Investment Management

Bond Portfolio Management

Road Map

- Bond markets
- Bond Yields
- Bond Pricing:
  - Zero-coupon bonds, Coupon bonds
- Price sensitivity
  - Duration and Convexity
- The term structure of interest rates
Bonds

- Promised stream of future cash flows
  - fixed interest payments ("coupons")
  - fixed dates for coupon payments and repayment of principal
- Failure to meet a coupon payment immediately puts the bond into default
- Issued by governments and companies
- Example: UK government “gilts” can be
  - “short-dated” (less than 5 years)
  - “medium-dated” (5 to 15 years)
  - “long-dated” (more than 15 years)
The bond family

- **Domestic bonds**
  - issued on local market by *domestic* borrower
  - denominated in *local* currency
  - e.g. gilt issued in London by Bank of England
- **Foreign bonds**
  - issued on local market by *foreign* borrower
  - denominated in *local* currency
  - e.g. sterling bond issued in London by US bank
- **Eurobonds**
  - issued on *international market*
  - denominated in a currency *other than that of the country in which issuer is based*
  - e.g. sterling bond issued by US firm

The global bond market

<table>
<thead>
<tr>
<th>Currency</th>
<th>2004</th>
<th>Percent</th>
<th>2003</th>
<th>Percent</th>
<th>2002</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Dollars</td>
<td>16,615,955</td>
<td>42.77%</td>
<td>10,181,215</td>
<td>43.73%</td>
<td>9,712,596</td>
<td>44.86%</td>
</tr>
<tr>
<td>Euro</td>
<td>7,526,628</td>
<td>30.32%</td>
<td>7,671,654</td>
<td>30.37%</td>
<td>6,520,821</td>
<td>30.12%</td>
</tr>
<tr>
<td>Japanese Yen</td>
<td>3,992,936</td>
<td>16.09%</td>
<td>3,645,001</td>
<td>15.65%</td>
<td>3,283,443</td>
<td>15.16%</td>
</tr>
<tr>
<td>Pound Sterling</td>
<td>1,132,734</td>
<td>4.56%</td>
<td>994,104</td>
<td>4.27%</td>
<td>877,950</td>
<td>4.05%</td>
</tr>
<tr>
<td>Canadian Dollar</td>
<td>541,192</td>
<td>2.18%</td>
<td>500,263</td>
<td>2.15%</td>
<td>470,166</td>
<td>2.17%</td>
</tr>
<tr>
<td>Indian Rupee</td>
<td>154,546</td>
<td>0.62%</td>
<td>146,524</td>
<td>0.63%</td>
<td>127,511</td>
<td>0.59%</td>
</tr>
<tr>
<td>Australian Dollar</td>
<td>143,120</td>
<td>0.58%</td>
<td>127,661</td>
<td>0.55%</td>
<td>117,588</td>
<td>0.54%</td>
</tr>
<tr>
<td>Swedish Krone</td>
<td>111,616</td>
<td>0.45%</td>
<td>106,711</td>
<td>0.46%</td>
<td>97,348</td>
<td>0.45%</td>
</tr>
<tr>
<td>Korean Won</td>
<td>110,370</td>
<td>0.44%</td>
<td>99,680</td>
<td>0.30%</td>
<td>46,000</td>
<td>0.21%</td>
</tr>
<tr>
<td>Danish Krone</td>
<td>86,643</td>
<td>0.34%</td>
<td>86,314</td>
<td>0.37%</td>
<td>90,448</td>
<td>0.42%</td>
</tr>
<tr>
<td>Taiwanese Dollar</td>
<td>81,020</td>
<td>0.33%</td>
<td>71,387</td>
<td>0.31%</td>
<td>64,611</td>
<td>0.30%</td>
</tr>
<tr>
<td>Swiss Franc</td>
<td>79,663</td>
<td>0.32%</td>
<td>68,680</td>
<td>0.29%</td>
<td>58,550</td>
<td>0.27%</td>
</tr>
<tr>
<td>All Other</td>
<td>247,119</td>
<td>0.99%</td>
<td>215,201</td>
<td>0.92%</td>
<td>184,589</td>
<td>0.85%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>24,821,550</strong></td>
<td><strong>100.00%</strong></td>
<td><strong>23,284,123</strong></td>
<td><strong>100.00%</strong></td>
<td><strong>21,662,020</strong></td>
<td><strong>100.00%</strong></td>
</tr>
</tbody>
</table>

**Annual Growth Rate**: 6.69% 7.54% 10.36%

Government bonds

The global bond market (cont'd)
Corporate bonds

Participating investors and bond ratings

- Bond ratings are provided by three major agencies (Fitch Investor Services, Moody’s and Standard and Poor’s)
- Bond ratings is important in order to assess the probability of default of a specific issuer (public or private)
  - Can the issuer service its debt in a timely manner over the life of a bond?
- Ratings have impact on bonds marketability and effective interest rates (risk premia)
- Ratings are provided in letter format ranging from AAA to D (notations may change from agency to agency)
Bond pricing

- How do we price a bond? Once again we rely on a present-value model
  - As for the stock case [cfr lecture on equity analysis] we have to discount expected cash flows (i.e. coupon payments and principal) by an appropriate discount rate \((r)\). Therefore:

\[
BV = \text{present value of coupons} + \text{present value of par value}
\]

\[
BV = \sum_{t=1}^{T} \frac{C_t}{(1 + r)^t} + \frac{ParValue}{(1 + r)^T}
\]

- Note that there is only one interest rate \((r)\) appropriate to discount cash flows. Different in practice

Example: 30-year maturity bond with par value $1,000 paying 60 semiannual coupon payments of $40 each. Annual interest rate (discount rate) is 8% (=4% every six months).

\[
BV = \sum_{t=1}^{60} \frac{40}{(1 + 0.04)^t} + \frac{1000}{(1 + 0.04)^{60}} = 904.94 + 95.06 = $1,000
\]

- What about if the annual discount rate increases to 10%?
There are different measures of bond yields and they convey different information:

- **Current yield**, that is the equivalent of the dividend yield in the stock market
  \[ CY_t = \frac{C_t}{P_t} \]
  where \( C_t \) is the annual coupon payment and \( P_t \) is the current market price of the bond. It can be interpreted as current income as a percentage of bond’s price.
  - This measure however only consider the cash income provided by the bond and ignores prospective capital gains or losses

- **Yield to maturity**, that is the rate that equates the present value of bond’s payment to its price. Hence it considers:
  - coupon interest (paid at the coupon rate)
  - capital gain (or loss) due to a change in price
Yields to maturity (YTM)

- Example: suppose a 30-year bond paying 8% coupon (semiannually) is currently selling at $1,276.76. Its yield to maturity can be calculated as:

\[
1276.76 = \sum_{i=1}^{60} \frac{40}{(1 + r)^i} + \frac{1000}{(1 + r)^{60}} \rightarrow r = 0.03
\]

- YTM are generally expressed on an annualized basis. The example above implies an annual yield to maturity of 0.06 (=0.03 x 2) or 6%. (i.e. bond equivalent yield)

- Bond equivalent yields do not consider the effect of compounding (i.e. interests are not reinvested). We have to calculate the effective yield to maturity:

\[
1\times (1.03)^2 = 0.0609 \quad \text{or} \quad 6.09\%
\]

Realized yields

- Yields to maturity are rate of returns obtained over the full life of the bond if all coupons are reinvested at bond’s yield to maturity
  - What about if the bond is sold prior to its maturity?
  - What about if the reinvestment rate is different from the bond’s yield to maturity?

- The correct measure to employ is in this case the realized (compound) yield

\[
y_{\text{realized}} = \left( \frac{\text{end-of-period wealth}}{\text{initial wealth}} \right)^{\frac{1}{T}} - 1
\]

- Example: Consider a 2-year bond selling at par value ($1,000) paying an annual coupon of 10%. (The yield to maturity is in this case also equal to 10%). What if the reinvestment rate is equal to 8% at the end of the first year?
Realized yields (cont’d)

\[
y_{\text{realized}} = \left[ \frac{1210}{1000} \right] - 1 = 1.10 - 1 = 0.10
\]

\[
y_{\text{realized}} = \left[ \frac{1208}{1000} \right] - 1 = 1.099 - 1 = 0.099
\]

Realized yields: caveats

- Realized yields need additional information not required by other yield measures. They require:
  - estimate of the expected future selling price (par in the other yield measures)
  - estimate of reinvestment rate for the coupon flows prior to the liquidation of the bond (not required in the other measures)
- Also note that when reinvestment rate = yield to maturity
  \( \Rightarrow \) realized yield = yield to maturity
Price sensitivity

- Return on a bond has two components
  - coupon interest (paid at the coupon rate)
  - capital gain (or loss) due to a change in price
- Price of a bond changes with
  1. time
  2. unexpected changes in interest rates

Change in bond price with time

- Example: pure discount bond or zero-coupon bond (=a bond that does not pay coupons) with 3 years to maturity (assume discount rate \( r = 10\% \) constant over the 3 years)

  \[
  \begin{array}{cccc}
  t = 0 & t = 1 & t = 2 & t = 3 \\
  P = 100 & P = 100/(1+0.10) & P = 100/(1+0.10)^2 & P = 100/(1+0.10)^3 \\
  P = 100 & P = 90.91 & P = 82.64 & P = 75.13 \\
  \end{array}
  \]
Change in price of a pure discount bond with time

Figure 14.7
The price of a 30-year zero-coupon bond over time at a yield to maturity of 10%.
Price equals $1,000(1.10)^T$, where $T$ is time until maturity.

Change in price of a coupon bond with time

Figure 14.6
Prices over time of 30-year maturity, 6.5% coupon bonds. Bond price approaches par value as maturity approaches.

- **Premium bonds:** coupon rate > market rate
- **Discount bonds:** coupon rate < market rate
Change in price due to unexpected changes in interest rates

- Example: again a pure discount bond
- Assume that discount rate increases from 10% to 11%:

<table>
<thead>
<tr>
<th>TTM=3</th>
<th>TTM=2</th>
<th>TTM=1</th>
<th>TTM=0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P ) ( (r = 10%) )</td>
<td>75.13</td>
<td>82.64</td>
<td>90.91</td>
</tr>
<tr>
<td>( P ) ( (r = 11%) )</td>
<td>73.12</td>
<td>81.16</td>
<td>90.09</td>
</tr>
<tr>
<td>( \Delta P )</td>
<td>-2.01</td>
<td>-1.48</td>
<td>-0.82</td>
</tr>
</tbody>
</table>

Interest rate sensitivity

- We have seen that
  - interest rates are inversely correlated with bond prices
  - interest rates fluctuate over time
- Interest rates rises and falls imply bond capital losses and gains. Even bonds are risky investments!
- Bond prices sensitivity to interest rate changes is of great concern to investors
Interest rate sensitivity: example

What do we learn?

1. Bond prices and yields are inversely correlated
2. Yield to maturity rises and falls impact differently on bond price changes (falls > rises)
3. Long-term bonds are more sensitive to interest rate changes (A vs. B)
4. Interest rate risk is less than proportional to bond's maturity (A vs. B)
5. Interest rate risk is inversely related to bond's coupon rate (B vs. C)
6. Bond price sensitivity is inversely related to the yield to maturity at which the bond is currently selling (C vs. D)
**Interest rate sensitivity (cont’d)**

- Bond maturity is crucial to define the impact of interest rate changes on bond prices (i.e. B vs. C)
- However it is not enough...

### Table 16.1 Prices of 8% Coupon Bond (coupons paid semiannually)

<table>
<thead>
<tr>
<th>Yield to Maturity (APR)</th>
<th>T = 1 Year</th>
<th>T = 10 Years</th>
<th>T = 20 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>1,000.00</td>
<td>1,000.00</td>
<td>1,000.00</td>
</tr>
<tr>
<td>9%</td>
<td>900.94</td>
<td>934.95</td>
<td>927.69</td>
</tr>
<tr>
<td>Change in price (%)</td>
<td>0.94%</td>
<td>0.50%</td>
<td>9.00%</td>
</tr>
</tbody>
</table>

*Equals value of bond at a 9% yield to maturity divided by value of bond at the original 8% yield, minus 1.

### Table 16.2 Prices of Zero-Coupon Bond (semiannual compounding)

<table>
<thead>
<tr>
<th>Yield to Maturity (APR)</th>
<th>T = 1 Year</th>
<th>T = 10 Years</th>
<th>T = 20 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>8%</td>
<td>924.56</td>
<td>406.39</td>
<td>258.29</td>
</tr>
<tr>
<td>9%</td>
<td>915.73</td>
<td>414.54</td>
<td>171.53</td>
</tr>
<tr>
<td>Change in price (%)</td>
<td>0.96%</td>
<td>9.15%</td>
<td>37.46%</td>
</tr>
</tbody>
</table>

*Equals value of bond at a 9% yield to maturity divided by value of bond at the original 8% yield, minus 1.

---

**Duration**

- What definition of maturity we should use?
- Effective maturity = average maturity of coupon bonds promised cash flows
- Effective maturity is generally called bond’s duration
  - that is the weighted average of the times to each coupon or principal payment made by the bond, where the weight is the present value of the payment divided by the bond price
Duration (cont’d)

- Duration
  \[ D = \sum_{t=1}^{T} t \times w_t \]
  \[ w_t = \frac{CF_t / (1 + y)^t}{P} \]

- Duration of coupon-paying bond
  - weighted sum of maturities of pure discount bonds
  - weights are equal to the proportion of the bond price represented by the present value of the individual coupons
  - maturities of the pure discount bonds are equal to the coupon maturities

Factors affecting duration

- Size of coupon, C
  - the larger the coupon, the more important are the intermediate cash-flows relative to the cash-flow at maturity
  - hence ... duration decreases if coupon increases

- Maturity, T
  - the longer the maturity, the further in the future the face value is re-paid
  - hence ... duration increases with maturity

- Interest rate, y
  - the higher the rate at which future cash-flows are discounted, the less important are distant cash-flows
  - hence ... duration decreases if the interest rate rises

- Duration of a zero-coupon bond = time to maturity
Duration and bond prices

- Duration is a key concept in bond portfolio management
  - It is a summary of the portfolio effective maturity
  - It measures the interest rate sensitivity of a bond portfolio
  - It is an essential tool to immunize bond portfolio against interest rate risk

\[
\frac{\Delta P}{P} = -D \times \left[ \frac{\Delta (1 + y)}{1 + y} \right] \quad \text{or} \quad \frac{\Delta P}{P} = -\frac{D}{1 + y} \Delta y = -D \times \Delta y \quad \text{where} \quad \Delta (1 + y) \cong \Delta y
\]

Duration: example

- Consider a 2-year maturity, 8% coupon bond making semiannual payments (4% semiannually); selling at \( P = \$964.54 \) for a yield to maturity (i.e. discount rate) of 10% (5% semiannually)

<table>
<thead>
<tr>
<th>years</th>
<th>( t )</th>
<th>( CF_t )</th>
<th>( \frac{CF_t}{(1+y)^t} )</th>
<th>( w = \frac{[CF_t/(1+y)^t]}{P} )</th>
<th>( w \times t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t=0.5 )</td>
<td>0.5</td>
<td>1000*0.04=40</td>
<td>40/(1+0.05)^2=38.09</td>
<td>( \frac{40}{964.54} = 0.039 )</td>
<td>0.039*.05</td>
</tr>
<tr>
<td>( t=1.0 )</td>
<td>1.0</td>
<td>40</td>
<td>40/(1+0.05)^3=36.28</td>
<td>36.28/964.54=0.038</td>
<td>0.038</td>
</tr>
<tr>
<td>( t=1.5 )</td>
<td>1.5</td>
<td>40</td>
<td>40/(1+0.05)^4=34.55</td>
<td>0.036</td>
<td>0.054</td>
</tr>
<tr>
<td>( t=2.0 )</td>
<td>2.0</td>
<td>1040</td>
<td>1040/(1+0.05)^4=855.61</td>
<td>0.887</td>
<td>1.774</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( P = 964.54 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{\Delta P}{P} = -\frac{D}{1 + y} \Delta y = -\frac{1.885}{1.10} \Delta y = -1.714 \times \Delta y
\]
Convexity

\[ \frac{\Delta P}{P} = -D \Delta y \]

- Duration postulates that the relationship between interest rate changes and bond price changes is linear. However we know that the relationship is indeed nonlinear (rule 2)
- Therefore, albeit useful, duration only approximates the impact of interest rate changes on bond prices
- Duration can account for the changes in bond prices provided there are only small changes in the discount rate. For larger shifts in the discount rate, need to include a term for convexity (cfr. Footnote 2, p. 359)

\[ \frac{\Delta P}{P} = -D \Delta y + \frac{1}{2} \times CONV \times (\Delta y)^2 \]

Convexity (cont’d)

Figure 16.4 Bond price convexity. 30-year maturity, 8% coupon bond; initial yield to maturity = 8%
Convexity (cont'd)

Do investors like convexity?

- Convexity is a desirable characteristic of a bond.
- Large convexity implies:
  - larger gains when interest rates fall
  - smaller losses when interest rates rise
- Convexity comes at a cost. Investors have to accept lower yields or pay more to have bonds with greater convexity.
Passive bond management

- Passive bond management takes bond prices as given and seek to control only the risk of the fixed-income portfolio
- Interest rates changes constitute the major source of risk to bond portfolios
- Two techniques for protecting the value of a fixed-income portfolio against changes in interest rates
  - Indexing, a technique that attempts to replicate the performance of a given bond index (similar to stock market indexation)
  - Immunization, which eliminates the sensitivity of a bond portfolio to changes in interest rates by offsetting price risk against re-investment risk (matching portfolio’s duration with liabilities’ duration)

Recall: bond pricing

- How do we price a bond? Once again we rely on a present-value model
  - As for the stock case [cfr lecture on equity analysis] we have to discount expected cash flows (i.e. coupon payments and principal) by an appropriate discount rate \( r \). Therefore:
    \[
    BV = \text{present value of coupons} + \\
    \text{present value of par value}
    \]
    \[
    BV = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{ParValue}{(1+r)^T}
    \]
  - Note that there is only one interest rate \( r \) appropriate to discount cash flows. Different in practice
Yield curve

- The assumption of a single discount rate is restrictive
- In general yields are different at different maturities and, for the same maturities, over different periods of time
- Yields to maturity have been obtained under the assumptions that:
  - bonds are held until maturity
  - cash flows are reinvested at the computed yield to maturity
- The second assumption is in contrast with the evidence that rates change over time and across maturities
- Investors at any point in time require a different rate of return for flows at different times (e.g. returns required are different if zero-coupon bonds are offered at 2-, 5- or 10 years maturity)
- The rate of return used to discount flows at a certain maturity is called **spot rate** (or short rate)

Yield curve (cont’d)

- The relationship between the yield to maturity (=spot rates) and the term to maturity is called **yield curve** (or the term structure of interest rates)
Yield curve (cont’d)

A rising yield curve is formed when the yields on short-term issues are low and rise consistently with longer maturities and flatten out at the extremes.

A declining yield curve is formed when the yields on short-term issues are high and yields on subsequently longer maturities decline consistently.

A flat yield curve has approximately equal yields on short-term and long-term issues.

A humped yield curve is formed when yields on intermediate-term issues are steeper than those on short-term issues and the yields on short-term issues are again steeper than those for the short term and then level out.

Yield to Maturity (Percent)

- The same applies to the calculations of duration and convexity.
- The yield curve is generally not flat. Hence, estimation of the term structure of interest rate is crucial to retrieve \( y \) at different maturities.
Interpreting the yield curve

What factors change the yield curve?

- **Underlying macroeconomic forces** drive movements of the yield curve
- When macroeconomic variables are now generally incorporated into yield curve models we find that:
  - **Inflation** is highly correlated with the **level** of the yield curve, **real activity** (GDP) is related to the **slope** of the yield curve
  - **Inflation and real activity** are likely to determine the **shifts** in the yield curve
  - **Monetary policy shocks** affect the **slope** of the yield curve, **GDP shocks** affect the **curvature** of the yield curve while **inflation shocks** affect the **level** of the yield curve
Readings

- BKM
  - Chapter 14,
  - Chapter 16 (up to 16.2 included)

- Other readings (optional)
  - BKM, Chapter 15