Investment Management

Portfolio Performance Measurement

Road Map

- Measures of returns: more in detail
- Measures of risk: a brief summary
- Composite measures (Treynor’s, Sharpe’s, Jensen’s, Information ratio)
- Benchmark selection and Style analysis
The problem

- How do we measure the performance of a portfolio manager?
  - against some agreed benchmark?
  - by comparing with other portfolio managers?
- How do we compute the average return achieved by a portfolio manager over a period of time?
  - complicated by intermediate inflows and outflows of cash outside the portfolio manager’s control
- Which risk measure to employ?
  - How do we adjust portfolio return for risk? (total risk or nondiversifiable risk?)

Single period example

- Consider a stock paying a dividend of $2 annually that currently sells at $50. Assume that the stock is sold at the end of the next period at $53.
- The rate of return is
  \[
  HPR_{t\rightarrow t+1} = \frac{53 - 50 + 2}{50} = 0.10
  \]
- Alternatively, we calculate the rate of return which equates the present value of all cash in-flows with the (initial) cash out-flow
  \[
  50 = \frac{53 + 2}{(1 + IRR)} \rightarrow IRR = 0.10
  \]
Intermediate cash flows

- Calculating returns is more complicated if cash is added to, or withdrawn from, the portfolio before the end of the holding period.
- Example:
  - at the beginning of Year 1, an investor a shares in a stock selling at $50
  - at the end of Year 1, the stock is trading at $53, and the investor buys an additional share of the stock
  - at the end of Year 2, the stock price is $54, and the investor sells his complete holding
  - the stock pays a dividend of $2/share at the end of each of the two years

  What is the investor’s annual return?

Value-weighted returns (IRR)

- The rate which solve the following equation is called Internal rate of return (IRR), or dollar-weighted return:

  \[
  50 + \frac{53}{(1 + IRR)} = \frac{2}{(1 + IRR)^2} + \frac{(108 + 4)}{(1 + IRR)^3}
  \]

  \[IRR = r = 0.071 \text{ or } 7.1\%\]
Time-weighted return

- It does not take into account the size of the investment in each period.

\[
\begin{array}{c|c|c}
0 & 1 & 2 \\
\hline
50 & 53 & 54 \\
\hline
r_1 & r_2 \\
\end{array}
\]

\[
r_1 = \frac{(53 - 50) + 2}{50} = 0.100
\]

\[
r_2 = \frac{(54 - 53) + 2}{53} = 0.057
\]

- Time-weighted return is the average of the returns \((r_1, r_2)\) over the two years. What kind of average?

Arithmetic vs geometric average

- Arithmetic average of \(N\) returns:

\[
r_{A} = \frac{\sum_{i=1}^{N} r_i}{N}
\]

- unbiased estimate of expected future returns on portfolio (only if the expected returns do not change over time)

- Geometric average of \(N\) returns:

\[
r_{G} = \left[ \prod_{i=1}^{N} (1 + r_i) \right]^{\frac{1}{N}} - 1
\]

- is the fixed return the portfolio would need to have earned each year in order to match actual performance
- good measure of past performance
- geometric average ≤ arithmetic average. Then, geometric average is a downward biased estimate of future performance
Arithmetic vs geometric average (cont’d)

- Approximation: \( r_G \simeq r_A - \frac{1}{2} \sigma^2 \), where \( \sigma \) is the standard deviation of returns (the rule holds exactly when returns are normally distributed)

<table>
<thead>
<tr>
<th>Table 24.1 Average Annual Returns by Investment Class, 1926-2002</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Arithmetic Average</strong></td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>Common stocks of small firms</td>
</tr>
<tr>
<td>Common stocks of large firms</td>
</tr>
<tr>
<td>Long-term Treasury bonds</td>
</tr>
<tr>
<td>U.S. Treasury bills</td>
</tr>
</tbody>
</table>

Arithmetic vs geometric average (cont’d)

- Which one should we use then?
- In general if our focus is on
  - past performance → geometric average
  - future performance, we should amend the value of the arithmetic average (since it is biased when future returns are changing over time)

\[
r_{\text{mod}} = r_A \times \left(1 - \frac{H}{T}\right) + r_G \times \left(\frac{H}{T}\right)
\]

where \( H \) is the forecasting horizon (i.e. \( H \) periods ahead) and \( T \) is the sample period used in the estimation
And what about risk?

- Average returns are not enough
- Returns must be adjusted for risk in order to compare them meaningfully
- Where should we look at? Early practice stated that we should *compare investment with similar risk characteristics* (such as high-yield bonds, growth stock equities etc.) or *comparison universe*
- When investments are grouped, then evaluate them individually (for example ‘the best out 100 managers’, the ‘10th performing manager’ etc.)

And what about risk? (cont’d)

![Box plot of rate of return for different periods and comparison groups](image)
And what about risk? (cont’d)

- Problems:
  - within the same universe we are not really looking at the effective investment strategies (portfolio managers may focus on selected sub-groups). The assumption of same risk among securities may be too restrictive
  - It does not tell us if portfolio managers have accomplished individual objectives and satisfied investment constraints

- Solution: composite portfolio performance measures

Treynor performance measure

- Treynor (1965) developed the first measure of portfolio performance that included risk
- The measure applies to all investors regardless of their risk preferences
  - it is based on the definition of the SML (see Lecture 3 on the CAPM)
  - it applies to fully diversified portfolios
- Treynor showed that each rational investor would like to achieve the largest the slope of SML or in turn maximize:

\[ T_p = \frac{(\bar{r}_p - \bar{r}_f)}{\beta_p} \]

where \( \bar{r}_p, \bar{r}_f \) are the average return on portfolio \( P \) and the risk-free rate during a specified period of time and \( \beta_p \) is the systematic risk associated with portfolio \( P \)
Treynor performance measure (cont’d)

<table>
<thead>
<tr>
<th>portfolio</th>
<th>( \bar{r}_p )</th>
<th>( \beta_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>0.12</td>
<td>0.90</td>
</tr>
<tr>
<td>X</td>
<td>0.16</td>
<td>1.05</td>
</tr>
<tr>
<td>Y</td>
<td>0.18</td>
<td>1.20</td>
</tr>
</tbody>
</table>

\[
T_M = \frac{0.14 - 0.08}{1} = 0.060
\]
\[
T_W = \frac{0.12 - 0.08}{0.90} = 0.044
\]
\[
T_X = \frac{0.16 - 0.08}{1.05} = 0.076
\]
\[
T_Y = \frac{0.18 - 0.08}{1.20} = 0.083
\]
Treynor performance measure

- In essence the Treynor measure is a risk premium per unit of systematic risk (beta)
- Question: Why is beta a measure of systematic risk?
- Recall from lecture 3 that in the Single index model

\[ R_i = \alpha_i + \beta_i R_M + \epsilon_i \]
\[ \sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(\epsilon_i) \]

- Since the measure applies to fully diversified portfolios the second component is close to zero, therefore the relevant measure of risk is only function of beta

Sharpe performance measure

- Sharpe (1966) developed another measure of portfolio performance to evaluate mutual funds
- Sharpe proposed (jointly with his work on the CAPM):

\[ S_P = \frac{\bar{r}_P - \bar{r}_f}{\sigma_P} \]

where \( \bar{r}_P, \bar{r}_f \) are the average return on portfolio \( P \) and the risk-free rate during a specified period of time and \( \sigma_P \) is a measure of total risk for portfolio \( P \) during the same period
- The Sharpe measure is an risk premium return earned per unit of total risk
Sharpe performance measure (cont’d)

Treynor vs Sharpe

- The difference is only in the measure of risk adopted.
- When is it appropriate to use them?
  - use Treynor when you compare fully diversified portfolios
  - use Sharpe when you compare poorly diversified (concentrated or focused) portfolios
- Treynor = Sharpe (will provide you with the same ranking) when evaluating fully diversified portfolios (the only source of risk is the systematic risk)
- Treynor > Sharpe when evaluating poorly diversified portfolios (differences in rank come directly from differences in diversification)
Jensen performance measure

- Jensen’s (1968) measure (or Jensen’s alpha) shares the same theoretical foundations with the previous measures (it is based on the CAPM)
- The measure is calculated as the average return over and above the one predicted by the CAPM (or single index model):

\[
\alpha_p = \overline{r}_p - \left[ \overline{r}_f + \beta_p \left( \overline{r}_M - \overline{r}_f \right) \right]
\]

- Superior portfolio managers who forecast market turns or pick consistently undervalued securities can earn higher risk premia than the market risk premium (active portfolio management)
  - \( \alpha > 0 \) denotes an (average) outperformance of the benchmark
  - \( \alpha < 0 \) denotes an (average) underperformance of the benchmark

Jensen performance measure (cont’d)
Jensen performance measure (cont’d)

- Refinements: in the investment industry Jensen’s alpha are calculated on period-by-period basis. Therefore the formula becomes

\[
\alpha_{P,t} = r_{P,t} - \left[ r_{f,t} + \beta_{P,t} \left( r_{M,t} - r_{f,t} \right) \right]
\]

- This formula does not provide you with the average performance but the current performance at the end of each period (i.e. month, quarter, year etc.)

- If we do not believe in the CAPM

\[
\tilde{\alpha}_{P,t} = r_{P,t} - \left[ r_{f,t} + \beta_{1P,t} \left( r_{M,t} - r_{f,t} \right) + \beta_{2P,t} SMB_{t} + \beta_{3P,t} HML_{t} \right]
\]

\[
\hat{\alpha}_{P,t} = r_{P,t} - \left[ r_{f,t} + \sum_{j} F_{j,t} \right]
\]

Using Treynor and Jensen measures
Using Treynor and Jensen measures (cont’d)

\[ \alpha_p = 0.02 \quad T_p = \frac{0.11}{0.90} = 0.120 \]
\[ \alpha_q = 0.03 \quad T_q = \frac{0.19}{1.60} = 0.118 \]
\[ \alpha_q > \alpha_p \quad \text{but} \quad T_p > T_q \]

- When comparing two portfolio managers with fully diversified portfolios Jensen’s alpha could provide us with misleading results. Treynor measures are more appropriate.

Jensen measures: caveat

- Always bear in mind that Jensen’s alphas are estimates (therefore there is some uncertainty associated with their values).
- Example: Assume that we have to evaluate a portfolio managers and we have got this monthly data
  \[ \hat{\alpha} = 0.2\%, \quad \hat{\beta} = 1.2, \quad \sigma(e) = 2\% \]
- When we estimate alphas we want to know whether or not they are significant (conditional on the sample of observations). We have to compute t-statistics.
- Statistical rules tell us
  \[ \hat{\alpha}(\alpha) = \frac{\hat{\sigma}(e)}{\sqrt{N}}; \quad t(\alpha) = \frac{\hat{\alpha}}{\hat{\sigma}(\alpha)} = \frac{\hat{\alpha}}{\hat{\sigma}(e)/\sqrt{N}} \]
  \[ t(\alpha) = \frac{0.2}{2/\sqrt{N}} = \frac{0.2\sqrt{N}}{2} \]
Jensen measures: caveat

- In order to have our t-statistic significant at 5%, its value must exceed 1.96. Hence

\[ \frac{0.2\sqrt{N}}{2} > 1.96 \Rightarrow N > 384 \]

- This implies that in order to prove his/her true skills, the portfolio manager will have to produce this performance (on average) over 384 months (or 32 years)!
- **Statistical inference makes performance evaluation extremely difficult.** It is complicated to assess the quality of ex-ante decisions using ex-post data

Information ratio

- Strictly related to the previous measures of performance is the information ratio (or appraisal ratio)
- It is an average return in excess of any predefined benchmark (CAPM, FF or Multifactor model) divided by the portfolio’s specific risk (or tracking error)

\[ IR_p = \frac{\alpha_p}{\sigma(e_p)} \]

- It is a ratio between the **benefit** of a non-perfect diversification (\( = \alpha \)) and its **cost** (\( = \sigma(e) \)).
- Values of IR > 0.5 denote a good performance, values of IR > 1.0 denote an exceptional performance. However how many portfolio managers achieve such values?
Information ratio (cont’d)

Measure for measure ...

- Jensen’s alpha can be used to compare the performance of a portfolio with that of a risk-adjusted benchmark
  - it accounts for systematic risk
- Treynor or Sharpe measures can be used to compare the performance of two or more portfolios with one another
  - Treynor measure gives the excess return per unit of systematic risk (fully diversified portfolios)
  - Sharpe measure gives the excess return per unit of total risk (poorly diversified portfolios)
... and for good measure

Benchmark problem?

- Problem: what is market portfolio (Roll, 1987)? How can we measure it? How do we select the proper benchmark?
  - In general market portfolios are proxied by market indices (e.g. FTSE100, S&P500 etc.)
  - However, different market indices or different benchmarks may yield different results (cfr. Lecture 3)
- One alternative: *Style analysis* (Sharpe, 1992)
  - can be used for obtaining an *estimate of the portfolio's investment style and exposures to certain asset classes*
Style analysis

Table 24.9 Sharpe's Style Portfolios for the Magellan Fund

<table>
<thead>
<tr>
<th>Regression Coefficient</th>
<th>Bills</th>
<th>Intermediate bonds</th>
<th>Long-term bonds</th>
<th>Corporate bonds</th>
<th>Mortgages</th>
<th>Value stocks</th>
<th>Growth stocks</th>
<th>Medium-cap stocks</th>
<th>Small stocks</th>
<th>Foreign stocks</th>
<th>European stocks</th>
<th>Japanese stocks</th>
<th>TOTAL</th>
<th>R-squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>0.03</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.26</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.83</td>
</tr>
<tr>
<td>Europe</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.26</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.83</td>
</tr>
<tr>
<td>Japan</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.26</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.83</td>
</tr>
<tr>
<td>T-bill</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.26</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.83</td>
</tr>
</tbody>
</table>

*a* Regressions are constrained to have non-negative coefficients and to have coefficients that sum to 100%.


Style analysis (cont’d)

<table>
<thead>
<tr>
<th>US equities</th>
<th>International equities</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABT</td>
<td>New Perspective Templeton World Vanguard</td>
</tr>
<tr>
<td>a</td>
<td>0.26</td>
</tr>
<tr>
<td>USA</td>
<td>89%</td>
</tr>
<tr>
<td>Europe</td>
<td>1%</td>
</tr>
<tr>
<td>Japan</td>
<td>0%</td>
</tr>
<tr>
<td>T-bill</td>
<td>10%</td>
</tr>
<tr>
<td>R²</td>
<td>0.83</td>
</tr>
</tbody>
</table>
Style analysis (cont'd)

![Diagram showing style analysis]

Readings

- BKM
  - Sections 24.1 and 24.5
- Other readings (optional)