Exercise 1

1. A credit card company charges an annual interest rate of 15%, which is effective only if the interest on the outstanding debts is paid in monthly instalments. Otherwise, the interest charges are compounded with the borrowings. What would be the effective annual rate of interest if £Q are borrowed at the beginning of the year and repaid with interest at the end of the year?

2. How long will it take to double your investment if you receive an annual rate of interest of 5% and if the interest is compounded with the principal?

3. With reference to the relevant Taylor-series expansions, demonstrate the following approximations

$$\ln(1+x) \simeq x, \qquad e^x \simeq 1+x,$$

which are valid when x is small. Calculate the value of n in question 2 using the relevant approximation in the denominator.

4. Using the expression $x = e^y = 10^z$, find a formula that will enable you to convert between $\log_e(x)$ and $\log_{10}(x)$, commonly denoted by $\log x$ and $\ln x$, respectively—the latter being described as a natural logarithms or Naperian logarithms.

5. Let $S_n = 1 + r + r^2 + \cdots + r^{n-1}$, which is a partial sum of n terms of an geometric progression. Show that $S_n + r^n = 1 + rS_n$ and thence derive an expression of S_n in terms of r.

6. Annual payments of £M must be made over a period of n years to redeem a mortgage. The present value of this stream of payments is

$$M(\delta + \delta^2 + \cdots + \delta^n),$$

where $\delta = (1+r)^{-1}$ is the rate of discount and r is the rate of interest. The present value of the stream of payments must equate to the value L of the loan.

If the loan was for £150,000 and the rate of interest was fixed a 5% for the entire period, what should be the size of the annual payment in order to redeem the loan in 20 year's time?