i, c_i)\). Let \( x \in [0, B_i] \) be the politician’s share of the bribe. If the politician and the public agent reach an agreement on sharing the bribe, their respective payoffs are

\[
W = S(q_i) - (t_i + c_i q_i) - B_i + \mu x, \\
V_j = t_i + B_i - x + (c_i - c_j) q_i; \quad j = i \text{ or } j \neq i.
\]

However, should the politician and the public-agent not be able to reach an agreement, their respective disagreement payoffs, \( d_P \) and \( d_A \), are

\[
d_P = S(q_i) - (t_i + c_i q_i),
\]
\[ d^A_j = t_i + (c_i - c_j) q_i. \]

The net surplus from this relationship equals \((W - d^P) + (V_j - d^A_j) = x_i (\mu - 1)\), which is positive only when the politician is venal i.e. \(\mu > 1\). The Nash Bargaining solution, \(x_i\), is found by maximizing the product \((W - d^P) (V_j - d^A_j)\), hence

\[ x_i \in \arg \max (\mu x - B_i) (B_i - x) \]

It is straightforward to check that the solution, \(x_i\), is given by

\[ x_i = \left( \frac{1 + \mu}{2\mu} \right) B_i \]  

(2.5)

and the public-agent’s share \(B_i - x_i\) equals

\[ B_i - x_i = \left( \frac{\mu - 1}{2\mu} \right) B_i. \]  

(2.6)

**Lemma 1**: In the event that hard evidence of bribes is found, a decent politician (\(\mu \leq 1\)) returns all bribes to the consumers, while a venal politician (\(\mu > 1\)) conceals the evidence for a share in the bribe. Furthermore, the politician’s share of the bribe is decreasing in the degree of venality, \(\mu\), with \(x_N \rightarrow B_i\) as \(\mu \rightarrow 1\) and \(x_i \rightarrow B_i/2\) as \(\mu \rightarrow \infty\).

In Lemma 1 the decent politician’s decision to eschew a corrupt deal with the public-agent is an endogenous one. It is harder to bribe a less venal politician, hence, the politician’s share of bribes is decreasing in the degree of venality.

### 2.8. The Public-agent’s Bribery Decision

When the politician is decent (\(\mu \leq 1\)), she confiscates the public-agent’s bribe. In this case the public agent’s expected bribe, \(E[B^A_i]\), is \(\rho (0) + (1 - \rho) B_i\), hence

\[ E\left[B_i^A\right] = (1 - \rho) B_i. \]  

(2.7)

When the politician is venal (\(\mu > 1\)), given (2.6), the expected bribe of a public-agent, who accepts the contract \((t_i, q_i, c_i)\), is \(E\left[B_i^A\right] = \frac{\mu - 1}{2\mu}\rho B_i + (1 - \rho) B_i\), hence

\[ E\left[B_i^A\right] = \left(1 - \frac{\rho}{2} (1 + \mu^{-1})\right) B_i. \]  

(2.8)

**Definition 2**: From (2.7) and (2.8), the generic expression for the public-agent’s expected bribe, when she accepts the contract \((t_i, q_i, c_i)\), is \(E\left[B_i^A\right] = \Phi (\mu, \rho) B_i\). When the politician is decent \(\Phi (\mu, \rho) = (1 - \rho) \geq 0\). When the politician is venal, \(\Phi (\mu, \rho) = 1 - \frac{\rho}{2} (1 + \mu^{-1}) \geq 0\).
From Definition 2, $\Phi(\mu, \rho) \geq 0$, hence, in the absence of any additional penalties above the confiscation of the bribe, the public-agent always accepts bribes$^8$.

3. The Full Information equilibrium

Under full information, the public-agent cannot misrepresent her type. Denote the equilibrium contract for a public-agent of type $c_i$ under full information by $(t_{ij}^*, q_{ij}^*)$ where $j = D, V$ indexes the type of the politician; decent and venal respectively and $i = H, L$ refers to the public-agent’s type.

3.1. The Generic Problem

The generic problem facing the politician is

$$\left(q_{ij}^*, t_{ij}^*\right) \in \arg\max W^j(q_{ij}, t_{ij}) = E[U] + \mu E[B_i^j] \quad j = D, V$$

Subject to:

$$E[V_i] = t_i + E[B_i^A] \geq 0 \quad \text{(Individual Rationality Constraint)}$$

$$p(q_i) \geq c_i \quad \text{(Feasibility Constraint)}$$

The feasibility constraint ensures that the public-agent does not make any per-unit losses, it is omitted for the time being but the solution is subsequently checked against it. The individual rationality constraint, which ensures that the public-agent receives at least the reservation utility, binds under full information, because rents to the public-agent must be given by sacrificing valuable consumer welfare, hence, $t_i = -E[B_i^A]$. Notice that the term $(c_i - c_j)q_i$ does not appear in the individual rationality constraint because types cannot be misrepresented under full information.

$^8$If fines are high enough then the decent (and perhaps some forms of venal politicians) can also stamp out corruption. We are sceptical about raising fines for corruption to an appreciable degree, for the following reasons. First, it is a legal requirement that comparable offences for fraud be punished in a comparable manner. Indeed, for comparable cases of fraud, for instance, tax evasion, the fine is only about 0.5 of the evaded tax payment. Second, legal practice is very concerned with Type I and Type II errors. High fines might make legal mistakes unacceptably expensive. Third, high fine are more difficult to collect and involve complex complementary legal positions on bankruptcy law. Fourth, agent’s behaviour might be distorted in important ways that are not modelled in this paper. Fifth, if the politician is venal then high fines will raise his bargaining power. Hence, we are reluctant to push the case for high fines.
3.2. Decent Politician ($\mu \leq 1$)

With probability $\rho$ the politician discovers hard evidence of the bribe and returns the bribe to the consumers so $E\left[B_i^C\right] = \rho B_i$, $E\left[B_i^P\right] = 0$ while $\Phi(\mu, \rho) = 1 - \rho$ (from (2.7)), hence, the generic problem in subsection 3.1 reduces to the following unconstrained problem.

$$ q_i^{D*} \in \arg \max W^D = S(q_i) - cq_i $$

The first order condition gives

$$ p(q_i^{D*}) = c_i. \quad (3.2) $$

It is obvious that the solution is first best, $q_i^{D*} = q_i^{FB}$, and the feasibility constraint is satisfied. The intuition is that with probability $\rho$ the bribe gets confiscated and returned back to consumers while with probability $1 - \rho$, transfers to the public-agent are reduced by the amount of the bribe so net bribes paid equal $B_i - B_i [\rho + (1 - \rho)] = 0$. The result is recorded without proof in Proposition 1.

**Proposition 1**: The corruptibility of the public-agent is irrelevant under the regime of a decent politician and the outcome is first best.

Proposition 1 shows that relative to the corruptibility of a public-agent, the corruptibility of the politician is of first order importance. Under full information, bribery never occurs if the politician is decent. This contrasts with Shleifer and Vishny (1993) where bribery can occur even when the politician is decent, because output is not contractible.

3.3. Venal Politician ($\mu > 1$)

The venal politician never returns any bribes to the consumers so $E\left[B_i^C\right] = 0$. Since hard evidence of bribes is found only with probability $\rho$ so $E\left[B_i^P\right] = \rho x_i$ and $E\left[B_i^A\right] = B_i \Phi(\mu, \rho)$ where $\Phi(\mu, \rho)$ is given in (2.8). Hence, the generic problem in section 3.1 can be written as the following unconstrained problem:

$$ q_i^{V*} \in \arg \max W^V (q_i, c_i) = S(q_i) - c_i q_i - \xi B(q_i, c_i) \quad (3.3) $$

where $\xi$, the venal politician’s private marginal cost of a unit of bribes is

$$ \xi = \frac{\rho}{2 \mu} (1 + \mu) (1 - \mu) < 0. \quad (3.4) $$

$\xi$ is decreasing in $\mu$ and $\rho$. The first order condition to the problem is

$$ \frac{\partial W^V}{\partial q_i} = 0 \iff \frac{\partial B(q_i^{V*}, c_i)}{\partial q_i} = \frac{p(q_i^{V*}) - c_i}{\xi} \quad (3.5) $$
while the second order condition is
\[
\frac{\partial^2 W^V}{\partial q_i^2} = p'(q_i) - \xi \frac{\partial^2 B_i}{\partial q_i} \leq 0
\]
which holds if \(\frac{\partial^2 B}{\partial q_i} \leq 0\). Substituting \(\frac{\partial B_i}{\partial q_i}\) in (3.5) one obtains:
\[
\ell_i^{V^*} = p(q_i^{V^*}) = c_i + \frac{\xi}{1 - \xi} q_i^{V^*} p'(q_i^{V^*}) \quad ; \quad i = H, L
\]

**Proposition 2** : The contracted output is intermediate between the first best and the monopoly level i.e. \(q_i^M < q_i^{V^*} < q_i^{FB}\) \(i = H, L\) and there are positive bribes in equilibrium, \(B_i(q_i^{V^*}) > 0\). The contracted output is decreasing and bribes are increasing in (1) the politician’s degree of venality, \(\mu\), and (2) efficiency of the monitoring technology, \(\rho\).

Proposition 2 shows that corruption takes the form of shortages of public output relative to the first best, however, the contracted output exceeds that produced by a private unregulated monopolist. Some well known examples of shortages include the former Soviet Union (Shleifer and Vishny (1992)), the recent experience of transition economies (Levine and Satarov (2000), UNDP (1997)), and the “License Raj” in India (Bardhan (1984)).

More venal politicians (high \(\mu\)) contract lower output because that is the direction of increasing bribes when \(q_i^M < q_i^{V^*}\) (see Remark 1), hence, shortages worsen under their regime. This result can change under asymmetric information, as will be shown in section 4 below. A more efficient auditing technology (high \(\rho\)) decreases \(\xi\), the venal politician’s private marginal cost of a unit of bribes, hence, creating greater shortages and bribes.

Under full information, transfers are negative in the regime of a venal politician because \(t_i = -\Phi(\mu, \rho) B_i \leq 0\). One interpretation of negative transfers is the sale of public offices, for instance, ‘tax farming’. Another interpretation is more natural in corrupt regimes, competition for government jobs by paying up-front bribes (negative transfers) is pervasive in several countries; see for instance Krueger (1974) and Shleifer and Vishny (1993).

### 4. Equilibrium Under Asymmetric Information

The full information allocation is not incentive compatible under asymmetric information. Denote the contracts designed for the efficient and the inefficient types respectively by \((q_L, t_L)\) and \((q_H, t_H)\)\(^9\). Dropping superscripts, the efficient type’s expected payoff from accepting her full information contract is \(EV_L(q_L, t_L) = 0\), but, under asymmetric information on the public-agent’s type, by accepting the inefficient type’s contract, its expected payoff is \(EV_L(q_H, t_H) = t_H + \Phi(\mu, \rho) B_H + \Delta c q_H\), where \(\Delta c = c_H - c_L > 0\). On substitution

\[^9\]The actual contracts are \((q_L, t_L, c_L)\) and \((q_H, t_H, c_H)\). However, since the official price always equal the corresponding marginal cost, these contracts are used in an abbreviated form as \((q_L, t_L)\) and \((q_H, t_H)\)
of $t_H = -\Phi (\mu, \rho) B_H$, check that $EV_L(q_H; c_L) = \Delta c q_H > 0$. Hence, the efficient type has an incentive to misrepresent her type.

4.1. The Generic Problem

The politician chooses type contingent contracts to maximize expected welfare $Z^j$, where

$$Z^j = \nu W^j (q_L, t_L) + (1 - \nu) W^j (q_H, t_H) \ ; \ j = D, V \quad (4.1)$$

subject to the following four constraints

$$EV_H (q_H, t_H) = t_H + \Phi (\mu, \rho) B_H \geq 0 \quad (IR_H)$$

$$EV_L (q_L, t_L) = t_L + \Phi (\mu, \rho) B_L \geq 0 \quad (IR_L)$$

$$EV_H (q_H, t_H) \geq EV_L (q_L, t_L) \Leftrightarrow t_H + \Phi (\mu, \rho) B_H \geq t_L + \Phi (\mu, \rho) B_L - \Delta c q_L \quad (IC_H)$$

$$EV_L (q_L, t_L) \geq EV_H (q_H, t_H) \Leftrightarrow t_L + \Phi (\mu, \rho) B_L \geq t_H + \Phi (\mu, \rho) B_H + \Delta c q_H \quad (IC_L)$$

where $W^j$ is defined in (3.1). The ‘individual rationality’ constraints $IR_H$ and $IR_L$ ensure that each of the types gets at least its reservation utility, while the ‘incentive compatibility constraints’ $IC_H$ and $IC_L$ ensure that none of the types chooses the contract intended for the other type. The solution to this problem is well known\textsuperscript{10}. Essentially, $IC_L$ and $IR_H$ bind and their satisfaction ensures satisfaction of $IR_L$. The constraint $IC_H$ is ignored for the time being; it can be checked later that it holds. From the binding $IR_H$ constraint, one obtains

$$t_H = -\Phi (\mu, \rho) B_H. \quad (4.2)$$

Substituting $t_H$ into the binding $IC_L$ constraint, the latter can be rewritten as

$$t_L = -\Phi (\mu, \rho) B_L + q_H \Delta c \quad (4.3)$$

**Definition 3**: The information rent of the efficient public-agent, type-$c_L$, equals $q_H \Delta c > 0$.

\textsuperscript{10}See Fudenberg and Tirole (1990a) or Laffont and Tirole (1993).
Furthermore, by adding the two IC constraints one gets $\Delta c (q_L - q_H) \geq 0$ which implies that $q_L \geq q_H$ i.e. incentive compatibility requires that the contracted output of the efficient type is higher. Substituting $t_H$ and $t_L$ from (4.2) and (4.3) into the objective function, one derives the unconstrained optimization problem of the politician, written below.

$$Z^j = \nu \left( S(q_L) - c_L q_L - [1 - \Phi (\mu, \rho)] B_L - \Delta c q_H + E [B'_L] + \mu E [B''_L] \right) + (1 - \nu) \left( S(q_H) - c_H q_H - [1 - \Phi (\mu, \rho)] B_H + E [B'_H] + \mu E [B''_H] \right)$$

where $j = D, V$ refers to the type of the politician. Denote the optimal solution under asymmetric information as $(q^j_L, t^j_L), (q^j_H, t^j_H)$. Since $t^j_L$ and $t^j_H$ can be found as residuals from (4.2) and (4.3), attention will be focussed on finding $q^j_L$ and $q^j_H$.

4.2. Decent Politician

With probability $\rho$ the politician discovers hard evidence of the bribe and returns the bribe to the consumers so $E [B^C] = \rho B, E [B^P] = 0$ and $E [B^A] = \Phi (\mu, \rho) B \geq 0$ where $\Phi (\mu, \rho) = (1 - \rho)$ (from (2.7)); substituting in (4.4), the politician’s unconstrained problem is

$$(q^D_L, q^D_H) \in \text{arg max } Z^D = \nu [S(q_L) - c_L q_L] + (1 - \nu) [S(q_H) - c_H q_H] - \nu q_H \Delta c$$

From the first order conditions, the optimal contracted output for types $c_L$ and $c_H$ respectively, is given by

$$p^D_L = p(q^D_L) = c_L \quad (4.5)$$

$$p^D_H = p(q^D_H) = c_H + \frac{\nu}{1 - \nu} \Delta c. \quad (4.6)$$

Since (4.5) is identical to (3.2) so $q^D_L = q^*_L = q^*_{FB}$, thus, the decent politician always requires the efficient type to produce the first best output\(^\text{11}\). However, (4.6) and (3.2) are not identical and so in general $q^D_H \neq q^*_H$.

**Definition 4** The ‘expected’ information rent per unit of revenue is defined as $\nu \frac{\Delta c}{1 - \nu}$.\(^\text{16}\)

We will denote $\eta$ as the elasticity of demand for the public output evaluated at $q^*_M$, where as before, $q^*_M$ is the output produced by a private unregulated monopolist.

\(^\text{11}\) It is easy to check that the omitted IC constraint is satisfied by the solution. Substituting (4.2) and (4.3) into the IC constraint one gets $\Delta c (q_H - q_L) \leq 0$, which is true because $q_H \leq q_L$.\(^\text{16}\)
Proposition 3: Under asymmetric information, the decent politician contracts $q_H^D: q_H^D < q_H^{D*} = q_H^{FB}$. Furthermore, $q_H^D$ is lower (greater) than $q_H^M$ as the “expected information rent per unit of revenue” is greater (lower) than the inverse of the elasticity of demand. If $q_H^D < q_H^M$ bribes are increasing in $q_H^D$ while if $q_H^D > q_H^M$ bribes are decreasing in $q_H^D$.

If expected information rents per unit of revenue are high enough then the politician creates shortages to limit information rents of the efficient type. The relevant comparison of information rents is with the inverse of the elasticity of demand. The intuition is that if elasticity is higher then the sacrifice in consumer surplus associated with shortages is lower. Under asymmetric information, even a decent politician might contract an output that is lower relative to the optimal output produced by an unregulated private monopolist i.e. it is possible that $q_H^D < q_H^M$. Recall that under full information (see section 3) even the venal politician does not contract output below that produced by a private unregulated monopolist i.e. $q_H^D > q_H^M$.

Although there is no distortion of output for an efficient public-agent in equilibrium, nevertheless, the efficient type is paid a “honesty allowance” or “information rent” in return for her honesty. Bardhan (1997) provides several examples of the empirical relevance of this result. Historically, imperial China used the policy of paying an extra allowance called the “yang-lien yin” (money to nourish honesty) to district magistrates. Robert Clive used a similar policy to reduce corruption in the East India Company. Hong-Kong and Singapore have successfully used incentive payments to reduce corruption; see for instance Klitgaard (1988) and Rose-Ackerman (1999). Although, incentive payments accord more naturally with an agency theoretic explanation, for instance Rose-Ackerman (1999), Mookherji (1997), Mookherji and Png (1995), Besley and McLaren (1993) and Klitgaard (1988), the essence of the result is unchanged in an adverse selection model.

Proposition 4: Shortages in contracted output worsen as (1) $\Delta c$ increases, and (2) $\nu$ increases. However, the affect on the magnitude of bribes in equilibrium depends on whether $q_H^D \geq q_H^M$; increasing when $q_H^D > q_H^M$ and decreasing when $q_H^D < q_H^M$.

An increase in $\Delta c$ or in $\nu$ increases expected information rents, which the politician attempts to reduce by creating shortages. Since the affect on the magnitude of bribes depends on whether the contracted output $q_H^D \geq q_H^M$, these results illustrate an important trade-off faced by anti-corruption programmes, namely, a possible conflict between efficiency (movement of output towards the first best) and bribery when $q_H^D < q_H^M$.

4.3. Venal Politician

The venal politician never returns any bribes to the consumers, thus, $E \left[ B^{C} \right] = 0$, $E \left[ B^{P} \right] = \rho x_i$ and $E \left[ B^{A} \right] = B \Phi (\mu, \rho)$ where $\Phi (\mu, \rho) = 1 - \frac{\rho}{\mu} (1 + \mu^{-1})$ (from equa-
tion (2.8)); substituting in (4.4), the venal politician’s unconstrained problem is to choose \((q_L, q_H)\) to maximize the following expression

\[ Z^V = \nu [S(q_L) - c_Lq_L - \xi B_L] + (1 - \nu) [S(q_H) - c_Hq_H - \xi B_H] - \nu q_H \Delta c \] (4.7)

As defined in (3.4), \(\xi = \frac{\rho^2}{2\mu} (1 + \mu) (1 - \mu) < 0\), is the venal politician’s private marginal cost of a unit of bribes. From the first order conditions, the optimal contracted output for types \(c_L\) and \(c_H\), respectively, is given by:

\[ p_V^L = p(q_V^L) = c_L + \frac{\xi}{1 - \xi} q_V^L p'(q_V^L) \] (4.8)

\[ p_V^H = p(q_V^H) = c_H + \frac{\xi}{1 - \xi} q_V^H p'(q_V^H) + \frac{\nu \Delta c}{(1 - \nu)(1 - \xi)} \] (4.9)

Comparing (3.7) with (4.8) and (4.9) it follows that \(q_V^L = q_V^{L*}\), however, \(q_V^H \neq q_V^{H*}\). The comparative static properties for a venal politician under full information, stated in Proposition 2, continue to hold for \(q_V^L\) but those for \(q_V^H\) are affected by the presence of the last term in (4.9). The discussion below is organized under three heads.

4.3.1. Shortages and Bribes

Under full information, when the politician is decent, higher-order corruption, in the terminology of Rose-Ackerman (1999) is more crucial, lower-order corruption becomes irrelevant in such a setting (see Proposition 1). However, under asymmetric information, lower-order corruption is relevant, as shown below in Lemma 2

**Lemma 2**: The venal politician contracts \(q_V^H < q_V^{H*}\) and there are positive bribes in equilibrium. \(q_V^H\) is greater (lower) than \(q_M^H\) as the “price markup per unit of revenue” is greater (lower) than the “expected information rent per unit of revenue”. Shortages in contracted output worsen as \(\Delta c\) or \(\nu\) increase. The effect on bribes depends on whether \(q_V^H \geq q_M^H\).

Under asymmetric information, an increase in either \(\nu\) or in \(\Delta c\) increases expected information rents. The venal politician responds, just as the decent politician, by contracting the inefficient type to supply an even lower quantity. The effect on bribes, however, depends on whether the contracted output is below (in which case bribes decrease) or above (in which case bribes increase) the monopoly level. If \(q_V^H > q_M^H\) then an increase in contracted output towards the first best is also accompanied by a reduction in bribes. However, when \(q_V^H < q_M^H\) a reduction in bribery is accompanied by a movement in output away from the first best; clearly in this case, there is trade-off in reducing bribery.
The tradeoff between bribery and shortages raises interesting welfare issues. For instance it is likely, depending on the parameter values, that an increase in welfare arising from a reduction in bribes overweighs the decrease in welfare arising from increased shortages. Some positive level of corruption can then be welfare enhancing. The existing literature typically ignores the tradeoff between production efficiency and bribery.

Under full information, the venal politician creates greater scarcities and bribes relative to the decent politician and the contracted output, \( q^*_H \), always lies below that produced by a private unregulated monopolist, \( q^M_H \). However, as shown in Lemma 2, under asymmetric information \( q^j_H \leq q^M_H \); \( j = D, H \). Furthermore, bribes are increasing in contracted output up to \( q^M_H \) and decreasing in contracted output thereafter. Since, the venal politician cares about bribes, in addition to limiting information rents, while the decent politician only cares about the latter, the relative contracted output under each of these types of politicians depends on which side of \( q^M_H \) the originally contracted output lies on. This is formally shown in Lemma 3 below.

**Lemma 3**: When the contracted output lies above \( q^M_H \), then \( q^V_H < q^D_H \). However, when the contracted output lies below \( q^M_H \), then \( q^D_H < q^V_H \).

Lemma 3 shows that, unlike the full information case, the venal politician does not necessarily create greater shortages relative to the decent politician. Many forms of public output where scarcity rents have been documented, form an important prerequisite for private investment/activity in the economy; these include, for instance, industrial licenses, export-import licenses and public credit. Thus, it is plausible to conjecture that scarcities, relative to the first best output, can potentially reduce private investment and by implication, growth; see for instance Mauro (1995, 1997). The implication of Lemma 3 in this context is that investment and growth might be greater under the regime of a decent politician when output is high \( (q^M_H < q_H) \), however, at low levels of output \( (q^M_H > q_H) \), it might be higher in the regime of a venal politician.

### 4.3.2. Auditing Technology

**Venal Politician**: Several widely advocated anti-corruption measures, documented for instance in Rose-Ackerman (1999), recommend an improvement in the auditing technology in order to reduce the incidence of corruption. However, when the politician is venal, the model predicts that this policy recommendation will be unsuccessful in reducing bribes. Furthermore, there are important, and hitherto unrecognized, implications for production efficiency; these issues are formalized in Proposition 5.

**Proposition 5**: If \( q^V_H < q^M_H \) then \( \frac{\partial q^V_H}{\partial \rho} > 0 \) and \( \frac{\partial B(q^V_H)}{\partial \rho} > 0 \). However, if \( q^M_H < q^V_H \), then \( \frac{\partial q^M_H}{\partial \rho} < 0 \) and \( \frac{\partial B(q^V_H)}{\partial \rho} > 0 \).
An increase in $\rho$ enables the venal politician to detect hard information about bribes more often. Hence, if the venal politician were to distort output to increase bribes, her expected bribes would increase. This increase in the marginal benefit of creating bribes (by distorting the contracted output) results in greater output distortions and bribes in equilibrium. Using Remark 1 it is obvious that the equilibrium contracted output moves towards the unregulated monopoly output, $q_M^H$, from both directions; this essentially is the result in Proposition 5.

Although, an improvement in the monitoring technology always leads to an increase in bribes, however, when $q_H^V < q_H^M$ (through an increase in contracted output) it enhances allocative efficiency, while if $q_H^V > q_H^M$ it (through a decrease in contracted output) reduces allocative efficiency. Hence, at high output levels an improvement in monitoring technology is unambiguously bad if the politician is venal, while at lower output levels ($q_H^V < q_H^M$) it can be welfare enhancing.

**Decent Politician:** Recall from (4.5) and (4.6) that the first order conditions for a decent politician are independent of $\rho$ and, hence, the contracted output in this case is independent of an improvement in the monitoring technology. This result can change if taxes are distortionary or there are secrecy costs involved in exchanging bribes.

Consider, for instance, that there are distortionary costs of taxation so that in order to raise a unit of tax revenues, one needs to raise $1 + \tau$ units of tax revenue ($\tau$ being the distortionary cost per unit). Check that in this case, the first order condition (4.6) is modified to

$$p^D_H = p(q_H^D) = c_H - \tau (1 - \rho) \frac{\partial B_H(q_H^D)}{\partial q_H} + \frac{\nu}{1 - \nu} \Delta c.$$  \hspace{1cm} (4.10)

Implicitly differentiating (4.10) it can be checked that sign of $\frac{\partial q_H^D}{\partial \rho} = - \text{sign of } \frac{\partial B_H(q_H^D)}{\partial q_H}$. The intuition hinges on the choice between two alternative sources of paying the public agent: directly by raising distortionary taxes or indirectly through bribes, so that her individual rationality constraint is satisfied. The marginal benefit of creating bribes (through output distortions) is high if $\rho$ is low so that the expected bribe received by the public agent, $E[B^A_i]$, is high and, therefore, a greater amount of distortionary taxes are offset. Hence, if $\rho$ is high the marginal benefit of creating bribes is low and output then is distorted towards lower bribes. Using Remark 1, this direction is away from $q_M^H$ in both directions; this is what is implied by sign of $\frac{\partial q_H^D}{\partial \rho} = - \text{sign of } \frac{\partial B_H(q_H^D)}{\partial q_H}$.

Exactly the same calculation would then apply to the case of a venal politician. However, Proposition 5 would continue to illustrate the additional influence on contracted output when the politician is venal; one would merely be using a new benchmark to
compare the two kinds of politicians. For this reason, we omit further discussion of the distortionary taxation case.

**Relation to the literature:** The bribe increasing aspect of better auditing technology is similar to Proposition 1 in Laffont and Guessan (1999) who interpret an increase in $\rho$ as an increase in “competitiveness”. The interpretation of $\rho$ as competitiveness is best seen by imagining that the responsibility for monitoring the public-agent rests with some auditing supervisors and a proportion $\rho$ are honest. However, unlike Laffont and Guessan (1999), the intuition here is that when the politician is corrupt, then an increase in $\rho$ reduces the private marginal cost of bribes to the venal politician.

Tanzi and Davoodi (1997) show empirically that greater distortions, relative to the first best, are associated with weaker auditing technologies (low $\rho$). However, in Acemoglu and Verdier (2000) the number of public-agents employed by the government (which corresponds to the contracted output) increases following a decrease in $\rho$. Proposition 5 is able to reconcile these conflicting results; at low levels of output ($q_H < q^M_H$) one gets the Tanzi-Davoodi result while at higher levels of output ($q_H > q^M_H$) one gets the Acemoglu-Verdier result. The tradeoff between production efficiency and bribery is again central to this explanation.

### 4.3.3. Degree of Politician’s Venality

Under full information, if the politician is venal, the contracted output is decreasing and bribes are increasing in the degree of venality (or corruptibility) of the politician, $\mu$ (see Proposition 2). This section extends that result to asymmetric information.

**Proposition 6:** If $q^V_H < q^M_H$ then $\frac{\partial q^V_H}{\partial \mu} > 0$ and $\frac{\partial B_H(q^V_H)}{\partial \mu} > 0$. However, if $q^M_H < q^V_H$, then $\frac{\partial q^M_H}{\partial \mu} < 0$ and $\frac{\partial B_H(q^M_H)}{\partial \mu} > 0$.

Proposition 6 shows that the full information result survives if $q^M_H < q^V_H$. In the complementary case ($q^V_H < q^M_H$), bribes (and contracted output) are increasing in $\mu$. The tradeoff between allocative efficiency and bribery creates similar welfare issues to those discussed following Proposition 5.

### 5. The Dynamic Game (Venal Politician)

Suppose now that the politician can offer the public-agent a long-term contract that lasts for two periods. Long-term contracts are generally not renegotiation proof. For instance, the optimal static contracts under asymmetric information, $(q^j_L, t^j_L)$ and $(q^j_H, t^j_H)$ for $j = D, V$, repeated over the two periods are not renegotiation-proof because the choice of the first
period contract reveals the type of the public-agent. To minimize notation we denote by $EV_i$ the expected payoff of a type $c_i$, $i = L, H$, public-agent when she accepts the contract $(q_i, t_i)$. At the beginning of the second period, the politician can offer to renegotiate the contract of the inefficient type by contracting it to produce the first best output $q^{i*}_H$, while maintaining $EV_H = 0$. In the second period, the efficient type will be offered a contract that does not contain any information rents so, $EV_L = 0$. Letting $0 < \delta$ be the discount factor, the intertemporal rent earned by the efficient type is thus $q_H \Delta c + 0 * \delta = q_H \Delta c$. However, if the efficient type pools with the inefficient type, the politician does not update beliefs and so, intertemporal rents equal $q_H \Delta c(1 + \delta)$, an improvement of $\delta q_H \Delta c$ over the separating equilibrium. Hence, the static contracts are not-renegotiation-proof.

The decent politician is a simpler and special case of a venal politician, hence, the discussion in this section focusses only on the latter.

5.1. Description of the Dynamic Game

Suppressing all superscripts and subscripts, write the contract $(q, t)$ as $t(q)$. The “official price” specified in the contract equals $c$ and the feasibility constraint requires $p(q) \geq c$. The timing of the contracts is summarized in Figure 5.1.

![Figure 5.1: Basic Representation Of The Dynamic Game](image)

Based on the prior belief, $\nu = \nu_1$, that the public-agent is of type $c_L$, the politician offers a long-term contract $(t_1(q_1), t_2^L(q_1, q_2))$ where $t_1(q_1)$ specifies “first period” transfers conditional on the first period contracted output $q_1$ and $t_2^L(q_1, q_2)$ is the “null contract” for the “second period”, conditional on the first and the second period contracted outputs, $q_1$ and $q_2$. Conditional on the contracts chosen in the first period, the politician updates her prior beliefs, using Bayes rule, at the end of the first period, to $\nu = \nu_2$. In the second period, the politician can offer a new, renegotiated, contract $t_2^R(q_1, q_2)$, to the public-agent. However, given the long-term nature of the original contract, if the new contract is rejected by the public-agent, the politician is committed to implementing the null contract.
\( t_2(q_1, q_2) \). Since the politician and the public-agent are rational and forward looking, the politician might as well offer a renegotiation-proof contract in the first period itself, in anticipation of future renegotiation.

5.2. Characterization of Renegotiation-Proof Contracts

Suppose that the second period rents promised to types \( c_L \) and \( c_H \) in the original long-term contract by a politician of type \( j = D, V \) are respectively \( EV_{jO}^L \) and \( EV_{jO}^H \); there is no loss in generality by normalizing \( EV_{jO}^H = 0 \).

In designing the second period contract, conditional on the existence of the null contract, the politician chooses \((q_L, t_L, EV_L)\) and \((q_H, t_H, EV_H)\) in order to maximize

\[
Z^j(\nu_2) = \nu_2 W^j(q_L, t_L) + (1 - \nu_2) W^j(q_H, t_H)
\]

subject to the following three constraints

\[
EV_H = t_H + \Phi(\mu, \rho) B_H \geq V_H^O \equiv 0 \quad (IR_H)
\]

\[
EV_L = t_L + \Phi(\mu, \rho) B_L \geq t_H + \Phi(\mu, \rho) B_H + (c_H - c_L) q_H \quad (IC_L)
\]

\[
EV_L \geq EV_L^O \quad (RP)
\]

where \( Z^j \) is as defined in (4.1). Denote the optimal renegotiation-proof contracts for types \( c_L \) and \( c_H \) by \( (q_L^R, t_L^R, EV_L^R) \) and \( (q_H^R, t_H^R, EV_H^R) \) respectively; superscript \( R \) signifies renegotiation while \( j = D, V \) denotes the type of the politician. With the exception of the third constraint, the second period optimization problem is identical to the one considered in Section 4.1, except that the \( IR_H \) constraint now requires that type \( c_H \) must not be given lower rents relative to those promised in the null contract. For the renegotiated contract to be attractive to the efficient type, its rents should be no lower relative to those in the null contract; this constitutes the renegotiation-proof constraint, denoted by \( RP \). As in Section 4, the \( IC_L \) constraint is omitted for the time being; it is easy to check ex-post that it is not violated by the solution.

**Definition 5**: The rents promised in the null contract are renegotiation-proof if \( EV_{jO}^L = EV_{jO}^R \) and \( EV_{jO}^L = EV_{jO}^R \).

Check that \( t_H \) enters negatively in the objective function, so the \( IR_H \) constraint binds, thus \( EV_H^O = EV_H^R = 0 \). Substituting \( t_H = -\Phi(\mu, \rho) B_H \) from \( IR_H \) into \( IC_L \), one can rewrite the latter as \( EV_L = t_L + \Phi(\mu, \rho) B_L \geq q_H \Delta c \). Unlike in Section 4, there is no guarantee that the \( IC_L \) constraint will bind because of the presence of the \( RP \) constraint.
Substituting \( t_H = -\Phi (\mu, \rho) B_H \) into the objective function, and letting \( \kappa_1 \) and \( \kappa_2 \) be the Lagrangian multipliers on the \( IC_L \) and the \( RP \) constraints, one gets the following generic problem.

The politician chooses \( t_L, q_L, q_H \) and \( EV_L \) to maximize the Lagrangian expression:

\[
\Lambda = \nu_2 \left[ S (q_L) - (t_L + c_L q_L) - \left( 1 - \frac{\xi}{1 - \mu} \right) B_L \right] + (1 - \nu_2) \left[ S (q_H) - c_H q_H - \xi B_H \right] + \kappa_1 [t_L + \Phi (\mu, \rho) B_L - q_H \Delta c] + \kappa_2 \left( EV_L - EV_L^{JO} \right) \tag{5.2}
\]

For a venal politician \( \xi = \frac{\rho}{2\mu} (1 + \mu)(1 - \mu) < 0 \) and \( \Phi (\mu, \rho) = 1 - \frac{\rho}{\xi} (1 + \mu^{-1}) \) (these are defined in (3.4) and (2.8)). In the first instance, differentiate (5.2) with respect to \( t_L \) and \( q_L \) only; the first order conditions are

\[
\frac{\partial \Lambda}{\partial t_L} = -\nu_2 + \kappa_1 = 0 \tag{5.3}
\]

\[
\frac{\partial \Lambda}{\partial q_L} = \nu_2 \left[ p (q_L) - c_L - \left( 1 - \frac{\xi}{1 - \mu} \right) \frac{\partial B_L}{\partial q_L} \right] + \kappa_1 \Phi (\mu, \rho) \frac{\partial B_L}{\partial q_L} = 0 \tag{5.4}
\]

Substituting \( \kappa_1 = \nu_2 \) from (5.3) into (5.4) evaluated at the optimal renegotiation proof output contracted by the venal politician for the efficient type, \( q_L = q_L^{VR} \),

\[
p(q_L^{VR}) = c_L + \frac{\xi}{1 - \xi} q_L^{VR} p' (q_L^{VR}) \tag{5.5}
\]

The first order conditions (5.5), (4.8) and (3.7) are identical, hence, the contracted output of the efficient type is not distorted i.e. \( q_L^{VR} = q_L^V = q_L^{V^*} \). Substituting \( q_L = q_L^{VR} = q_L^V \) and using the definition \( EV_L = t_L + \Phi (\mu, \rho) B_L \) in (5.2), the problem reduces to finding \( q_H \) and \( EV_L \) to maximize the following Lagrangian expression:

\[
L = \nu_2 \left[ S (q_L^V) - c_L q_L^V - \xi B_L (q_L^V) \right] + (1 - \nu_2) \left[ S (q_H) - c_H q_H - \xi B_H \right] + (\varphi_1 + \varphi_2 - \nu_2) EV_L - \varphi_2 EV_L^{JO} - \varphi_1 q_H \Delta c
\]

where \( \varphi_1 \) and \( \varphi_2 \) are respectively the Lagrangian multipliers on the \( IC_L \) and the \( RP \) constraints when \( q_L^{VR} = q_L^V \). The first order conditions with respect to \( q_H \) and \( EV_L \) can be simplified and written as

\[
p(q_H) = c_H + \frac{\xi}{1 - \xi} q_H p' (q_H) + \frac{\varphi_1 \Delta c}{(1 - \nu_2)(1 - \xi)} \tag{5.6}
\]

\[
\varphi_1 + \varphi_2 = \nu_2 ; \quad \varphi_1 \geq 0, \varphi_2 \geq 0 \tag{5.7}
\]
Since $EV_{OH}^O = EV_{OH}^R = 0$ as above, then using Definition 5, renegotiation-proofness of the null contract is guaranteed if one can find the conditions under which $EV_{OL}^O = EV_{OL}^R$; these conditions are stated in Lemma 4.

**Lemma 4**: The null contract is renegotiation-proof if $q^V_H(\nu_2) \Delta c \leq EV_{OL}^O \leq q^V_* \Delta c$.

where $q^V_H(\nu_2)$ is the optimal static contract under asymmetric information when $\nu = \nu_2$.

5.3. The Optimal Renegotiation-Proof Long-Term Contract

Attention is restricted to a menu of two long-term contracts, $X$ and $Y$, offered by the politician at the beginning of the game. The output contracted in contract $k = X, Y$ in time period $t = 1, 2$ is denoted by $q_k^t$. The time discount factor is $0 < \delta$. Type $c_L$ plays a mixed strategy and accepts contract $X$ with probability $\pi$ and contract $Y$ with probability $1 - \pi$ while type $c_H$ accepts contract $Y$ with probability equal to one. The basic structure of the long-term contract is described in Figure 5.2.

![Diagram](image)

**Figure 5.2: Description Of The Long Term Contract**

Several elements of the optimal contract can be constructed by the following heuristic arguments. Posterior beliefs are updated using Bayes rule. At the upper node, followed

---

12 Since the two contiguous time periods are not necessarily of equal length, thus, there is no presumption that $\delta$ is bounded above by 1.
13 Restricting the number of contracts to equal the number of types, and allowing the efficient type to randomize, does not compromise on generality, see Laffont and Tirole (1993).
by the first period choice of \( q_1^X \) the type of the public-agent is revealed to be \( c_L \), hence, posterior beliefs specify that \( \nu_2 = 1 \) and the second period choice of contracted output, \( q_2^X \), is thus optimally equal to the first best i.e. \( q_L^{V^*} \). At the lower node, followed by the first period choice of \( q_1^Y \), the posterior beliefs are given by \( \nu_2 = \frac{(1-\pi)\nu_1}{(1-\pi)\nu_1+\nu_1} \) and following this node, the intended contract for the efficient type, denoted by \( q_2^Y (c_L) \), is the first best contract \( q_L^{V^*} \).

When type \( c_L \) chooses contract \( Y \) then, in a renegotiation-proof contract, its total intertemporal information rents equal \( q_1^Y \Delta c + \delta EV_L^O \). Since type \( c_L \) randomizes between the two contracts, it must get identical rents from contract \( X \). Furthermore, given that type \( c_L \) produces the first best allocation in the second period of contract \( X \) and must be offered a fixed amount of intertemporal rents, \( q_1^Y \Delta c + \delta EV_L^O \), it is also optimal to contract the efficient output in the first period i.e. \( q_1^X = q_L^{V^*} \). Fixing \( \pi \) for the time being (the optimal \( \pi \) is found in section 5.4) the problem for the politician is to find the following. (1) Output allocations in contract \( Y \), i.e. \( q_1^Y \) and \( q_2^Y \) (henceforth, \( q_2^Y \)), the quantity contracted for type \( c_H \) in the second period of contract \( Y \), is denoted simply by \( q_2^Y \) because \( q_2^Y = q_L^{V^*} \).

(2) Second period expected rent \( EV_L^O \) (in the null contract) to offer to type \( c_L \). Formally, the politician chooses \( q_1^X \), \( q_2^Y \) and \( EV_L^O \) to maximize

\[
W^R = \nu_1 \left[ \pi W (q_L^{V^*}, c_L) + (1 - \pi) W(q_1^Y, c_L) \right] + (1 - \nu_1) W \left( q_1^Y, c_H \right) - \nu_1 \left[ q_1^Y \Delta c + \delta EV_L^O \right] + \delta \left[ \nu_1 W (q_L^{V^*}, c_L) + (1 - \nu_1) W (q_2^Y, c_H) \right] \tag{5.8}
\]

Subject to : \( q_2^Y (\nu_2) \Delta c \leq EV_L^O \leq q_H^Y (\nu_2) \Delta c \) \tag{RP}

The \( \text{RP} \) constraint derives from Lemma 4 and ensures that the solution is renegotiation-proof.

**Lemma 5** : The optimal contracted output is \( q_2^Y = q_H^Y (\nu_2) \) and the expected information rent in the null contract is \( EV_L^O = q_H^Y (\nu_2) \Delta c \).

Substituting \( q_2^Y = q_H^Y (\nu_2) \) in the objective function, and collecting terms corresponding to \( q_1^Y \), the optimal solution to \( q_1^Y \) can be found by maximizing

\[
L^R = \nu_1 (1 - \pi) W(q_1^Y, c_L) + (1 - \nu_1) W(q_1^Y, c_H) - \nu_1 q_1^Y \Delta c
\]

The first order condition with respect to \( q_1^Y \) can be simplified and written as

\[
p(q_1^Y) = \frac{1 - \nu_1}{1 - \nu_1 \pi} \left( \frac{c_L \nu_1 (1 - \pi) \nu_1}{1 - \nu_1} + c_H + \frac{\nu_1}{(1 - \xi)(1 - \nu_1)} \Delta c \right) + \frac{\xi}{1 - \xi} q_1^Y p' (q_1^Y) \tag{5.9}
\]
Denote the solution to $q_Y^V$ derived from (5.9) by $\hat{q}_1 (\nu_1)$. Proposition 7 compares $\hat{q}_1 (\nu_1)$ to the solution in the static asymmetric information case, $q_H^V (\nu_1)$.

**Proposition 7**: Relative to the output contracted for the inefficient type in the static contract, $q_H^V (\nu_1)$, the magnitude of shortages decreases in each period of the renegotiation-proof dynamic contract i.e. $q_H^V (\nu_1) \leq \hat{q}_1 (\nu_1) = q_Y^V$ and $q_H^V (\nu_1) \leq q_Y^V (\nu_2) = q_H^V (\nu_2)$. Bribes can either increase or decrease depending on whether $\hat{q}_1 \gtrless q_H^M$. Furthermore, the contracted output $\hat{q}_1$ is increasing in the probability $\pi$ with which type $c_L$ chooses contract $X$.

The contracted output is increasing in $\pi$ because it reduces the expected information rents of the efficient type, hence, reducing the need to create shortages in order to limit such rents. One could compare the long-term contract analyzed above to a series of short-term (one period) contracts offered to a sequence of public-agents each of whom has private information about costs. Proposition 7 provides some guidelines for such a choice in the presence of corruption. The long-term contract unambiguously dominates the short-term contract in terms of reducing the output distortion ( $\hat{q}_1$ is relatively closer to the first best as compared to $q_H^V$). However, the long-term contract might have relatively lower bribes (if $q_H^M < \hat{q}_1$) or higher bribes (if $\hat{q}_1 < q_H^M$). In the former case, the long-term contract is unambiguously better in terms of both production efficiency and bribes while in the latter case, the choice between the two contracts depends on society’s preferences over efficiency and bribery. It is entirely possible that the cost in terms of greater bribery is found acceptable in return for the benefits of enhanced production efficiency.

In the hybrid equilibrium above, $0 \leq \pi \leq 1$; the two polar cases of the fully separating equilibrium ($\pi = 1$) and the fully pooling equilibrium ($\pi = 0$) are considered below for completeness.

5.3.1. Fully Separating Equilibrium

Substituting $\pi = 1$ in (5.9) one gets:

$$p (q_Y^V) = c_H + \frac{\xi}{1 - \xi} q_Y^V p' (q_Y^V) + \frac{\nu_1}{(1 - \nu_1) (1 - \xi)} \Delta c$$  \hspace{1cm} (5.10)

Comparing (5.10) and (4.9) it is obvious that $q_Y^V = q_H^V (\nu_1)$.

5.3.2. Fully Pooling Equilibrium

Substituting $\pi = 0$ in (5.9) one gets:

$$p (q_Y^V) = [c_L \nu_1 + c_H (1 - \nu_1)] + \frac{\xi}{1 - \xi} q_Y^V p' (q_Y^V) + \frac{\nu_1}{1 - \xi} \Delta c$$  \hspace{1cm} (5.11)

Comparing (5.11) with (4.9) and using $\Delta c > 0$ it is straightforward to show that $p (q_Y^V) < p (q_H^V (\nu_1))$ and so $q_Y^V > q_H^V (\nu_1)$. 

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5.4. Determining the Optimal $\pi$

The optimal dynamic contracts have so far been worked out conditional on a given value of $\pi$. This subsection examines some properties of the optimal $\pi$. Essentially, the politician substitutes the optimal contracts in (5.8) and since the lower limit of the associated constraint binds, maximizes the following expression with respect to $\pi$:

$$W(\pi) = \nu_1 \left[ \pi W \left( \hat{q}_L^V, c_L \right) + (1 - \pi) W \left( \hat{q}_I, c_L \right) \right] + (1 - \nu_1) W \left( \hat{q}_I, c_H \right) - \nu_1 \left[ \hat{q}_I \Delta c + \delta q_H^V (\nu_2) \Delta c \right] + \delta \left[ \nu_1 W \left( \hat{q}_L^V, c_L \right) + (1 - \nu_1) W \left( q_H^V (\nu_2), c_H \right) \right]$$

Noting that $\hat{q}_I$ and $q_H^V$ depend on $\pi$, and using the envelope theorem, the first order condition, upon simplification, can be written as:

$$\frac{\partial W(\pi)}{\partial \pi} = \left( \hat{q}_I + \pi \frac{\partial \hat{q}_I}{\partial \pi} \right) \left[ (1 - \xi) c_H (1 - \nu_1) - \nu_1^2 \Delta c \right] - \nu_1^2 \Delta c \delta \frac{\partial q_H^V (\nu_2)}{\partial \pi} \leq 0 ; \pi \geq 0$$

(5.12)

Let the solution to (5.12) be given by $\pi = \pi^\ast$. Using Lemma 2 it is easy to check that $\frac{\partial \hat{q}_I}{\partial \pi} = \frac{\partial \hat{q}_I}{\partial \nu} \frac{\partial \nu}{\partial \pi} > 0$ and $\frac{\partial q_H^V}{\partial \pi} = \frac{\partial q_H^V}{\partial \nu} \frac{\partial \nu}{\partial \pi} > 0$, thus, a necessary condition for an interior solution to $\pi$ is that the term in the second braces in (5.12) be positive. Using this information, it is straightforward to check, using the implicit function theorem that $\frac{\partial \pi^\ast}{\partial \delta} < 0$ i.e. the probability of separation of the efficient type is decreasing in the discount factor $\delta$. Proposition 8 below describes how $\pi^\ast$ responds to $\delta \in (0, \infty)$; recall that the lengths of the two periods are not necessarily identical so that $\delta$ is not bounded above.

**Proposition 8**: A sufficient condition for the “fully pooling equilibrium” ($\pi^\ast = 0$) to be optimal is that $\delta \rightarrow \infty$. There exists some critical $\delta = \delta_C > 0$ such that the “fully separating equilibrium” ($\pi^\ast = 1$) is optimal for $\delta \leq \delta_C$. The hybrid equilibrium is optimal for intermediate values of $\delta$.

Since the public-agent of type $c_L$ is indifferent between the two contracts $X$ and $Y$, what assurance does one have that the public-agent will randomize according to $\pi^\ast$? A simple interpretation is that when there are a large number of type $c_L$ public-agents, a proportion $\pi^\ast$ are offered the contract $X$ and another proportion $1 - \pi^\ast$ are offered the contract $Y$. A more detailed discussion of the issue using purification theorems falls outside the scope of the paper, see for instance Fudenberg and Tirole (1990).

6. Conclusions

This paper considers a hierarchical relation between a possibly venal politician and a privately informed public-agent contracted to supply output/services on behalf of the former
to final consumers. The politician contracts the public-agent to supply a certain output at an official price, but the latter can choose to receive a bribe to clear the market i.e. charge a price in excess of the official price. The politician uses a monitoring technology that unearths the incidence of bribery with some positive probability. In the event that the bribe is discovered, a venal politician is willing to hide evidence of the bribe if the public-agent shares the bribes with her. The paper provides information-theoretic microfoundations to this classical problem of scarcity rents. Furthermore, it generates a set of plausible and potentially testable theoretical predictions.

In general, the equilibrium is characterized by shortages and bribes. The contracted output can be smaller or greater relative to that produced by a private unregulated monopolist. The paper reconciles apparently conflicting results on the affect of an improvement in the auditing technology on the size of the public sector. An important insight of the paper is that anti-corruption reforms, such as an improvement in the auditing technology, face important trade-offs in enhancing allocative efficiency on the one hand and changes in equilibrium bribes on the other. The size of the public sector can be symptomatic of alternative degrees of corruption. Relative to the static case, in the dynamic renegotiation-proof equilibrium, shortages fall but bribes can either increase or decrease. This suggests important determinants of the choice between offering short-term and long-term contracts to public-agents.

Future research can incorporate political institutions and electoral procedures. Political competition among parties could possibly offset some of output distortion that arises on account of corruption. For instance, a party might contest the election on the platform that its candidates have a relatively lower degree of venality, $\mu$, as compared to the opposition. A plausible model along these lines would have to consider a host of contractibility, credibility and coordination issues. Other interesting extensions of the model would be to examine the relationship between corruption and growth, to consider a judicial system that could possibly punish corrupt politicians, and lobbying by consumer groups to influence the contracted output by directly engaging in side transactions with the politician.

7. Appendix

Proof of Lemma 1: Check that the net surplus from Nash bargaining, $W + V_j - d^P - d^A_j$, equals $x(\mu - 1)$. If the politician is decent ($\mu \leq 1$) then the net surplus is non-positive so there are no gains from the corrupt transaction between the politician and the public-agent. Thus, decent politicians (endogenously) refrain from such corrupt deals. Clearly, for a venal politician ($\mu > 1$) there is net positive surplus to be shared between the two parties. Using (2.5) check that (1) $\frac{\partial x}{\partial \mu} = -B_i/2\mu^2 \leq 0$, hence, the venal politician’s share of the bribe is decreasing in the degree of venality,
and (2) in the limit \( x_i \to B_i \) as \( \mu \to 1 \) and \( x_i \to B_i/2 \) as \( \mu \to \infty \). □

**Proof of Proposition 2**: Since \( \xi < 0 \) it follows from (3.7) that \( p(q_i^{V*}) > c_i \), hence, \( q_i^{V*} < q_i^{FB} \) and \( B_i(q_i^{V*}) = [p(q_i^{V*}) - c_i] q_i^{V*} > 0 \). Recall from Remark 1 that \( B_i(q_i) \equiv \Pi^M(q_i, c_i) \) where \( \Pi^M \) is the profit function of a unregulated monopolist such that \( \frac{\partial \Pi^M(q_i, c_i)}{\partial q_i} = 0 \). Using this in conjunction with the first order condition (3.5) and recalling that \( \xi < 0 \) it follows that \( \frac{\partial \Pi^M(q_i^{V*}, c_i)}{\partial q_i} < \frac{\partial \Pi^M(q_i^M, c_i)}{\partial q_i} \). Given that the second order condition requires concavity of the bribe function (and by implication that of \( \Pi^M \)) it follows that \( q_i^M < q_i^{V*} \).

Implicitly differentiating (3.7) with respect to \( \xi \):

\[
\frac{\partial q_i^{V*}}{\partial \xi} = \left( \frac{\partial^2 W}{\partial q_i^2} \right)^{-1} \frac{q_i^{V*} p'(q_i^{V*}, c_i)}{(1 - \xi)^2} > 0 \tag{7.1}
\]

It can be checked that:

\[
\frac{\partial \xi}{\partial \mu} = \frac{-\rho}{2} \left( 1 + \mu^{-2} \right) < 0 \tag{7.2}
\]

\[
\frac{\partial \xi}{\partial \rho} = \frac{1}{2} \left( 1 + \mu^{-1} \right) (1 - \mu) < 0 \tag{7.3}
\]

Using (7.2) and (7.3) in conjunction with (7.1) implies that \( \partial q_i^*/\partial \mu < 0 \) and \( \partial q_i^*/\partial \rho < 0 \). Finally, since \( \frac{\partial B_i}{\partial q_i} < 0 \) from (3.7), the claims on the magnitude of bribes follow by using Remark 1. □

**Proof of Proposition 3**: Comparing (4.6) and (3.2), \( p(q_H^D) > p(q_H^{D*}) \), and since \( p' < 0 \), therefore, \( q_H^D < q_H^{D*} \). The problem of a private unregulated monopolist is to maximize \( \Pi^M(q_H) = q_H \{ p(q_H) - c_H \} \) and the solution \( q_H^M \) satisfies the first order condition

\[
p(q_H^M) = c_H - q_H^M p'(q_H^M). \tag{7.4}
\]

Comparing (4.6) and (7.4) it is evident that \( p(q_H^D) \geq p(q_H^M) \) as \( \frac{\nu}{1 - \nu} \frac{a c}{p q} \geq \frac{1}{\eta} \). Since \( p' < 0 \) thus \( q_H^D \leq q_H^M \) as \( \frac{\nu}{1 - \nu} \frac{a c}{p q} \geq \frac{1}{\eta} \). The last part of the Proposition, on the magnitude of bribes, follows by using Remark 1 and noting that \( \Pi^M(q_H) \) is concave with an optimizing choice given by \( q_H^M \). □

**Proof of Proposition 4**: The proof follows by using \( p' < 0 \) and implicitly differentiating (4.6) with respect to \( \Delta c \) and \( \nu \). The effect on bribes uses Remark 1, the concavity of \( \Pi^M(q_H) \) and the fact that \( q_H^M \) is the optimizing choice. □
**Proof of Lemma 2**: Rewriting (4.9)

\[
\frac{\partial B_H(q_H^V)}{\partial q_H} = \frac{1}{\xi} \left( p(q_H^V) - c_H - \frac{\nu \Delta c}{1 - \nu} \right).
\]

The feasibility condition requires \( p(q_H^V) - c_H \geq 0 \) and since \( \xi < 0 \), thus

\[
\frac{\partial B_H(q_H^V)}{\partial q_H} \geq 0 \quad \text{as} \quad \frac{\partial B_H(q_H^V)}{\partial q_H} \equiv \frac{\nu q_H^V \Delta c}{1 - \nu - \nu p q_H^V}.
\]

Since \( \frac{\partial B_H(q_H^M)}{\partial q_H} = 0 \) it follows that \( q_H^V \) is greater (lower) than \( q_H^M \) as the price markup per unit of revenue, \( \frac{\nu q_H^M \Delta c}{p q_H^M} \), is greater (lower) than the “expected information rent per unit of revenue”, \( \nu (1 - \nu - \nu q_H^M \Delta c) \), as claimed. Implicitly differentiating (4.9) with respect to \( \Delta c \) and \( \nu \), check that \( q_H^V \) is decreasing in each argument. The affect on bribes follows from Remark 1, the concavity of \( B \equiv \Pi^M \) and the fact that \( q_H^M \) is the optimizing choice.

**Proof of Lemma 3**: Comparing the first order conditions (4.6) and (4.9):

\[
p (q_H^V) - p (q_H^D) = \frac{\xi}{1 - \xi} \left( q_H^V p' (q_H^V) + \frac{\nu \Delta c}{1 - \nu} \right) \quad (7.5)
\]

By definition \( \frac{\partial B_H(q_H^V)}{\partial q_H} = p(q_H^V) - c_H + q_H^V p' (q_H^V) \). Using this definition and (4.6) in (7.5) one gets

\[
p (q_H^V) - p (q_H^D) = \xi \frac{\partial B_H(q_H^V)}{\partial q_H} \quad (7.6)
\]

Since \( p' (q) < 0 \) by assumption, and \( \xi < 0 \) for a venal politician, therefore, (7.6) implies that

\[
q_H^V \geq q_H^D \quad \text{as} \quad \frac{\partial B_H(q_H^V)}{\partial q_H} \geq 0
\]

The statement in the lemma now follows by noting that if \( q_H^M > q_H^V \) then \( \frac{\partial B_H(q_H^V)}{\partial q_H} > 0 \) while if \( q_H^M < q_H^V \) then \( \frac{\partial B_H(q_H^V)}{\partial q_H} < 0 \).

**Proof of Proposition 5**: Differentiating (4.9) with respect to \( q_H \), the second order condition is

\[
\frac{\partial^2 Z^V}{\partial q_H^2} = p'(q_H^V) - \frac{\partial B_H(q_H^V)}{\partial q_H} \xi \leq 0.
\]

Implicitly differentiating (4.9) with respect to \( \rho \),

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\[ \frac{\partial q_H^V}{\partial \rho} = \left( -\frac{\partial^2 Z^V}{\partial q_H^2} \right)^{-1} \frac{1}{2\mu (1 - \xi)^2} (1 + \mu) (\mu - 1) \left( q_H^V p' \left( q_H^V \right) + \frac{\nu \Delta c}{1 - \nu} \right). \] (7.8)

Using the definition \( \frac{\partial B_H(q_H^V)}{\partial q_H} = p \left( q_H^V \right) - c_H + q_H^V p' \left( q_H^V \right) \) and (4.6) in (7.8):

\[ \frac{\partial q_H^V}{\partial \rho} = \left( -\frac{\partial^2 Z^V}{\partial q_H^2} \right)^{-1} \frac{1}{2\mu (1 - \xi)^2} (1 + \mu) (\mu - 1) \frac{\partial B_H(q_H^V)}{\partial q_H}. \] (7.9)

For a venal politician, \( \mu > 1 \) and \( \xi < 0 \), hence, the sign of \( \frac{\partial q_H^V}{\partial \rho} \) is identical to the sign of \( \frac{\partial B_H(q_H^V)}{\partial q_H} \). If \( q_H^V < q_H^M \) then \( \frac{\partial B_H(q_H^V)}{\partial q_H} > 0 \) and so, \( \frac{\partial q_H^V}{\partial \rho} > 0 \). However, when \( q_H^M < q_H^V \), \( \frac{\partial B_H(q_H^V)}{\partial q_H} < 0 \) and so \( \frac{\partial q_H^V}{\partial \rho} < 0 \). ■

**Proof of Proposition 6** : Implicitly differentiate (4.9) with respect to \( \mu \),

\[ \frac{\partial q_H^V}{\partial \mu} = \left( -\frac{\partial^2 Z^V}{\partial q_H^2} \right)^{-1} \frac{\rho}{2(1 - \xi)^2} \left( 1 + \mu^{-2} \right) \left( q_H^V p' \left( q_H^V \right) + \frac{\nu \Delta c}{1 - \nu} \right). \] (7.10)

Using the definition \( \frac{\partial B_H(q_H^V)}{\partial q_H} = p \left( q_H^V \right) - c_H + q_H^V p' \left( q_H^V \right) \) and (4.6) in (7.8):

\[ \frac{\partial q_H^V}{\partial \mu} = \left( -\frac{\partial^2 Z^V}{\partial q_H^2} \right)^{-1} \frac{\rho}{2(1 - \xi)^2} \left( 1 + \mu^{-2} \right) \frac{\partial B_H(q_H^V)}{\partial q_H}. \] (7.11)

For a venal politician \( \xi < 0 \), hence, the sign of \( \frac{\partial q_H^V}{\partial \mu} \) is identical to the sign of \( \frac{\partial B_H(q_H^V)}{\partial q_H} \). If \( q_H^V < q_H^M \) then \( \frac{\partial B_H(q_H^V)}{\partial q_H} > 0 \) and so, \( \frac{\partial q_H^V}{\partial \mu} > 0 \). However, when \( q_H^M < q_H^V \), \( \frac{\partial B_H(q_H^V)}{\partial q_H} < 0 \) and so \( \frac{\partial q_H^V}{\partial \mu} < 0 \). ■

**Proof of Lemma 4** : Renegotiation-proofness requires that \( EV_L^O = EV_L^R \), so the null contract will not be renegotiation-proof when the RP constraint (see section 5.2) does not bind.

[Case-I] RP constraint does not bind

In this case \( EV_L^O < EV_L^R \) and the null contract is not renegotiation-proof. Since the RP constraint does not bind, the complementary slackness condition implies that \( \varphi_2 = 0 \), which when substituted in (5.7) gives \( \varphi_1 = \nu_2 \). Substituting \( \varphi_1 = \nu_2 \) in (5.6) at the optimal solution \( q_H^{VR} \) one gets:

\[ p \left( q_H^{VR} \right) = c_H + \frac{\xi}{1 - \xi} q_H^{VR} p' \left( q_H^{VR} \right) + \frac{\Delta c \nu_2}{1 - \nu_2 (1 - \xi)}. \] (7.12)

The first order condition (7.12) is identical to (4.9), hence, \( q_H^{VR} = q_H^V (\nu_2) \) but with prior beliefs replaced by the updated beliefs. If the first constraint (the IC_L constraint) binds,
then \( \varphi_1 > 0 \) and \( EV^R_L = t^V_R + \Phi(\mu, \rho) B_L(q^V_R) = q_H \Delta c \). If it does not bind then \( \varphi_1 = 0 \) and since \( \varphi_2 = 0 \), using (5.7) one gets \( \nu_2 = 0 \). But \( \nu_2 > 0 \), hence, ruling out this case. From the discussion above, in Case-I, the status of the two constraints is \( EV^O_L < EV^R_L \) and \( EV^O_L = q_H^V(\nu_2) \Delta c \) respectively. So, the null contract is not renegotiation-proof if \( EV^O_L < q_H^V(\nu_2) \Delta c \), therefore, a necessary condition for renegotiation-proofness is that \( q_H^V(\nu_2) \Delta c \leq EV^O_L \).

[Case-II] \textit{RP constraint binds}

When the \textit{RP} constraint binds i.e. \( EV^O_L = EV^R_L \), then the original contract is renegotiation-proof and the complementary slackness condition implies that \( \varphi_2 > 0 \). Using the same reasoning as in Case-I, the first constraint binds and so \( \varphi_1 = 0 \). Thus, (5.7) implies that \( \varphi_1 = \nu_2 - \varphi_2 \) which when substituted into the first order condition (5.6) gives

\[
p(q_H) = c_H + \frac{\xi}{1 - \xi} q_H p'(q_H) + \frac{\Delta c}{(1 - \nu_2)(1 - \xi)} (\nu_2 - \varphi_2) \tag{7.13}
\]

Letting the solution to (7.13) be \( q_H = \bar{q}_H \) and comparing (7.12) and (7.13) one gets \( p(\bar{q}_H) \leq p(q_H^V) \) which implies that \( q_H^V(\nu_2) \leq \bar{q}_H(\nu_2) \). This case is valid till the first constraint just ceases to be binding in which case \( EV^R_L > \bar{q}_H \Delta c \) and, therefore, \( \varphi_1 = 0 \) from the complementary slackness condition. Substituting \( \varphi_1 = \nu_2 - \varphi_2 = 0 \), one gets:

\[
p(q_H) = c_H + \frac{\xi}{1 - \xi} q_H p'(q_H) \tag{7.14}
\]

Comparing to the first order condition under full information (3.7), the solution in this case is \( q_H^V_R = q_H^V \). Hence, \( q_H^V \) sets a upper limit on \( \bar{q}_H(\nu_2) \) for the null contract to be renegotiation-proof.

Summarizing the two cases, when \( q_H^V(\nu_2) \Delta c \leq EV^O_L \leq q_H^* \Delta c \), then the original contract is renegotiation-proof.

\textbf{Proof of Lemma 5:} The optimization problem is considered in two steps.

Step-I: Ignore the \textit{RP} constraint and use the definition \( \xi_c \Delta c = EV^O_L \), to substitute out \( EV^O_L \) from the objective function. Writing terms involving only \( q^Y_2 \), the politician’s objective is to choose \( q^Y_2 \) in order to maximize

\[
-\nu q^Y_2 \Delta c + (1 - \nu_1) \left[ S (q^Y_2) - c_H q^Y_2 - \xi B_H (q^Y_2) \right]
\]

The first order condition with respect to \( q^Y_2 \) can be simplified and written as

\[
p(q^Y_2) = c_H + \frac{\xi}{1 - \xi} q^Y_2 p'(q^Y_2) + \frac{\nu_1}{(1 - \nu_1)(1 - \xi)} \Delta c \tag{7.15}
\]

Comparing (7.15) with (4.9) it is obvious that the unconstrained (because the \textit{RP} constraint has been ignored) solution is \( q^Y_2 = q^Y_H(\nu_1) \).
Step-II: On account of Bayesian updating, following the first period choice of $q^Y$, priors are revised downwards i.e. $\nu_2 < \nu_1$. From Lemma 2, $\frac{\partial q_H^Y}{\partial \nu} < 0$, thus, the solution to the unconstrained problem, $q_H^Y(\nu_1)$, is lower than the lower limit of the constraint, $q_H^Y(\nu_2)$. Therefore, the solution to the constrained problem is also $q_2^Y = q_H^Y(\nu_2)$. ■

**Proof of Proposition 7**: Using the first order condition (5.9):

$$p\left(\hat{q}_1(\nu_1)\right) \leq c_H + \frac{\nu_1}{(1-\xi)(1-\nu_1)} \Delta c + \frac{\xi}{1-\xi} q^p \left(\cdot; \frac{1-\nu_1}{1-\nu_1(1-\xi)} \leq 1\right)$$

$$\leq c_H + \frac{\nu_1}{(1-\xi)(1-\nu_1)} \Delta c + \frac{\xi}{1-\xi} q^p \left(\cdot; \frac{\xi \nu_1(1-\pi)}{1-\nu_1} \geq 0\right)$$

$$= p\left(q_H^Y(\nu_1)\right) \quad \text{(Using (4.9))}$$

Since $p\left(q_H^Y(\nu_1)\right) \geq p\left(\hat{q}_1(\nu_1)\right)$ thus $\hat{q}_1(\nu_1) \geq q_H^Y(\nu_1)$. Lemma 5 shows that the output contracted in the second period of a dynamic contract is $q_2^Y = q_H^Y(\nu_2)$. It is easy to show that $q_H^Y(\nu_1) < q_H^Y(\nu_2)$ because $\nu_2 < \nu_1$ (on account of Bayesian updating) and $\frac{\partial q_H^Y}{\partial \nu} < 0$ from Lemma 2. From Remark 1 we know that whether greater shortages lead to an increase or decrease in bribes depends on whether the contracted output is greater or lower than $q^Y$, the output produced by an unregulated private monopolist.

The second order condition requires $\frac{\partial^2 L^R}{\partial q_1^Y \partial^2} \leq 0$. Implicitly differentiating (5.9) with respect to $\pi$ one gets:

$$\frac{\partial \hat{q}_1}{\partial \pi} = \left(-\frac{\partial^2 L^R}{\partial (q_1^Y)^2}\right)^{-1} \frac{\nu_1 \Delta c}{(1-\nu_1(1-\xi))^2} \left(1 - \nu_1 + \frac{\nu_1}{1-\xi}\right) > 0$$

and this proves the second claim in the proposition. ■

**Proof of Proposition 8**: From (5.12), the coefficient of $\delta$ is negative, so the “fully pooling equilibrium” ($\pi^* = 0$) is always optimal when $\delta \to \infty$. In the case of a “fully separating equilibrium” ($\pi^* = 1$) check that $\frac{\partial V(\pi)}{\partial \pi} |_{\pi=1} \geq 0$ if $\delta \leq \delta_C$ where

$$\delta_C = \left(\hat{q}_1 + \pi \frac{\partial \hat{q}_1}{\partial \pi}\right) [(1-\xi) c_H (1-\nu_1) - \nu_1^2 \Delta c]$$

$$\frac{\nu_1^2 \Delta c \frac{\partial q_H^Y(\nu_2)}{\partial \pi}}{\nu_1^2 \Delta c \frac{\partial q_H^Y(\nu_2)}{\partial \pi}} > 0.$$  \hspace{1cm} (7.16)

The computation of $\delta_C$ in (7.16) completes the proof. ■

**References**


