LECTURE 4

Fisheries

Models of population growth may be used in the management of renewable resources such as in the regulation of fisheries. Here the object is to achieve an optimal rate of extraction which allows the fish populations to regenerate themselves rapidly in spite of their losses. This is not simply a matter of ensuring that fishing does not lead to the extinction of fish populations. Even when it does not threaten the survival of the fish, overfishing will certainly cause wasted effort.

Unfortunately, it is true that, in practice, we do have to guard against the danger of extinction through over-intensive fishing in which the effort devoted to the pursuit goes far beyond optimal levels and is, therefore, largely self-frustrating. The problem, which is widely acknowledged, arises when there is open access to the fish stocks. Whilst it may be in the common interest of all the exploiters to reduce the intensity of fishing, it will not be in the interest of any individual to do so if the only consequence is to allow a competitor to take a greater share of the meagre supplies.

Fishing for a Given Harvest

A sustainable rate of fishing is one which leads to an equilibrium state in which the rate of extraction h is equal to the rate dy/dt at which the population regenerates itself. Recall that, in the case of the logistic model, the natural growth rate of the population, which is the difference between the birth rate and the death rate, is given by

$$\frac{dy}{dt} = \rho y \left(1 - \frac{y}{\gamma}\right). \tag{1}$$

Here the factor ρy represents the rate of growth of a population which is following an exponential path. The factor $1 - y/\gamma$ represents the constraint which stauches the exponential grow as the population size y progresses toward γ, which is the carrying capacity of the environment.

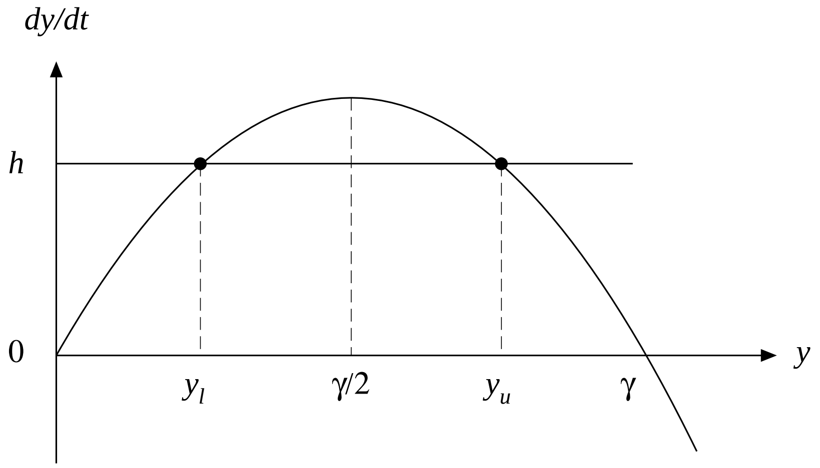

Figure 1. The harvest h may be obtained with least effort from a large population y_u or with greater effort from a depleted population y_l.

The steady-state condition, which prevails when the rate of harvesting h equals the rate of regeneration, gives rise to a simple quadratic equation of the form

$$\begin{aligned} 0 &= \rho y\left(1 - \frac{y}{\gamma}\right) - h, \quad \text{or, equivalently,} \\ 0 &= \rho y^2 - \gamma\rho y + \gamma h. \end{aligned} \tag{2}$$

The solution is

$$y = \frac{\gamma\rho \pm \sqrt{\gamma^2\rho^2 - 4\gamma\rho h}}{2\rho}. \tag{3}$$

The nature of the solution depends upon the rate of extraction h. If $h > \gamma\rho/4$, then there is no real-valued solution; and the implication is that the proposed rate of extraction would lead to the extinction of the fish stock.

If $h = \gamma\rho/4$, then the solution has the unique value of $y = \gamma/2$. Then the value of h is the maximum sustainable rate of extraction, and the corresponding value of y is also the size of the fish population which has the highest absolute rate of growth in the absence of fishing.

If $h < \gamma\rho/4$, then there are two solutions which correspond to two distinct ways in which the fish harvest h might be obtained. The first way is via a low expenditure of effort applied to a relatively large stock of fish y_u. The second way is via a large expenditure of effort applied to a depleted stock of fish y_l. This signifies that over-fishing has already occurred.

To fish invariably at the maximum sustainable rate of $h = \gamma\rho/4$ would, of course, be very perilous. If ever the population should fail to regenerate itself fully, or if the maximum rate of extraction were exceeded momentarily, then the population would be driven to extinction. However, disaster might be averted if an ever-increasing effort were required in order to sustain the maximum rate of extraction in the face of a declining fish stock. For the effort might become uneconomic before the fish were imperilled by extinction. In effect, the maximum sustainable rate of extraction represents an unstable equilibrium.

The same problem of instability arises in the case the over-fishing where a harvest of h is being extracted from a population of size y_l. It can be see from the diagram that, if the population falls below y_l, then the rate dy/dt at which it regenerates itself will fall below the rate of extraction h, and therefore the stocks will head for extinction. On the other hand, if the stocks that are left after harvesting exceeds y_l, then, according to this model, they will go on increasing until y_u is achieved. In effect, y_u is a stable equilibrium whereas y_l is an unstable equilibrium.

However, it is reasonable to suppose that, for every fish population, there is a threshold level $\kappa > 0$ below which growth is negative and the population heads for extinction. This threshold might be crossed even by moderate efforts in fishing. Matters of this sort require careful analysis in the light of facts which, by the very nature of fishing, are hard to come by.

The conclusion is that a stable equilibrium can only be reached if the rate of extraction is less than the maximum rate, ie. if $h < \gamma\rho/4$ in the case of the simple logistic model, and if this is extracted from a population $y_u > \gamma/2$.

Fishing with a Constant Effort

In a situation where only limited resources can be devoted to fishing. it is appropriate to imagine the size of the harvest is governed by the overall effort devoted to fishing, denoted by e, and by the size of the stocks y. Then $h = h(y, e)$ and, for simplicity, we shall assume that $h = ey$, which implies that a given effort in fishing is rewarded by a harvest which is directly proportional to the size of the fish population. In these circumstances, the condition for equilibrium, which equates the size of the harvest to the rate at which the stocks are regenerated, is

$$0 = \rho y\left(1 - \frac{y}{\gamma}\right) - ey. \tag{4}$$

Solving this equation for the size of the fish stocks gives

$$y = \gamma\left(1 - \frac{e}{\rho}\right), \tag{5}$$

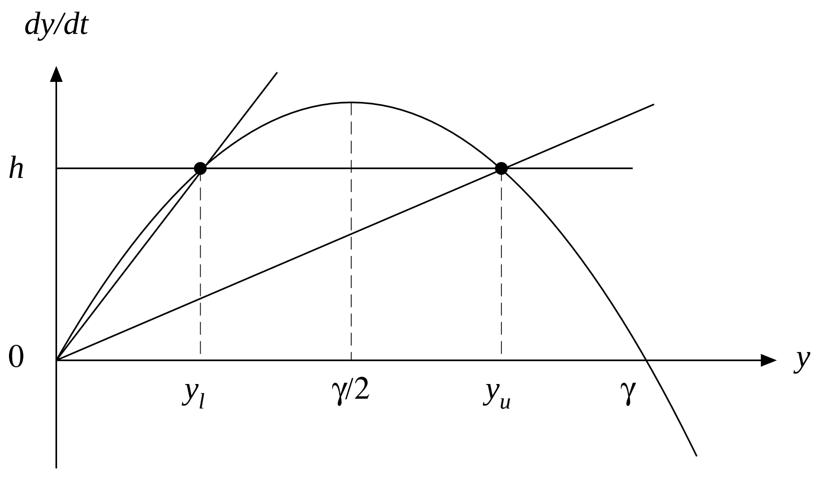

Figure 2. Excessive effort devoted to fishing may be self-frustrating. The harvest h obtained from the effort e is determined by intersection of the harvest function $h = ey$ with the growth function $dy/dt = f(y)$. The diagram shows that a given harvest h can be obtained from two different levels of effort.

whence the harvest which is obtained from the given effort is

$$(6) \qquad h = ey = e\gamma\left(1 - \frac{e}{\rho}\right).$$

If the effort is great, then the harvest may, for a while, exceed the natural growth rate of the population. In that case, the condition $e > \rho$ will prevail; and it is evident from equation (5) that the condition $y > 0$ cannot be sustained. That is to say, the fish stocks will be driven to zero.

It is straightforward to discover the level of fishing effort e_m which will lead to maximum sustainable rate of extraction h_m. The condition for maximising the harvest, which is obtained from equation (6), is that

$$(7) \qquad \frac{dh}{de} = \gamma\left(1 - \frac{2e}{\rho}\right) = 0,$$

which gives

$$(8) \qquad e_m = \frac{\rho}{2}, \qquad y_m = \frac{\gamma}{2} \qquad \text{and} \qquad h_m = \frac{\gamma\rho}{4}.$$

These results are in accordance with what we have discovered already; and the opportunity may be taken to reiterate the point that a policy of aiming for the maximum harvest is a perilous one which is liable to endanger the fish stocks.

Profit Maximisation under Controlled Access

Let us now reveal nature of the profit-maximising solution which might be achieved under a regime which is capable of determining at will the exact rate of harvesting h. It is often imagined that this must coincide with the maximum sustainable yield. However, there are economic arguments which suggest that a quite different rate might materialise.

For a start, the costs of fishing have to been considered. These are liable to rise as the fish stock fall for the reason that, when their density is reduced, the fish are liable to be harder to locate and catch. This suggests that a lowering of h might be desirable in order to increase the density of the fish and to lower the costs of fishing, even at the expense of foregoing some revenue from sales.

On the other hand, a strong preference for immediate profits at the expense of future ones might encourage an unsustainable rate of extraction leading to the extinction of the stocks.

In order to investigate these matters, let us preserve the assumption that the size of the harvest is governed by the overall effort devoted to fishing, denoted by e, and by the size of the stocks y; and let us continue to assume, for simplicity, that the size of the harvest is $h = ey$. Then a given effort in fishing is rewarded by a harvest which is directly proportional to the size of the fish population.

The profits π from fishing are the difference between the revenues $r = r(h)$ and the costs $c = c(e)$. Assuming a fixed price $\bar{p}$ and a fixed cost $\bar{w}$ per unit of effort, we have

$$\pi = r - c = \bar{p}ey - \bar{w}e. \tag{9}$$

Here the size of the fish stocks y which determine the size of the harvest $h = ey$ are themselves the result of the effort devoted to fishing. In order to express the profits as a function of this effort, we substitute the expression $y = y(e)$ from equation (5) into equation (9) to give

$$\pi = \bar{p}e\gamma\left(1 - \frac{e}{\rho}\right) - \bar{w}e. \tag{10}$$

The profit-maximising effort, which is found from the first-order condition $d\pi/de = 0$ is

$$e = \frac{\rho}{2} - \frac{\rho\bar{w}}{2\gamma\bar{p}}. \tag{11}$$

On putting this back into equation (5), it is found that

$$y = \frac{\gamma}{2} + \frac{\bar{w}}{2\bar{p}}. \tag{12}$$

The implications of this equation are as one might expect. With $\bar{p}, \bar{w} > 0$, which is to say with nonzero costs and prices, the size of the fish stocks associated with a profit-maximising exploitation of the fisheries will be greater that the size of the stocks which are associated with maximum sustainable rate of exploitation. The corresponding rate of extraction is less than the maximum rate.

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