PUT–CALL PARITY

Upper Bounds Let $p_{\tau|0}$ be the current price at time $t = 0$ of a put option effective at time τ, and let $c_{\tau|0}$ be the current price of a corresponding call option. Also, let $K_{\tau|0}$ be the strike price of the options at the time τ of their expiry, and let S_0 and S_τ be the spot prices at time $t = 0$ and $t = \tau$, respectively.

The current spot price is an upper bound on the price of a call option:

$$c_{\tau|0} \leq S_0.$$

To understand this, observe that, by paying $c_{\tau|0}$, one has acquired the opportunity either of owning the stock at a later date, if $K_{\tau|0} < S_\tau$, or or foregoing ownership, if $K_{\tau|0} > S_\tau$. If $c_{\tau|0} \geq S_0$, then one would have the possibility of owning the stock for certain both now and at time τ at the lesser cost of S_0.

The value of a put option can never exceed the present value of the strike price:

$$p_{\tau|0} \leq K_{\tau|0}e^{-\tau}$$

Either the put option becomes worthless, if $S_\tau \geq K_{\tau|0}$, or else it serves to secure a payment K_τ at time τ, which has a discounted present value of $K_\tau e^{-\tau}$ at time $t = 0$. If $p_{\tau|0} > K_\tau e^{-\tau}$, it would be better to have $K_\tau e^{-\tau}$ for certain at time $t = 0$ instead of spending it on a put option.

The Put-Call Parity Consider the circumstances of two portfolios at time $t = 0$, which, as we will show, have equivalent values. The first portfolio C consists of one call option on a unit of stock, which is valued at $c_{\tau|0}$ and which has a strike price of $K_{\tau|0}$, together with a cash sum of $K_{\tau|0}e^{-r\tau}$, which will yield $K_{\tau|0}$ when invested at a riskless compound rate of return of r. The second portfolio P consists of one put option valued at $p_{\tau|0}$ together with a unit of the stock with a spot price of S_0.

At time τ, the portfolio C will be worth

$$K_{\tau|0} + \max(S_\tau - K_{\tau|0}, 0) = \max(S_\tau, K_{\tau|0}).$$

At worst, the option will not be exercised and the funds $K_{\tau|0}$ will be retained, whereas, at best, an asset worth $S_\tau > K_{\tau|0}$ will be acquired at the cost of the strike price of $K_{\tau|0}$.

At time τ, the portfolio P will be worth

$$S_\tau + \max(K_{\tau|0} - S_\tau, 0) = \max(S_\tau, K_{\tau|0}).$$

At worst, the option will not be exercised and the asset worth S_τ will be retained, whereas, at best, the sum of $K_{\tau|0} > S_\tau$ will be received for the delivery of an asset worth S_τ.

The values of these two portfolios, which are equal at time τ, must also be equal at time $t = 0$. It follows that

$$c_{\tau|0} + K_{\tau|0}e^{-r\tau} = p_{\tau|0} + S_0.$$

This is the formula for the put–call parity of European options.