FUTURES: MARKING TO MARKET

The holder of a futures contract will be required to deposit with the brokers a sum of money described as the margin, which will be calculated at a percentage of the current spot price S_0 of the asset. At the end of each day of trading, the margin will be adjusted to reflect the gains or losses of the contract holder.

Should the cumulated losses reduce the margin to below a certain threshold level, described as the maintenance margin, then extra funds will be called for to maintain its level.

The process of adjusting the margin account is described as marking to market. Its effect is to ensure that, at the end of any day of futures trading, when the daily settlements have been made, there will be no outstanding obligations.

This will allow the position of the contract holder to be closed without further losses or gains, thereby virtually eliminating the risk of a default on the contract.

Let $F_{\tau|t}$ be the value at time t of a futures contract that is to be settled at time τ, and let δ_t be the gains or loses realised on day t, which is a sum that must be paid to the brokers or paid by them to the contract holder.

Day	*Futures Price*	*Gain or Loss*
t	$F_{\tau\|t}$	δ_t
0	$F_{\tau\|0}$	—
1	$F_{\tau\|1}$	$\delta_1 = F_{\tau\|1} - F_{\tau\|0}$
2	$F_{\tau\|2}$	$\delta_2 = F_{\tau\|2} - F_{\tau\|1}$
$\vdots$	$\vdots$	$\vdots$
τ	$F_{\tau\|\tau} = S_\tau$	$\delta_\tau = S_\tau - F_{\tau\|\tau-\|1}$

It can be assumed that $F_{\tau|t} \rightarrow S_\tau$ as $t \rightarrow \tau$, which is to say that the futures price converges to the spot price as the delivery time approaches.

The settlement on the final day, which is $\delta_\tau = S_\tau - F_{\tau|\tau-1}$, will be a negligable amount. At that time, the cumulated total of the adjustments is

$$\sum_{t=1}^{\tau} \delta_t = S_\tau - F_{\tau|0},$$

which is the difference between the contract price and the spot price on the date of delivery.

Observe, however, that the contract could be closed at any time t prior to the delivery date. Then the cumulated total of the adjustments would be

$$\sum_{j=1}^{t} \delta_j = F_{\tau|t} - F_{\tau|0}.$$

OPTIONS

Options give their holders rights to buy (if they are *call options*) or rights to sell (if they are *put options*). The party owning the right to buy in a call option or the right to sell in a put option is on the *long* side of the contract. The party with the corresponding liabilities is on the *short* side of the contact.

	Call	Put
Short	*liability to sell*	*liability to buy*
Long	*right to buy*	*right to sell*

A party who is long on a call option or short on a put option will profit if prices rise. (They have bullish expectations.)

A party who is short on a call option or long on a put option will profit if prices fall. (They have bearish expectations.)

We denote the contract date by $t = 0$ and the expiry date by $t = \tau$. The price agreed in the contract is the *exercise price* denoted $K_{\tau|0}$ and the *spot price* that prevails at the time of expiry is S_τ. The price of a call option, or its premium, will be denoted by $c_{\tau|0}$

The **call option** will be exercised only if

$$S_\tau > K_{\tau|0},$$

when it will be worth more than what will be paid for it. The call option will have proved profitable to the holder only if

$$S_\tau > K_{\tau|0} + c_{\tau|0}e^{r\tau}$$

which is when the value of what is called for exceeds what is paid for it plus the cost, up to the present time, of holding the option.

The price of a put option, or its premium, will be denoted by $p_{\tau|0}$.

The **put option** will be exercised only if

$$S_\tau < K_{\tau|0},$$

which is when the asset possessed by the option holder is worth less on the open market that can be claimed for it under the terms of the contract.

The put option will have prove profitable to the holder only if

$$K_{\tau|0} > S_\tau + p_{\tau|0}e^{r\tau},$$

which is when the price that is paid for it exceeds its current value plus the cost, up to the present time, of holding the option.

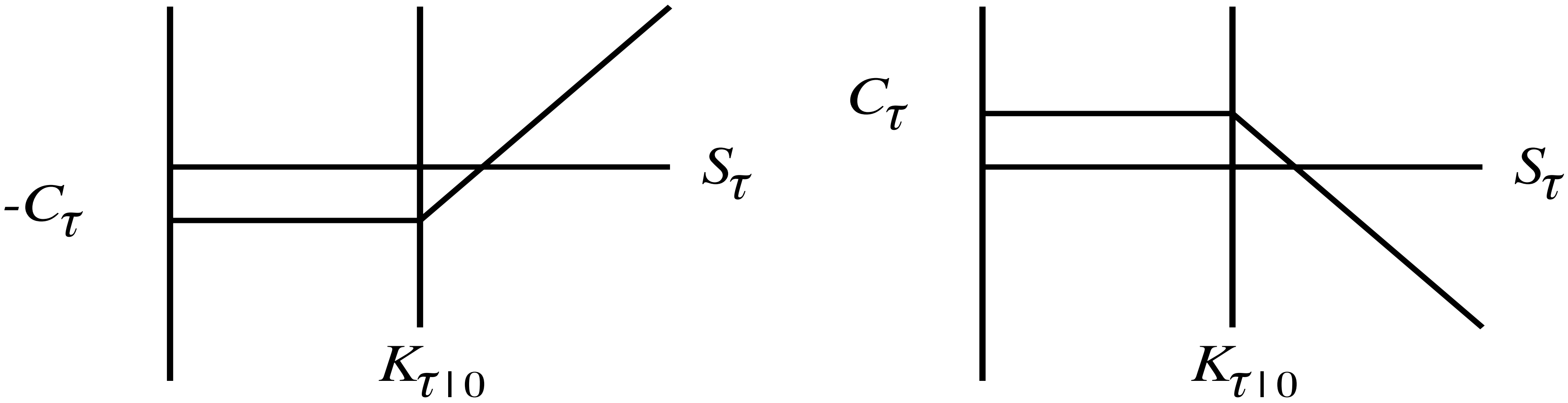

Figure 1. The profits of a **call option** written at time $t = 0$ and with a date of expiry of $t = \tau$. The premium evaluted at time $t = \tau$ is $C_\tau = c_{\tau|0}e^{r\tau}$, the strike price is $K_{\tau|0}$ and the spot price on expiry is S_τ. On the left are the profits of the option holder (in the long position) and on the right are the profits of the writer (in the short position).

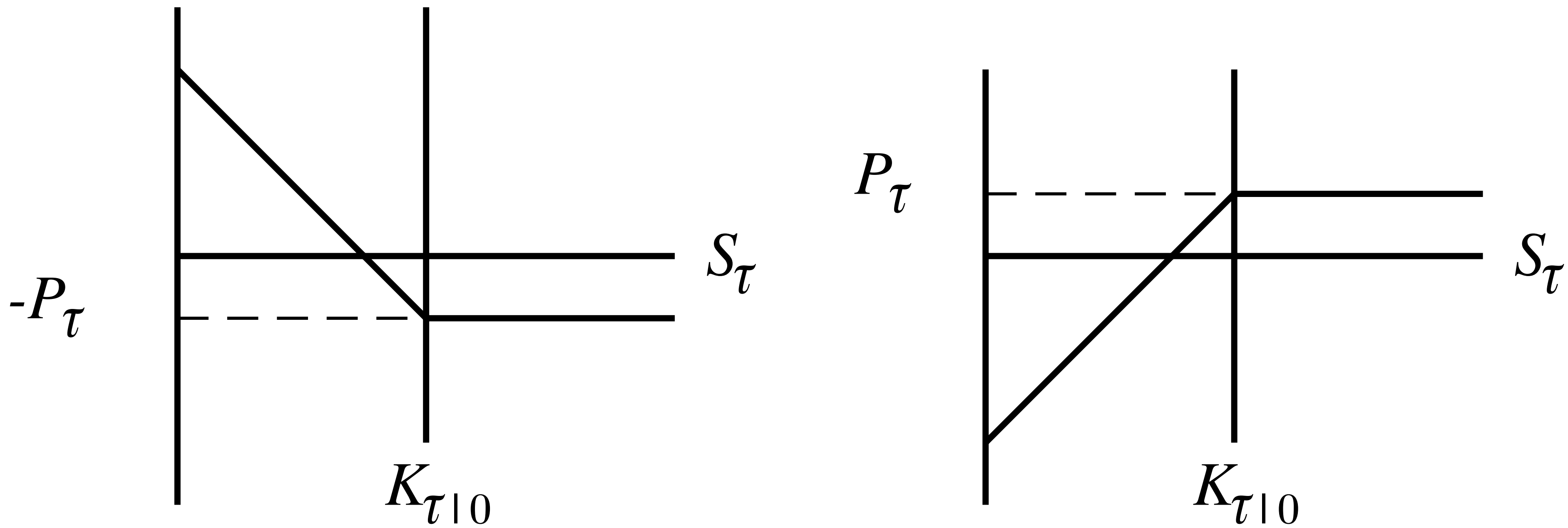

Figure 2. The profits of a **put option** written at time $t = 0$ and with a date of expiry of $t = \tau$. The value of the premium is $P_\tau = p_{\tau|0}e^{r\tau}$, the strike price is $K_{\tau|0}$ and the spot price on expiry is S_τ. On the left are the profits of the option holder (in the long position) and on the right are the profits of the writer (in the short position).

UPPER BOUNDS FOR OPTION PREMIA

$p_{\tau|0}$ denotes the price at time $t = 0$ of a put option effective at time τ.

$c_{\tau|0}$ denotes the current price of a corresponding call option.

$K_{\tau|0}$ denotes the strike price of the options at the time τ of their expiry,

S_0 and S_τ are the spot prices at time $t = 0$ and $t = \tau$, respectively.

The current spot price is an upper bound on the price of a call option:

$$c_{\tau|0} \leq S_0.$$

By paying $c_{\tau|0}$, one acquires the opportunity either of owning the stock at a later date, if $K_{\tau|0} < S_\tau$, or of foregoing ownership, if $K_{\tau|0} > S_\tau$.

If $c_{\tau|0} \geq S_0$, then one has the possibility of owning the stock for certain both now and at time τ at the lesser cost of S_0. Therefore, the cost of the option cannot exceed S_0.

The value of a put option cannot exceed the present value of the strike price:

$$p_{\tau|0} \leq K_{\tau|0}e^{-\tau}$$

Either the put option becomes worthless, if $S_\tau \geq K_{\tau|0}$, or else it serves to secure a payment K_τ at time τ, which has a discounted present value of $K_\tau e^{-\tau}$ at time $t = 0$.

If $p_{\tau|0} > K_\tau e^{-\tau}$, it would be better to have $K_\tau e^{-\tau}$ for certain at time $t = 0$ instead of spending it on a put option. Therefore, the cost of the option cannot exceed $K_\tau e^{-\tau}$.

THE PUT-CALL PARITY

Portfolio C consists of one call option on a unit of stock, valued at $c_{\tau|0}$ and with has a strike price of $K_{\tau|0}$, together with a cash sum of $K_{\tau|0}e^{-r\tau}$, which will yield $K_{\tau|0}$ when invested at a riskless compound rate of return of r.

At time τ, the portfolio C will be worth

$$K_{\tau|0} + \max(S_\tau - K_{\tau|0}, 0) = \max(S_\tau, K_{\tau|0}).$$

At worst, the option will not be exercised and the funds $K_{\tau|0}$ will be retained, whereas, at best, an asset worth $S_\tau > K_{\tau|0}$ will be acquired at the cost of the strike price of $K_{\tau|0}$.

Portfolio P consists of one put option valued at $p_{\tau|0}$ and a unit of the stock with a spot price of S_0.

At time τ, the portfolio P will be worth

$$S_\tau + \max(K_{\tau|0} - S_\tau, 0) = \max(S_\tau, K_{\tau|0})$$

At worst, the option will not be exercised and the asset worth S_τ will be retained. At best, the sum of $K_{\tau|0} > S_\tau$ will be received for the delivery of an asset worth S_τ.

The values of these two portfolios, which are equal at time τ, must also be equal at time $t = 0$. It follows that

$$c_{\tau|0} + K_{\tau|0}e^{-r\tau} = p_{\tau|0} + S_0.$$

This is the formula for the put–call parity of European options.