BANDPASS FILTERING AND WAVELETS ANALYSIS

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Econometric models have traditionally depended of linear time-invariant structures. Structural changes have been accommodated by postulating structural breaks in which one locally invariant structure is succeeded by another, or by allowing for instantaneous switching between alternative regimes.

To accommodate structural changes of a more varied nature, it is appropriate to pursue a wavelets analysis.

In a wavelets analysis, a temporal data sequence is decomposed into its frequency-specific components, of which the amplitudes can vary through time. Thus, a wavelets analysis reveals the structure of the data in both its time and its frequency dimensions.

A Dyadic Mosaic

A dyadic mosaic provides a framework for a set of basis functions that will serve to analyse a function that varies both in time and frequency. In the diagram, the height of a cell corresponds to a bandwidth in the frequency domain, whereas its width denotes a temporal duration.

Centred on each cell, but liable to extend beyond its temporal boundaries, there is a localised function described as a wavelet. In the dyadic scheme, the uppermost frequency band of $[\pi/2, \pi]$ covers half the frequency range and it is populated by $T/2$ wavelets separated one from the next by two sampling intervals.

The remaining lower half of the frequency range, which is the interval $[0, \pi/2]$ and which is the subject of successive subdivisions, is populated by $T/2$ scaling functions.

The next frequency band in the mosaic diagram, which is the interval $[\pi/4, \pi/2]$, covers half of this remaining range, and it is populated by $T/4$ wavelets separated by multiples of 4 intervals.

The successive subdivision of the remaining frequency range continues in the same manner, until a final band is reached that stretches the width of the sample, or else the divisibility of T by increasing powers of 2 is at an end.

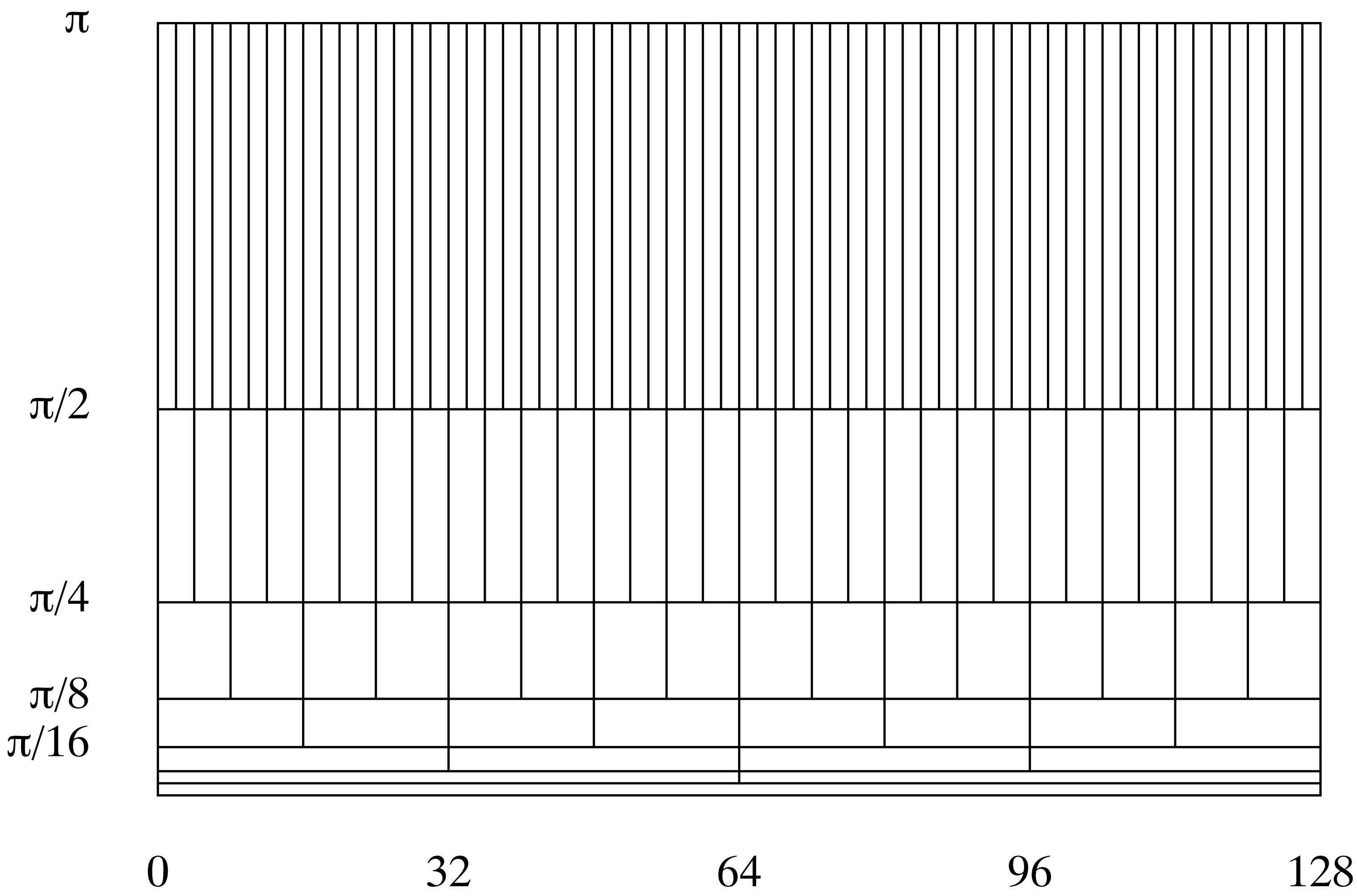

Figure 1. The partitioning of the time–frequency plane according to a multiresolution analysis of a data sequence of $128 = 2^7$ points.

Music with a Dyadic Structure: Pachelbel's Canon

Pachelbel's Canon combines the techniques of a canon and a ground bass. There are three voices engaged in the canon, but there is a fourth voice, on the baseline, which plays an independent and wholly repetative part.

Pachelbel's canon, which spans four octaves, is constructed of just 12 variations, mostly four bars in length:

(bars 3–6) quarter notes

(bars 7–10) eighth notes

(bars 11–14) sixteenth notes

(bars 15–18) leaping quarter notes, rest

(bars 19–22) 32nd-note pattern on scalar melody

(bars 23–26) staccato, eighth notes and rests

(bars 27–30) sixteenth note extensions of melody with upper neighbour notes

(bars 31–38) repetitive sixteenth note patterns

(bars 39–42) dotted rhythms

(bars 43–6) dotted rhythms and 16th-note patterns on upper neighbour notes

(bars 47–50) syncopated quarter and eighth notes rhythm

The Need for a Non-Dyadic Analysis

The dyadic wavelets scheme is applicable to a wide range of natural and artificial phenomena. Its octave frequency structure is ideally suited, for example, to the analysis of canonical baroque music, when played in a strict tempo.

However, a dyadic scheme is less appropriate to the analysis of mechanical vibrations or to the analysis of economic data displaying regular seasonal fluctuations. In such cases, a different set of frequency bands may be required to correspond to the spectral structure of the data.

For an example of a statistical data series that requires a more flexible form of wavelet analysis, we might consider the familiar monthly airline passenger data of Box and Jenkins (1976), depicted in Figure 2.

The monthly series comprises $T = 144 = 3^2 \times 2^4$ data points. The detrended series, which is obtained by taking the residuals from fitting a quadratic function to the logarithms of the data, is shown in Figure 3.

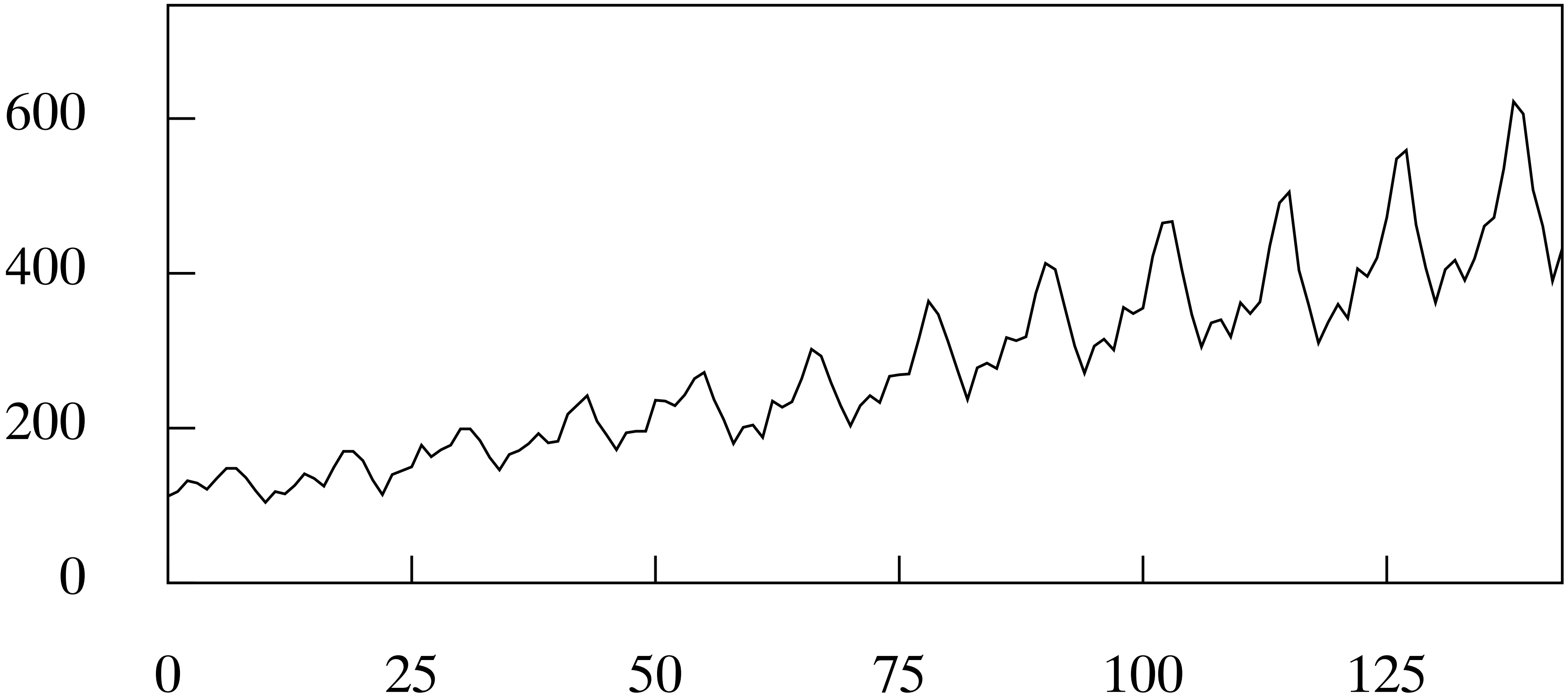

Figure 2. International airline passengers: monthly totals (thousands of passengers) January 1949–December 1960: 144 observations.

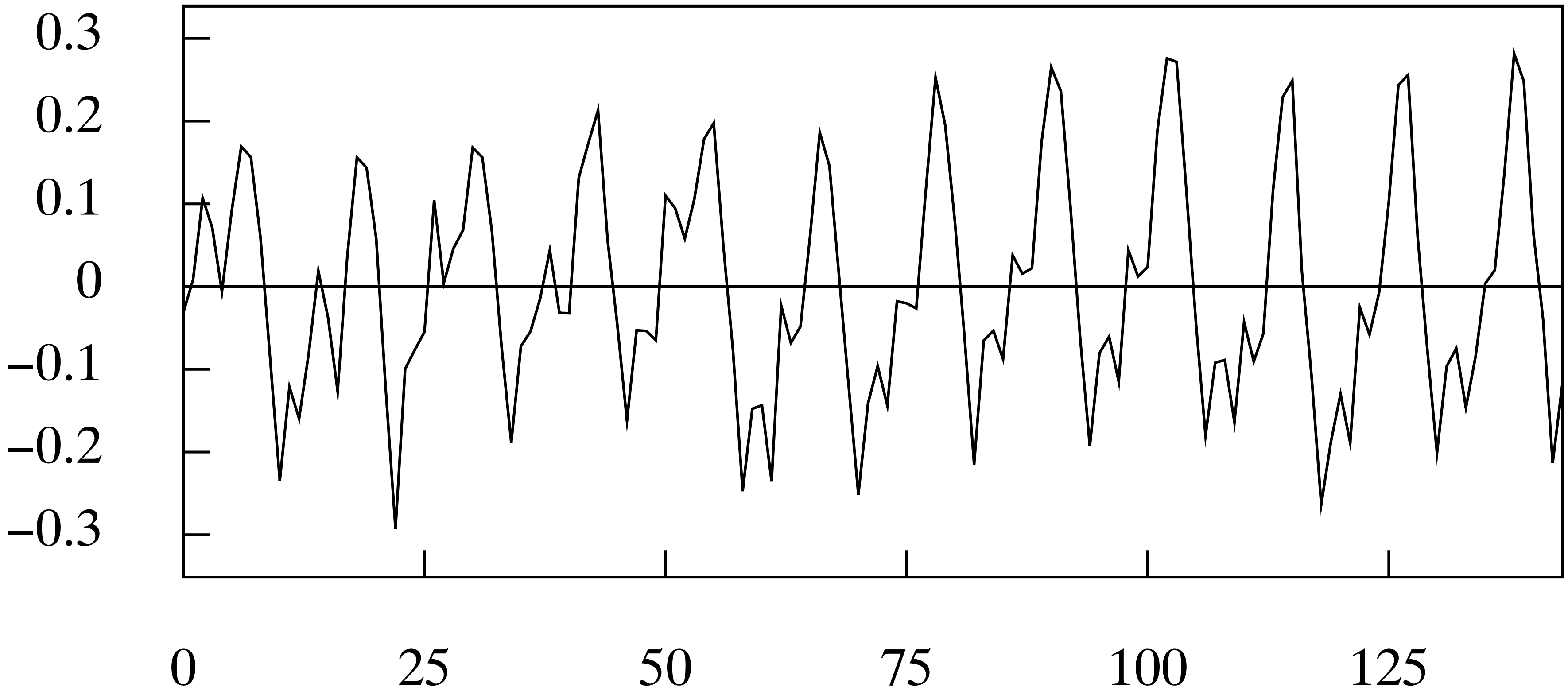

Figure 2. The seasonal fluctuation in the airline passenger series, represented by the residuals from fitting a quadratic function to the logarithms of the series.

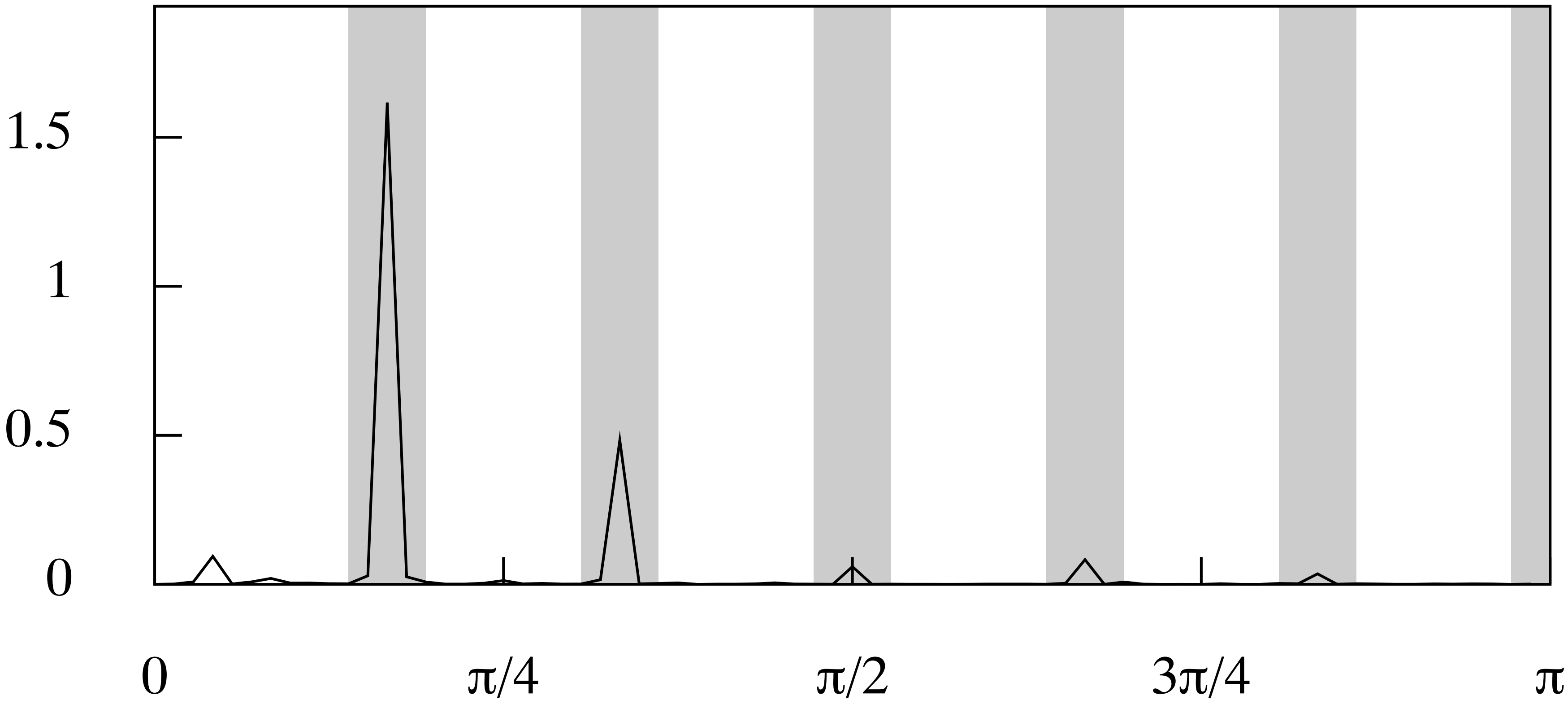

Figure 3. The periodogram of the seasonal fluctuations in the airline passenger series.

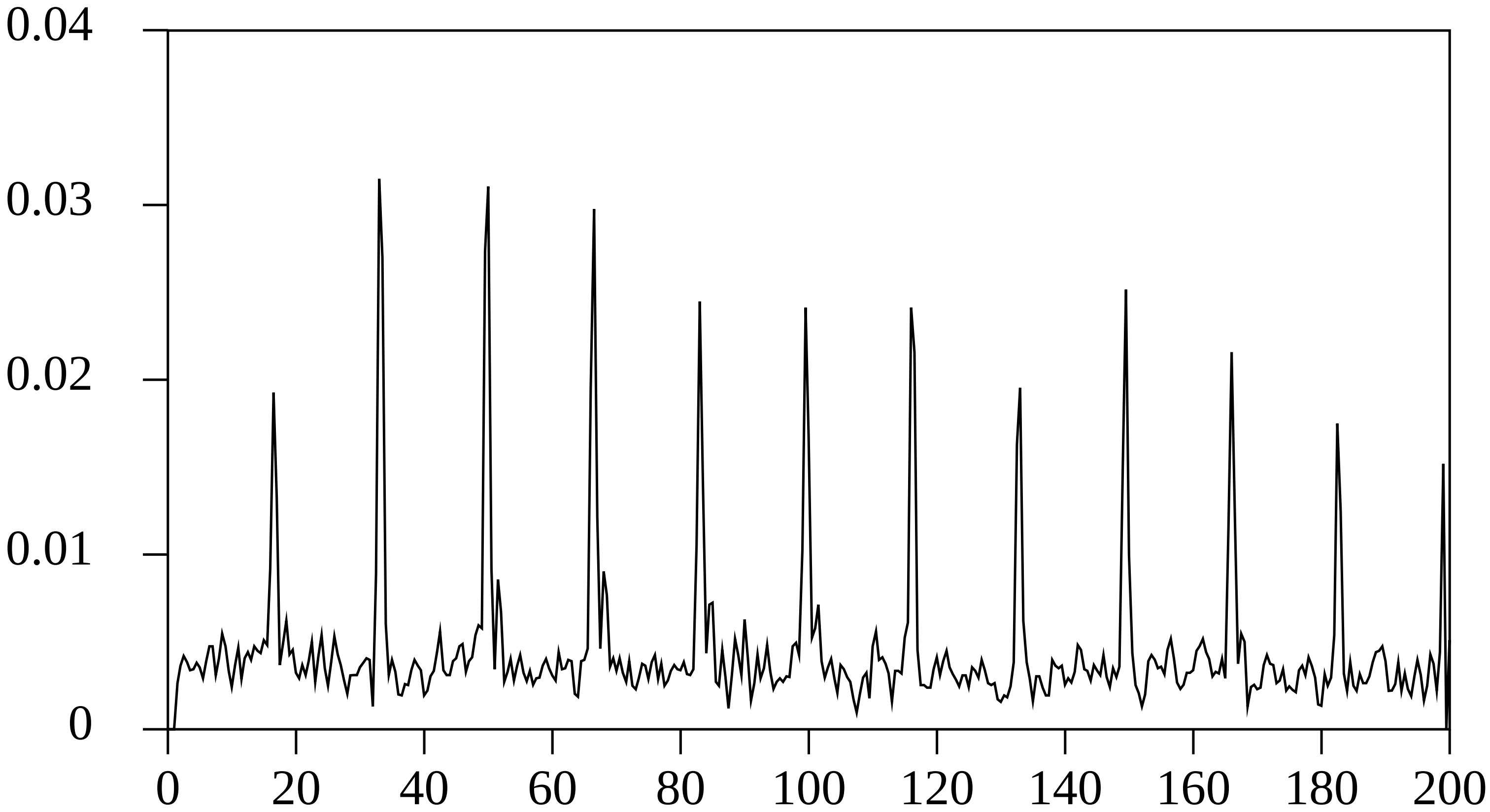

Figure 4. The power spectrum of the vibrations transduced from the casing of an electric motor in the process of a routine maintenance inspection. The units of the horizontal axis are hertz. The first peak at 16.6 hertz corresponds to a shaft rotation speed of 1000 rpm. The prominence of its successive harmonics corresponds to the rattling of a loose shaft.