LECTURE 1

Metaphors for Time-Series Analysis

Newton's Laws of Motion

(1) $$y = y_0 + vt,$$

(2) $$y = y_0 + v_0 t + \frac{1}{2}at^2,$$

The Wiener Process

(3) $$Z(\tau) = Z(a) + \int_a^\tau dZ(t).$$

(4) $$E\{Z(\tau) - Z(a)\} = 0.$$

(5) $$E\{dZ(s)dZ(t)\} = \begin{cases} 0, & \text{if } ds \cap dt = \emptyset; \\ \sigma^2 dt, & \text{if } ds = dt. \end{cases}$$

(6) $$\begin{aligned} V\{Z(\tau) - Z(a)\} &= \int_{s=a}^\tau \int_{t=a}^\tau E\{dZ(s)dZ(t)\} \\ &= \int_{t=a}^\tau \sigma^2 dt = \sigma^2(\tau - a). \end{aligned}$$

The First-Order Random Walk

$$y_t = y_{t-1} + \varepsilon_t. \tag{7}$$

$$E(\varepsilon_t) = 0 \quad \text{and} \quad V(\varepsilon_t) = \sigma^2 \quad \text{for all } t. \tag{8}$$

$$y_t = y_0 + \big\{\varepsilon_t + \varepsilon_{t-1} + \cdots + \varepsilon_1\big\}. \tag{9}$$

$$E(y_t) = y_0 \quad \text{and} \quad V(y_t) = t \times \sigma^2. \tag{10}$$

$$y_t - y_{t-1} = \varepsilon_t. \tag{11}$$

$$y_{t+h} = y_{t+h-1} + \varepsilon_{t+h} \tag{12}$$

Forecasting a Random Walk

$$E(y_{t+h}|\mathcal{I}_t) = \begin{cases} \hat{y}_{t+h|t}, & \text{if } h > 0; \\ y_{t+h}, & \text{if } h \leq 0. \end{cases} \tag{13}$$

$$\begin{aligned} E(y_{t+h}|\mathcal{I}_t) &= \hat{y}_{t+h|t} = \hat{y}_{t+h-1|t}, \\ E(y_{t+1}|\mathcal{I}_t) &= \hat{y}_{t+1|t} = y_t. \end{aligned} \tag{14}$$

Orstein–Ulenbeck Model of Brownian Motion

$$mV(\tau) - mV(a) = m\int_a^\tau dV(\tau) = p(\tau - a). \tag{15}$$

A Second-Order Random Walk

$$\begin{aligned} y_t &= y_{t-1} + v_t \\ &= y_{t-1} + v_{t-1} + \varepsilon_t \\ &= 2y_{t-1} - y_{t-2} + \varepsilon_t \end{aligned} \tag{16}$$

Forecasting a Second-Order Random Walk

$$y_{t+h} = 2y_{t+h-1} - y_{t+h-2} + \varepsilon_{t+h}. \tag{17}$$

$$\begin{aligned} E(y_{t+h}|\mathcal{I}_t) &= \hat{y}_{t+h|t} = 2\hat{y}_{t+h-1|t} - \hat{y}_{t+h-2|t}, \\ E(y_{t+1}|\mathcal{I}_t) &= \hat{y}_{t+1|t} = 2y_t - y_{t-1}, \end{aligned} \tag{18}$$

$$\hat{y}_{t+h} = \alpha + \beta h \quad \text{with} \quad \alpha = y_t \quad \text{and} \quad \beta = y_t - y_{t-1}, \tag{19}$$

Motion in a Viscous Fluid: An IAR Process

$$m\frac{dv}{dt} = p - cv, \tag{20}$$

$$v_t - v_{t-1} = \varepsilon_t - \beta v_{t-1}. \tag{21}$$

$$v_t = \phi v_{t-1} + \varepsilon_t, \tag{22}$$

$$y_t = (1+\phi)y_{t-1} - \phi y_{t-2} + \varepsilon_t. \tag{23}$$

Random Walk with Added Noise

$$\begin{aligned} y_t &= \xi_t + \eta_t, \\ \xi_t &= \xi_{t-1} + \nu_t, \end{aligned} \tag{24}$$

$$\begin{aligned} y_t - y_{t-1} &= \xi_t - \xi_{-1} + \eta_t - \eta_{t-1} \\ &= \nu_t + \eta_t - \eta_{t-1}. \end{aligned} \tag{25}$$

$$\nu_t + \eta_t - \eta_{t-1} = \varepsilon_t - \mu\varepsilon_{t-1}, \tag{26}$$

$$y_t = y_{t-1} + \varepsilon_t - \mu\varepsilon_{t-1}. \tag{27}$$

Forecasting A Random Walk with Added Noise

$$E(y_{t+h}|\mathcal{I}_t) = \hat{y}_{t+h|t} = \hat{y}_{t+h-1|t}. \tag{28}$$

$$\begin{aligned} E(y_{t+1}|\mathcal{I}_t) = \hat{y}_{t+1|t} &= y_t - \mu\varepsilon_t \\ &= y_t - \mu(y_t - \hat{y}_{t|t-1}) \\ &= (1-\mu)y_t + \mu\hat{y}_{t|t-1}. \end{aligned} \tag{29}$$

$$\begin{aligned} \hat{y}_t &= (1-\mu)(y_t + \mu y_{t-1} + \cdots + \mu^{t-1}y_1) + \mu^t\hat{y}_0 \\ &= (1-\mu)\{y_t + \mu y_{t-1} + \mu^2 y_{t-2} + \cdots\}, \end{aligned} \tag{30}$$

An Ideal Harmonic Oscillator

$$m\frac{d^2y}{dt^2} + hy = 0. \tag{31}$$

$$y(t) = 2\rho\cos(\omega_n t - \theta), \tag{32}$$

Damped Oscillatory Motion

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} + hy = 0, \tag{33}$$

$$y(t) = 2\rho e^{\gamma t}\cos(\omega t - \theta), \tag{34}$$

Quasi-Cyclical Stochastic Processes

$$\begin{aligned} y_t &= \phi y_{t-1} - y_{t-2} + \varepsilon_t \\ &= 2\cos\omega_n y_{t-1} - y_{t-2} + \varepsilon_t, \end{aligned} \tag{35}$$

$$y_t = 2\rho\cos\omega_n y_{t-1} - y_{t-2} + \varepsilon_t, \tag{36}$$

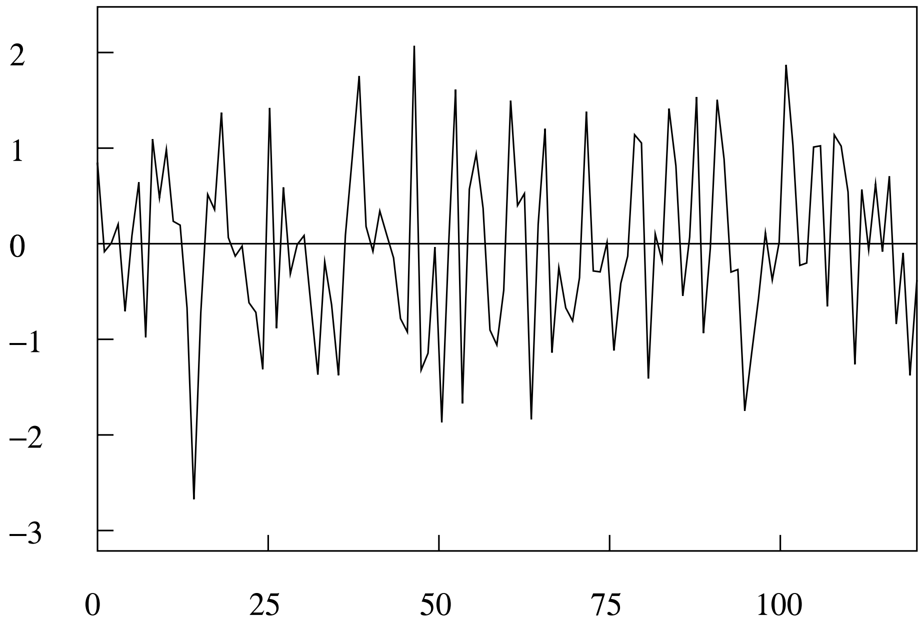

Figure 1. A sequence generated by a white-noise process.

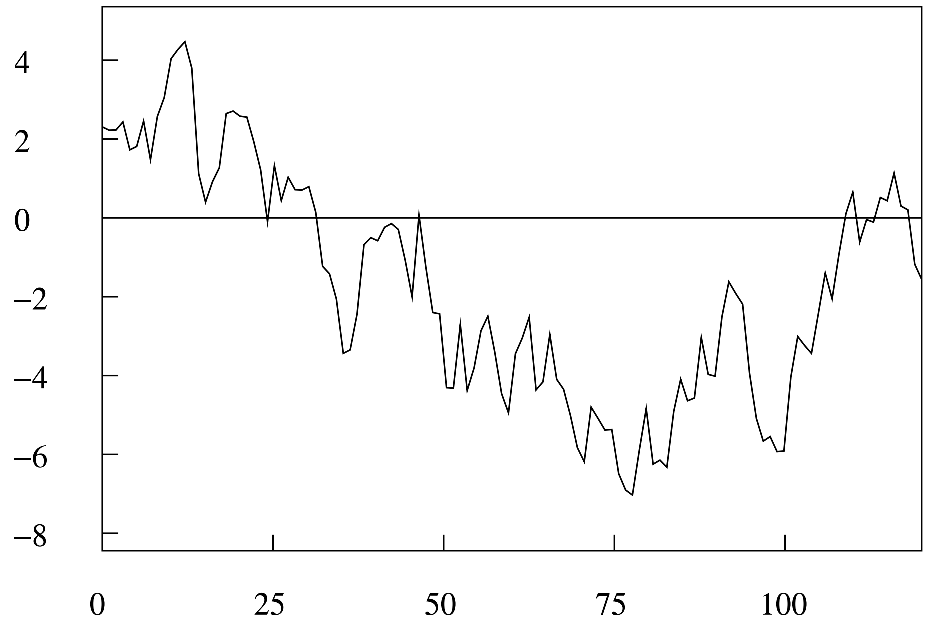

Figure 2. A first-order random walk

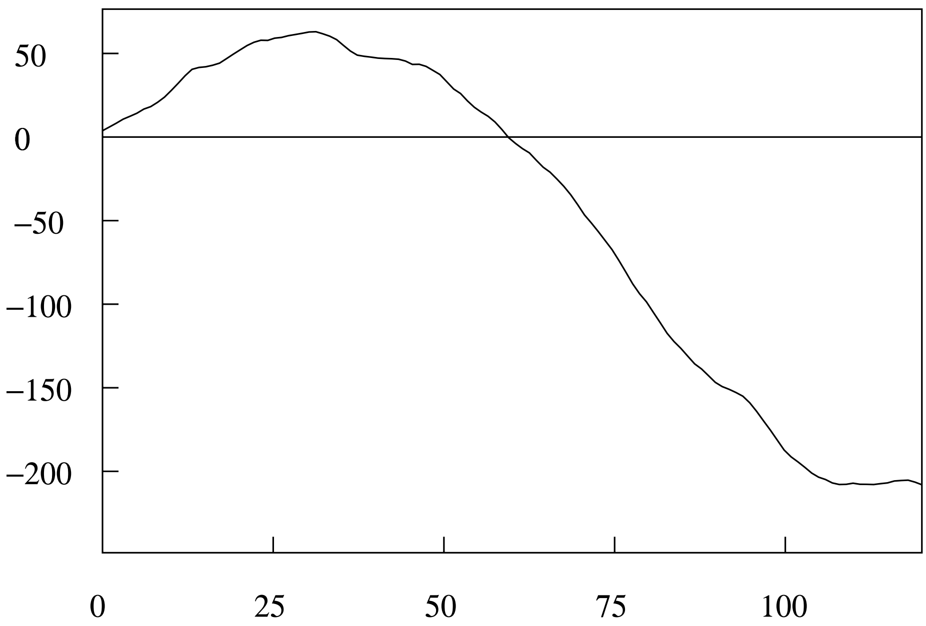

Figure 3. A second-order random walk

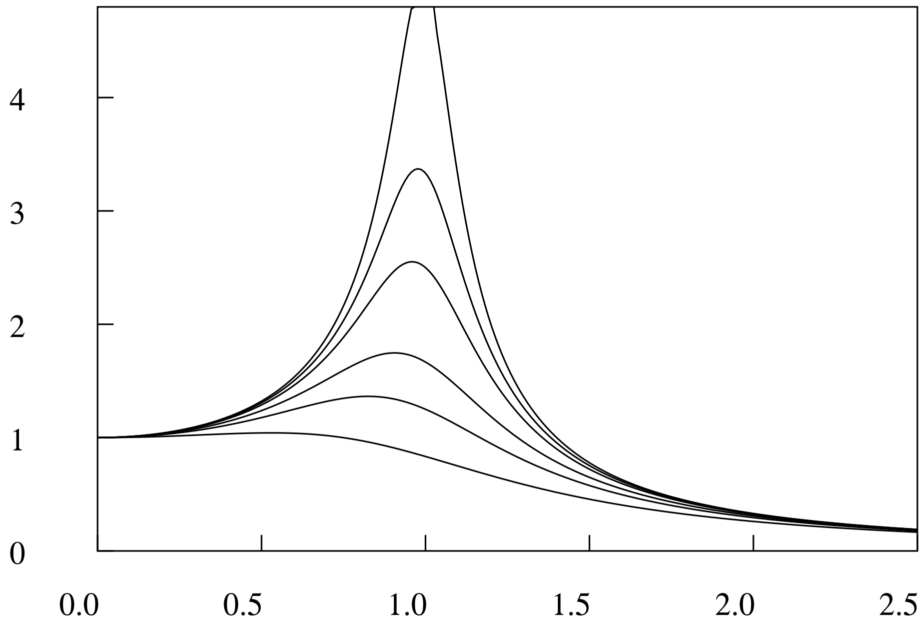

Figure 4. The frequency response of a second-order system with various damping ratios. On the vertical axis is the gain in the amplitude of the sinusoidal motion. On the horizontal axis is the relative frequency ω/ω_n of the forcing function. The six curves, from the highest to the lowest, correspond to the damping ratios $\zeta = 0.1, 0.15, 0.2, 0.3, 0.4, 0.6$.

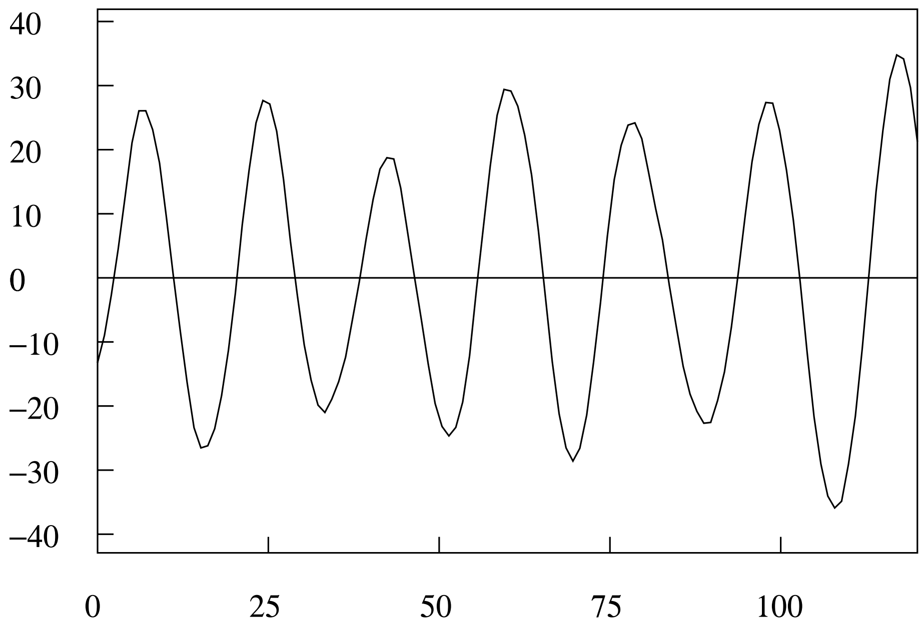

Figure 5. A quasi-cyclical sequence generated by an AR(2) process.